PHYSICS
2001
Revised

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BEAK PUBLICATIONS

Pacific Palisades

ISBN 0 -9712898 - -6

Beak Publications
P. 0. Box 333
Pacific Palisades, CA 90272

To

Patricia

partner for life

Preface to the 2009 edition

PHYSICS

Introduction: For roughly 400 years physics has been the flagship in the advance of science, but there is still much confusion about what physics really is. Until 100 or so years ago, most physicists would have said it was an attempt to *understand* the real world. As they probed deeper into matter, atoms, particles etc. they tried to *visualize* the *cause and effect* relationships between the various levels of abstraction. After observing *what* happened in some physical process, they tried to understand *why* it worked that way. Great swaths of phenomena were ultimately amalgamated by this process.

Roughly 100 years ago, because of the increasing difficulty of explaining the lower atomic and particle levels, some physicists adopted the easy answer that it is only necessary to *describe* the world, to say *what* happened in an overall way, and the *visualization* and cause and effect of the *why* were not required in physics.

At present, perhaps 95% of all physicists have adopted the defeatist attitude that atomic and particle levels *cannot* be visualized or given a cause and effect basis in the same way that the levels above them are. Instead, the mathematics of *ensemble* statistics called Quantum Physics is touted as the fundamental basis; and the divisions of "Classical" and "Modern" have arisen to define the old and new approaches.

The Silver Thread: Against a background of numerous ingenious experiments and endless smaller insights, starting with Copernicus and Galilei, the laws of motion and particle structure have been developed in a few giant steps. First, by Newton, who extended the work on falling bodies and the Solar system to a theory of gravitation, gave solid definitions of space and time and stated most clearly the laws of motion. Newton was followed by Faraday and Maxwell, who synthesized the visualization of electric and magnetic fields into a theory of electromagnetism that included a simple unification of E&M and light propagation. This theory still describes about 95% of all that goes on in the world.

By the late 1800's, Poynting had produced his field energy flow theorem. Also, Fitzgerald and Lorentz had given an ad hoc cause and effect basis for why experiments like the Michelson-Morley, done in a constant velocity inertial laboratory, could not measure the velocity.

In 1899, based on many similar experiments, Poincare' deduced a new physical principle:

> Electro-optical phenomena depend only on relative motion of material bodies, radiation sources, and electro-optical apparatus.

In 1900 he said that precise observations cannot reveal more than relative displacements, implying that detection of datum ether motion in inertial labs was not possible. His position can be summed up by:

> Identical *experiments*, carried out in *all* inertial labs, yield identical results.

In April 1904, Lorentz published the complete transformations, from one moving lab to another, for Maxwell's *equations*. At that time, those transformations were thought to show that Maxwell's *equations* had exactly the same form in *all* inertial systems. Finally, in September 1904, Poincare' named this new principle *The Principle of Relativity*, which, paraphrased, stated:

> For any observer in uniform motion of translation relative to a "fixed" observer, the *laws* of physics must be the same as for those of the "fixed" observer.

In a paper written to explain moving particle behavior and electro-optics, the *Principle of The constancy of the velocity of light* was added to the Poincare'/Lorentz *Principle of Relativity* by Einstein in 1905 (to his credit, he did not propose space-time. That was the mathematician Minkowski's mistake). In that same year, Einstein also invented the idea of the *photon*, with $E = h\nu$, to explain the photo-electric effect; and he elaborated on the equivalence of mass and energy, represented by $E = mc_0^2$. Finally, in 1916, he published the *General Theory of Relativity*, which geometrized space. He and many others toiled for years to unify the Theory of Gravitation and E&M. They did not succeed. In spite of this, when Quantum Physics was adopted by most physicists as the basis of physics, Einstein, Planck and a few others maintained their conviction that the basis was not statistical but *deterministic*.

For about 100 years, the statisticians have failed to develop a viable theory of particle structure. Almost all advances have been as a result of experiment. In spite of this, the statisticians cling to their faith in the *ensemble statistics* basis, and it is quite common to see

one of their many expositions in which the premise is that "Einstein was wrong".

Oh, to be so wrong ! : Of course Einstein was wrong (but not about determinism). So were Copernicus, Galilei, Newton, Faraday, Maxwell and all the other physicists. There are two reasons why *all* physicists are wrong. First, because every human is born with a *tether* that prevents seeing a great way into the future. All are limited by their own experience, so that even the greatest thinkers don't throw past some limit. Second, because it is in the nature of physics that *a theory cannot be proven*, it can only be proven *wrong* (usually by experiment). So, literally thousands of theories are proposed and rejected. Only the very few that still appear to fit the facts are given strong credence. The practitioner of physics spends a lifetime being wrong.

Who's right: The present work presents strong evidence that Einstein was right about *determinism*. The "classical" problems are solved deterministically. A unified field theory results. It is relatively simple, but involves an unfamiliar fluid. It is *visualizable* and the *cause and effect* connections are apparent at all levels. It is *intuitive* and *paradox free*. It is Main Line Physics. Let's get to work.

May, 2009

ACKNOWLEDGEMENT

In addition to the continued help from those listed in the earlier editions, G. Ialongo gave much aid in developing programs for the particle analysis.

Preface to the 2001 Version

This work began in the middle 40's of the 20th century, motivated by the repugnance for teaching about fields and particles in terms of vague symbols and non-visualizable processes. The utility of a fluid medium to describe the world became apparent quickly. Showing, in rigorous detail, the connection between the ether and the physical processes observed came slowly and essentially unsupported because of the tabu against the ether. Some of the field equations were particularly elusive. After 42 years, a substantial portion of the visualization was complete and about half of the field equations were available; so because time was running out, the 1989 version was produced. Lack of the ℓ-wave equations necessitated writing it from the top down, with a big gap at the bottom.

In the past 11 years, one version of the final ℓ-wave equations has been found, permitting the picture to be displayed formally from the bottom up. Many of the chapters have been rearranged, chopped up, and regrouped. This is how it was visualized 57 years ago.

ACKNOWLEDGEMENTS:

Many of those mentioned in the 1989 version continued to support and encourage this work. Since then, Professor A. H. Huffman contributed a number of cogent observations about the 1989 version that have improved the present volume. Professor Franco Selleri of the Bari group, through both his technical writing and his role as coordinator of a worldwide, international group of investigators (who approach this problem from a more open minded viewpoint), continues to empower those seeking reasonable answers (including the present writer), particularly in the areas of wave-particle duality and relativity.

Several, much closer to the writer, contributed so much to this new version that there is no way to credit them except to say that there were times when, without their help, the work just could not have continued. Their contributions appear throughout.

Robert L. Kirkwood	Demetrius J. Margaziotis
Lee O. Heflinger	Thomas Hudspeth
Bernard H. Mueller	Robert S. Margulies

Finally, no one knows better than my wife Patricia what living with a seemingly never ending task for 56 years means.

 2001

Preface to the 1989 Version

Modern physics books are honeycombed with strange if not bizarre philosophical Ideas. The student is told to abandon his intuition, cause and effect and the deterministic world. He is offered, instead, a statistical numbers game, a space and time done with mirrors, and an abstract view of the world comfortable only to pure mathematicians. The work presented in this volume returns to the simple physical approach to physics prevalent in the year 1900, augmented by 90 years of additional experimental data and understanding.

Physics is an intuitively understandable deterministic description of the real world with no paradoxes. It consists of cause and effect explanations of observed phenomena in successively lower levels of abstraction. At the bottom is a remaining unexplained metaphysical base. The goal is a visualizable unified field theory of particles, energy, charge, electricity and magnetism, gravitation and nuclear forces. The following outlines in considerable detail such a description of the world.

The qualitative deterministic picture is almost complete. The quantitative level is varied. Chapters 1 through 7 are well known material, slightly rearranged and modified. Chapters 8 and 9 (The Electron) are reasonably rigorous. Chapters 10 (Rods, Clocks and Plumb Bobs), 11 (Mechanics) and 12 (The Atom) are solidly rigorous. On the other hand, Chapters 13 (The Nucleus) and 14 (The Particles) are highly speculative, but based on ideas from 8 through 12. Chapters 15, 16 and 17 are rigorous where possible.

One thing should be clear from the reading:

Fuzzy atoms are out,
Determinism is In.

ACKNOWLEGMENTS:

The writer is indebted to a large group of family, friends and acquaintances who helped to maintain the atmosphere of discussion and curiosity so essential to a study of fundamentals. In addition to the numerous authors quoted in the text, many others, through books and journals, had a strong influence. Even those who disagreed with the concepts presented here, and they were abundant, often unwittingly contributed.

A few gave special support and encouragement. One of the earliest was Arthur Kohl, with his stimulating discussions. Bernard G. King and the writer studied portions of this problem together, and the atomic picture developed from our early gropings. Much later, David B. Langmuir and his research organization provided an environment conducive to the intellectual excitement so necessary to a sustained

pursuit of an active life in Science. Still later, a similar kind of condition was maintained by Eric J. Woodbury.

In direct support of the final writeup, Roy Dixon, through correspondence and discussions, provided insight to several problems in fluid mechanics. Professor Edgar Reich gave mathematical aid related to chapter 12, and Professor G. Mollenstedt donated much published material on the Aharanov-Bohn effect. Professor Dimitri J. Margaziotis read some of the chapters and also provided considerable aid in the computer-printer phase of the effort. Finally, Robert L. Kirkwood, a life-long friend deeply immersed in the quest for the ether equations, read the whole manuscript. He was instrumental in a number of corrections that improved the work substantially. Since he is convinced of the fundamental premise, but has his own views regarding the details and approaches, he cannot be considered responsible for any part of the work that might be found wrong in the future.

Finally, I wish to express my deep felt gratitude to my wife Patricia for continued concern and help directly affecting the task over a period of years. To top it off, she typed the manuscript; a formidable job

1989

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CHAPTER 1

FUNDAMENTALS

1.1 **Introduction**: Physics is the intuitive visualization of the real world that replaces superstition with paradox free, cause and effect explanations of observed phenomena. It describes qualitatively and quantitatively successively lower levels of abstraction, at the bottom of which is a remaining, unexplained *metaphysical* base. Viewing the universe as a gigantic mechanism, the goal of this work is to establish the few fundamental laws to which all parts of the complex structure conform.

Physics *explains* observed phenomena, in terms of *cause* and *effect*, to at least one level of abstraction deeper. An explanation is called a theory until it has been shown to agree with all other known phenomena to the extent that any change or improvement in it will yield only minute differences; then it is called a law. To qualify as physics, a theory or explanation must satisfy the *intuition*, i.e. conform to deeply rooted instincts. It must not be accepted on the basis of its "beauty" or its "elegance", but strictly on its "utility". Utility is not measured in terms of computational ease or numbers or details of results but in the range of levels of abstraction that can be intuitively unified. A long view of physics shows that as deeply fundamental laws are discovered, they tend to unify widely diverse phenomena at lower and lower levels of abstraction. *The present work gives a set of equations in intuitively understandable physical variables that appear to be able to account for all observed physical phenomena in a simple way.*

1.2 **Physics and Philosophy**: Early societies did not distinguish between philosophy and physics; but gradually the two disciplines were separated until, by the end of the nineteenth century, the overlap was small. Philosophers continued to ponder such things as good and evil, right and wrong, the beginning and end of time, and the existence or non existence of the "real" world. Physicists narrowed their scope to attack the following problem: given the existence of the "real" world, what are the laws that control it and determine its nature? By making this separation, considerable progress in physics was achieved. The backbone of this progress was the coupling of observed phenomena to *cause* and *effect* explanations. An experimentalist observed what happened. Cause and effect were used to explain *why* it happened. To have physics, the second step is unavoidable. Even a philosopher can observe what has happened, although he may not be as skilled as an experimental physicist in

controlling it. However, unless the question "why" is answered, without paradoxes, physical science is not satisfied. In physics, a paradox is anathema.

Completely different rules apply in philosophy. Who knows whether anyone will ever understand the meaning of the beginning or end of time, for example. Speculation in several directions at once is encouraged, and paradoxes only add to the fun. Instead of answering questions, philosophers take particular delight in confounding the issue. One of their favorite ploys consists of responding to a question with a "proof" that the question cannot be asked. In another form, an object offering considerable difficulty to being understood is "proven" not to exist. This is completely understandable as a normal reaction of a human mind facing an *unanswerable* question. No such reaction can be allowed a physicist.

1.3 **Cause and Effect**: Physicists and philosophers recognize that the concept of cause and effect arises from everyday experience. The simple act of dropping a stone is a good example. Day after day, the same stone will drop in very much the same way, straight down. On a very windy day, however, the stone may move with the wind as it drops. In answer to the question, "*Why* does the stone move sideways as it falls?", the answer is "*Because* of the wind". In common usage, the wind is the *cause* of the observed change, and the moving sideways is the *effect*. Dropping the stone under water produces a different effect, because of the presence of the water, which causes the observed change. It is this elementary form of macroscopic cause and effect that is the essence of physics, the simple listing of significant conditions that must be followed by related sequences of change.

Even in microscopic environments, the same approach can be taken. In the case of the stone, although the wind was an invisible cause, it could be sensed. However, as measuring techniques improve, more and more subtle conditions can be listed as causes for sequences of change in environments that are insensible to humans but not to instruments. For example, the answer to the question, “Why did the stone fall *down* in the first place ?” comes under this category.

