Symbolic Representation of Alternating Electric Waves

Dr. Eric Dollard
Borderland Sciences, P.O. Box 429, Garberville, CA 95440-0429, U.S.A.
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I. ALGEBRAIC & GRAPHICAL REPRESENTATION OF ALTERNATING ELECTRICAL ENERGY

Let:
$e_{|}$ = e.m.f. consumed by the consumption of magnetic lines of induction, or magnetic energy.
$e_{||}$ = e.m.f. consumed by the storage of magnetic energy.
$i_{|}$ = m.m.f. consumed by the consumption of dielectric lines , or dielectric energy.
$i_{||}$ = m.m.f. consumed by the storage of dielectric energy.

$\dot{E} = e_{|} - je_{||}$, total e.m.f. consumed by the alternating current, volts complex.
$\dot{I} = i_{|} + ji_{||}$, total m.m.f. consumed by the alternating currect, amperes complex, where $-j$ is 90° lead and $+j$ is 90° lag.

Thus
$e_{|}$ = the voltage drop of effective series resistance due to consumption of magnetic energy,

$$e_{|} = I_0 R, \quad \text{volts, real.} \tag{1}$$

$e_{||}$ = the voltage drop of effective series reactance due to storage of magnetic energy,

$$e_{||} = I_0 X, \quad \text{volts, leading reactive.} \tag{2}$$

Hence, the total consumption of e.m.f., or voltage drop is

$$\dot{E} = e_{|} - je_{||}, \text{volts, complex} \tag{3}$$

$$\dot{E} = I_0(R - jX), \tag{4}$$

$$\dot{E} = I_0 Z. \tag{5}$$

Z is the effective series impedance of the A.C. circuit.
The e.m.f. measured by a voltmeter across Z is

$$\dot{E}_0 = |\dot{E}| = \sqrt{e_{|}^2 + e_{||}^2}. \text{ volts} \tag{6}$$

$i_{|}$ = the current drop of effective shunt conductance due to consumption of dielectric energy,

$$i_{|} = E_0 G. \text{ amperes, real} \tag{7}$$

$i_{||}$ = the corrent drop of effective shunt suceptance due to storage of dielectric energy,

$$i_{||} = E_0 B. \text{ amperes, lagging, reactive} \tag{8}$$

Hence, the total consumption of m.m.f., per current drop is

$$\dot{I} = i_{|} + ji_{||}, \text{ amperes, complex} \tag{9}$$

$$\dot{I} = E_0(G + jB), \tag{10}$$

$$\dot{I} = E_0 Y. \tag{11}$$

Y is the effective shunt admittance of the A.C. circuit. The current measured by an ampere through Y is

$$I = |\dot{I}| = \sqrt{i_{|}^2 + i_{||}^2}. \text{ amperes} \tag{14}$$

Because, in any alternating current circuit, the magnetic energy is discharging during the time in which the dielectric energy is charging, the e.m.f. produced by the discharge of magnetic energy is in phase opposition and rotating in opposite direction to the m.m.f. consumed by the charge of dielectric energy.

Thus,
$-je_{||}$ represents clockwise rotation of e.m.f. $e_{||}$,
and
$+ji_{||}$ represents counterclockwise rotation of m.m.f. $i_{||}$.

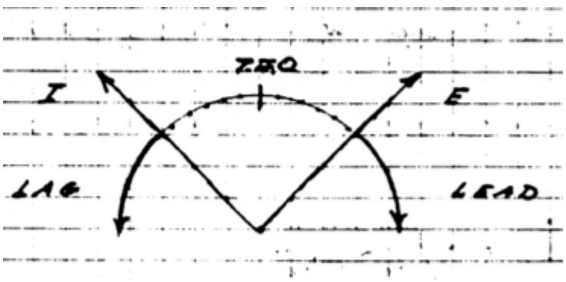
The effective values of impedance Z and admittance Y can only be directly taken from measurement of the circuit constants if the effective velocity of the A.C. wave is that of light. In many electrical networks the velocity is much different than the velocity of light due to the existence of transverse and longitudinal energy flow.

Thus, the circuit constants R, G, X, and B must be multiplied by a wave factor that represents the ratio of transverse to longitudinal energy flow. The derivation of this wave factor is beyond the scope of this paper and the circuit constants R, G, X, B will be those of the electromagnetic field relationships involved and not the conventional schematic representation of the A.C. circuit.

Since the alternating power is the product of clockwise rotation vector $\dot{E}$, and counter clockwise rotation vector $\dot{I}$, that is $\dot{E}$&$\dot{I}$ rotating in opposite directions, the power rotates at twice the frequency of $\dot{E}$ or $\dot{I}$.

The physical meaning is, if an incandescent lamp, which the brilliance of is a source of alternating e.m.f., the lamp glows with the same brilliance on the positive and negative half cycles of the e.m.f.. Thus the brilliancy of the lamp pulsates at twice the frequency of the e.m.f or current.

For the proper understanding of alternating electric power, it is of importance to investigate the product of the opposite rotation vectors $\dot{E}$ and $\dot{I}$.



Algebraically, it is

$$\begin{aligned}\dot{P} &= \dot{E}\dot{I} \quad \text{Volt amperes, complex}\\ \dot{P} &= (e_{|} - je_{||})(i_{|} + ji_{||})\\ &= e_{|}i_{|} + je_{|}i_{||} - je_{||}i_{|} - j^2 e_{||}i_{||}\end{aligned}$$

Since the factor $-j^2$ represents 360° rotation

$$-j^2 = +1,$$

hence

$$\dot{P} = \dot{E}\dot{I} = (e_{|}i_{|} + e_{||}i_{||}) + j(e_{|}i_{||} - e_{||}i_{|}) \tag{15}$$

The product $\dot{P} = \dot{E}\dot{I}$ consists of two parts:
the real part of useful energy flow

$$P_1 = (e_{|}i_{|} + e_{||}i_{||}) \quad \text{Watts;} \tag{16}$$

the imaginary part of reactive energy flow

$$P_j = (e_{|}i_{||} - e_{||}i_{|}) \quad \text{Vars.} \tag{17}$$

The real part will be distinguished by the subscript 1, the imaginary part by the subscript j.

