Detection of thirteen resonances of radio waves from particularly intense lightning discharges

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[1] Thirteen resonances of radio waves from particularly intense lightning discharges in the Earth’s atmosphere are detected in the frequency range from 5–90 Hz. The inferred frequency dependent relative wave propagation velocities are in excellent agreement with theoretical predictions. The full sequence of the resonance frequencies may be used to monitor the global ionospheric D-layer height variability resulting from space weather phenomena. Citation: Füllekrug, M. (2005), Detection of thirteen resonances of radio waves from particularly intense lightning discharges, Geophys. Res. Lett., 32, L13809, doi:10.1029/2005GL023028.

1. Background

[2] Particularly intense lightning discharges in the troposphere transmit radio waves at extremely-low frequencies [Sentman, 1995], which propagate with little attenuation within the Earth’s atmosphere [Chapman and Jones, 1964] and exhibit constructive interference, denoted Earth-ionospheric cavity (or Schumann) resonance. This electromagnetic resonance phenomenon in a thin spherical shell geometry was predicted theoretically [Schumann, 1952] prior to experimental confirmation [Balser and Wagner, 1960]. These first observations reported five resonance frequencies from 7.8 to 32.5 Hz based on the spectral analysis of 2 hour long recordings of naturally occurring radio noise. The determined center frequencies of the resonances were supported by numerical model computations [Madden and Thompson, 1965], which were undertaken to infer a mean global conductivity profile of the lower ionosphere. A similar investigation used radio waves from intense lightning discharges with a comparison to numerical model computations and reported the detection of six resonances from 7–38 Hz [Jones and Kemp, 1970; Ogawa et al., 1979]. Eight distinct resonances were detected in the frequency range from 7–52 Hz by use of 24 hour long recordings of radio noise [Sentman, 1987], supported by analytical modeling based on Maxwell’s equations [Sentman, 1990, 1996]. These results were experimentally confirmed by use of radio noise recordings during an effective time interval of 2.6 days [Füllekrug and Fraser-Smith, 1996]. It may be concluded that further averaging of radio noise recordings can not detect additional resonances, since the latter measurements were undertaken at an extremely quite site in an electromagnetic environmental protection area [Fraser-Smith et al., 1992]. On the other hand, recent theoretical model calculations indicated the possible existence of up to eleven resonances [Nickolaenko and Hayakawa, 1998]. If confirmed, this finding would explain in a natural way the conundrum that only the radio noise in the frequency range from 5–100 Hz (and not from 5–50 Hz, as eight globally observable resonances do imply) exhibits a diurnal variation with universal time, which is similar to the diurnal variation of the atmospheric electric field (Carnegie curve, since both variations are thought to result from global lightning activity [Holzer and Deal, 1956; Füllekrug et al., 1999]). It may therefore be speculated that resonances do indeed exist in the frequency range from 52–100 Hz, but that they remain undetected as a result of the limited bandwidth of the measurement instruments and/or the statistical analysis methods used. This letter reports the detection of five additional resonances in the frequency range 55–90 Hz by use of broadband recordings of radio waves from particularly intense lightning discharges.

2. Observations and Modeling

[3] The radio waves are recorded with induction coil magnetometers in Silberborn, Germany, during April 1998. The location was chosen for its particularly quiet electromagnetic environment with little interference from power line radiation at 50 Hz and higher harmonic frequencies. The original recordings are corrected for the instrumental response and the remaining narrow band interference from power line harmonic radiation is removed during the digital pre-processing of the data. One second long time intervals of the broadband (0.5–200 Hz) waveforms from 52510 globally triangulated particularly intense lightning discharges are extracted [Füllekrug and Constable, 2000], transformed to the frequency domain and averaged into one spectrum for all source receiver distances from 2–18 Mm [Füllekrug, 2000]. Seven resonances are immediately apparent in the resulting spectrum (Figure 1, left panel). Six additional resonances are detected upon enlargement of the graphical display (Figure 1, right panel). The magnetic field amplitudes at the resonance frequencies decrease with increasing frequency such that the resonances at higher frequencies are more difficult to detect. For comparison of the observational results with theoretical model calculations, the short pulse approximation of the normal mode expansion with frequency dependent ionospheric heights

\[
B_0(\omega, \theta) = \frac{Q l}{4\pi a^2 h_1(\omega) c_0} \sum_n \frac{2n + 1}{(\omega - \omega_n)(\omega + \omega_n)} P^l_n(\cos \theta)
\]  

(1)
is used [Sentman, 1996; Füllekrug, 2000]. In this approach, the theoretical spectral magnetic field amplitude \( B_t(\omega, \vartheta) \) is related to the intensity of the lightning discharge, the geometric spreading of the radio wave and the ionospheric transfer function. The intensity of the lightning discharge is characterised by the charge moment \( Q_l \), which describes the amount of charge \( Q \) lowered from cloud to ground within a vertical lightning channel of length \( l \). The geometric spreading of the radio wave is described with the associated Legendre polynomials \( P_n^m(\cos \vartheta) \) of degree \( n \) and order \( m = 1 \) at an angular distance \( \vartheta \) from the lightning discharge on a spheroidal Earth with radius \( a \). The ionospheric transfer function is characterised by the frequency dependent conduction boundary \( h_1(\omega) \approx 50 \) km, where the displacement and conduction current become equal, and the complex modal frequency

\[
\omega_\text{c} = \sqrt{n(n+1) \frac{c}{a} \sqrt{\frac{h_1(\omega)}{h_2(\omega)}} \left[ 1 - i \frac{\pi}{4} \left( \frac{s_1}{h_1(\omega)} + \frac{s_2}{h_2(\omega)} \right) \right]}
\]