The common sense separation of events into before and after, corresponding to cause and effect, has misled philosophers into a morass of irrelevant distinctions between whether one "produces" the other or one is "followed" by the other, etc. In physics, the position is taken that there is a continuous sequence of change in any situation. The laws of physics determine the sequence and, given a set of conditions, a particular sequence "must" follow. Cause and effect has its deepest meaning in the differential change in a region about a point in a differentially short time. It is the confusion between

macroscopic and microscopic cause and effect that leads to the philosophical problem, which has meaning only at the most microscopic levels of abstraction.

1.4 **Measurement**: Philosophically, the problem of the validity of experiment and the meaning of cause and effect are more complex. Science has contributed to the belief that, in the process of observation, when the initial conditions are more carefully controlled, the results are more accurately predictable. Physics, in particular, is based upon experiment; repetitive observation under controlled conditions. All experiments are limited in accuracy and control, but the laws of physics are deduced from experiment on the assumption that there is in operation the principle of identical environments, which can be stated: *Any Measurements made in identical environments, performed with identical instruments, will yield identical results*[1]. Whether or not such experiments can ever be carried out is a moot point; but such is the basis of physics. The principle of identical environments (P.I.E) applies anywhere, any time; e.g. in inertial systems, accelerating systems, rotating systems, etc. Without the P.I.E. there could be no physics.

Viewed as a giant machine, the universe has innumerable microscopic parts all moving in ways too hopelessly complex to control in any experiment. The success of a controlled experiment rests on the facts that, *first*, only those parts of the universe very close to the experiment and only the conditions most recent in time have any strong influence on it; *second*, all but a few of the local conditions can be ignored because they are effectively balanced or averaged out; and *third*, it is possible to separate the few controlled changes so that they can be related to their most immediate influences. It is the combination of the principle of identical environments and these three observational facts that permit experiment to be the basis of explanation of the operational laws of the real world.

The universe functions according to its basic laws independently of being observed or measured. That a small part of it insists that the functioning of the universe depends in any way on the ability of that small part to measure it is just as absurd as men's early insistence that the earth was the center of the universe. Relative to the number of parts in the whole machine, the total number of measurements ever to be made on it will be negligible. Physics is not based on the idea

1. J.E.McGuire & M.Tammy, Certain Philosophical Questions: Newton's Trinity Notebook, Cambridge University Press (1983); J.C.Maxwell, Matter and Motion, p13, Dover Publications, N.Y.; R.L.Kirkwood, On The Theory of Relativity, Thesis, Stanford U. Phys. Dept. (1950).

that a thing exists only when it is directly measurable. Physics is based upon *knowing* backed up by a sufficient number of indirect and direct measurements. The temperature of the sun, for example, is not measured directly, but is deduced from observations and what is *known* about suns.

1.5 **Determinism vs. Statistics**: Anyone who has done experimental work realizes that measurement is far from perfect. A number of difficulties must be faced by experimenters. This has led to a serious division among physicists; one group assuming that the world is basically deterministic and the other that the basis is statistical. *In the present discourse, it is assumed that the basis is deterministic and that statistics only enters as a practical expedient.*

When measuring the motion of an electron, for example, since there are no known entities small enough to measure it without interacting with it, the results of the interaction must be included in the measurement. This in no way says that if not measured, the electron could not have existed or moved deterministically; and it in no way prevents one from *knowing* what that motion would have been in principle. To do this, it is only necessary to determine what electrons are made of and the rules of motion of their constituent parts; all governed, of course, by cause and effect.

A second example is that of a boat driven by a jet engine mounted on the boat's superstructure. In smooth water, the boat's path is easily predicted deterministically. If the water is choppy, even though its every wavelet is moving deterministically, as a practical matter, the boat's motion will best be described by some average path on which is superimposed a statistical component of motion due to the chop.

Finally, consider a drop of water inside a sealed tin can sitting on a fire. It is totally impractical to solve the deterministic, many body problem of the steam molecules banging around in the can. The best that can be done is to get averages for pressure, temperature, etc. Here, statistics is indispensable; but the fundamental physics is still deterministic.

It is both practical and good physics to apply statistical mechanics to ensembles of small machines such as gases, plasmas, liquids and solids. That is what quantum mechanics is, the study of ensembles of microscopic machines. It can and will reveal only limited information about individual particles and their substructures; and it will provide even less information about the more important deterministic laws of particles and fields.

1.6 **Levels of Abstraction**: Rigid lines between levels of abstraction need not be drawn. Some levels commonly accepted are: Macroscopic--distribution and motion of galaxies and stars;

standard--planetary environment including temperature, pressure, etc.; molecular--kinetic theory etc.; atomic--chemical properties, spectroscopy; sub-atomic--particles and radiation. The question, "What are the particles made of?", leads to a lower, sub-particle level. Of all these levels, the highest and the lower few are the frontiers of physics. The central levels are those where the solid gains of the last 400 years have been made, and about which it can be claimed that "physical understanding" has been achieved. This means that the mind can *explain* what it observes in non-metaphysical terms.

A typical example is the process of evaporation. In earliest times, when fluid resting on a surface was observed to disappear gradually, several explanations were offered. One was that "spirits drank it". Another was that it seeped through the substance below. Still another was that the elementary particles of the fluid were moving at high speed and they gradually escaped. Any one of these theories was possibly valid until tied in with other phenomena. Only one is an explanation of what actually happens when evaporation takes place. The latter will be an imperfect explanation until tied in with all other phenomena; but as this is done, it is gradually recognized as the *correct* explanation.

1.7 **Imagination, Theory and Intuition**: Before the separation of philosophy and physics, the whole physical world was explained philosophically in terms of spirits, gods, etc. Of course, this was both non-intuitive and unpredictable; no cause could be sensed, and no process controlled. Little by little these metaphysical explanations were replaced by *physical* explanations. The concept of *explaining* by identifying and relating levels of abstraction replaced superstition. When a very complex set of phenomena is observed, the appearance may be compounded from individual phenomena from several levels of abstraction. Physicists of the nineteenth century recognized this and unified broad areas of physics by identifying various levels and their interrelationships. These areas where *physical understanding* has been reached can be described as predictable and intuitive.

A process is *explained* physically when it is described both intuitively and correctly. To say that a description satisfies the intuition means that when it is explained, even to a relatively uninitiated person, he can understand the cause and effect relationship at least on the same level or one level of abstraction deeper. Correctness of a description is related to how many diverse phenomena are tied into the observed process without conflict or paradox. Sometimes, it is easier to determine the correctness of a theory than to develop the cause and effect understanding that the intuition demands. An example of this was the choice of the Copernican system over the Ptolemaic in describing planetary motion.

With limited information, the intuition might have preferred Ptolemy, and neither theory could have been eliminated on the basis of disagreement with known facts. Later, as more observations were made, data on eclipses, retrogression in planetary motion, etc. made it clear that Copernicus was correct. The earth did in fact go around the sun. The intuition was now more satisfied with Copernicus because of the additional cause and effect relationships brought out by the new data. Later, the concept of universal gravitation and the inverse square law strengthened the intuition in this regard, even though they were deduced from the assumption of the Copernican system's correctness in a bootstrap type jump.

Lower levels of abstraction present some difficulty in trying to find the correct explanation of what actually happens because, *at the present level of understanding, the physical knowledge and metaphysical ideas there are not yet well separated*. Added to this is the problem of trying to visualize what is happening. Ultimately, it is the visualization that must be completed to make an intuitive and correct description.

When an observed phenomenon is to be explained, several different steps must be taken, although not always in the same order. Generally there are some data from experiments. The *imagination* is used to suggest a tentative theory based on cause and effect. The theory is both a *visualization* and a mathematical formalism. It is regarded as correct only if all the data is in agreement with the formalism and the visualization is intuitively satisfying.

In experiments performed on large ensembles, the visualization can often be fairly crude (i.e. basically incorrect in detail) and still yield accurate statistical results; so the intuition is satisfied with less than an exact visualization of all the details of the phenomenon. However, when the theory attempts to describe the structure of an *individual* elementary particle, for example, the *intuition* plays a much more important role in rejecting incorrect theories.

In formulating deterministic visualizations, serious errors and omissions occur as the result of using *models*. Several types are commonly employed. The simplest of these is the crude analogy, such as visualizing a molecule as a marble. Much more common is the *mathematical* model. This usually consists of an entity, described by a solution of an equation, very restricted in its resemblance to the real world but considered valuable because of the difficulty of getting even such a restricted solution to a difficult mathematical problem. Anyone who has struggled with this type of problem will agree that a partial solution is better than no solution at all. Nevertheless, the worst physics done today is involved in the trap of the mathematical model.

The *point* charge is a perfect example of a model and its problems. The *point* charge is a mental and mathematical construct that has some of the properties of the electron, for example. But, to think of the electron as a point charge is very bad physics. The electron should be thought of as the electron; a real entity of infinite extent with changeable shape, position, and orientation, but concentrated enough in one region to give the appearance of being small in interactions with other entities. The *point* charge model of the electron is an approximate representation of a few of its properties. Nevertheless, practitioners can be heard to discuss it as if it were one kind of electron and another model were another kind of electron, etc. There is only one kind of electron, and the job of physics is to explain or describe it as completely as possible as a single entity. When this is done, the result is not a "model" but the real electron. Use of the word abstraction to describe the various levels should not be construed as meaning that the entities and processes described are models. When an explanation is essentially complete, it describes what is actually happening or the entity that is actually there.

In the case of the point charge electron model, the infinities and other considerations clearly force the intuition to reject the theory as physically incorrect. This is the role that the intuition plays in physics. It cannot pronounce a visualization correct, the experiments must do that; but it can reject incorrect visualizations.

1.8 **Metaphysics**: The metaphysics necessary as a base on which to describe the physical world consists of a small number of statements that will be taken on faith and will be unexplainable in the context of the levels of abstraction above that base. Even so, they will not disturb the intuition any more than the concept of absolute space does. In this regard, even the more recent idea of a self enclosed curved space cannot prevent one from intuitively feeling that it must "be" somewhere, meaning that the mind feels more natural with an infinitely extending space. In this sense the base will be intuitive.

One of the major frontiers of physics today is at the sub-particle level of abstraction. Great difficulty is being experienced in trying to determine and visualize what particles are composed of. Not the least of this difficulty results from the failure to have a good visualization of the levels of abstraction just above, i.e. the atomic level and that of gravitic and electromagnetic fields. Partly this is the result of the needless abandoning of a deterministic visualization at those levels, replacing it with a statistical one. In addition, the interference of many metaphysical concepts introduced at too high a level of abstraction causes some disturbing paradoxes to obscure the real picture. The abandonment of the fundamental tenets of physics under these circumstances essentially precludes the solution of this sub-particle problem. To have any hope of solving it, the resumption of proper

physical approaches must be made, and the metaphysics must be pushed down farther below the levels of abstraction being contemplated.

At present there is confusion about the nature of *space* because of a failure to distinguish between space itself and the measurement of it using material rods. The latter are made of particles which change size in motion, leading to flexible rods. The structure and behavior of the rods are part of the theory used to visualize phenomena. The nature of space itself is not part of the theory but of the inexplainable metaphysical base. There is also a similar confusion about the nature of *time*. Clocks are made of particles, and their measured times are dependent on their motion and on the manner in which they are set. Thus, the behavior of clocks is part of the theory. The nature of time itself is not part of the theory, but of the metaphysical base.

1.9 **Ether**: For several centuries, one of the most powerful aids to *visualizing* physical processes was the idea that space was filled with a physical medium, called the ether, that was the seat of all electromagnetic and possibly gravitic phenomena. Unfortunately, in 1905, before anyone could come up with the correct properties of the ether, Einstein published his **special theory of relativity**[1], in which he stated, "...a 'luminiferous ether' will prove to be superfluous...". This was followed by the introduction, in 1908 by the mathematician Minkowski[2], of the strange blend of mathematics and philosophy called "space-time". Subsequently, the ether was abandoned as an aid to visualization; and, with the advent of quantum theory, visualization itself was abandoned by most investigators. This distinctly "non-physics" approach to physics persists today on the basis of Einstein's "proof" that the ether does not exist.

Few physicists are now aware of the fact that Einstein not only knew that special relativity did **not** "prove" the non-existence of the ether[3]; but that he used the ether, as late as 1924, as the basis for general relativity[4]. He continued to do so, even though he failed to establish the correct characteristics of the ether.

In contemporary physics, the idea of "empty space" has given way to the "vacuum", an entity with ever increasing real properties[5]. That

1. A.Einstein, Annalen der Physik, **17,** 891 (1905); translation reprinted in The Principle of Relativity, p37, Dover Publications, New York.
2. H.Minkowski, "Space and Time", a speech delivered at Cologne in 1908; translation reprinted in The Principle of Relativity, p75, Dover Publications, N. Y.
3. A.Einstein, "Ether and the Theory of Relativity" an address delivered at Leyden in 1920; Translation printed in Sidelights on Relativity, p1, Dover Publications, N. Y.
4. ....., "Uber den Aether", Verh.Schweiz.Naturf.Ges., **105**, p85 (1924).
5. Physics Through the 1990s, Elementary Particle Physics, p71, National Academy, Washington, D.C. (1986).

these properties are those of the ether will be assumed here, and the description of its other properties will be given in the following. The immense task of trying to visualize and analyze the real world will not be burdened here by ruling out the ether for no good reason.

