3. *The Nature of Ferromagnetism*

Heisenberg's Theory

Heisenberg's theory of ferromagnetism attributes the ferromagnetic state to an alignment of electron spins in atoms due to exchange forces. In wave-mechanical terms, the probability that an electron in one atom will change places with an electron in an adjacent atom is given by an exchange integral which is positive or negative according to the ratio of the radius of the relevant electron shell r and the atomic spacing d. In general, this integral is negative since the attractions between the atomic nuclei and the electrons are greater than the repulsions between the nuclei and between the electrons. It is positive when there exists a certain ratio d/r of the distance between the adjacent atoms of the crystal and the radius of the electron shells containing the uncompensated electron spin. Slater (1930) presents the data:

Metal	Fe	Co	Ni	Cr	Mn	Gd
d/r	3·26	3·64	3·94	2·60	2·94	3·10

The conclusion drawn from this is the empirical presumption that, for ferromagnetism to exist, d/r must be greater than 3·0 but not much greater.*

As Bates (1951, c) points out, in Heisenberg's theory the exchange forces depend upon the alignment of the electron spins but the forces between the spins themselves are not responsible for the ferromagnetic state. Ferromagnetism is presumed to be due to interaction forces between the atoms because these forces apparently have a common feature if the ratio d/r has an approximately common value, evidently greater than 3·0 but not much greater. However, is this a

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* In this chapter the symbols r and d are used to denote dimensions in an atomic lattice, whereas in the remainder of this book they are used consistently to denote dimensions of the space–time lattice. The coincidental feature that the ratio d/r is slightly more than 3 in the space–time lattice is deemed to be fortuitous.

sufficient explanation of the ferromagnetic state? Also, accepting that the exchange forces do have values coming within certain limits which are conducive to the ferromagnetic state in the ferromagnetic substances, what really is the link between these forces and the intrinsic magnetism? How do the electron spins get aligned and why is it that so few electrons in each atom have their spins set by action of the exchange forces? Why is Heisenberg's theory so vague in its quantitative account of the ferromagnetic state? Also, since in Chapter 2 it has been argued that ferromagnetism is not primarily associated with electron spin, as is popularly believed, but is in fact due to the orbital motion of electrons, how is this to be related to Heisenberg's theory?

The Cause of Ferromagnetism

In considering the nature of ferromagnetism, the idea that magnetic energy is a negative quantity, presented in the previous chapter, has immediate significance. Magnetism may have a tendency to become the preferred state and ferromagnetism will result if the other forms of energy which go with this magnetic state can be fully sustained by the source of magnetic energy itself. This is simple physics without recourse to exchange integrals defining probabilities of electron interchanges between atoms.

On this point of negative magnetic energy, it is appropriate to note that it is included as a negative term in magnetic domain theory where the equilibrium states of magnetic domain formation are evaluated (see Kittel, 1949). "The minus sign merely indicates that we have to supply heat in order to destroy the intrinsic magnetization."*

To say that energy has to be supplied to *destroy* intrinsic magnetism is to say that energy is needed to restore the undisturbed state of the field medium (the aether) since the disturbance, which is magnetism, has yielded energy and needs it back to be restored to normal. If ferromagnetism, meaning an alignment of the magnetic moments of adjacent atoms in a crystal, needs other energy to sustain it, such as strain energy, this other form of energy can participate in the return to the demagnetized state. But the question of whether a substance is or is not ferromagnetic must depend upon the ratio of the available energy from the magnetic source and the sustaining

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* See page 7.

energy needed, as by the strain. If this ratio is greater than unity, there is ferromagnetism. Otherwise there is no ferromagnetism.

Why is iron ferromagnetic to the exclusion of so many other elements? The answer to this question is that it so happens that in the atomic scale iron is positioned to have properties for low interaction forces between atoms, with a significant alignment of certain electron states. In addition, iron is strong enough to withstand the effects of these forces, which are many tons per square inch and do approach the normal breaking stresses of metallic crystals. Further, iron, as well as nickel and cobalt, does happen to have a rather high modulus of elasticity so that the energy needed to sustain the strain is relatively low.