Thus, the total power of the A.C. circuit, $\dot{P}$, is in symbolic representation,

$$\dot{P} = P_1 + P_j \quad \text{Volt amperes, complex.} \tag{18}$$

just as the symbolic representation of voltage and current as complex quantities does not only give the mere intensity but also the direction, or phase

$$\dot{I} = i_{|} + ji_{||} \quad \text{vectorial amperes.}$$

of magnitude(as read on ammeter scale)

$$I = |\dot{I}| = \sqrt{i_{|}^2 + i_{||}^2} \quad \text{absolute amperes.}$$

and direction or phase

$$\theta = \tan^{-1}\frac{i_{||}}{i_{|}} \quad \text{radians.}$$

So does the double frequency vector product $\dot{E}\dot{I}$ denote more than the mere power as determined by the product of voltmeter and ammeter readings, by giving its two components;

$$P_1 = (e_{|}i_{|} + e_{||}i_{||}) \quad \text{Watts,} \tag{16}$$

$$P_j = (e_{|}i_{||} - e_{||}i_{|}) \quad \text{Vars.} \tag{17}$$

The true energy volt-amperes and the reactive energy volt-amperes respectively.

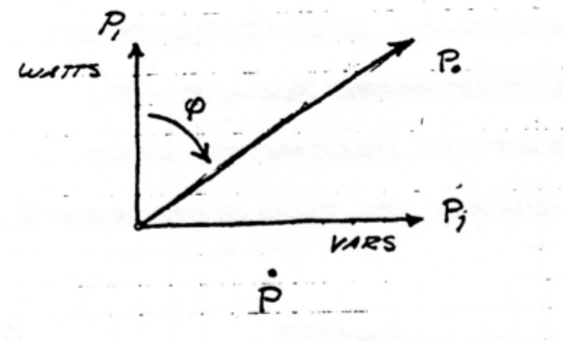
Hence,

$$\dot{P} = \dot{E}\dot{I} = P_1 + jP_j \quad \text{Vectorial V.A..}$$

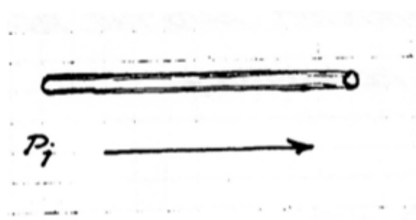
$$P_0 = EI = \sqrt{P_1^2 + P_j^2} \quad \text{Absolute V.A..}$$

$$\varphi = \tan^{-1}\frac{P_j}{P_1} \quad \text{radians.}$$

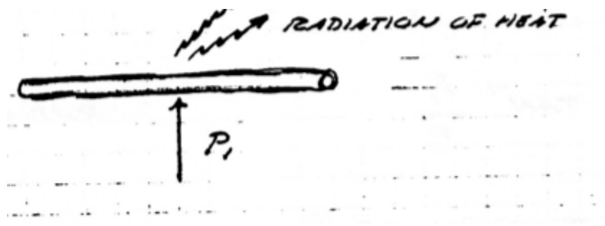
That is, the true power P_1, and the reactive power P_j are the two rectangular components of the apparent power P_0, where P_0 is the product of the voltmeter and ammeter readings.



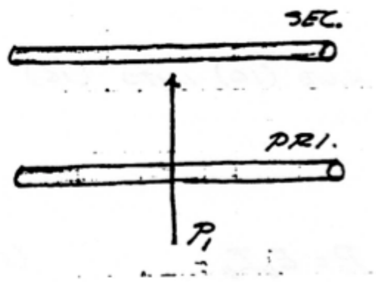
An approximate physical representation is, the reactive power flows along the axis of the conductor,



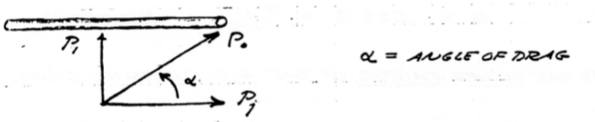
The real power flows into the conductor sand is covered to heat



or the real power is delivered to an adjacent conductor, such as the rotor winding of a motor or a secondary winding of a transformer.



Thus, in the flow of power along the conductor, a certain amount of drags into the conductor where it is converted into heat, or carried off to another conductor via mutual induction.



Hence, the reactive power P_j is the carrier of electromagnetic energy, and the real power is that taken off to the side to perform useful work. Thus the reactive power is as important as the real power in an electric circuit since without it energy cannot be conveyed. This is illustrated by the fact that the power factor of a good transmission line is zero.

Substituting equations (4) and (10) into (16) gives the real power as,

$$P_1 = (e_| i_| + e_{||} i_{||}) \qquad P_0 = E_0 I_0, \tag{16}$$

$$= [P_0(RG + XB)]^* \quad \text{Watts}, \tag{19}$$

where $*$ denotes transmission lines only and denoting

$$a = (RG + XB) \quad \text{percent}, \tag{20}$$

$$a = \cos\theta = \frac{P_1}{P_0} \quad \text{percent}, \tag{21}$$

as the <u>power factor</u> of the A.C. circuit gives

$$P_1 = aP_0 = aEI \quad \text{Watts}. \tag{22}$$

That is, the product of the voltmeter reading, E, and of the ammeter reading, I, multiplied by the power factor, a, gives the amount of the real power flow, P_1. P_1 is the rate at which energy is removed from the electric circuit by heat loss or the production of mechanical energy.

Substituting equations (4) and (10) into (17) gives reactive power as

$$P_j = (e_| i_{||} - e_{||} i_|) \tag{17}$$

$$= [P_0(RB - XG)]^* \quad \text{Vars}, \tag{23}$$

where $*$ denotes transmission lines only and denoting

$$b = (RB - XG) \quad \text{percent}, \tag{24}$$

$$b = \sin\theta = \frac{P_j}{P_0} \quad \text{percent}, \tag{25}$$

as the <u>induction factor</u> of the A.C. circuit gives

$$P_j = bP_0 = bEI \quad \text{Vars}. \tag{26}$$

That is, the product of the voltmeter reading, E, and of the ammeter reading, I, multiplied by the induction factor, b, gives the amount of reactive power flow, P_j. P_j is the rate at which energy is carried along the electrical circuit by self-induction, or if the circuit is at the end of the line, the rate at which energy is bounced back into the line.

Hence, the total power in terms of equations (14) and (23) is

$$\dot{P} = P_0\left[(RG + XB) + j(RB - XG)\right] \tag{27}$$

volt-amperes complex t.e.m. wave.