1.10 **Main Line Physics**: Until 1900, physics was making magnificent strides towards a total unification of the various subfields in its domain into one overall discipline based on a relatively small number of fundamental principles, which in turn were to be encompassed in a single fundamental entity, the ether. Mechanics and Heat had been combined in the Kinetic Theory; Light, Electricity and Magnetism had been combined in Electromagnetic Theory; and it only remained to bring these together with Gravitation and Physical Chemistry, the latter of which now includes also nuclei and particles. All of this was to be done with a complete visualization at all levels of abstraction and based on cause and effect. Special and General Relativity caused a retreat from this position. Quantum Physics excited a rout. The present claim is that the final unification is now nearing completion under Quantum Physics. However, the going is slowing.

The old way of looking at physics is called "Classical", the new way "Modern". This division exists only in the field of philosophy. There is only one *physics*. It accepts all well tested phenomena, uses any mathematical tool that has utility, reconciles all phenomena through cause and effect, unifies the explanations of *why* in the various levels of abstraction and keeps the minimum metaphysical base as far down in the layers of abstraction as possible. Where no explanation can be found, the plain fact that "I don't know yet" will be appended. The following chapters on Main-Line Physics have been written using this process.[1]

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1. R.H.Dishington, Physics, Beak Publications, Pacific Palisades, CA (1989).

CHAPTER 2

THE UNIFIED FIELD

2.1 **Introduction**: Looking out, the world gives the impression of being a vast empty room in which, at a few scattered places, something special exists. This something special appears to take on various forms, sometimes as waves moving at high speed, sometimes as particles either moving or at rest, generally following an observable sequence of events. In earlier times, these various configurations were given separate names, such as mass or energy, although finally it was realized that they were all just energy in different forms. This energy appeared to be conserved in passing from one form into another. However, as the idea of energy was extended, and its conservation became well established, the nature of energy became more abstract. Ultimately, because one form of energy seemed to be purely wave-like, the feeling gradually developed that there was a more fundamental substance called the ether in which energy was a configuration or a form of motion. This concept will be accepted here, and will be the basis for the development of the unified field theory.

The first step in constructing a unified field theory is to find the equations of motion of the ether as seen by an *absolute observer*; i.e. one who sees the whole universe at once and whose measurements are not affected by any energy configuration observed. When these equations of motion are found, they can then be rewritten to apply for any observer, using flexible rods and clocks made of particles, moving in any arbitrary way.

In principle, an ideal theory of the world would answer all questions with no remaining metaphysical base, an unlikely possibility. The next best goal is a theory with a minimum of unanswered questions, and that is achieved by pushing the unanswered questions lower and lower in the layers of abstraction, until the metaphysical base is all that remains.

2.2 **The Metaphysical Base**: No attempt will be made here to choose a metaphysical base and argue in its defense. That is not the job of physics. The job of physics is to explain the various levels of abstraction in such a way that the metaphysical base is pushed down to the lowest possible level. Whatever metaphysical base then results will be accepted. If properly done, that metaphysical base will be as inexplainable as the beginning and end of time; but all the levels above will be intuitive and simple, and no paradoxes will extend up into those levels of abstraction. The metaphysical base that has

resulted from preparing the present work is:

1. The existence of the *real* world is *assumed*.

2. Measurement, carried out by experimenters, is in no way necessary for the real world's existence or functioning.

3. Space is absolute - a place.

4. Time is the sequence of events, not as they are measured, but as they occur.

5. Space is filled with a substance called ether, whose properties are unknown except for the few to be discussed in this work.

Here, space is Newton's *absolute* space[1], an unwarpable, euclidean place of large extent; exactly as the intuition indicates. Time is Newton's *absolute* time[1], the sequence of events, not as they are measured but as they occur; again in agreement with the intuition. The *fact* that space is filled with ether is part of the metaphysical base, but the ether's properties are not. They are clearly part of the theory; since, until they are correctly specified there could be alternative ways to describe the ether. In the present theory, the ether is the lowest level of abstraction. Speculation beyond its equations of motion, questions related to its sub-structure, will not be dealt with here, being regarded as meaningless. Certainly, no description of its nature using the words particle, mass, charge, energy or momentum is permissible. All of these properties are *derivable* from the ether itself. The five items listed earlier, then, represent the total metaphysical base that results from the *specific* theory developed here.

2.3 **The Perfect Continuum, Ether**: Once the properties of the ether are specified, the unified theory is complete; i.e. the theory and a statement of the ether's properties are synonymous. A complete description of the ether has two parts; visualizable definitions of certain of its physical characteristics, and a few formal equations giving a shorthand description of the relationships between those physical properties. This compact theory will be given in full in the present chapter. The remainder of the book will consist of examples

1. I.Newton, Principia Mathematica, (1686); translated by F.Cajori, University of California Press, p6 (1946).

of how the theory describes selected phenomena, and a demonstration of how the world's structure is built up from the ether.

To start, *some* of the ether's important properties are:

a. The ether is a conserved, compressible fluid.

b. At any one absolute time t, at every absolute space point x,y,z, the ether has a positive absolute density ϕ_a that varies from point to point.

c. Over large volumes, there is an average or datum ether density ϕ_d.

d. The incremental ether density ϕ at each point is defined as $\phi_a - \phi_d$, which can be positive or negative.

e. At any one absolute time t, at every absolute space point x,y,z, the ether has three velocity components V_x, V_y, V_z.

f. In regions of space remote from energy (matter) $\phi_a = \phi_d$ and $\phi = 0$.

g. An observer considering a large region free of energy will be called an *absolute* observer if the datum ether has zero velocity everywhere as seen by him.

h. Questions related to other observers moving at constant velocity relative to the absolute observer are trivial and will be dealt with later under rods and clocks.

i. If energy in the form of waves or particles is introduced into the region observed, the absolute observer sees a four variable field ϕ, V_x, V_y, V_z which varies with x, y, z and t ($\phi = V_x = V_y = V_z = 0$ at ∞).

j. The laws of physics can be written as non-linear, partial differential field equations relating the field variables and x, y, z and t.

k. These intrinsic equations can be written in a form that is valid for any observer.

l. They and their solutions represent all of the properties of the ether now known.

m. They represent all of the physics of the known world, and all of the physics of the unknown world.

The *mathematical* theory of fields involves functions that are single valued (uniform), finite, continuous, and have continuous derivatives. None of the examples of *physical* fields usually discussed satisfies these requirements rigorously; since, if analyzed microscopically, such things as the density of a gas or the velocity of a fluid cease to have meaning because of the particulate structure of matter. Consequently, field mathematics is only applicable to these "fields" as an approximation, albeit a very good one in most cases. However, the match between the field mathematics and the physics in the case of the ether is exact; because the ether is a perfect continuum. It is not composed of elements of the periodic table, it is basic. It is a frictionless fluid, of great compressibility, but has no mass, i.e., it has no inertia or linear momentum per se and is not directly affected by gravitation. Therefore, it does not obey Newton's laws or any of their derivatives such as the Navier-Stokes equation, etc. Dynamically it will remain unspecified until later, kinematically it behaves as a perfectly compressible fluid, a continuum.

In succeeding sections, the ether will be described primarily by its absolute density ϕ_a and its velocity $\mathbf{V}$. An alternative representation for the density is possible because of the assumption that the average density over all space is a constant ϕ_d. Far out from all matter and energy, the *ether* density is visualized as actually having the datum value ϕ_d. Consequently, at any point where ether has space and time variations it is possible and often considerably more convenient to use the incremental density,

$$\phi = \phi_a - \phi_d \quad . \tag{2.3.1}$$

The fact that ϕ_d is a constant ensures that space and time derivatives of ϕ and ϕ_a are equal. An important difference between ϕ and ϕ_a is that ϕ_a has only positive values, representing the actual density in space, whereas ϕ can be positive or negative as ϕ_a is greater or less than the average value ϕ_d. When referring to the *ether*, the word *density* will be applied to both ϕ_a and ϕ throughout, and it will be left to the reader to keep in mind the difference between them.

It is clear from the preceding, that the ether has only a few simple properties, each of which is visualizable. All that is required to complete its description is a set of formal relationships that connect these various properties.

2.4 **Observer Systems**: Field equations are written by an observer describing what happens in a laboratory. The ether field equations are best investigated in a laboratory that, at first, is free of all electromagnetic and gravitic fields; e.g. it could be out in space, floating along uninfluenced by external fields. If the laboratory is

equipped with an external jet engine, it immediately allows the discovery of one fundamental property of particles of matter composed of ether distortions. Anyone who has driven a car or ridden on a merry-go-round knows that acceleration relative to the ether is absolute; i.e. matter "knows" when it is *accelerating* relative to the ether. Thus, if the laboratory is pushed by its external jet engine, the laboratory moves relative to the ether; and the laboratory observer, standing with feet fixed to the floor, "feels" the acceleration as the ether goes by and his speed increases.

Although the final ether equations could be written for an arbitrarily moving laboratory (even accelerating), they are far simpler if written by an observer whose laboratory is not accelerating, i.e. whose laboratory's speed and direction of motion, relative to the ether, are not changing. All constant velocity laboratories, moving at arbitrary speeds, are defined as *inertial* systems, in which observers standing still in each room feel no acceleration. While, today, it is fashionable to formulate field equations using the theory of transformations between the various inertial systems, *all of physics can be discovered by a single observer in any one inertial laboratory*. Later on, it will be shown that identical experiments have the same results in any two inertial laboratories; but the *derivation* and *visualization* of the physics is far simpler for one particular inertial observer called the absolute observer.

The absolute observer is one whose laboratory is at rest relative to the datum ether, so that before particles and fields are introduced inside, the ether in the laboratory is homogeneous, isotropic, and at rest. The laws of physics are discovered by introducing a particle, such as an electron; or an electric field, such as that between the plates of a charged capacitor; or two small masses suspended close to each other; etc. Various controlled experiments are conducted with these objects and their behavior is observed. Then the generalization of all the experiments is synthesized using the well known mathematics of field representation (see Appendix A). This consists of simple forms that describe spatial changes with time.

Although the forms used to represent the fields are usually quite simple, the actual experiments or calculations related to them, using the equations, usually require the establishment of a coordinate system inside the laboratory (see Appendix B). This is done by visualizing three sets of imaginary surfaces, fixed relative to the laboratory walls, ceiling and floor, which allow a different set of three numbers to identify each point in the room. In the case of the ether, the density ϕ_a and fluid velocity $\mathbf{V}$ are specified at each space point; but also, other more complicated quantities, related to ϕ_a and $\mathbf{V}$, can be written in the form of scalars, vectors or dyadics (See Appendices A and C) and specified at each point.

Most experiments of interest involve fields that are changing with time, so, in several places about the room, the laboratory has clocks that have been set by one of several possible methods. *The time is considered to be the same at every point in the laboratory, whether or not a clock is located there*. Choice of the method of clock setting involves certain subtleties that are discussed in detail in chapter 8.

Once a coordinate system and a laboratory time have been established, the field can be described by partial differential equations for the field variables like ϕ_a, $\mathbf{V}$, etc. in terms of the independent variables x, y, z, and t, for example (see Appendices D & F). The ether field equations for the absolute observer's system, as obtained through the preceding process, will now be presented.

2.5 **The Conservation Law**: The study of fluid motion is logically divided into two parts, kinematics and dynamics. Kinematics is a *geometrical* description of the possible motions resulting from the fact that any fluid occupies space as it moves about, and a particular part of it cannot be in two places at once. Dynamics deals with the laws of cause and effect governing a *particular* fluid's motion. *Kinematics is, therefore, the same for all fluids, whereas the dynamics of each different fluid can be different* (see Appendix E).

The most fundamental *kinematic* equation of motion of a conventional fluid is the continuity equation, which relates the change in density at a point to the flow of the fluid towards or away from that point. Defining the ether *flow* vector as $\phi_a\mathbf{V}$ and $\nabla\bullet(\phi_a\mathbf{V})$ as the divergence of the flow vector at the point in question (see Appendix A), the continuity equation is written,

$$\frac{\partial\phi_a}{\partial t} = -\nabla\bullet(\phi_a\mathbf{V}) \quad . \tag{2.5.1}$$

It says that the time rate of increase of density ϕ_a at a fixed point is equal to the negative of the divergence of the flow vector $\phi_a\mathbf{V}$ at that point.

The intuitive meaning of the continuity equation is perfectly clear. If the divergence of the flow away from a point is net positive, then the density of the fluid at that point must be decreasing and vice versa. Only if fluid is being created or destroyed at a point is it possible to violate this relationship. Therefore, Eq.(2.5.1) is the formal expression for the conservation of ether.

2.6 **Static Ether Concentrations**: The ether can be distorted into particles, and propagates both transverse, *t*, and longitudinal, ℓ, waves. However, the concept that certain particles are composed of regions of condensed and rarefied ether is complicated by the frictionless fluidity of the medium, which would immediately flow to

thin the condensed regions and fill in the attenuated regions. So, it is clear that no static ether configurations exist. In the case of so called "static" fields, the stability is the result of a combination of bulk displacements held together by some dynamic action in the ether. Thus, the designation "static" field, as commonly used, implies both bulk displacements and active, dynamic ether motion.

2.7 **Longitudinal Waves**: The motion of transverse ether waves is well understood through electromagnetic theory. These *t*-waves are generated by moving charged particles, and they carry *energy* in two forms; antenna and photon radiation.

No longitudinal radiation appears in electromagnetic theory. Nevertheless, ℓ-waves are more prevalent and potent than *t*-waves. The sometimes strange physical effects they produce are not yet recognized as being caused by ℓ-waves; and the theory of ℓ-waves is sadly lacking any large *experimental* base. Since all energy transfer by waves is presently observed to be carried by t-waves, the ℓ-waves appear to be *energyless*; which accounts for their lack of observability.