Why does the ferromagnetic property disappear as temperature is increased through the Curie point? There are the conventional explanations for this in the standard works on magnetism (such as that of Smart, 1966). A simple alternative answer which appeals to the writer is that, since the modulus of elasticity does decrease rather rapidly with increase in temperature, by the right amount, the strain energy needed to sustain magnetism increases to cross the threshold set by the ratio mentioned above. This threshold is at the Curie point.

If ferromagnetism is so closely related with internal strain, and if this internal strain is high, and if at high strain the modulus of elasticity becomes non-linear, all of which are logical, then, at least in some ferromagnetic substances, there should be significant changes in the modulus of elasticity at the Curie point. This is found to be the case. The phenomenon has been discussed by Döring (1938).

It is shown below how the elements of a theory of ferromagnetism can be based on the above argument. The analysis is simplified by the expedient of regarding the Bohr theory of the atom as applicable. This merely serves to allow easy calculation of the stresses mentioned.

Stress Energy Analysis due to Orbit-Orbit Interactions in a Ferromagnetic Crystal Lattice

In view of the different account of the gyromagnetic ratio given above, the ferromagnetic state can be regarded as due to electrons in an orbital motion, rather than a mixture of spin and orbit actions. The electron in orbit traversing a circular loop at a steady speed will be taken seriously, notwithstanding the wave-mechanical aspects and the accepted improbability of such steady motion in an atom. The purpose of this is to facilitate the approximate calculations presented here. Offset against this also, one can argue that the Principle of Uncertainty, as used in wave mechanics, may well only have meaning when viewing events in atoms on a statistical basis. This principle is no warranty that, in some atoms, those of certain size, arranged in certain crystal configurations and under certain energy conditions, just one electron could not defy the principle, *as viewed by an electron in an adjacent atom*, and actually be in a harmonious state of motion with such electrons in adjacent atoms. The motion of electrons in atoms is not random. Statistically, wave mechanics helps us to understand the systematic behaviour of atomic electrons, but they are a mere mathematical tool used for this purpose and not a set of laws which a particular electron has to obey. If, energetically, it suits the electron to move steadily in an orderly orbit, it will do so. Such is the premise on which the model to be studied is based, and with it the Bohr theory of the atom will be used.

Imagine two adjacent atoms arranged in a crystal lattice with their electron orbits aligned along the crystal direction linking the particles. This is illustrated in Fig. 3.1. Only one electron per atom is taken to be in this state. The atoms are spaced apart by a distance *d*. Each atom has a nuclear charge *Ze*, an electron system depicted as a cloud, shown shaded, of charge $e-Ze$, and a single electron of charge $-e$ describing a circular orbit of radius r and velocity v. In effect, it is assumed that one electron in each atom has adopted a motion in strict accordance with Bohr's theory, whereas the other electrons form, statistically, a charge centred on the nucleus, but not screening the orbital charge from the electric field set up by the nucleus.

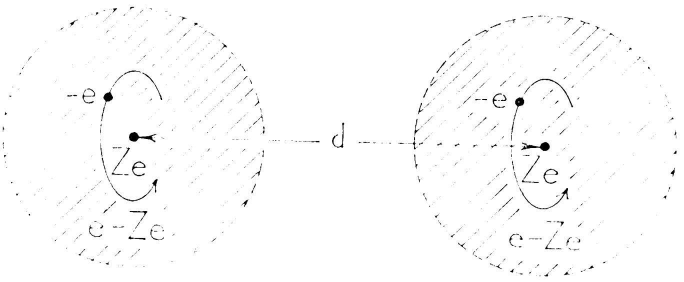

Fig. 3.1

The orbital electrons are taken to move in synchronism in view of their mutual repulsion. Then the following components of interaction force between the two atoms may be evaluated in terms of the radius r of the electron orbits and the velocity v of the electrons.

(a) Between the orbital electrons: e^2/d^2 repulsive,
(b) Between the orbital electrons: $(ev/c)^2/d^2$ attractive,
(c) Between the remaining atoms: e^2/d^2 repulsive,
(d) Between the orbital electrons and the atoms: $2e^2d/(r^2+d^2)^{3/2}$ attractive.

