APPENDIX II

Magnetic Field Angular Momentum Analysis

Referring to Fig. 1, consider two charges q_1 and q_2 in close association at O moving at right angles at velocities v_1 and v_2 respectively. The frame of reference is that in which magnetic field reaction is induced. That is, the velocities are measured in the E frame, in the sense in which this term is used in Chapter 4. Thus, the magnetic field induced at a point P distant OP from O may be expressed as the vector sum of two components H_1 due to q_1 and H_2 due to q_2,

Let q_1 be taken as moving along the axis Ox.

Let q_2 be taken as moving along the axis Oy.

Take axes Ox, Oy and Oz as orthogonal.

Let the angles θ, φ, ε, η be as shown.

The magnetic field at P due to q_1 is:

$$H_{1y} = +(q_1 v_1/c) \sin\varepsilon \sin\eta/(OP)^2 \text{ in the } y \text{ direction}$$
$$H_{1z} = -(q_1 v_1/c) \sin\varepsilon \cos\eta/(OP)^2 \text{ in the } z \text{ direction}$$

The magnetic field at P due to q_2 is:

$$H_{2z} = -(q_2 v_2/c) \sin\theta \sin\varphi/(OP)^2 \text{ in the } z \text{ direction}$$
$$H_{2x} = +(q_2 v_2/c) \sin\theta \cos\varphi/(OP)^2 \text{ in the } x \text{ direction}$$

Now, imagine that the field due to q_1 exists but that the field due to q_2 has only just been established by q_2 having been suddenly accelerated from rest to assume the velocity v_2. This means that the magnetic field energy density at P changes from $(H_{1y}^2 + H_{1z}^2)/8\pi$ to $[H_{1y}^2 + H_{2x}^2 + (H_{1z} + H_{2z})^2]/8\pi$ as the wave passes. At the point Q it may be shown that the same effect produces a change of magnetic field energy density from $(H_{1y}^2 + H_{1z}^2)/8\pi$ to $[H_{1y}^2 + H_{2x}^2 + (H_{2z} - H_{1z})^2]/8\pi$.

The point now to note is that there is a component of energy density which has to be added in equal measure at P and Q by the passage of the wave. This is $(H_{2z}^2 + H_{2x}^2)/8\pi$. Also, there is a component to be added at P and an exactly equal component to be subtracted at Q. It is:

$$\frac{1}{4\pi}(H_{1z}H_{2z}) \tag{10}$$

As was discussed in Chapter 2, mutual magnetic energy is equal and of opposite magnitude to the mutual dynamic electric field energy. Indeed, the two sum to zero. Electric field energy has mass properties. This follows from the discussion of the velocity-dependence of mass in Chapter 1. We need not think in terms of the motion

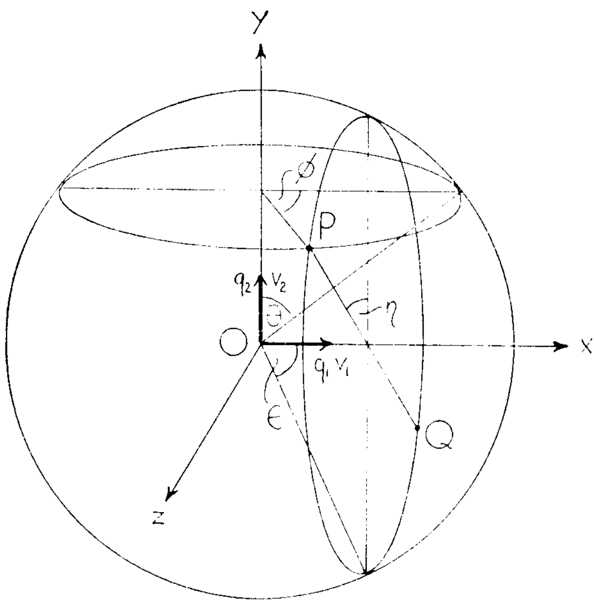

Fig. 1

of magnetic energy. Consequently, in considering the motion of energy and its mass properties, expression (10) represents the energy density of the electric field which has to move from P to Q as the wave passes through these points. This is a measure of the mass *redistribution* in the field. The main energy terms, that is the non-interaction terms, are related to the self energies of the moving charges. The faster they move, the greater their dynamic electric field energies. Hence, the greater their masses, as explained in Chapter 1. Interaction itself does not augment mass in the system shown in Fig. 1. Interaction means repositioning of mass. The passage of the wave can result in angular momentum being imparted to the field energy.

To calculate the angular momentum of this field reaction we note that mass is moving around the wave region about the axis Oz.

Movement from P to Q is through an arc subtending the angle 2ε at radius OP but projected by multiplication by $\cos\eta$. This movement is completed in the time taken for the wave to cross the region contributing to the energy interchange. Let w be the angular velocity of the energy transfer. Then the projected velocity moment is $w(OP)^2\cos\eta$, and, since w is $2\varepsilon/\mathrm{dt}$, where dt is the time taken by the transfer, this velocity moment is:

$$2\varepsilon(OP)^2\cos\eta/dt \tag{11}$$

The radial thickness of the region under study is cdt and an elemental volume at P or Q can be formed by multiplying cdt by $2\pi(OP)\sin\varepsilon$ and $(OP)d\varepsilon$. Thus, the elemental energy being transferred between these volumes at P and Q is found, from (10), as:

$$\tfrac{1}{2}(OP)^2cdt(H_{1z}H_{2z})\sin\varepsilon\, d\varepsilon \tag{12}$$

We divide this by c^2 to obtain mass and multiply the result by (11) to determine the angular momentum as:

$$\frac{1}{c}(H_{1z}H_{2z})(OP)^4\varepsilon\sin\varepsilon\cos\eta\, d\varepsilon \tag{13}$$

Substituting now the originally stated values of H_{1z} and H_{2z} gives:

$$(q_1q_2v_1v_2/c^3)\varepsilon\sin^2\varepsilon\sin\theta\cos\varphi\cos^2\eta\, d\varepsilon \tag{14}$$

It may be seen from Fig. 1 that:

$$\cos\varepsilon = \sin\theta\cos\varphi \tag{15}$$

From (14) and (15) the elemental field angular momentum given by (14) is obtained in terms of ε and η. When averaged for all values of η, $\cos^2\eta$ becomes $\frac{1}{2}$. Thus the total angular momentum may be found by evaluating:

$$\tfrac{1}{2}(q_1q_2v_1v_2/c^3)\int_0^{\pi/2}\varepsilon\sin^2\varepsilon\cos\varepsilon\, d\varepsilon \tag{16}$$

This is:

$$\left(\frac{\pi}{12}-\frac{1}{9}\right)q_1q_2v_1v_2/c^3 \tag{17}$$

Consideration shows that if v_1 and v_2 are not at right angles, as shown in Fig. 1, the expression has to be multiplied by the sine of the angle between them. Thus (17) is a measure of the maximum angular reaction between the charges.