Substituting a and b gives

$$\dot{P} = P_0\,(a + jb) \tag{28}$$

or in trigonometric form

$$\dot{P} = P_0\,(\cos\theta + j\sin\theta)\,. \tag{29}$$

Thus

$$\dot{P} = \gamma_0 P_0 \quad \text{volt-amperes} \quad \text{complex}, \tag{30}$$

where

$$\gamma_0 = (a + jb) = (\cos\theta + j\sin\theta), \tag{31}$$

$$|\gamma_0| = \sqrt{a^2 + b^2} = 1,$$

$$\varphi = \tan^{-1}\frac{b}{a}.$$

The factor γ_0 is called the wave factor of the A.C. circuit, and is a complex quantity consisting of the real part a, the circuit power factor, and the imaginary part b, the circuit induction factor. The magnitude of γ_0 is always equal to unity, that is, 100%, as it represents all factors.

Substituting into (27) dividing out power,

$$\gamma_0 = (a + jb) \quad \text{percent, complex,} \tag{31}$$

$$\gamma_0 = (RG + XB) + j(RB - XG), \tag{32}$$

$$\gamma_0 = ZY \quad \text{percent, complex.} \tag{33}$$

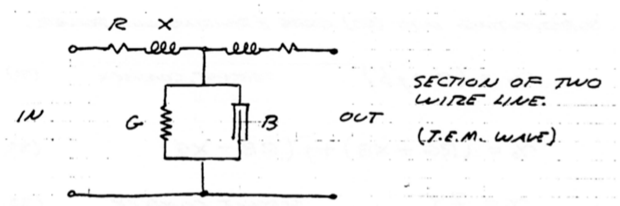
Equation (32) is known as the wave equation of the A.C. circuit. This equation is the fundamental equation for the investigation of alternating current phenomena.

Thus, the product of the voltmeter reading and the ammeter reading, P_0, multiplied by the wave factor, γ_0, gives the amount of vectorial power flow, $\dot{P}$.

$$\gamma_0 P_0 = P_1 + jP_j.$$

Breaking the wave equation down into its components will help in the understanding of its practical significance.

$$\gamma_0 = ZY = (RG + XB) + j(RB - XG)$$
$$= ZY = (R - jX)(G + jB).$$

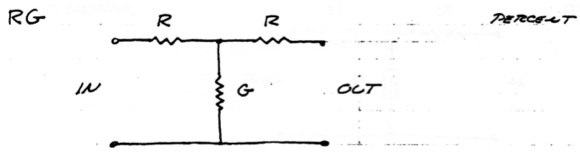


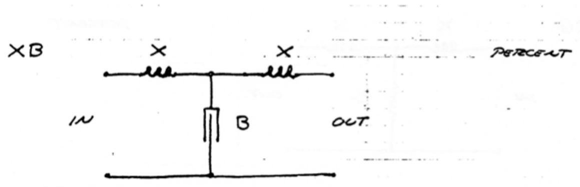
total circuit element↑, RG, XB, RB, and XG.

$X = \omega L = 2\pi FL$ Ohms = Henry per second
$B = \omega C = 2\pi FC$ Mhos = Faraday per second
$\omega L = 2\pi F$
angular velocity of rotation in radians per second
F = cycles per second

RG is the rate of energy consumption with respect to distance along the A.C. circuit, and is independent of the frequency of the A.C. wave.

XB is the rate of nervy exchange between the magnetic fields stored energy and the dielectric fields stored energy, that is, the rate at which the discharging magnetic field charges the dielectric field and vis. vis. Thus XB is the natural frequency





of oscillation of the circuit, and determines the velocity of the wave along the circuit.

RB is the time rate of the draining of dielectric stored energy into the circuit resistance, and is called the time constant of dielectric energy.

XG is the time rate of the draining of magnetic stored energy into the circuit conductance, and is called the time constant of magnetic energy.

Consequently, in any alternating current circuit there exist for distinct flows of energy:

$+e_{|}i_{|}$, a function of RG, represents the flow of energy out of the circuit, and is independent of the frequency or phase of the applied A.C. wave.

$+e_{||}i_{||}$, a function of XB, represents the pulsation of energy between the magnetic and dielectric fields surrounding the circuit.

$+e_{|}i_{||}$, a function of RB, represents the flow of energy into the dielectric field, through circuit resistance, R.

$-e_{||}i_{|}$, a function of XG, represents the flow of energy out of the magnetic field through circuit conductance G.

II. SYMBOLIC & GRAPHICAL ANALYSIS OF SPECIAL CASES

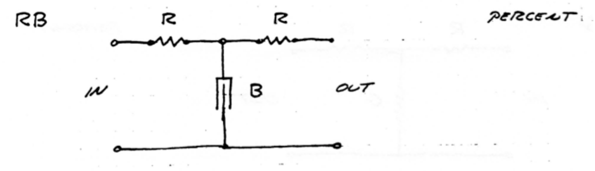
1) The wattless reactive power is absent, or the total power is real, if

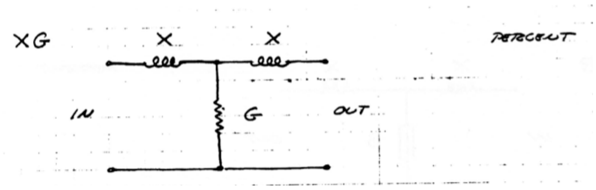
$$P_j = 0$$
$$(e_{|}i_{||} - e_{||}i_{|}) = 0.$$

Hence

$$e_{|}i_{||} = e_{||}i_{|}.$$

That is, the rate of dielectric dissipation is equal to the rate of magnetic dissipation of energy.



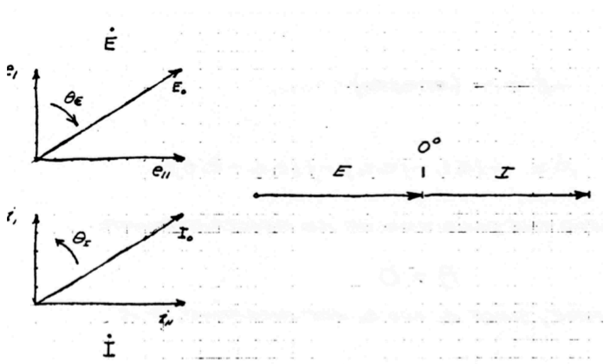


Thereby,

$$\frac{e_{\|}}{e_{|}} = \frac{i_{\|}}{i_{|}},$$

or

$$\tan(\dot{E}) = \tan(\dot{I}).$$



That is, $\dot{E}$ and $\dot{I}$ are in phase conjunction.

Substituting $P_j = 0$ into the equation for γ_0 gives

$$\gamma_0 = (a + jb) = +1 + j0$$
$$\gamma_0 = +1.$$

Thus the wave factor becomes the power factor, which is 100%. The induction factor is zero.