These force components are, simply, the electrostatic and electrodynamic interaction forces between the two electrical systems defined. If the last term is expanded, then, neglecting high order terms in r/d, since r is less than d for all cases and very much less for most, it becomes $2e^2/d^2-3e^2r^2/d^4$ . . . Combining the force components, the total force between the two atoms becomes, approximately, $(v/c)^2-3(r/d)^2$ times e^2/d^2, as an attractive force.

On Bohr theory:

$$v/c=aZ/n \tag{3.1}$$

where a is the Fine Structure Constant $7{\cdot}298\ 10^{-3}$, and n is the quantum number of the electron level in the atom. Also:

$$r=n^2r_H/Z \tag{3.2}$$

where r_H is $5{\cdot}29\ 10^{-9}$ cm.

It follows that as Z increases, the attractive force component diminishes and the repulsive force component increases. The zero force state occurs when:

$$Z^4/n^6=3r_H{}^2/a^2d^2\simeq 4{,}000$$

if d is $2\ 10^{-8}$ cm. This gives, for $n=2$, $Z=23$. For iron, $Z=26$, and it so happens that the measured value of the effective value of n is 2·2. This represents the number of Bohr Magnetons per atom applicable to iron in its state of intrinsic magnetization.

The above calculation is merely to demonstrate that the approach being pursued may prove profitable.

To develop the theory on more realistic, though still approximate, terms, the transverse forces have to be taken into account in stress energy considerations. The force in the lateral sense between two atoms in the crystal lattice will be effectively all electrodynamic. The electrostatic action of the orbital electron of one atom will, on average, tend to act from a point close to the nucleus when its action on the other atomic nucleus is considered. It follows from the law of electrodynamics developed in Chapter 2, that the force $(ev/c)^2/d^2$ will act in the lateral sense. This will create components of stress energy precluding the total stress energy from passing through zero as Z increases. This will make the ferromagnetic state less likely to occur and very much will depend upon the value of the related magnetic field energy and stress energy.

To proceed, the stress in the substance will be taken to be of the order of $1/d^2$ times the elemental force just deduced. This is taking into account only forces between adjacent atoms in a cubic lattice. The actual force will be greater than this, perhaps by a factor of two or three. Although there are many atoms interacting, when the spacing doubles the forces are reduced in inverse square proportion. Further, the harmonious nature of the electron motions may not be seen as such for interactions over large distances. In travelling a distance d of $2\ 10^{-8}$ cm at velocity c of $3\ 10^{-10}$ cm/sec, the electrodynamic action, for example, involves a transmission time of $0{\cdot}67\ 10^{-18}$ seconds. In this time, for $Z=23$ and $n=2$, equation (3.1) shows that the electron may move $1{\cdot}7\ 10^{-9}$ cm. This is slightly more than one quarter of a revolution. This really means that this approach to explaining ferromagnetism requires a redefinition of the synchronous state assumed in Fig. 3.1. In fact, since energy considerations are involved, the mutual repulsion forces between the electrons in orbit urge maximum separation, subject to the propagation velocity. This velocity may be different from c, but this does not matter. We take it that synchronism exists as viewed by each individual atom. This means that electrons in adjacent atoms are out-of-phase in their motion as viewed from remote positions. It also means that atoms not adjacent to the one under study will be seen by that atom to have orbital electrons also out-of-phase. There is an exception for successive atoms along the magnetization direction and transverse to it along the crystal axis, because the effective value of d increases in integral steps. From considerations such as this, it may be shown that the prime term is the energy due to interaction with atoms adjacent in the crystal lattice directions. The energy will be greater than this only provided the surrounding atoms are seen to be in synchronism and make a significant contribution to the energy required. If these atoms are out of synchronism, they may add to, or subtract from, the energy, but, overall, should have little effect.