The generation of ℓ-waves is impossible to avoid. Just as a stone dropped into a quiescent pool of water causes a set of circular waves to leave the point of impact, *any disturbance of the ether at any point immediately produces high frequency, spherical ℓ-waves that move outward*. These waves are essentially like those visualized by Huygens[1]. When they come in contact with particles, more waves are generated; so that the ether, everywhere, is traversed by ℓ-waves, moving in all directions, caused by all the interacting particles in the universe. Unlike waves in elastic media, the velocities of ether ℓ-waves and *t* –waves are the same, and equal to the velocity of light.

Non-linearity is a major factor in writing the equations of motion of the ether; and for this reason the equations that establish the energyless ℓ-wave amplitudes are different from those giving the bulk flow properties, which bulk motions involve energy in one way or another. Thus, in writing the field equations, the bulk ether distortions must be distinguished from the ℓ-waves, so the incremental density and velocity are separated into two components,

$$\phi = \underset{\text{bulk}}{\bar{\bar{\phi}}} + \underset{\ell\text{-wave}}{\dot{\phi}} \quad , \qquad \mathbf{V} = \underset{\text{bulk}}{\bar{\bar{\mathbf{V}}}} + \underset{\ell\text{-wave}}{\dot{\mathbf{V}}} \quad , \qquad (2.7.1)$$

where the double bar indicates a constant (time average) or slowly varying bulk ether deformation or a *t*-wave, and the sub-dot indicates a rapidly oscillating, periodic, zero time average, longitudinal wave.

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1. C.Huygens, Treatise on Light, (1690); translation by S.P.Thompson, Dover Publications, p16 ff (1962).

In some situations, a problem can be solved using either the bulk or the ℓ-wave equations separately; but more often than not, a close meshing of both is required to explain the physical phenomena.

2.8 **Separation Equations**: Eqs.(2.7.1) represent two of a set of separation equations that allow working with one or the other of the two components, bulk or ℓ-wave. As an example, consider the flow vector $\phi_a \mathbf{V}$, i.e. the density, velocity product that specifies the ether current density or flow density at each point in space. If separated into time average and periodic components, it is written,

$$\phi_a \mathbf{V} = \overline{\overline{\phi_a \mathbf{V}}} + \{\phi_a \mathbf{V}\}_. \quad . \tag{2.8.1}$$

Its components can be found simply by using Eq.(2.3.1) and Eqs.(2.7.1) to give,

$$\phi_a = \phi_d + \bar{\phi} + \phi_. = \bar{\phi}_a + \phi_. \quad , \tag{2.8.2}$$

which is then combined with Eqs.(2.7.1). The result is,

$$\phi_a \mathbf{V} = (\bar{\phi}_a + \phi_.)(\overline{\mathbf{V}} + \mathbf{V}_.) \quad .$$

Carrying out the multiplication,

$$\phi_a \mathbf{V} = \bar{\phi}_a \overline{\mathbf{V}} + \phi_. \mathbf{V}_. + \bar{\phi}_a \mathbf{V}_. + \phi_. \overline{\mathbf{V}} \quad , \tag{2.8.3}$$

where the first RHS term is non-periodic, the last two terms are periodic, and $\phi_. \mathbf{V}_.$ can have both periodic and non-periodic components given by,

$$\phi_. \mathbf{V}_. = \overline{\overline{\phi_. \mathbf{V}_.}} + \{\phi_. \mathbf{V}_.\}_. \quad . \tag{2.8.4}$$

Combining Eqs.(2.8.4) and (2.8.3),

$$\phi_a \mathbf{V} = \bar{\phi}_a \overline{\mathbf{V}} + \overline{\overline{\phi_. \mathbf{V}_.}} + \bar{\phi}_a \mathbf{V}_. + \phi_. \overline{\mathbf{V}} + \{\phi_. \mathbf{V}_.\}_. \quad . \tag{2.8.5}$$

Comparing Eqs.(2.8.1) and (2.8.5), the separated components of $\phi_a \mathbf{V}$ are seen to be,

$$\overline{\overline{\phi_a \mathbf{V}}} = \bar{\phi}_a \overline{\mathbf{V}} + \overline{\overline{\phi_. \mathbf{V}_.}} \quad , \qquad \text{(bulk)} \tag{2.8.6}$$

and,

$$\{\phi_a \mathbf{V}\}_. = \bar{\phi}_a \mathbf{V}_. + \phi_. \overline{\mathbf{V}} + \{\phi_. \mathbf{V}_.\}_. \quad . \qquad (\ell\text{ - wave}) \tag{2.8.7}$$

Using the same procedure, the separation equations for $\mathbf{a}$, the acceleration of the ether at a point, are found to be,

$$\bar{\bar{\mathbf{a}}} = \frac{\partial \bar{\bar{\mathbf{V}}}}{\partial t} + \bar{\bar{\mathbf{V}}} \bullet \nabla \bar{\bar{\mathbf{V}}} + \overline{\overline{\mathbf{V}_{\cdot} \bullet \nabla \mathbf{V}_{\cdot}}} \quad , \qquad \text{(bulk)} \qquad (2.8.8)$$

and,

$$\mathbf{a}_{\cdot} = \frac{\partial \mathbf{V}_{\cdot}}{\partial t} + \bar{\bar{\mathbf{V}}} \bullet \nabla \mathbf{V}_{\cdot} + \mathbf{V}_{\cdot} \bullet \nabla \bar{\bar{\mathbf{V}}} + \left\{ \mathbf{V}_{\cdot} \bullet \nabla \mathbf{V}_{\cdot} \right\}_{\cdot} \quad . \quad (\ell\text{-wave}) \qquad (2.8.9)$$

Even more useful is the separation of complete equations. For example, the ether conservation law of Eq.(2.5.1) can be expressed as,

$$\nabla \bullet \left((\bar{\phi}_a + \phi_{\cdot})(\bar{\bar{\mathbf{V}}} + \mathbf{V}_{\cdot}) \right) = -\frac{\partial \bar{\phi}_a}{\partial t} - \frac{\partial \phi_{\cdot}}{\partial t} \quad ,$$

or when expanded out,

$$\nabla \bullet (\bar{\phi}_a \bar{\bar{\mathbf{V}}}) + \nabla \bullet (\bar{\phi}_a \mathbf{V}_{\cdot}) + \nabla \bullet (\phi_{\cdot} \bar{\bar{\mathbf{V}}}) + \nabla \bullet (\phi_{\cdot} \mathbf{V}_{\cdot}) = -\frac{\partial \bar{\phi}_a}{\partial t} - \frac{\partial \phi_{\cdot}}{\partial t} \quad .$$

Here, the first term is clearly a time average quantity and the next two are simply periodic; but the term involving $\phi_{\cdot}\mathbf{V}_{\cdot}$ can be decomposed into a time average and a zero time average component, as before. Thus, the separation evolves with one more term than given above, so that,

$$\underbrace{\nabla \bullet (\bar{\phi}_a \bar{\bar{\mathbf{V}}}) + \nabla \bullet (\overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}}) + \frac{\partial \bar{\phi}_a}{\partial t}}_{\text{Time Average}} = \underbrace{-\nabla \bullet (\bar{\phi}_a \mathbf{V}_{\cdot}) - \nabla \bullet (\phi_{\cdot} \bar{\bar{\mathbf{V}}}) - \nabla \bullet \left\{ \phi_{\cdot} \mathbf{V}_{\cdot} \right\}_{\cdot} - \frac{\partial \phi_{\cdot}}{\partial t}}_{\text{Zero Time Average}} \qquad (2.8.10)$$

In Eq.(2.8.10) *the two sides can be instantaneously equal only if both are independently equal to zero.* In effect, then, the ℓ-waves obey a conservation law that can be used independently of the bulk conservation law. This is expressed by,

$$\nabla \bullet (\bar{\phi}_a \bar{\bar{\mathbf{V}}}) + \nabla \bullet (\overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}}) + \frac{\partial \bar{\phi}_a}{\partial t} = 0 \quad , \qquad \text{(Bulk)} \qquad (2.8.11)$$

and,

$$\nabla \bullet (\bar{\phi}_a \mathbf{V}_{\cdot}) + \nabla \bullet (\phi_{\cdot} \bar{\bar{\mathbf{V}}}) + \nabla \bullet \left\{ \phi_{\cdot} \mathbf{V}_{\cdot} \right\}_{\cdot} + \frac{\partial \phi_{\cdot}}{\partial t} = 0 \quad . \quad (\ell\text{-wave}) \qquad (2.8.12)$$

One important result comes immediately from the separation equation for $\overline{\overline{\phi_a \mathbf{V}}}$. In many "static" fields, there is *no net ether flow*,

i.e. $\overline{\overline{\phi_a \mathbf{V}}} = 0$. Nevertheless, *in such cases*, from Eq.(2.8.6),

$$\overline{\mathbf{V}} = -\frac{\overline{\overline{\phi_. \mathbf{V}_.}}}{\overline{\phi}_a} \quad ; \qquad (\overline{\overline{\phi_a \mathbf{V}}} = 0) \qquad (2.8.13)$$

which indicates that *even if no ether flow occurs, there can be a time average velocity component, if* $\overline{\overline{\phi_. \mathbf{V}_.}}$ *is not zero.* Furthermore, from Eq.(2.8.8), *even if the time average velocity* $\overline{\mathbf{V}}$ *is zero, there can be a non-zero time average acceleration,*

$$\overline{\mathbf{a}} = \overline{\overline{\mathbf{V}_. \bullet \nabla \mathbf{V}_.}} \quad . \qquad (\overline{\mathbf{V}} = 0) \qquad (2.8.14)$$

For convenience, the separation equations are listed in Table 2.8.1.

TABLE 2.8.1

SEPARATION EQUATIONS

$$\phi = \overline{\phi} + \phi_. \qquad \mathbf{V} = \overline{\mathbf{V}} + \mathbf{V}_. \qquad \mathbf{a} = \overline{\mathbf{a}} + \mathbf{a}_.$$

$$\phi_a = \phi_d + \phi = \phi_d + \overline{\phi} + \phi_. = \overline{\phi}_a + \phi_.$$

$$\overline{\mathbf{a}} = \frac{\partial \overline{\mathbf{V}}}{\partial t} + \overline{\mathbf{V}} \bullet \nabla \overline{\mathbf{V}} + \overline{\mathbf{V}_. \bullet \nabla \mathbf{V}_.} \quad , \qquad \mathbf{a}_. = \frac{\partial \mathbf{V}_.}{\partial t} + \overline{\mathbf{V}} \bullet \nabla \mathbf{V}_. + \mathbf{V}_. \bullet \nabla \overline{\mathbf{V}} + \{\mathbf{V}_. \bullet \nabla \mathbf{V}_.\}_.$$

$$\overline{\overline{\phi_a \mathbf{V}}} = \overline{\phi}_a \overline{\mathbf{V}} + \overline{\overline{\phi_. \mathbf{V}_.}} \quad \text{(bulk)} \quad , \quad \{\phi_a \mathbf{V}\}_. = \overline{\phi}_a \mathbf{V}_. + \phi_. \overline{\mathbf{V}} + \{\phi_. \mathbf{V}_.\}_. \quad (\ell\text{-wave})$$

$$\overline{\mathbf{V}} = -\frac{\overline{\overline{\phi_. \mathbf{V}_.}}}{\overline{\phi}_a} \quad \text{if} \quad (\overline{\overline{\phi_a \mathbf{V}}} = 0) \quad , \qquad \overline{\mathbf{a}} = \overline{\overline{\mathbf{V}_. \bullet \nabla \mathbf{V}_.}} \quad \text{if} \quad (\overline{\mathbf{V}} = 0)$$

$$\nabla \bullet (\overline{\phi}_a \overline{\mathbf{V}}) + \nabla \bullet (\overline{\overline{\phi_. \mathbf{V}_.}}) + \frac{\partial \overline{\phi}_a}{\partial t} = 0 \quad , \quad \text{(Bulk)}$$

$$\nabla \bullet (\overline{\phi}_a \mathbf{V}_.) + \nabla \bullet (\phi_. \overline{\mathbf{V}}) + \nabla \bullet \{\phi_. \mathbf{V}_.\}_. + \frac{\partial \phi_.}{\partial t} = 0 \, , \quad (\ell\text{-wave})$$

2.9 **Vorticity and Angular Persistence**: In Section 2.3, the ether was described as a frictionless fluid that has no inertia or linear momentum, per se. There are situations, however, where it gives the appearance of having *angular* momentum. Wherever a steady state, closed circulation of ether occurs, since it is frictionless, the angular rotation will persist forever unless acted upon externally. This *angular persistence* gives the appearance of angular momentum, although it has no relationship to mass or inertia as commonly understood. A more complete discussion of this effect will appear later on, since it is closely related to difficulties with the conventional concept of magnetic energy.

2.10 **Units and Universal Constants**: All of the equations developed here are written in Heaviside-Lorentz units, because these units more closely represent the fundamental physics of the ether than do other more popular systems. The latter introduce constants which have no basic role in ether theory. Since the standardization of practical units, H-L units have all but disappeared from the literature, so a table of conversion factors is included in Appendix G. The units for ether density $(\text{ether}/\text{cm}^3)$ have been called "descartes" here, because no name for them had been established in the past, and descartes seemed an appropriate name.