If, however, the circuit produces more real energy than it consumes, such as the secondary circuit of a motor or transformer, or a system involving co-generation, the back e.m.f. of series impedance becomes forward e.m.f.

$$\dot{E} = -(e_{|} - je_{\|})$$

and

$$\dot{P} = -(e_{|}i_{|} - je_{\|}i_{\|}) - j(e_{|}i_{\|} - e_{\|}i_{|}).$$

But, for the condition of no reactive power

$$P_j = 0.$$

Therefore, since e_{mid} and $e_{\|}$ are negative, it is

$$-e_{|}i_{\|} = -e_{\|}i_{|}.$$

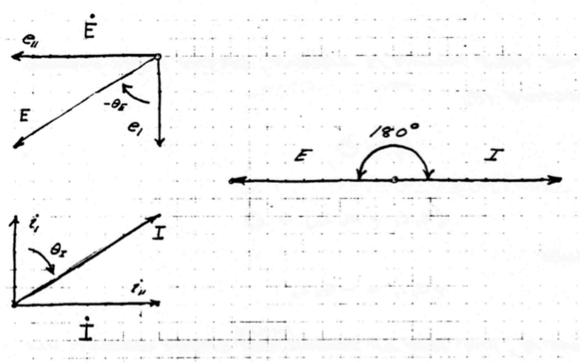
That is, the rate of dielectric charge is equal to the rate of magnetic charge of energy.

Thereby,

$$\frac{-e_{\|}}{-e_{|}} = \frac{i_{\|}}{i_{|}}.$$

or

$$\tan(-\dot{E}) = \tan(\dot{I}).$$



That is, $\dot{E}$ and $\dot{I}$ are in phase opposition.

Substituting $P_j = 0$ and $-P_1$ into the equation for γ_0 gives

$$\gamma_0 = (a + jb) = -1 + j0$$
$$\gamma_0 = -1.$$

Thus the wave factor, which is the power factor in the case of zero reactive power, now is negative 100%. The induction factor remains zero.

2) The true power is absent, or the total power reactive if

$$P_1 = 0$$
$$(e_{|}i_{|} + e_{\|}i_{\|}) = 0.$$

Hence

$$e_{|}i_{|} = -e_{\|}i_{\|}.$$

That is, the flow of power in and out of the circuit via RG is equal and in opposition to the flow of pulsing stored energy between magnetic and dielectric forms.

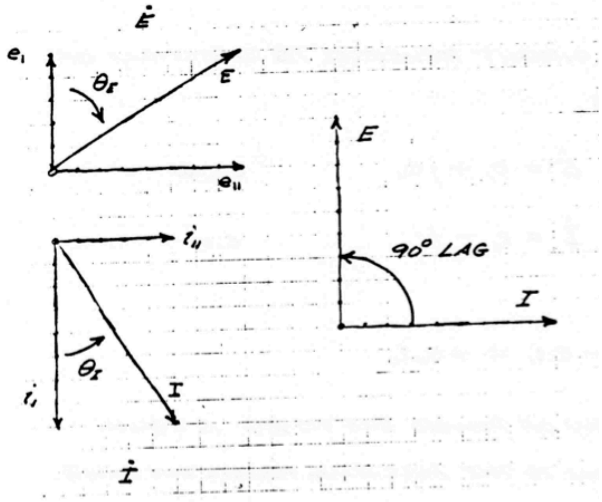
Thus,

$$\frac{e_{\|}}{e_{|}} = \frac{-i_{|}}{i_{\|}}.$$

or

$$\tan(\dot{E}) = -\cot(\dot{I}).$$

– that is, $\dot{E}$ and $\dot{I}$ are in phase lag quadrature.



Substituting $P_1 = 0$ into the equation for $gamma_0$ gives

$$\gamma_0 = (a + jb) = 0 + j$$
$$\gamma_0 = +j.$$

Thus, the wave factor becomes the induction factor for the condition of zero real power, and is 100%. The power factor is zero.

If, however, the circuit consumes less reactive power than it produces, such as with capacitor loads, synchronous condensers, etc. The alternating wave of the A.C. circuit reverses its direction of rotation, that is,

$$\dot{E} = e_{|} + je_{\|}. \quad \text{c.c.w}$$
$$\dot{I} = i_{|} - ji_{\|}. \quad \text{c.w.}$$

Hence

$$-e_{|}i_{|} = +e_{\|}i_{\|}.$$

or

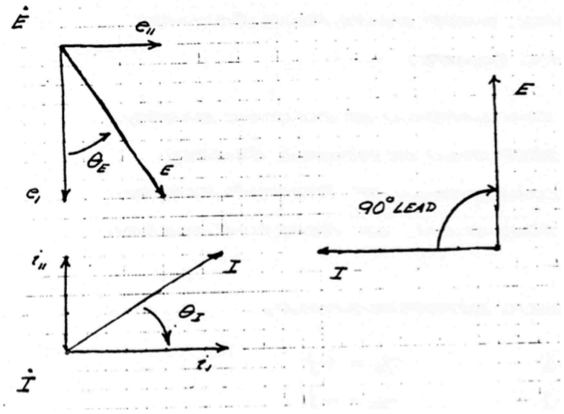
$$-\cot(\dot{E}) = \tan(\dot{I}).$$

That is, $\dot{E}$ and $\dot{I}$ are in phase lead quadrature.

Substituting reactive energy production $-P_j$ and $P_1 = 0$ into the equation for γ_0 gives

$$\gamma_0 = (a + jb) = 0 - j$$
$$\gamma_0 = -j.$$

Thus the wave factor, which is the induction factor for the condition of zero power flow, is negative 100%. The power factor is zero.



Consequently, there aexists four distinct classes of A.C. power:

1A) The consumption of electric energy.
1B) The production of electric energy.
2A) The consumption of reactive energy.
2B) The production of reactive energy.

or in symbolic representation,

$\gamma_0 = 1$	$\gamma_0 = -j$
$\gamma_0 = -1$	$\gamma_0 = -j$
real	reactive

and the power is

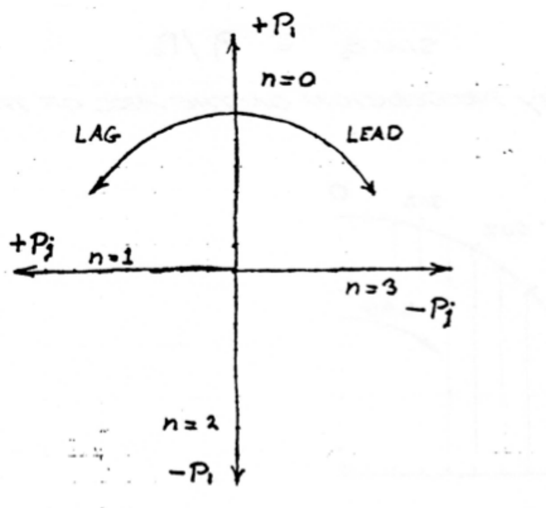
$$\dot{P} = +P_1 \quad \dot{P} = +P_j$$
$$\dot{P} = -P_1 \quad \dot{P} = P_j.$$