[Greifinger and Greifinger, 1978; Sentman, 1990; Füllekrug, 2000], where \( h_2(\omega) \approx 100 \) km is the ionospheric height where the radio waves are reflected and \( s_1 \approx s_2 \approx 2.5 \) km are scale heights, which determine the exponential increase of the ionospheric conductivity in the atmosphere. The theoretical magnetic field spectrum is shown for comparison with the observed spectrum in Figure 1. The

**Figure 1.** Thirteen resonances are detected in the observed magnetic field spectrum of radio waves from particularly intense lightning discharges (solid lines). The observations are supported by the theoretical magnetic field spectrum calculated with the short pulse approximation of the normal mode expansion with frequency dependent ionospheric heights (dashed lines). A simple scaling law can be used to approximate the decrease of the spectral amplitudes with frequency of the theoretical magnetic field spectrum (right panel, inset, dashed dotted line).

**Figure 2.** The transfer function between the observed and the approximated magnetic field spectrum (*) and the transfer function between the theoretical and the approximated magnetic field spectrum (x) reveal the full sequence of thirteen resonance frequencies from 5–90 Hz. The center frequencies of the resonances (o) are determined from the relative maxima of the harmonically interpolated transfer functions (solid and dashed lines) with a frequency resolution of 10 mHz.
observed and the theoretical magnetic field spectrum both exhibit the same decrease of the amplitudes with frequency and the resonant structure of the spectrum. It is interesting to note that the modeling predicts a fourteenth resonance at about 90 Hz, which is not apparent in the observations. As a result of the decrease of the magnetic field amplitudes with frequency and the limited spectral resolution of 1 Hz, it is not possible to directly determine the resonance frequencies from the observed spectrum. Consequently, a detrending (pre-whitening) technique needs to be applied to remove the (red) spectral behaviour from the observed spectrum and an interpolation technique needs to be applied to increase the frequency resolution of the spectrum prior to the determination of the resonance frequencies. In first order approximation, the theoretical magnetic field spectrum can be described with a simple scaling law \( B_\delta(\omega) = B_\delta(\omega) \) [Lanzerotti et al., 1990; Fraser-Smith et al., 1991], where \( B_\delta \approx 500 \text{ pT} \) is the best fitting magnetic field amplitude for the scaling law (Figure 1, right panel, inset). The transfer function between the theoretical \((B_i)\) and the approximated \((B_\delta)\) magnetic field spectrum \( T_i = B_i/B_\delta \) is then ideally \( T_i(\omega) \equiv 1 \) at the resonance frequencies. The transfer function between the measured \((B_m)\) and the approximated \((B_\delta)\) magnetic field spectrum \( T_m = B_m/B_\delta \) is calculated for comparison. Both transfer functions are interpolated harmonically to a frequency resolution of \( \Delta f = 10 \text{ mHz} \), which corresponds to an extension of the analysis time interval to 100 s without adding information (zero padding). This method unambiguously reveals the full sequence of thirteen resonance frequencies from 5–90 Hz (Figure 2). No fourteenth resonance can be distinguished from the observations. The center frequencies of the resonances are determined from the relative maxima of the interpolated transfer functions. These center frequencies are related to the frequency dependent relative wave propagation velocity \( v(\omega) = \sqrt{(h_1(\omega)/h_2(\omega))} \) of the radio wave, which is determined from the observed resonance frequencies \( \omega_n \) and theoretical calculations by use of Equation 2. The relative wave propagation velocities of the newly discovered resonances exhibit an excellent agreement with the theoretical predictions (Figure 3). This excellent agreement makes it possible to infer a mean global ionospheric conductivity profile [Füllekrug et al., 2002], which is compared to a mean one scale height conductivity profile [Tran and Polk, 1979a, 1979b], based on scattered rocket measurements of the atmospheric conductivity [Mechtly et al., 1972; Widdel et al., 1976].

3. Conclusions

[4] Thirteen resonances of radio waves from particularly intense lightning discharges are detected in the frequency range 5–90 Hz. The observed resonance frequencies are used to determine relative wave propagation velocities in excellent agreement with theoretical predictions. The relative wave propagation velocities are subsequently used to infer a mean global ionospheric conductivity profile. The entire analysis is based on one second long observations of radio waves from particularly intense lightning discharges, which occur about once per minute [Füllekrug and Constable, 2000; Sato and Fukunishi, 2003]. It seems therefore plausible that a network of measurement instruments around the globe could simultaneously detect the radio waves from individual lightning discharges and determine the resonance frequencies in real time to infer a mean global ionospheric conductivity profile, which may be determined with a time resolution of one minute to monitor the global ionospheric D-layer height variability resulting from space weather phenomena.

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References

Lanzerotti, L., C. Maclennan, and A. Fraser-Smith (1990), Background magnetic spectra: \( \sim 10^{-5} \) to \( \sim 10^{-7} \) Hz, Geophys. Res. Lett., 17(10), 1593–1596.


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