EPR and Bell Revisited

Horace Heffner October 2004

Assume, as did Einstein, Podolski, and Rosen (EPR), the state of conjugate entangled particles is set at the time of the creation of the conjugates, at the moment of entanglement.[1] EPR maintained that entangled particles in effect carry hidden variables, or an equivalent of a computer program, that determines how they will act when observed. Assume there are, as in Alain Aspect's experiment designed to examine this assumption, three independent quantum values involved. That is to say there are three axes of spin observation, in which a particle is in either a clockwise or counterclockwise spin state upon observation. Unfortunately, spin can only be observed in one axis, not all three at the same time. However, Bell figured out how to see if the quantum variables were set before measurement, i.e. how to see if a hidden variable was involved. The situation is shown in Table 1, below.

i	A	B	C	D	E	F
1	0	0	0	1	1	1
2	0	0	1	1	1	0
3	0	1	0	1	0	1
4	0	1	1	1	0	0
5	1	0	0	0	1	1
6	1	0	1	0	1	0
7	1	1	0	0	0	1
8	1	1	1	0	0	0

Key:

i - possible combination (row) number
A, B, C - Alice's possible observation axes
D, E, F - Bob's possible observation axes

Table 1 - Possible observations by Alice and Bob

Table 1 assumes that when an entangled particle pair is created that all three quantum variables, i.e. spins, are set (by Mother Nature) at that time and carried as "hidden variables". Columns A, B and C are possible spins observed by a sender Alice in orthogonal axes A, B and C, and are denoted "o" for clockwise spin and "1" for counterclockwise spin. Columns D, E, and F are the corresponding spins observed by receiver Bob in the axes A, B and C. It is assumed there is no error in the detection of the spins or the transmission of the hidden variables. As the variables are independent, and it is well known from observation of single particles that the spin probability of clockwise spin being observed in any axis is 0.5, we see that there are exactly 8 equally probable combinations, possibilities denoted 1 - 8 in column i.

Bell suggested that sender Alice and receiver Bob, for each particle pair, select a column at random (Alice from A, B or C, and Bob from D, E, or F) and observe the spin. That's all there is to the experiment.

To see the expected results, look at Table 2.

a	b	matches
A	D	8/8
A	E	4/8
A	F	4/8
B	D	4/8
B	E	8/8
B	F	4/8
C	D	4/8
C	E	4/8
C	F	8/8

Table 2 - Expected results

In Table 2 column "a" indicates the axis Alice chooses to observe. Column "b" indicates the axis Bob chooses to observe at the same time. We can determine the probability of a match by comparing the two columns of equally probable outcomes shown in table in Table 1. By "match" here we mean the observation of opposed, i.e. conjugate, spins. For example, the first row of Table 2 has the entries, A, D, and 8/8. This means that when Alice chooses axis A, i.e. column A, and Bob coincidentally also chooses a parallel axis to A, i.e. column D, then both will always observe complimentary spins. We get 8 out of 8 matches. This is the principle of, the definition of in this case, entanglement. When we look at row 2 of Table 2, we have the entries A, E, 4/8. This is because there are only 4 possible ways out of 8 outcomes, each equally probable, that a match occurs. Summing up the entries in Table 2, we see that there are 9*8 = 72 possible outcomes to the observation of a single entangled pair, and there 3*(8+4+4) = 48 possible matches. There is thus a 2/3 probability of a match for a given particle pair.

That is all there is to it! If there are hidden variables, then there will be a 2/3 probability of a match.

The Aspect experiment actually yields a 1/2 probability of a match. It was deduced from this result there is no hidden variable involved.

This is quite amazing. If the (thought) experiment data for 7200 trials is tabulated in the format of Table 2, we might expect it to look something like the idealization shown in Table 3.

a	b	matches
A	D	800/800
A	E	200/800
A	F	200/800
B	D	200/800
B	E	800/800
B	F	200/800
C	D	200/800
C	E	200/800
C	F	800/800

Total matches 3600
Total trials 7200
Match probability 0.5

Table 3 - Idealized experimental results

The amazing thing that has happened is a reduction of the probability of a match when Alice and Bob have chosen differing axes to observe. We know they get a match 100 percent of the time when choosing the same axes, i.e. in the combinations A D, B E, and C F. The matches in the differing axes have in effect been "discorrelated", to coin a term, reduced in matching by 50 percent from what they should be if programmed by hidden variables. The computer programs inside the particle pairs appear to have no means to accomplish this discorrelation without knowing what choice of axis was made by both Alice and Bob. Since Alice and Bob can chose the axis to observe the last moment, the computer programs would have to communicate faster than light to do their work.