Along the direction of magnetization, there will be a stress F_x given by:

$$F_x = (e^2/d^4)[(v/c)^2 - 3(r/d)^2] \tag{3.3}$$

In the orthogonal directions, there will be forces F_y and F_z, both given by:

$$F_y = F_z = (e^2/d^4)(v/c)^2 \tag{3.4}$$

From (3.3) and (3.4):

$$F_x = F_y - F_o \tag{3.5}$$

where:

$$F_o = 3(e^2/d^4)(r/d)^2 \tag{3.6}$$

In terms of Young's Modulus Y and Poisson's Ratio σ, the strain energy density is:

$$E = \frac{1}{2Y}[F_x^2 + F_y^2 + F_z^2 - 2\sigma F_x F_y - 2\sigma F_y F_z - 2\sigma F_x F_z]$$

and, if σ is approximated as 1/3, from (3.4), (3.5) and (3.6):

$$E = \frac{1}{2Y}[F_y^2 - \tfrac{2}{3}F_o F_y + F_o^2] \tag{3.7}$$

From equations (3.1), (3.2), (3.4), (3.6) and (3.7), it is possible to evaluate $2YE/e^4$ as a function of Z for different values of n, provided d is known. The value of d depends upon the nature of the crystal, the atomic weight and the density of the substance. Consistent with the degree of approximation involved in deriving (3.7), it seems feasible to assume that d changes linearly with increasing Z. It will be taken as the cube root of the atomic weight divided by the density, and referred to two substances, say, iron and lead, for which Z is 26 and 82 respectively. The crystal lattice will be taken to be simple cubic, even though iron is body-centred, with lattice dimension $2{\cdot}8\ 10^{-8}$ cm. The value of d, derived as indicated, is given by:

$$d = (1{\cdot}93 + 0{\cdot}0143Z)\ 10^{-8} \tag{3.8}$$

The plot of $2YE/e^4$ is shown in Fig. 3.2 for $n = 1$, 2, 3 and 4. In the same figure, along the abscissa, the short lines indicate those atoms for which the atomic susceptibility has been found to exceed 10^{-4}. The broken lines indicate the values of $2YE_{\text{mag}}/e^4$, plotted for different values of n and on a base value of Y of $2\ 10^{12}$ cgs units. E_{mag} is the magnetic energy density, evaluated as $2\pi n^2/d^6$ times the value of the Bohr Magneton (in cgs units) squared. The Bohr Magneton is $9{\cdot}274\ 10^{-21}$.

The pattern of the high susceptibility atoms has a grouping matching the minima of the strain energy curves. This encourages the strain analysis approach to explaining ferromagnetism. The minima of the strain energy curves corresponds to the increased likelihood of ferromagnetism, though this latter state can only occur if the magnetic energy (being negative) exceeds in magnitude the strain energy. Of importance here is the fact that the strain energy density and the magnetic energy density are of the same order of magnitude, thus making select states of ferromagnetism feasible in some materials but not in others. The strain energies of the order of 10^7 ergs per cc, correspond to stresses of tens of tons per square inch. This means that selectivity for the ferromagnetic state may also depend on the rupture strengths of the materials; ferromagnetism clearly being more likely in strong materials of high Young's Modulus.

Discussion of New Theory

Theoretically, ignoring the error factor in the under-estimation of the strain energy, the curves show that a simple cubic crystal of oxygen ($Z=8$), if it could exist and if its Young's Modulus was $2\ 10^{12}$ or higher, would be ferromagnetic. For $n=1$, the prospect of a ferromagnetic state has to be ruled out for other atoms, except possibly carbon. Diamond has an extremely high Young's Modulus, some five times that assumed for the comparison curve. However, with $Z=6$, carbon, to be ferromagnetic, would have to sustain very high internal stresses and these probably preclude ferromagnetism. For $n=2$, iron, nickel and cobalt have to be given favoured consideration. They all have a relatively high Young's Modulus, some 50% higher than for copper, for example. They are all strong enough to sustain stresses accompanying the ferromagnetic state. Note that for Fe, Co, Ni and Cu, Z is 26, 27, 28 and 29 respectively. The broken curve in Fig. 3.2 has to be placed 20% or so higher for Fe, Ni and Co and the same amount lower for Cu. Fig. 3.2, therefore, explains why iron is ferromagnetic and copper non-ferromagnetic. Of course, in applying the curves in Fig. 3.2, it should be noted that