In the following, a number of universal constants appear, some familiar and some newly defined. The velocity of light with respect to the datum ether is the well known constant, c_0. If the ether density ϕ_a is greater or less than ϕ_d, in some large region, the velocity of light c will be greater or less than c_0. The datum density ϕ_d is also a universal constant. Several others will be defined and discussed, later on, where they appear in the development. Table 2.10.1 gives the values of these constants. Since the accepted values of fundamental constants are regularly adjusted, as measurements gradually improve, the values here are not to be regarded as final, nor even up to date; but they are accurate enough to allow proper exposition of the theory.

The important thing to notice about Table 2.10.1 is that the constants presented are divided into two sets, basic and derived. The basic constants appear in the fundamental field equations, whereas the derived constants only appear in the solutions of those equations. The only exception to this is c_0, which is a derived constant that is used instead of ϕ_d, the basic quantity. The use of c_0, instead of ϕ_d, in the field equations eliminates the requirement for a fifth basic constant to adjust for the units used.

TABLE 2.10.1

UNIVERSAL CONSTANTS

Basic	Derived
$a = 7.954945 \times 10^{-24}\ \text{cm}^2/\text{sec}$	$c_0 = 2.99792458 \times 10^{10}\ \text{cm/sec}$
$b = 1.428438 \times 10^{26}\ \text{sec/cm}^2$	$e = 1.7026924 \times 10^{-9}$ hlc
$\phi_d = 8.9875517 \times 10^{20}$ descartes	$h = 6.6260755 \times 10^{-27}$ erg-sec
$D = 2.7346139 \times 10^{7}\ \text{cm/sec}$	$G = 8.3850240 \times 10^{-7}\ \frac{\text{dyne-cm}^2}{\text{gm}^2}$

2.11 **The Function of ℓ-Waves**: The way in which ℓ-waves contribute to the world's structure can be understood best by considering the process of pair production, i.e. the generation of an electron/positron pair using a high energy photon colliding with, say, a neutron. From the ether point of view, the pair of particles is produced by removing some ether from one region and depositing it in another, so that the slightly depleted region (electron) is separated from the slightly compressed region (positron). In this case, the fluid ether would ooze out of the positron and flow into the electron until nothing remained but the datum. In order for the electron and positron to be "stable" particles, something else must prevent this oozing. During pair production, an energyless, longitudinal *sustaining* wave is set up that goes out of the electron and into the positron to hold their bulk displacements of ether in place. These frictionless ℓ-waves persist as long as the electron and positron remain separate particles.

The ℓ-wave is a permanent part of each particle, and its first function is to stabilize and *sustain* the bulk distortion that constitutes the part of the particle that is directly measurable in the laboratory. A second function is to establish the particle's gravitic field. Third, the ℓ-wave Doppler shift properties of a particle in motion determine many of what are conventionally called its quantum mechanical characteristics. Finally, many of the mysteries, such as the double-slit and Aharonov-Bohm experiments, can be explained by the ℓ-waves. Therefore, *the ℓ-wave is the fundamental ether property that most controls the phenomena that appear in experiments with particles and fields*; even though the bulk properties are the ones usually measured.

For this reason, *the ℓ-wave equations are regarded as the basic equations of the ether*; for once the fields $\phi_{\cdot}$ and $\mathbf{V}_{\cdot}$ are known, the bulk configurations $\bar{\bar{\phi}}$ and $\bar{\bar{\mathbf{V}}}$ can be found from them. Then, from the $\bar{\bar{\phi}}$ and $\bar{\bar{\mathbf{V}}}$ distributions come the physical definitions and visualizations of charge, energy, inertia, momentum, etc.

2.12 **The ℓ-Wave Equations**: As mentioned in Section 2.7, there is no large experimental base to aid in discovering the basic ℓ-wave equations. The number of experiments, past and present, on the *bulk* characteristics of particles and fields is so great as to be uncountable; yet, essentially all ℓ-wave measurements made, to date, are indirect and are recognized for what they are by only a few dissident physicists. The consequence is that the ℓ-wave equations presented here are the result of much circuitous relating of ostensibly unrelated facts, considerable guess work, and a remaining uncertainty. Until a solid groundwork of ℓ-wave experiments is available, the formal description of the ether presented here must be used with reservation and care. Some part of it will undoubtedly be modified in the future. Nevertheless, the *picture* it has generated will probably endure.

The most desirable form of equation would be one that gives $\mathbf{V}_{\cdot}$ directly. Then the continuity equation could be used to get $\phi_{\cdot}$. Because of the conditions described in the preceding paragraph, such an equation is not known at this time. What has been achieved is an equation for a scalar velocity potential $\bar{\bar{\eta}}$ from which $\mathbf{V}_{\cdot}$ can be found. Here, again, *the non-linearity of the ether requires the use of two separate ℓ-wave equations; one for standing ℓ-waves and one for traveling ℓ-waves.*

The <u>TRAVELING</u> ℓ-wave equation takes the form,

$$\nabla^2\bar{\bar{\eta}} - \frac{1}{c_0^2}\frac{\partial^2\bar{\bar{\eta}}}{\partial t^2} - \frac{1}{\bar{\bar{\eta}}}\left(\left(\nabla\bar{\bar{\eta}}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial\bar{\bar{\eta}}}{\partial t}\right)^2\right) = \pm\frac{c_0\omega}{\phi_d D}\nabla\bullet\,\overline{\overline{\phi_{\cdot}\mathbf{V}_{\cdot}}} \quad , \qquad (2.12.1)$$

where the sign on the RHS is,

$$+ \text{ if } \overline{\overline{\phi_{\cdot}V_{\cdot}}} \le 0 \qquad , \qquad - \text{ if } \overline{\overline{\phi_{\cdot}V_{\cdot}}} \ge 0 \qquad (2.12.2)$$

and, for ℓ-wave sources at rest,

$$\bar{\bar{\eta}} = \overline{\overline{\mathbf{V}_{\cdot}^2}} \quad . \qquad (\ell\text{-wave sources at rest}) \qquad (2.12.3)$$

If the ℓ-wave sources are moving, the relationship between $\mathbf{V}_\cdot$ and $\bar{\bar{\eta}}$ must be modified. This will be discussed in a later section.

The angular frequency ω functions as a constant while solving Eq. (2.12.1) for $\mathbf{V}_\cdot$, and a separate conditional equation, to be discussed further on, is required to determine its value. Eq.(2.12.1) involves both $\mathbf{V}_\cdot$ and $\phi_\cdot$, so one other equation is needed to solve for them. This is the ℓ-wave continuity equation. Because $\bar{\bar{\phi}}_a$ is of the order of ϕ_d, which is immensely greater than $\bar{\bar{\bar{\mathbf{V}}}}$, $\phi_\cdot$ and $\mathbf{V}_\cdot$ in any practical situation, Eq.(2.8.12) for the full ℓ-wave continuity relationship can be approximated with extreme accuracy by,

$$\phi_d \nabla \bullet \mathbf{V}_\cdot + \frac{\partial \phi_\cdot}{\partial t} \cong 0 \qquad (2.12.4)$$

Eqs.(2.12.1) and (2.12.4) allow $\phi_\cdot$ and $\mathbf{V}_\cdot$ to be found for traveling ℓ-waves. Here, again, certain subtleties are involved, so that the examples given later should be consulted before attempting to solve Eqs.(2.12.1) and (2.12.4).

The STANDING ℓ-wave equation takes the form,

$$\nabla^2 \bar{\bar{\eta}} - \frac{1}{c_0^2} \frac{\partial^2 \bar{\bar{\eta}}}{\partial t^2} = 0 \quad , \qquad (2.12.5)$$

where $\bar{\bar{\eta}}$ is again defined by Eq.(2.12.3). The ℓ-wave continuity equation, Eq.(2.12.4), is again used to solve for $\phi_\cdot$.

Eqs.(2.12.1) and (2.12.5) are field *shape* determining equations; since, as stated before, ω is a constant. The latter sets the *scale* of the solution, and is dependent on the compression properties of the ether, to be discussed next.

2.13 **The Compression/Oscillation Equation**: If ether is compressed, so that ϕ_a exceeds ϕ_d in some small region, and then is allowed to expand, the surrounding ether interacts with the expanding ether and an oscillation is set up. Since it is frictionless, the oscillation and the resulting ℓ-waves continue unabated. The waves are attenuated with distance from the generating region, but the overall oscillation persists. This process is similar to a mass/spring system; but the ether is so non-linear that its compression/oscillation properties are quite different from familiar cases.

The mass/spring process is usually described by an equation such as,

$$\frac{d^2 \phi_\cdot}{dt^2} + \mathcal{F}(\phi_\cdot) = 0 \quad , \qquad (2.13.1)$$

where the non-linear function $\mathcal{F}(\dot{\phi})$ includes the mass/spring characteristics. If $\mathcal{F}(\dot{\phi})$ is a known function, then Eq.(2.13.1) can be solved for a frequency/density relationship of the form,

$$\omega = G(\dot{\phi}_m) \quad , \qquad (2.13.2)$$

where $\dot{\phi}_m$ is the initial, maximum incremental density, and $G(\dot{\phi}_m)$ is generally an increasing, monotonic function.

The exact form of $\mathcal{F}(\dot{\phi})$ for the ether must be determined empirically, and is not known at this time. However, measured particle characteristics have been used to obtain much insight as to the form of the function $G(\dot{\phi}_m)$. This will be discussed in detail in Chapter 4.

The three equations necessary to solve for ℓ-wave fields, then, are Eqs.(2.12.4), (2.13.2) and either Eq.(2.12.1), for traveling ℓ-waves, or Eq.(2.12.5), for standing ℓ-waves. They are the main ℓ-wave field equations, as they are now known. Until a larger program of ℓ-wave experiments is carried out, they represent the only formal method for determining $\mathbf{V}_\cdot$ and $\dot{\phi}$ directly. Aside from certain auxiliary equations involved in the solutions for particles, they have within them the ability to describe everything related to the structure of matter.

Next to be discussed is the equation that connects the solutions $\mathbf{V}_\cdot$ and $\dot{\phi}$ found from the ℓ-wave equations, to the bulk solutions for $\bar{\dot{\phi}}$ and $\bar{\mathbf{V}}$, which produce the directly measurable properties of matter.

2.14 **The Bridge Equation**: The bulk properties of matter are found using the "bridge" equation, which gives, quantitatively, just how much ether distortion a given ℓ-wave can *sustain*. It takes its simplest form for static fields, as expressed by,

$$\nabla\bar{\dot{\phi}} = b\,\overline{\dot{\phi}\mathbf{V}}_\cdot \quad ; \qquad \text{(static)} \qquad (2.14.1)$$

which indicates that, *if the phase angle between* $\dot{\phi}$ *and* $\mathbf{V}_\cdot$ *is* 90 *degrees, no gradient of the bulk incremental density* $\bar{\dot{\phi}}$ *can be sustained*. However, *if* $\dot{\phi}$ and $\mathbf{V}_\cdot$ *have an in phase component, then the gradient of* $\bar{\dot{\phi}}$ *depends linearly on the time average of* $\dot{\phi}\mathbf{V}_\cdot$. Once the ℓ-waves $\dot{\phi}$ and $\mathbf{V}_\cdot$ are known throughout a region, $\bar{\dot{\phi}}$ can be determined everywhere in that region by integrating Eq.(2.14.1).

The designation "static" field in Eq.(2.14.1) means that no bulk field quantity such as $\overline{\overline{\phi}}$, $\overline{\overline{\mathbf{V}}}$, $\overline{\overline{\phi_a \mathbf{V}}}$, etc. is changing with time. In cases where the bulk fields are time variable, the picture is complicated by the finite propagation time delay required to readjust the fields. So, in the general case, the bridge equation is written,

$$\nabla \overline{\overline{\phi}} + \frac{1}{c_0^2} \frac{\partial \overline{\overline{\phi_a \mathbf{V}}}}{\partial t} = b\, \overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}} \quad , \quad \text{(general)} \qquad (2.14.2)$$

and is known as the "retarded" form (as in retarded potential).

2.15 **Bulk Equations**: Here again there are two sets of bulk equations, microscopic and macroscopic. Unfortunately, they are almost identical in appearance; but they have completely different physical meanings and are applied in quite different ways. To emphasize the disparity between them, *only the microscopic equations will be presented in this chapter. Discussion of the macroscopic bulk equations and their applications will be delayed until Chapter 11.*

Whether to use the microscopic or macroscopic form of the bulk equations is determined by the type of "charge" distribution to be analyzed. Most everyday electromagnetic problems involve the separation and recombination of groups of whole charged particles, such as electrons and other particles composing atomic nuclei. In all these cases, the "charge" is a number *assigned* to each whole particle, and the charge density distribution is described in terms of how many whole charged particles per cubic centimeter act at each point. *In these cases, the fundamental nature of "charge" is not considered, and the macroscopic equations are used.*

When the problem to be studied concerns the internal structure of particles, and an internal distribution of charge density that integrates throughout the particle to give the whole particle "charge" used in the macroscopic cases, then *the fundamental nature of "charge" is of concern, and the microscopic bulk equations are used.*

Even when analyzing a microscopic case, there are two ways to write some of the equations, because of the nature of bulk measurements. For example, in a field where the ether flow vector at each point is $\overline{\overline{\phi_a \mathbf{V}}}$, the bulk equations can be written in terms of $\overline{\overline{\phi}}_a$ and $\overline{\overline{\phi_a \mathbf{V}}}$. However, $\overline{\overline{\phi}}$ is the quantity found directly from the bridge equation, not $\overline{\overline{\phi}}_a$; and in the laboratory, $\overline{\overline{\phi}}$ is measured rather than $\overline{\overline{\phi}}_a$; so it is often more convenient to write the bulk equations in terms

of $\bar{\bar{\phi}}$ and the flow vector $\bar{\bar{\phi}}\mathbf{u}$, where $\mathbf{u}$ is a velocity defined by the relationship,

$$\bar{\bar{\phi}}\mathbf{u} = \bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}} \quad . \tag{2.15.1}$$

The significance of Eq.(2.15.1) is that the actual density $\bar{\bar{\phi}}_a$ at each point, moving at the actual ether velocity $\bar{\bar{\mathbf{V}}}$ defines a definite amount of ether actually flowing through any given small area, perpendicular to the flow at the point, as represented by the flow vector $\bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}}$; so, if $\bar{\bar{\phi}}$ is used, instead of $\bar{\bar{\phi}}_a$, there must be an apparent or effective velocity $\mathbf{u}$ such that $\bar{\bar{\phi}}\mathbf{u}$ gives the *same net amount* of ether passing through the small area. Because $\bar{\bar{\phi}} \ll \bar{\bar{\phi}}_a$ (due to the large value of ϕ_d), $\mathbf{u} \gg \bar{\bar{\mathbf{V}}}$. In some rare cases, the velocity $\mathbf{u}$ can appear to be infinite; but, in those cases, reverting to $\bar{\bar{\phi}}_a$ and $\bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}}$ eliminates any problem.