Hence γ_0 is a result of

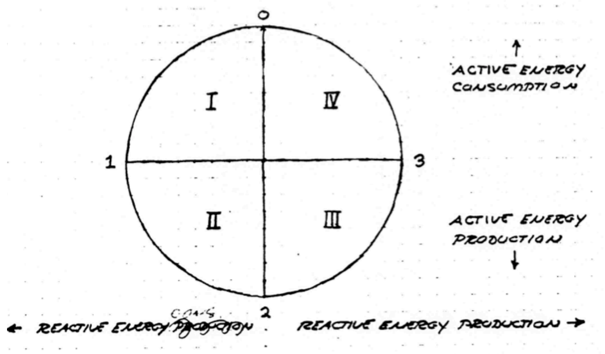
$$\sqrt[4]{+1} = k^n = \pm 1, \ \mp 1.$$

k^n is the quadrant operator.

Graphically, it is, k^n =



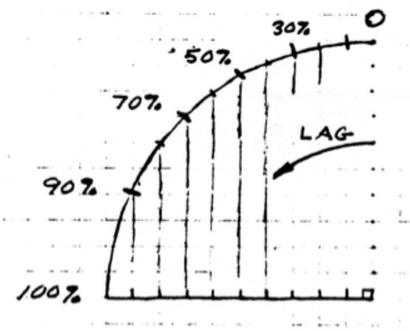
Thus, four quadrants,



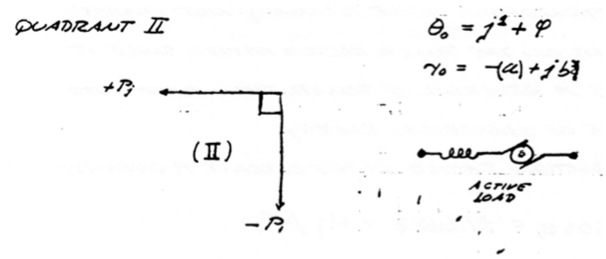
The induction factor of the circuit is given by

$$\sin\theta_0 = \frac{P_j}{P_0}.$$

and is given by projection on the arc of the quadrant



Thus, the corresponding induction meter scale section.

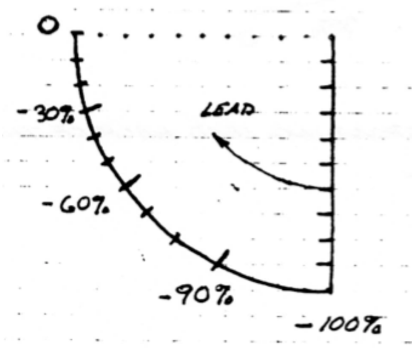


This quadrant represents production of real energy and the consumption of reactive energy, such as the combination of induced forward e.m.f. and reactive back e.m.f. of an alternator coil.

The power factor is given by

$$-\sin\theta_0 = \frac{-P_1}{+P_0}$$

and projection of the real power upon the arc of the quadrant gives

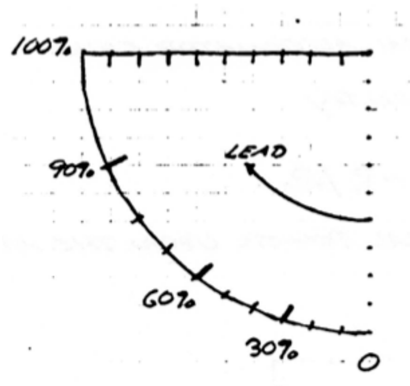


This portion of the curve is usually not marked by divisions on the power meter since it represents a reversal of power flow back into the source of electrical energy.

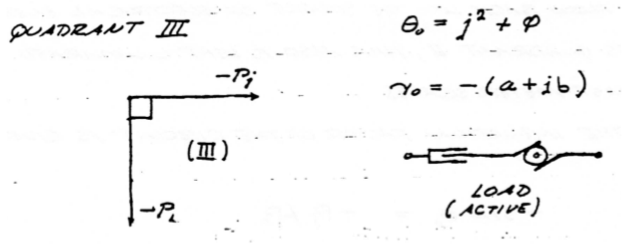
The induction factor of the circuit is given by

$$\cos\theta_0 = \frac{+P_1}{-P_0}$$

and projection of the reactive power upon the arc of the quadrant gives



Thus the corresponding lead half of the induction factor meter scale.

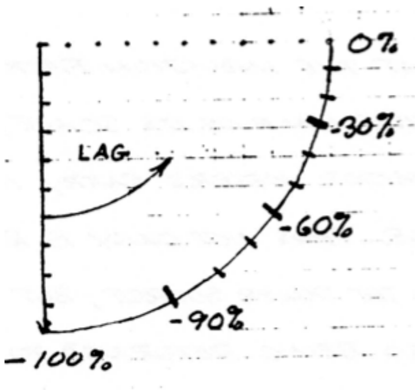


This quadrant represents the projection of both real and reactive energy, such as the combination of an alternator with negligible self induction and a capacitor.

The power factor is given by

$$\cos\theta_0 = \frac{-P_1}{-P_0}$$

and the projection of the real power upon the arc of the quadrant gives

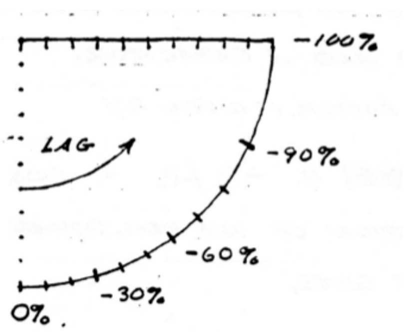


This portion of the curve also represents reversed power flow back into the source of electrical energy as does quadrant II, thus also is left unmarked on most meter scale.