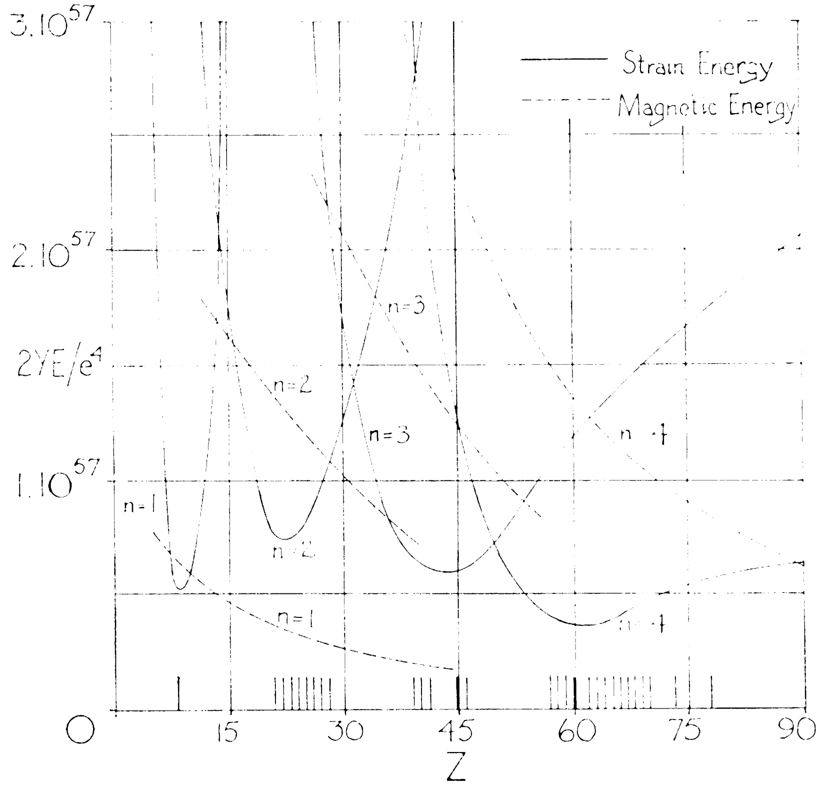

Fig. 3.2

the analysis has only been approximate. Perhaps, also, it was wrong to ignore the screening action of some of the electrons in inner shells or perhaps this, and an accurate evaluation of the strain energy allowing for surrounding atomic interaction, will shift the minima of the curves very slightly to the right. This would better relate the minima to the susceptibility data and permit a higher error factor in the strain energy evaluation. Note that if the strain energy is underestimated by much in Fig. 3.2, nickel is only marginally ferromagnetic. With $n=3$ and 4, the screening action of electrons will assume more importance and the evident prediction of a theoretical state of ferromagnetism in several substances shows some qualification of the actions to be necessary. It is significant that Gd with Z of 64, located near the minimum of the $n=4$ curve, is ferromagnetic. It may well be that the higher n and the higher Z, the more electrons there are in the shell which can be ferromagnetic. Then, the less likely it is for the synchronous action to remain as a preferred energy state. The interference from the effects of other electrons could well suppress this condition in the larger atoms.*

The understanding of ferromagnetism by its relation to stress properties may prove of interest in that it may be that under the very high pressures prevailing inside the earth, even materials which are not ferromagnetic at the surface may become ferromagnetic. Young's Modulus may then be of no importance and a compression modulus may be the factor which is deciding the state of balance between stress energy and magnetic energy.

Summary

In this chapter, it has been shown how the nature of ferromagnetism can be explained without recourse to wave mechanics. The law of electrodynamics developed in Chapter 2 and the principles of negative magnetic energy are applied successfully in the analysis. In the next chapter we will explain how the theory is reconciled with wave-mechanics. It will be shown that an electron can spend some time in a Bohr orbit and some time in its wave mechanical state. Thus, a factor has to be applied to lower the magnetic energy curves in Fig. 3.2, so limiting the elements in the ferromagnetic state still further.

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* In *Physical Review Letters*, v. 22, p. 1260, June 1969, E. Bucher *et al.* report the discovery that Pr and Nd, of atomic numbers $Z = 59$ and 60, respectively, are ferromagnetic in their face centred cubic phases.