2.16 **The Bulk Conservation Equation**: In Section 2.8, the equation for ether conservation, Eq.(2.5.1), was separated into its bulk and ℓ-wave components, given by Eqs.(2.8.11) and (2.8.12). More compact forms of these separated equations are,

$$\nabla\bullet\bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}} + \frac{\partial\bar{\bar{\phi}}_a}{\partial t} = 0 \quad , \qquad \text{(bulk)} \tag{2.16.1}$$

and

$$\nabla\bullet\{\phi_a\mathbf{V}\}_{\cdot} + \frac{\partial\phi_{\cdot}}{\partial t} = 0 \quad . \qquad (\ell\text{-wave}) \tag{2.16.2}$$

In Section 2.12, an extremely close approximation to Eq.(2.16.2) was introduced in the form of Eq.(2.12.4). No such approximation is needed for the bulk Eq.(2.16.1), however, because the bulk equations are written directly in terms of $\bar{\bar{\phi}}_a$ and $\bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}}$. Thus, in the light of the discussion in Section 2.15 regarding Eq.(2.15.1), the two forms of the bulk conservation law required are,

$$\nabla\bullet\bar{\bar{\phi}}_a\bar{\bar{\mathbf{V}}} + \frac{\partial\bar{\bar{\phi}}_a}{\partial t} = 0 \quad \text{and} \quad \nabla\bullet(\bar{\bar{\phi}}\mathbf{u}) + \frac{\partial\bar{\bar{\phi}}}{\partial t} = 0 \quad . \tag{2.16.3}$$

These kinematic equations can be used interchangeably. The remainder of the bulk equations describe the ether's dynamics.

2.17 **Bulk Ether Distortions**: If the ℓ-wave equations are solved and the bridge is used to determine $\overline{\overline{\phi}}$ everywhere, then the $\overline{\overline{\phi}}$ field represents a bulk distortion away from ϕ_d, the uniform datum. There are many ways to describe this distorted field. To establish a good visualization of some of these different descriptions, first consider a "static" $\overline{\overline{\phi}}$ field held in place by its ℓ-waves. Not only is the original field $\overline{\overline{\phi}}$, itself, a distortion; but, for example, its gradient $\nabla\overline{\overline{\phi}}$ is a different, coexistent distortion field. The product $\nabla\overline{\overline{\phi}}\bullet\nabla\overline{\overline{\phi}}$ is another different distortion field; as is any other function of $\overline{\overline{\phi}}$ such as $\nabla^2\overline{\overline{\phi}}$, for example. The reason it becomes important to recognize and identify these different, coexisting forms of distortion, all implicit in the original $\overline{\overline{\phi}}$, is that in various interactions between fields, such as an electron's field immersed in the field of a charged parallel plate capacitor, for example, each of these distortions accounts for a different effect in the interaction. A few distortions produce such unique and recognizable effects that they have been given special names. Each of these more important deformations will be discussed here in some detail.

The principal thing to keep in mind is that there is nothing present in the field but the $\overline{\overline{\phi}}$ distortion distribution of ether.

2.18 **Incremental Bulk Distortion**: One of the most directly measurable properties of electric fields is the electric potential. In practical units it is given in volts, and is measured with the common voltmeter. The corresponding measurement in H-L units is the hlvolt. Physically, what is being measured here is the incremental ether density distortion $\overline{\overline{\phi}}$. When a potential of 1 hlvolt is *measured from the datum level*, the density $\overline{\overline{\phi}}$ is 1 descartes. Thus, *the incremental bulk density is the physical definition of what has been named "electric potential"*. If this had been known at the time various electrical properties were being named, electric potential now probably would be called "ether density", and voltmeters would be called ether density difference meters. It is important to remember that the voltmeter does not measure the absolute ether density $\overline{\overline{\phi}}_a$, but either the incremental value $\overline{\overline{\phi}}$ relative to the datum, or the difference in $\overline{\overline{\phi}}$ between two regions.

2.19 "**Gradient Squared" Distortion**: In 1837, Faraday first recognized the importance of the "gradient squared" distortion, when he defined the "electric energy density" in an electrostatic field as,

$$\varepsilon_e = \tfrac{1}{2}(\nabla \overline{\overline{\phi}}_a)^2 = \tfrac{1}{2}(\nabla \overline{\overline{\phi}})^2 \quad , \quad \text{(static)} \qquad (2.19.1)$$

measured in ergs/cm^3. From his point of view, $\overline{\overline{\phi}}$ was the electric potential (hlvolts) in the field, but here it is clear *that the physical definition of "electric energy density" is the particular distortion in the ether described by* Eq.(2.19.1), where $\overline{\overline{\phi}}$ is measured in descartes (hlvolts).

If the fields are changing with time, the retardation due to the finite propagation velocity must be included, just as was done in the general bridge equation. Therefore, the general definition of electric energy density takes the (retarded) form,

$$\varepsilon_e = \frac{1}{2}\left(\left(\nabla \overline{\overline{\phi}}_a\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial \overline{\overline{\phi}}_a}{\partial t}\right)^2 \right) \quad ,$$

or, (general) (2.19.2)

$$\varepsilon_e = \frac{1}{2}\left(\left(\nabla \overline{\overline{\phi}}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial \overline{\overline{\phi}}}{\partial t}\right)^2 \right) \quad .$$

Here again, as in the case of the incremental bulk density $\overline{\overline{\phi}}$, the physical definition of electric energy density is a simple, physically visualizable property of the bulk ether density.

2.20 **Surrounding Function Distortion**: In Appendix A, the Laplacian $\nabla^2 \overline{\overline{\phi}}$ of a scalar field $\overline{\overline{\phi}}$ is shown to be the difference, at each point in space, between the average of $\overline{\overline{\phi}}$ in a differential volume surrounding the point and the value of $\overline{\overline{\phi}}$ at the point. This is a simple visualization of the physical meaning of the surrounding function in any scalar field. *In the bulk incremental ether density field $\overline{\overline{\phi}}$, the surrounding function distortion produces the unique effects attributed to "distributed charge density".* Formally, the definition of "distributed charge density" in a static field is,

$$\rho = -\nabla^2 \overline{\overline{\phi}}_a = -\nabla^2 \overline{\overline{\phi}} \quad , \quad \text{(static)} \qquad (2.20.1)$$

measured in hlcoulombs/cm^3. Lack of knowledge of the physical nature of charge resulted in the less convenient choice of sign in Eq.(2.20.1) by early investigators.

As before, the propagation delay time must be involved when the fields are changing with time, so the general expression for charge density is,

$$\rho = -\left(\nabla^2 \overline{\overline{\phi}}_a - \frac{1}{c_0^2} \frac{\partial^2 \overline{\overline{\phi}}_a}{\partial t^2} \right) \quad ,$$

or, (general) (2.20.2)

$$\rho = -\left(\nabla^2 \overline{\phi} - \frac{1}{c_0^2} \frac{\partial^2 \overline{\phi}}{\partial t^2} \right) \quad .$$

Eqs.(2.20.1) and (2.20.2) look familiar, because when written with the RHS and LHS reversed, they have the same appearance as the well known Maxwell wave equation for the scalar potential. It is of utmost importance here to understand the profound difference between Maxwell's macroscopic equations and the microscopic Eqs.(2.20.1) and (2.20.2). In the macroscopic equations, ρ is the known function, being a count of *whole charged particle* density at each point. From this given source distribution, the potential function is found by solving the macroscopic equation. The whole particles exert coulomb forces on each other. Quite the opposite is true in the microscopic case described by Eqs.(2.20.1) and (2.20.2). There, the known field function $\overline{\phi}$, obtained from the ℓ-waves and the bridge, is operated on to find the distribution of surrounding function distortion, ρ. *The elements of distributed charge* ρ *exert* ***<u>no</u>*** *force on each other.*

2.21 **The Bulk Ether Flow Equation**: There are many ether fields that cannot be derived by the previously given bulk distortion equations. Common examples are the fields of permanent magnets, solenoids and transformer coils. Other important examples are the neutrino and photon particles. Some of these, as well as the analysis of antenna radiation in the form of transverse waves, require what is called the ether flow equation, which takes the static form,

$$\rho \mathbf{u} = \nabla^2 \overline{\overline{\overline{\phi}}}_a \mathbf{V} = \nabla^2 (\overline{\phi} \mathbf{u}) \quad , \quad \text{(static)} \qquad (2.21.1)$$

measured as hlcoulombs/cm^2-sec. The form for time variable fields

becomes,

$$\rho\mathbf{u} = -\left(\nabla^2 \overline{\overline{\phi_a}}\mathbf{V} - \frac{1}{c_0^2}\frac{\partial^2 \overline{\overline{\phi_a}}\mathbf{V}}{\partial t^2} \right) \quad ,$$

or, (general) (2.21.2)

$$\rho\mathbf{u} = -\left(\nabla^2 (\bar{\bar{\phi}}\mathbf{u}) - \frac{1}{c_0^2}\frac{\partial^2 (\bar{\bar{\phi}}\mathbf{u})}{\partial t^2} \right) \quad .$$

Here again, the equations have the same form as the macroscopic flow equations reversed; but in this case problems are sometimes solved the same way microscopically and macroscopically, even though the same profound difference exists in their physical interpretations.

There is a whole class of flow problems where there are vortices present. In these cases, the incremental density has little effect, so $\bar{\bar{\phi}}$ is effectively zero, **u** is not defined, and the flow is handled using a reduced form of Eq.(2.21.2),

$$\nabla^2 \bar{\bar{\mathbf{V}}} - \frac{1}{c_0^2}\frac{\partial^2 \bar{\bar{\mathbf{V}}}}{\partial t^2} = 0 \quad . \qquad (2.21.3)$$

2.22 **Summary of the Unified Field Equations**: Eq.(2.21.3) is the last of the main equations constituting the unified field theory of the ether. For convenience, all of the key equations of this chapter are summarized in Table 2.22.1. The following chapters will consist of solutions of these equations for much of the basic structure of matter observed in the world. They will not, however, delve into the physics of solids, liquids, gases or plasmas. Those areas develop logically from the more fundamental structures that will be derived here, but are so voluminous as to prohibit inclusion. Moreover, the physics in those disciplines will be modified only slightly by the fundamental changes elaborated here. On the other hand, a number of, at present, mysterious phenomena will be examined on the basis of the ether theory.