However, perhaps Table 1 can be modified to restrict possible combinations. After all, the spin of one particle can not be measured in all three axes at once. It may be that spin direction in orthogonal axes are not independent variables. Suppose, for example, that spin in all three axes can not be the same at one time. We then have Table 4.

i	A	B	C	D	E	F
2	0	0	1	1	1	0
3	0	1	0	1	0	1
4	0	1	1	1	0	0
5	1	0	0	0	1	1
6	1	0	1	0	1	0
7	1	1	0	0	0	1

i - possible combination (row) number
A, B, C - Alice's possible observations
D, E, F - Bob's corresponding observations

Table 4 - Hidden variable table prior to weighting considerations

Tabulation of Table 4 still shows Bell's inequality to be in effect. The expected percent of total matches exceeds 50 percent. However, suppose Mother Nature now simply decides to weight each row's probability. This results in Table 5.

w	A	B	C	D	E	F	
g	0	0	0	1	1	1	Key:
h	0	0	1	1	1	0	
i	0	1	0	1	0	1	w - weight for given row (w nonnegative)
j	0	1	1	1	0	0	A, B, C - Alice's possible observations
k	1	0	0	0	1	1	D, E, F - Bob's corresponding observations
m	1	0	1	0	1	0	
n	1	1	0	0	0	1	Let T = (g+h+i+j+k+m+n+p)
p	1	1	1	0	0	0	

Table 5 - Prospective hidden variable table for observations by Alice and Bob

Table 6, below, is a tabulation of the entries in Table 5.

a	b	matches	
A	D	T/T	
A	E	(g+h+n+p)/T	
A	F	(g+i+m+p)/T	
B	D	(g+h+n+p))/T	Total possibilities = 9*T
B	E	T/T	Total matches = 3*T + 6*g + 6*p +
B	F	(g+j+k+p)/T	2*(h+i+j+k+m+n)
C	D	(g+i+m+p)/T	
C	E	(g+j+k+p)/T	Match probability desired .5
C	F	T/T	

Table 6 - Expected results based on Table 5

Here again the term "match" refers to spin orientations being conjugate, i.e. 1 and 0 or 0 and 1 in the columns chosen in Table 5.

We can immediately see the justification for throwing out rows 1 and 8, as their weights g and p have factors of 9. We thus set g=p=0, which throws out rows 1 and 8.

Further, even with g=p=0, to obtain the desired probabilities of 1/4 for the case

where differing axes are chosen, we obtain from Figure 6 the following equations:

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(h + n)/T = 1/4
(i + m)/T = 1/4
(j + k)/T = 1/4
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which says:

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4h + 4n = T
4i + 4m = T
4j + 4k = T
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and adding all the above:

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4(h+i+j+k+m+n) = 3T
4T = 3T
T=0
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so the weights all disappear in a flash! This in itself is sufficient to prove the impossibility of hidden variables, because T = (g+h+i+j+k+m+n+p) is the sum of nonnegative values, and T=0 implies all weights were chosen as zero and thus no data was transmitted.

Bell's inequality, however, rests on the overall observed hits. We have from Table 5 that T = h+i+j+k+m+n and from Table 6 that total possibilities = 9*T. Total matches from Table 6 is 3*T + 2*T = 5*T. We want (matches)/(possibilities) = 0.5. However, the ratio (matches)/(possibilities) = (5*T)/(9*T) = 5/9, *no matter what Mother Nature might chose for the weights.* If g and/or p are nonzero the ratio is worse than 5/9. Bell's inequality thus holds no matter how the 8 possible spin combinations are weighted. Faster than light communication is required to account for the experimental results.

An interesting thing shown by this analysis, as opposed to the amazing but well known *experimental observation* of the correlation of conjugate particles, is the herein demonstrated FTL statistical "discorrelation" effect that Mother Nature causes, in order to preserve a 50/50 spin state distribution for all observations on both ends, when Alice and Bob happen to choose nonparallel observation axes .

Footnotes:

[1] The Fabric of the Cosmos, Brian Green, 2004, Alfred A. Knopf, p 103 ff