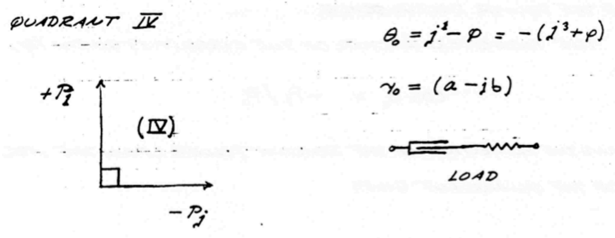
The induction factor of the circuit is given by

$$-\sin\theta_0 = \frac{-P_j}{-P_0}$$

and the projection of the reactive power upon the arc of the quadrant gives



It can be seen that the induction factor has now entered the negative half of its scale, 90° later than the power factor meters entry into its negative half scale. Thus, not only is real energy being returned to the power source, but reactive energy is now also being returned to the source of power.

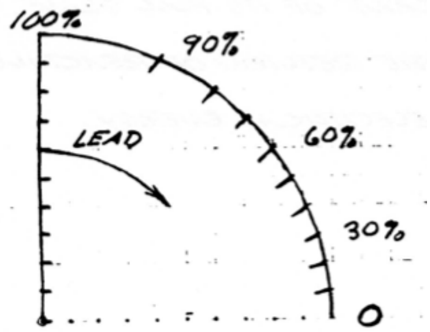


This quadrant represents the consumption of real energy and the production of reactive energy, such as a synchronous condenser.

The power factor is given by

$$\sin\theta_0 = \frac{+P_j}{+P_0}$$

and the projection of the real power upon the arc of the quadrant gives

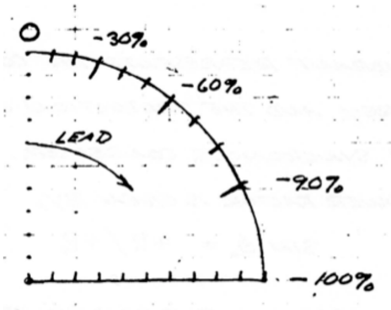


This curve will be recognized as the lead portion of the power factor scale.

The induction factor of the circuit is given by

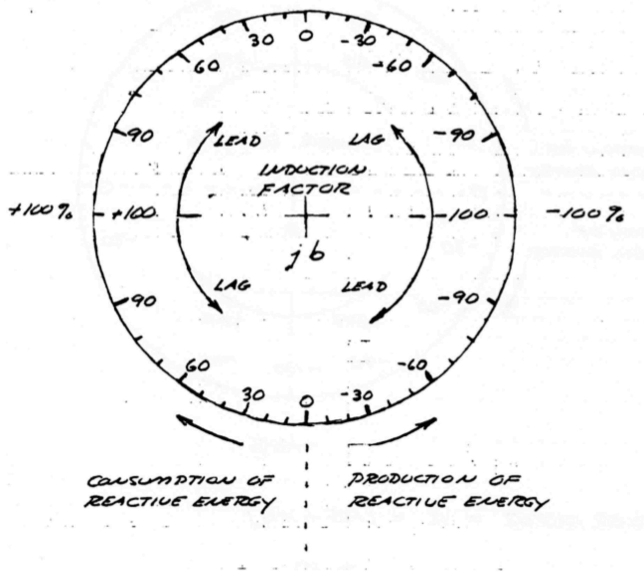
$$\cos\theta_0 = \frac{-P_j}{+P_0}$$

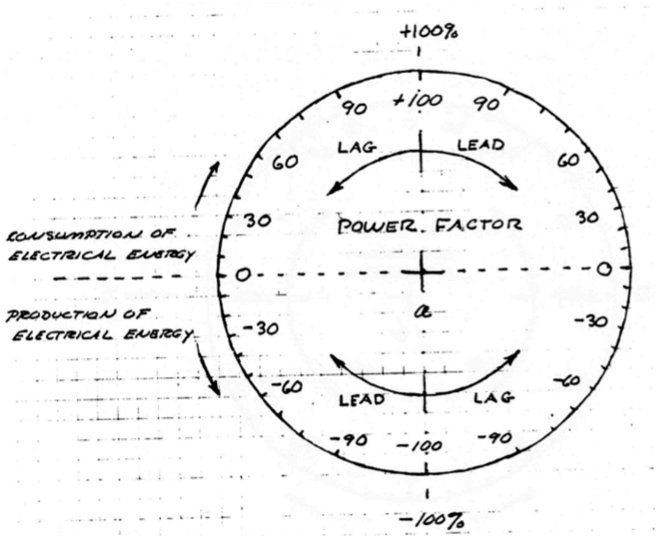
and the projection of the reactive power upon the arc of the quadrant gives



Thus, the induction factor scale is in the second negative quadrant of its full scale. Therefore, it is indicating the return of reactive energy into the source of electrical energy.

Having completed a complete cycle of alternating electrical energy, the complete meter scales are as follows:

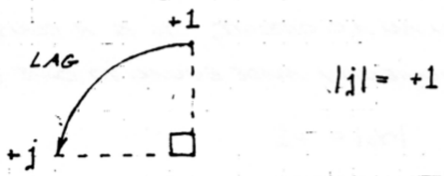




Hence, the physical significance of the equation of the electric wave,

$$\gamma_0 = a + jb = \cos\theta + j\sin\theta,$$

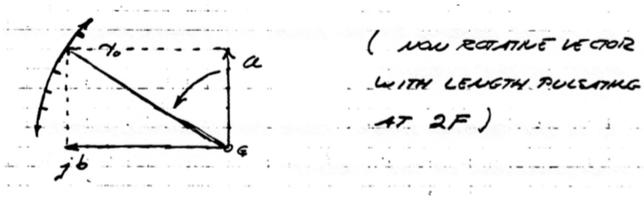
a is the reading taken from the power factor measurement of the circuit.
b is the reading taken from the induction factor measurement of the circuit.
$+j$ is that the induction factor scale is at right angles to the power factor scale.



Power factor = $a = (RG + XB) = \frac{P_1}{P_0}$
Ratio of real (active) power to apparent power.
Induction factor = $jb = j(RB - XG) = j\frac{P_j}{P_0}$
Ratio of reactive power to apparent power.

Multiplication by $+j$ means to rotate one quarter cycle(one quadrant) in the lag(c.c.w.) direction, and the magnitude of $+j$ is equal to $+1$.