TABLE 2.22.1

EQUATIONS OF THE UNIFIED FIELD

Maxwell's Macroscopic Equations

$$\nabla^2\bar{\bar{\phi}} - \frac{1}{c_0^2}\frac{\partial^2\bar{\bar{\phi}}}{\partial t^2} = -\rho \quad , \quad \nabla^2(\bar{\bar{\phi}}\mathbf{u}) - \frac{1}{c_0^2}\frac{\partial^2(\bar{\bar{\phi}}\mathbf{u})}{\partial t^2} = -\rho\mathbf{u} \quad , \quad \nabla\bullet\bar{\bar{\phi}}\mathbf{u} = -\frac{1}{c_0}\frac{\partial\bar{\bar{\phi}}}{\partial t}$$

Unified Microscopic Equations

ℓ-wave equations

$$\nabla^2\bar{\bar{\eta}} - \frac{1}{c_0^2}\frac{\partial^2\bar{\bar{\eta}}}{\partial t^2} - \frac{1}{\bar{\bar{\eta}}}\left(\left(\nabla\bar{\bar{\eta}}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial\bar{\bar{\eta}}}{\partial t}\right)^2\right) = \pm\frac{c_0\omega}{\phi_d D}\nabla\bullet\overline{\overline{\phi_\cdot\mathbf{V}_\cdot}} \qquad \text{(traveling)}$$

$$\nabla^2\bar{\bar{\eta}} - \frac{1}{c_0^2}\frac{\partial^2\bar{\bar{\eta}}}{\partial t^2} = 0 \qquad \text{(standing)} \quad , \quad \bar{\bar{\eta}} = \overline{\overline{\mathbf{V}_\cdot^2}}$$

$$\phi_d\nabla\bullet\mathbf{V}_\cdot + \frac{\partial\phi_\cdot}{\partial t} \cong 0 \qquad \text{(conservation)} \quad , \quad \omega = \mathcal{G}(\phi_{\cdot m}) \qquad \text{(compr./osc.)}$$

the bridge equation

$$\nabla\bar{\bar{\phi}} + \frac{1}{c_0^2}\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t} = b\,\overline{\overline{\phi_\cdot\mathbf{V}_\cdot}}$$

bulk equations

$$\bar{\bar{\phi}}\mathbf{u} = \overline{\overline{\phi_a\mathbf{V}}} \qquad \text{(apparent flow)} \quad , \quad \nabla\bullet(\bar{\bar{\phi}}\mathbf{u}) + \frac{\partial\bar{\bar{\phi}}}{\partial t} - 0 \qquad \text{(conservation)}$$

$$\varepsilon_e = \frac{1}{2}\left(\left(\nabla\bar{\bar{\phi}}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial\bar{\bar{\phi}}}{\partial t}\right)^2\right) \qquad \text{(gradient squared distortion)}$$

$$\rho = -\left(\nabla^2\bar{\bar{\phi}} - \frac{1}{c_0^2}\frac{\partial^2\bar{\bar{\phi}}}{\partial t^2}\right) \qquad \text{(surrounding function distortion)}$$

$$\rho\mathbf{u} = -\left(\nabla^2(\bar{\bar{\phi}}\mathbf{u}) - \frac{1}{c_0^2}\frac{\partial^2(\bar{\bar{\phi}}\mathbf{u})}{\partial t^2}\right) \qquad \text{(bulk flow)} \quad , \quad \nabla^2\overline{\overline{\mathbf{V}}} - \frac{1}{c_0^2}\frac{\partial^2\overline{\overline{\mathbf{V}}}}{\partial t^2} = 0 \qquad \text{(vortex)}$$

CHAPTER 3

THE ELECTRON

3.1 **Introduction**: The physical properties of the ether, described in Chapter 2, and its equations of motion and distortion, summarized in Section 2.22, represent the complete unified theory to be presented here. Whether or not it is the ultimate theory must be determined by comparing its results with all known phenomena. This is a big order that must be carried out by many investigators, who are now not even aware of the theory's existence. In this and the following chapters, a skeletal outline of the whole phenomenological structure to be matched will be surveyed. Surely it will take years and many experiments, not even conceived yet, before it can be said with any certainty that the theory is complete. However, this sketch is very encouraging.

To give the reader a feeling for how the theory is used, including some subtleties in its application, the electron is a perfect example; because it involves almost every basic phenomenon known, electricity, magnetism, gravitation, quantum effects, etc. Its study begins with the visualization of a simple, physical flow pattern that describes it.

3.2 **Visualization of the Electron**: The picture of the electron to be used here applies equally well to the positron, where some of the physical functions are reversed. The electron, at rest, is assumed to be a spherically symmetrical region where a small reduction of the central bulk ether density has been made, and which reduced density configuration is held in shape, and prevented from filling in, by an outgoing longitudinal wave. Figure 3.2.1 shows an electron greatly exaggerated in amplitude relative to ϕ_d .

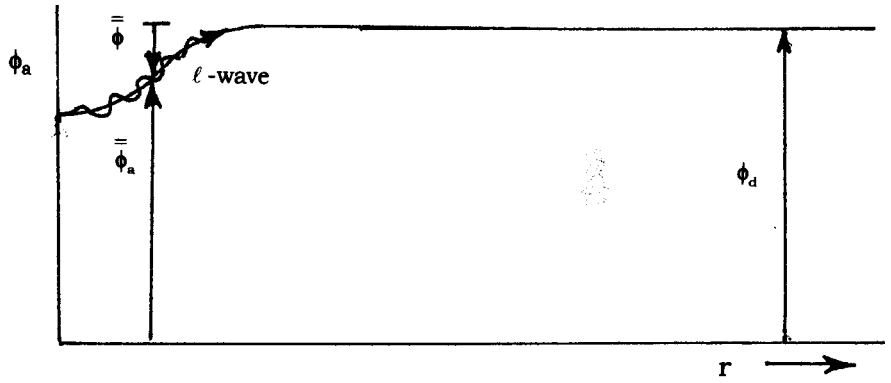

Figure 3.2.1 The electron ether density distribution.

The analysis begins with the solution of Eq.(2.12.1) for the traveling ℓ-wave. The bridge equation will then provide the bulk incremental density distribution $\bar{\bar{\phi}}$. From this, the distributed charge and electric energy density distributions ρ and ε_e will be obtained and integrated over all space to give q_e and E_0, the electron's charge and rest energy. Finally, its spin, magnetic moment and gravitic field will be computed.

Next, the electron will be considered moving at constant velocity, and its shape change, energy and charge will be evaluated. Certain of its quantum mechanical properties will be derived. Finally, some surprising conclusions will be drawn concerning its magnetic field and radiation properties.

3.3 **The Electron's Traveling ℓ-Waves**: First the *outgoing* ℓ-wave characteristics will be found. Since it is clear that, for an electron at rest, its fields are spherically symmetrical and drop off as some function of the radius r, measured from its center, a simple trial form for $\mathbf{V}_.$ can be written,

$$\mathbf{V}_. = \hat{\mathbf{r}}\frac{a\psi}{r}C \quad , \tag{3.3.1}$$

where,

$$C = \cos\omega\left(t - \frac{r}{c_0}\right) \quad , \tag{3.3.2}$$

and ω is as yet unspecified; a is the amplitude constant given in Table 2.10.1; and $\psi = \psi(r)$ is an unknown, monotonically increasing function to be found from Eq.(2.12.1).

From Eq.(3.3.1),

$$\nabla\bullet\mathbf{V}_. = \frac{a\psi}{r}\left(\left(\frac{1}{\psi}\frac{d\psi}{dr} + \frac{1}{r}\right)C + \frac{\omega}{c_0}S\right) \quad , \tag{3.3.3}$$

where,

$$S = \sin\omega\left(t - \frac{r}{c_0}\right) \quad , \tag{3.3.4}$$

Now, combining Eqs.(2.12.4) and (3.3.3),

$$\frac{\partial\phi_.}{\partial t} = -\frac{\phi_d a\psi}{r}\left(\left(\frac{1}{\psi}\frac{d\psi}{dr} + \frac{1}{r}\right)C + \frac{\omega}{c_0}S\right) \quad , \tag{3.3.5}$$

which can be integrated with respect to time, leading to,

$$\phi_{\cdot} = \frac{\phi_d a\psi}{c_0 r}\left(C - \frac{c_0}{\omega}\left(\frac{1}{\psi}\frac{d\psi}{dr} + \frac{1}{r}\right)S\right) \quad . \tag{3.3.6}$$

The product of Eqs.(3.3.6) and (3.3.1) results in,

$$\phi_{\cdot}\mathbf{V}_{\cdot} = \hat{\mathbf{r}}\frac{\phi_d a^2\psi^2}{c_0 r^2}\left(C^2 - \frac{c_0}{\omega}\left(\frac{1}{\psi}\frac{d\psi}{dr} + \frac{1}{r}\right)CS\right) \quad , \tag{3.3.7}$$

which leads directly to the time average,

$$\overline{\overline{\phi_{\cdot}\mathbf{V}_{\cdot}}} = \hat{\mathbf{r}}\frac{\phi_d a^2\psi^2}{2c_0 r^2} \quad , \tag{3.3.8}$$

and its divergence,

$$\nabla\bullet\overline{\overline{\phi_{\cdot}\mathbf{V}_{\cdot}}} = \frac{\phi_d a^2}{2c_0 r^2}\frac{d\psi^2}{dr} \quad . \tag{3.3.9}$$

Since $\overline{\overline{\phi_{\cdot}V_{\cdot}}} > 0$, from Eq.(2.12.2), the RHS of Eq.(2.12.1) will have a negative sign.

Together, Eqs.(2.12.3) and (3.3.1) yield,

$$\mathbf{V}_{\cdot}^2 = \frac{a^2\psi^2}{r^2}C^2 \quad \text{and} \quad \bar{\bar{\eta}} = \frac{a^2\psi^2}{2r^2} \quad . \tag{3.3.10}$$

Since this is a "static" case, Eq.(2.12.1) reduces to,

$$\nabla^2\bar{\bar{\eta}} - \frac{1}{\bar{\bar{\eta}}}\left(\nabla\bar{\bar{\eta}}\right)^2 = -\frac{c_0\omega}{\phi_d D}\nabla\bullet\overline{\overline{\phi_{\cdot}\mathbf{V}_{\cdot}}} \quad . \tag{3.3.11}$$

This illustrates one of the subtleties of equations in which both bulk quantities and their time derivatives appear. In Eq.(2.12.1), for example, $\bar{\bar{\eta}}$ is a time averaged quantity; yet, in some situations, there can be time derivatives of $\bar{\bar{\eta}}$. *The double bar notation indicates only a time average over the high frequency ℓ- wave cycles* and not over the bulk time variations. Only in a "static" case (where $\bar{\bar{\phi}}$, $\overline{\overline{\mathbf{V}}}$, $\overline{\overline{\phi_a\mathbf{V}}}$ do not change with time) can the time derivatives in Eq. (2.12.1), for example, be set equal to zero. This same dichotomy of bulk time variations and

ℓ-wave time averages comes up in many places in the theory, so caution should be the guide.

When $\overline{\overline{\eta}}$ from Eq.(3.3.10) is substituted into Eq.(3.3.11) and the indicated differentiations are carried out, Eq.(3.3.11) is reduced to an equation, for the unknown function $\psi(r)$ in $\mathbf{V}_\bullet$, of the form,

$$\frac{d^2\psi}{dr^2} - \frac{1}{\psi}\left(\frac{d\psi}{dr}\right)^2 + \left(\frac{2}{r} + \frac{\omega}{D}\right)\frac{d\psi}{dr} - \frac{\psi}{r^2} = 0 \quad . \qquad (3.3.12)$$

The simplest non-trivial solution of Eq.(3.3.12) is[1],

$$\psi = \varepsilon^{-D/\omega r} \quad . \qquad (3.3.13)$$

So, from Eq.(3.3.1),

$$\mathbf{V}_\bullet = \hat{\mathbf{r}}\frac{a}{r}\varepsilon^{-D/\omega r}C \quad , \qquad (3.3.14)$$

is the desired velocity ℓ-wave solution of Eq.(3.3.11). The corresponding density wave is obtained by substituting Eq.(3.3.13) into Eq.(3.3.6), with the result,

$$\phi_\bullet = \frac{\phi_d a}{c_0 r}\varepsilon^{-D/\omega r}\left(C - \frac{c_0}{\omega}\left(\frac{D}{\omega r^2} + \frac{1}{r}\right)S\right) \quad . \qquad (3.3.15)$$

3.4 **The Electron's Oscillation Rate, ω**: Normally, the angular frequency would be determined from the compression/oscillation Eqs.(2.13.1) and (2.13.2); but this requires an extended discussion that is better postponed until Chapter 4. Suffice it to say that the electron's oscillation rate is found to be, $\omega_e = 7.7634396\times10^{20}$ radians per second; and this value will be used for the electron in Eqs.(3.3.8), (3.3.9), and (3.3.13) through (3.3.15). In conjunction with the values for D and a, given in Table 2.10.1, all unknowns have now been eliminated from the equations just enumerated; so that the absolute values of $\mathbf{V}_\bullet$ and $\phi_\bullet$ are known, as are the bulk values for $\overline{\overline{\phi_\bullet\mathbf{V}_\bullet}}$ and $\nabla\bullet\left(\overline{\overline{\phi_\bullet\mathbf{V}_\bullet}}\right)$. The latter can now be used with the bridge equation to calculate the electron's bulk structure.

3.5 **The Electron's Incremental Bulk Density Structure**: Since the electron is at rest here, the static bridge Eq.(2.14.1) applies. With the aid of Eq.(3.3.8), the gradient of the incremental bulk density

1. B.Liebowitz, Phys.Rev. **64**, 294 (1943). Liebowitz suggested a similar cutoff function in a somewhat different context.

distortion can be written,

$$\nabla\overline{\overline{\phi}} = \hat{\mathbf{r}}\frac{\partial\overline{\overline{\phi}}}{\partial r} = b\,\overline{\overline{\phi}}\cdot\mathbf{V} = \hat{\mathbf{r}}\frac{\phi_d a^2 b}{2c_0 r^2}\varepsilon^{-2D/\omega r} \quad , \tag{3.5.1}$$

where the constants are those in Table 2.10.1. Next, $\overline{\overline{\phi}}$ can be found directly by integrating Eq.(3.5.1) as follows (see Appendix H),

$$\overline{\overline{\phi}} = \frac{\phi_d a^2 b}{2c_0}\int\frac{\varepsilon^{-2D/\omega r}}{r^2}dr + \phi_0 \quad ;$$

so,

$$\overline{\overline{\phi}} = \frac{\phi_d \omega a^2 b}{4c_0 D}\varepsilon^{-2D/\omega r} + \phi_0 \quad .$$

To find the integration constant ϕ_0, remember that as $r \to \infty$, $\overline{\overline{\phi}} \to 0$, which means that,

$$\phi_0 = -\frac{\phi_d \omega a^2 b}{4c_0 D} = -1.9233\times 10^3 \quad \text{des} \quad . \tag{3.5.2}$$

Therfore, the final form of the $\overline{\overline{\phi}}$ distribution for the electron is,

$$\overline{\overline{\phi}} = \phi_0\left(1-\psi^2\right) = \phi_0\left(1-\varepsilon^{-2D/\omega r}\right) \quad . \tag{3.5.3}$$

Figure 3.5.1 is a plot of this very simple particle.