Thus γ_0 is a vector consisting of a vertical component a, the power factor of the A.C. circuit, and a horizontal component, b, the induction factor of the circuit,



Since the length of the vector γ_0 is the ratios of a circle having no relation to the A.C. circuit, but is an arbitrary length, such as the diameter of meter scale, graph paper size, etc., and since γ_0 is a constant and is most conveniently made equal to one$(+1)$

$$|\gamma_0| = +1$$

and it flows that

$$a^2 + b^2 = 1,$$

$$\sin^2\theta + \cos^2\theta = 1.$$

Thereby,

$$a = \pm\sqrt{1 - b^2}, \quad \text{Power factor}$$

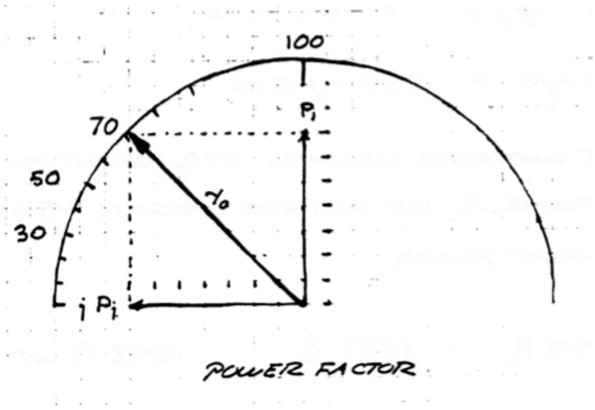
$$b = \pm\sqrt{1 - a^2}, \quad \text{Induction factor}$$

That is, if no induction factor meter exists, the induction factor may be calculated from the power factor, since the magnitude of the wave factor is always 100% or 1.

Thus, the wave factor γ_0 of the A.C. circuit points the direction of power flow with respect to the A.C. cycle.

III. EXAMPLES

1) An induction meter is delivering 12.5 horsepower($10kW$) to a mechanical load. The reactive energy of the motor's unused magnetic flux is $5kW$(6.25 HP).



Hence,

$$P_1 = 10kW, \quad \text{wattmeter}$$

$$P_j = 5kW, \quad \text{varmeter}$$

$$P_0 = \sqrt{10^2 + 5^2} = 11.2kW, \quad \text{voltmeter} \times \text{anmeter}$$

$$a = 89\%, \quad \text{Power factor meter}$$

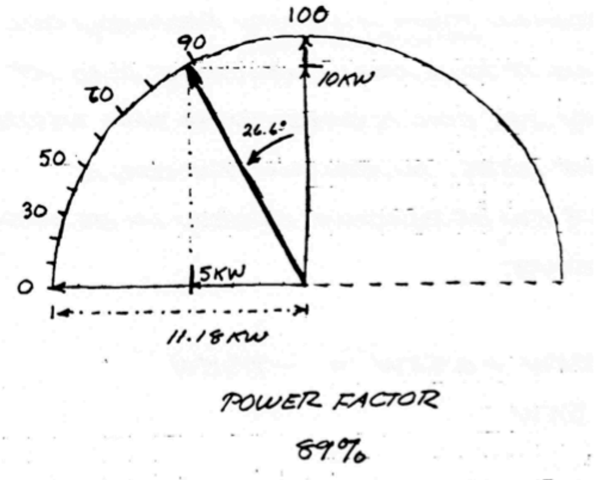
$$b = 45\%, \quad \text{Induction factor meter}$$

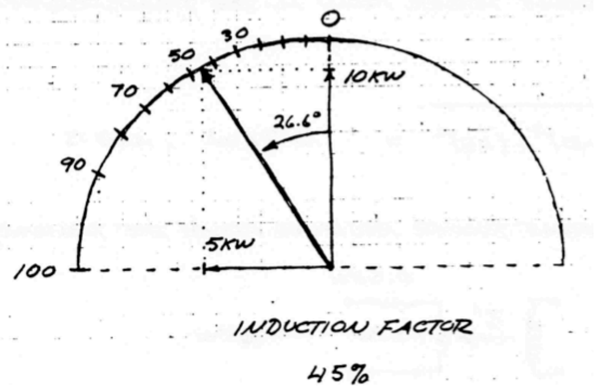
$$\theta_0 = \tan^{-1}\frac{P_j}{P_1} = 26.6^\circ \text{ Lag}$$

$$\gamma_0 = a + jb = 0.89 + j0.45$$

Thus the wattmeter indicates 89% of the total apparent power, P_0, the varmeter indicates 45% of the total apparent power.

$$\dot{P} = 89\%P_0 + j45\%P_0 = 100\%P_0 \ \text{ at } 26.6^\circ$$





2) A transformer coil is receiving $10kW$ of active power by induction from a nearly primary coil and delivering it to a non-inductive load at 95% efficiency. The coil is thus dissipating $0.5kW$ in heat lists. In addition, the coil is consuming $1.5kW$ of reactive energy in its magnetic leakage reactance.

Hence,

$$P_1 = -10kW + 0.5kW = -9.5kW$$

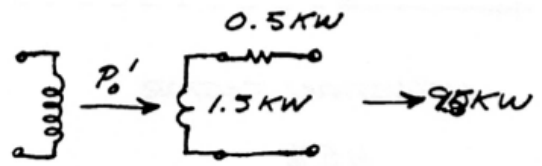
$$P_j = 1.5kW$$

$$P_0 = \sqrt{(-9.5)^2 + (1.5)^2} = 9.6kW$$

and is the apparent power flow in the transformer coil.

$$P_0' = \sqrt{(-10)^2 + (1.5)^2} = 10.2kW$$

and is the apparent power received from the primary coil.



The power factor of the transformer coil receiving induced energy is

$$a = P_1/P_0 = -9.5/9.6 = -98\%$$

The induction factor is

$$b = P_j/P_0 = 1.5/9.6 = +16\%$$

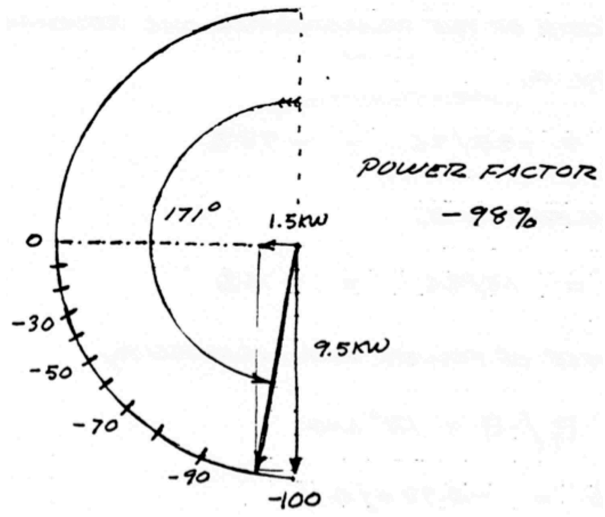
The phase angle of power flow is given by

$$\theta_0 = \tan^{-1}(P_j / - P_0) = 171°\text{Lag}$$

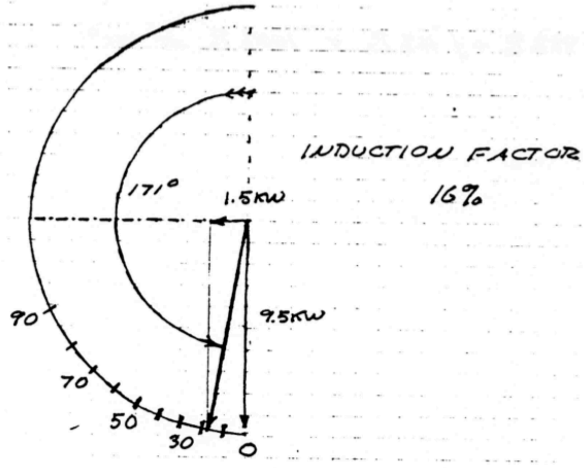
$$\gamma_0 = a + jb = -0.98 + j0.16$$

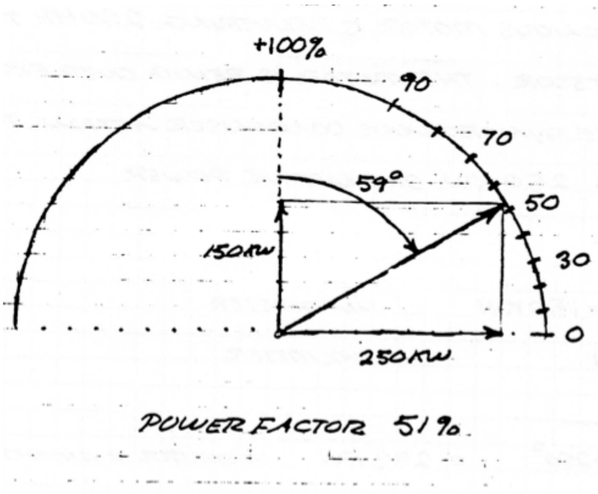
Thus, the vectorial power is

$$\dot{P} = -98\%P_0 + j16\%P_0 = 100\%P_0 \ \ at \ \ 171°$$



3) A synchronous motor is delivering 200HP to an air compressor. The motor is being over excited to facilitate synchronous condenser action, thereby generating $250kW$ of reactive power.





Hence,

$P_1 = 200HP = 150kW$ Wattmeter

$P_j = -250kW$ Varmeter

$P_0 = \sqrt{(150)^2 + (250)^2} = 283kW$ voltmeter × ammeter

$a = P_1/P_0 = 51\%$ active energy Power factor meter

$b = P_j/P_0 = 85\%$ reactive energy Induction factor meter

$\theta_0 = \tan^{-1}(-P_j/P_0) = 59°$Lead

$\gamma_0 = a - jb = 0.51 - j0.85$

Thus, the wattmeter is inducting 51% of the total apparent power. The war meter is indicating 85% of the total apparent power.

$$\dot{P} = 51\%P_0 - j85\%P_0 = 100\%P_0 \ \ at \ -59°$$

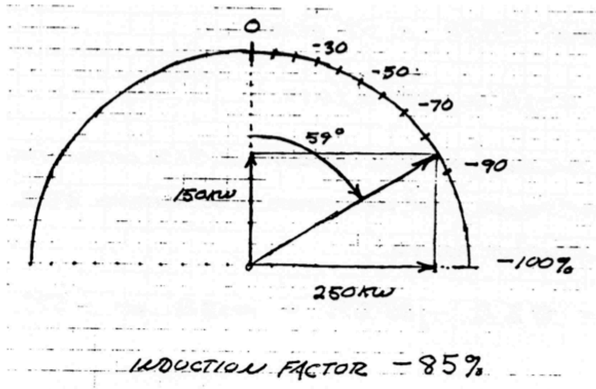
IV. SUMMARY

1) For the algebraic representation of A.C. power the e.m.f. $\dot{E}$ and m.m.f. $\dot{I}$ must be expressed in symbolic representation by real and imaginary quantities, $A_| \neq A_{||}$.

2) The vectors of e.m.f., $\dot{E}$, and m.m.f., $\dot{I}$, rotate in opposite directions as a result of consumption of m.m.f. coinciding with the production of e.m.f. during the A.C. cycle.

3) The effective impedance Z and effective admittance Y are a result of the electric field surrounding the circuit materials. In order for these values to represent the values measured by ohmmeters, etc., the ratio of transverse to longitudinal flow must be considered.

4) Alternating electric power is a result of two vectors of opposite rotation, $\dot{E}$ and $\dot{I}$, thus power pulsates at twice the frequency of volts or amperes.



5) A.C. power consists of two components, the real active energy, and the imaginary reactive energy.

6) A.C. power can be represented as a complex quantity with real and imaginary components in the same manner as $\dot{E}$ or $\dot{I}$ themselves.

7) Reactive power is as important as real power for the transmission and utilization of A.C. energy.

8) The power factor is used as a correction factor applied to the wattmeter reading to give the quantity of reactive power flow. The induction factor is used as a correction factor applied to the varmeter reading to give the quantity of active power flow. Both serve as correction factors to the volt-ampere reading to give the quantity of active and reactive power respectively.

9) The vector sum of the power factor and the induction factor is the wave factor of the A.C. circuit. The wave factor gives the direction of power flow.

10) In an A.C. circuit consuming active energy only, the e.m.f. $\dot{E}$ and m.m.f. $\dot{I}$ are in phase conjunction, (0°).

11) In an A.C. circuit consuming reactive energy only, the

e.m.f. $\dot{E}$ and m.m.f. $\dot{I}$ are in phase opposition, (180°).

12) In an A.C. circuit consuming reactive energy only, the e.m.f. $\dot{E}$ and m.m.f. $\dot{I}$ are in phase lag quadrant, (+90°).

13) In an A.C. circuit producing reactive energy only, the e.m.f. $\dot{E}$ and m.m.f. $\dot{I}$ are in phase lead quadrant, (−90°).

14) The four classes of power are represented by the algebraic symbol,

$$k^n = \sqrt[4]{+1} = +1, +j, -1, -j$$

And thus the cycle of alternating current is divided into four quartets or quadrants

I. First quarter cycle; consumption of active and of reactive energies.

II. Second quarter cycle; production of active energy and consumption of reactive energy.

III. Third quarter cycle; production of active and of reactive energies.

IV. Fourth quarter cycle; consumption of active energy and production of reactive energy.

15) The wattmeter, power factor meter and induction factor meter, together provide a complete representation of the A.C. wave.