Only two features of the electron are apparent from Figure 3.5.1. First, it is impressive that the electron is such a minute deformation.

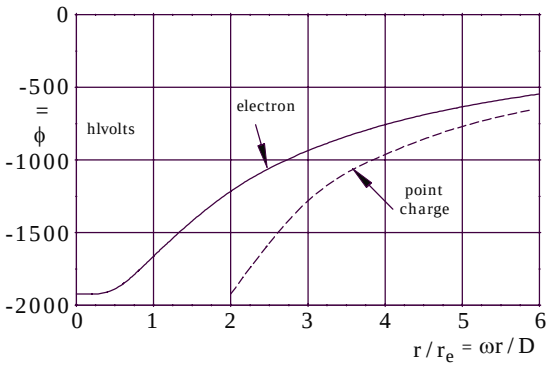

Figure 3.5.1 The bulk structure of the electron.

The greatest depletion of ether is at the center, and has the value ϕ_0; so that,

$$\frac{|\phi_0|}{\phi_d} = 2.1400 \times 10^{-18} \quad , \tag{3.5.4}$$

almost no depletion at all. Second, except for the inflection point at $\omega r/D = 1$, this extended distribution has no other clearly identifiable radius. As will be shown, the inflection point is significant in several respects, and its radius makes a very good choice for the effective radius of the electron, i.e.,

$$r_i = \frac{D}{\omega} = 3.522426 \times 10^{-14} \quad \text{cm.} \tag{3.5.5}$$

Eq.(3.5.3) can now be written in the more intuitive and useful form,

$$\bar{\bar{\phi}} = \phi_0 \left(1 - \varepsilon^{-2r_i/r}\right) \quad . \tag{3.5.6}$$

3.6 **The Electron's Electric Energy**: Now that the space distribution of incremental density $\bar{\bar{\phi}}$ is known, Eq.(2.19.1) can be employed to establish the electron's gradient squared distortion or electric energy density. Taking the gradient of $\bar{\bar{\phi}}$ in Eq.(3.5.6),

$$\nabla\bar{\bar{\phi}} = -\hat{\mathbf{r}}\,\frac{2\phi_0 r_i}{r^2}\,\varepsilon^{-2r_i/r} \quad . \tag{3.6.1}$$

When Eq.(3.6.1) is substituted into Eq.(2.19.1), the electric energy density is found to be,

$$\varepsilon_e = 2\,\frac{\phi_0^2\, r_i^2}{r^4}\,\varepsilon^{-4r_i/r} \quad , \quad \frac{\text{ergs}}{\text{cm}^3} \tag{3.6.2}$$

which is plotted in Figure 3.6.1. Apparently, ε_e is a smooth shell of distortion that peaks at $r = r_i$, which is one good reason for the choice of the $\bar{\bar{\phi}}$ inflection point as the electron's effective radius.

The total rest energy in the electron's field is found by integrating Eq.(3.6.2) over all space,

$$E_0 = \int_{\text{space}} \varepsilon_e \, \text{dvol} = 8\pi\phi_0^2\, r_i^2 \int_0^\infty \frac{\varepsilon^{-4r_i/r}}{r^2}\, dr \quad ,$$

with the result,

$$E_0 = 2\pi\phi_0^2\, r_i = 8.18711 \times 10^{-7} \quad . \quad \text{ergs} \tag{3.6.3}$$

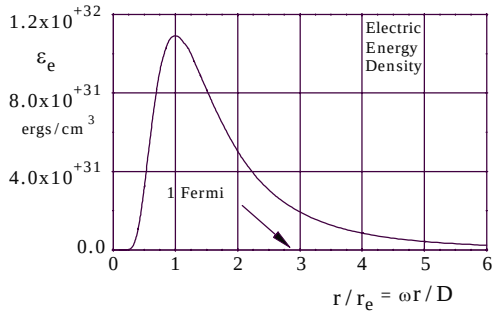

Figure 3.6.1 The electron's "gradient squared" distortion distribution.

The physical effects produced by this gradient squared distortion are numerous; so much so, that in the past some of the effects were attributed to "mass", a property thought related in some way to energy. Modern writers say that mass and energy are "equivalent" on the basis of the famous equation of Einstein ,et al,

$$m_0 = \frac{E_0}{c_0^2} \quad . \tag{3.6.4}$$

Here, the position is taken that they are *identical*, i.e. *one physical phenomenon, with two names, expressed in different units*. There is just one gradient squared distortion. It causes all the effects of electric energy and all the effects of mass, but the units identified with mass are c_0^2 larger than those identified with energy.

3.7 **Electron Charge**: The coexistent charge density is found from the surrounding function distortion Eq.(2.20.1) by first taking the divergence of Eq.(3.5.1), leading to,

$$\rho = -\nabla^2 \bar{\bar{\phi}} = -b\nabla \bullet \left(\overline{\overline{\phi . \mathbf{V}}} \right) \quad . \tag{3.7.1}$$

Then, substituting Eq.(3.6.1) into Eq.(3.7.1),

$$\rho = 4 \frac{\phi_0 r_i^2}{r^4} \varepsilon^{-2 r_i / r} \quad , \quad \frac{hlc}{cm^3} \tag{3.7.2}$$

which is plotted in Figure 3.7.1. Here, again, ρ is a smooth shell of distortion, but it peaks at $r = r_i / 2$, half the radius of peak energy

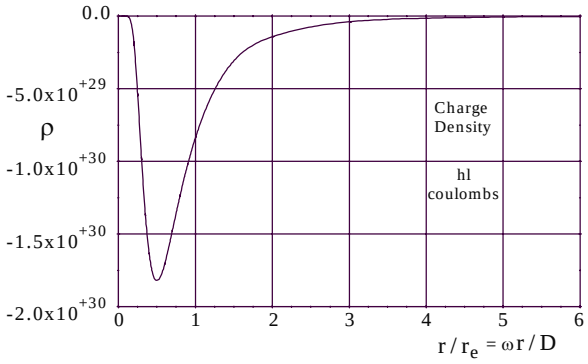

Figure 3.7.1 The electron's "surrounding function" distortion distribution.

density. It should be noted that the electron's charge density, ρ, is negative because ϕ_0 is negative.

The total electron charge is calculated by integrating Eq.(3.7.2) over all space,

$$q_e = \int_{space} \rho dvol = 16\pi\phi_0 r_e^2 \int_0^\infty \frac{\varepsilon^{-2r_e/r}}{r^2} dr \quad ,$$

with the result,

$$q_e = 8\pi\phi_0 r_e = -\frac{2\pi\phi_d a^2 b}{c_0} = -1.702692\times10^{-9} = -e \quad , \qquad (3.7.3)$$

Here, again, it should be noted that q_e is negative because ϕ_0 is negative.

Equations (3.5.6), (3.6.1), (3.7.2) and (3.7.3) apply equally well for the positron if the value of ϕ_0 in Eq.(3.5.2) is used without the negative sign. This is the result of an ingoing ℓ-wave. However, Eqs.(3.3.8) and (3.3.9) change sign for the positron.

3.8 **Electron Spin and Magnetic Moment**: In interaction processes between particles, a considerable amount of ether churning occurs; and when particles like the electron or positron are created from the splatter, they end up with a specific angular momentum or what is called spin. Since their charge density rotates with the ether, they have a magnetic moment as well. In Section 2.9 it was indicated that angular momentum in the ether is just the angular persistence of a vortex in a frictionless fluid. Thus, the first step in the determination of the spin angular momentum is to find the spin vortex velocity field. The vortex equation, Eq.(2.21.3) provides that information.

Calculation of the electron spin vortex is done by first ignoring the radial, outgoing ℓ-waves that sustain the bulk gradient. The incremental bulk density $\bar{\bar{\phi}}$ also has little effect on the circulating flow. The simplest case is that where the electron is at rest and the circulation is not changing with time. Eq.(2.21.3) then reduces to,

$$\nabla^2 \bar{\bar{\mathbf{V}}} = 0 \quad , \tag{3.8.1}$$

where, $\bar{\bar{\mathbf{V}}}$ represents only the unchanging velocity of circulation about one particular axis through the electron's center (see Figure 3.8.1). In spherical coordinates (r, θ, α),

$$\bar{\bar{\mathbf{V}}} = \hat{\boldsymbol{\alpha}} \bar{\bar{V}}_{\alpha} \quad , \tag{3.8.2}$$

and Eq.(3.8.1) leads to,

$$\nabla^2 \bar{\bar{V}}_{\alpha} - \frac{\bar{\bar{V}}_{\alpha}}{r^2 \sin^2 \theta} = 0 \quad . \tag{3.8.3}$$

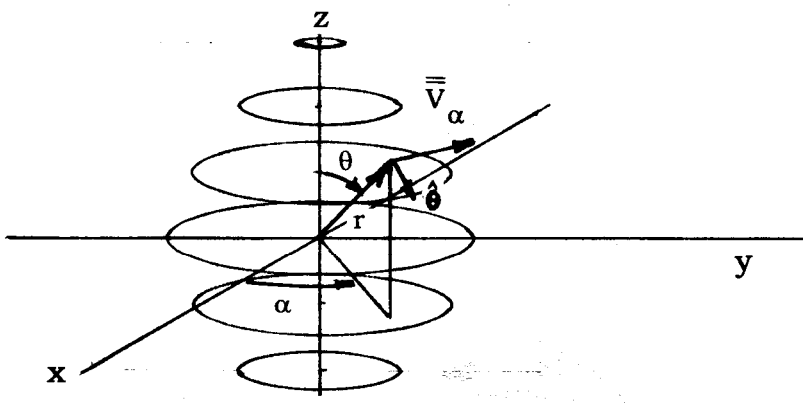

Figure 3.8.1 Spin flow.

Considering the flow as separable into r and θ dependent parts, let,

$$\bar{\bar{V}}_{\alpha} = \frac{\mathcal{R}(r)}{r^2} \mathcal{T}(\theta) \quad , \tag{3.8.4}$$

and the separated equations become,

$$\frac{d^2\mathcal{R}}{dr^2} - \frac{2}{r}\frac{d\mathcal{R}}{dr} + \left[1 - \frac{\ell(\ell+1)}{2}\right]\frac{2\mathcal{R}}{r^2} = 0 \quad ,$$

and, (3.8.5)

$$\frac{d^2\mathcal{T}}{d\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{d\mathcal{T}}{d\theta} + \left[\ell(\ell+1) - \frac{1}{\sin^2\theta}\right]\mathcal{T} = 0 \quad .$$

The simplest solution free of non physical attributes evolves from $\ell = 1$, with the result,

$$\mathcal{T} = \sin\theta \quad , \tag{3.8.6}$$

and,

$$\frac{d^2\mathcal{R}}{dr^2} - \frac{2}{r}\frac{d\mathcal{R}}{dr} = 0 \quad . \tag{3.8.7}$$

Eq.(3.8.7) is solved directly, with $\mathcal{R}$ expressed as,

$$\mathcal{R} = K_s + K_r r^3 \quad . \tag{3.8.8}$$

Eqs.(3.8.6) and (3.8.8) can now be combined with Eq.(3.8.4) to give the spin velocity field. For obvious physical reasons, the solution has two regions, inner and outer, where $\overline{\overline{V_\alpha}}$ of each matches at some radius δr_i. Thus,

$$\overline{\overline{V_\alpha}} = \begin{cases} K_s \dfrac{r}{(\delta r_i)^3}\sin\theta \quad , & \text{inside } \delta r_i \\ K_s \dfrac{1}{r^2}\sin\theta \quad , & \text{outside } \delta r_i \end{cases} \quad . \tag{3.8.9}$$

Customarily, angular momentum is found by integrating, over all space, the mass density at each radius times the velocity of that mass density; but considering what has been said about momentum and its meaning, i.e. particles have it but ether doesn't, it might be doubted that the same integration process would apply microscopically to the ether in electron spin. Nevertheless, it will be assumed here that the ether angular momentum can be made quantitative by the conventional approach, with minor modifications. In the light of Eq.(3.6.4), the spin diadic is,

$$S = \frac{\sigma}{2}(\mathbf{ij} - \mathbf{ji}) \quad , \tag{3.8.10}$$

where,

$$\sigma = \kappa \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{\varepsilon_e}{c_0^2}\,\overline{\overline{V_\alpha}}(r\sin\theta)\ \mathrm{dvol} \quad , \tag{3.8.11}$$

and ε_e is the energy density of Eq.(3.6.2). Here, κ is a scaling constant that allows for the fact that the angular persistence of a frictionless fluid is not easily made quantitative, yet a relationship must be found that connects it to the momentum-like effects it produces in interactions with particles and fields. After the proper

substitutions and integrations,

$$\sigma = \frac{\pi\kappa K_s \phi_0^2}{3c_0^2}\left[1 - \left(1 + \frac{4}{\delta}\right)\varepsilon^{-4/\delta} + \frac{64}{\delta^3}\,T\left(\frac{\delta}{4}\right)\right] \quad , \qquad (3.8.12)$$

where δ establishes the break point of maximum $\overline{\overline{V_\alpha}}$, and T(x) is the Truncation integral (see Appendix H). Conventionally, spin is taken as the vector of S, which is S_v or,

$$\boldsymbol{\sigma} = \mathbf{k}\,\sigma \quad . \qquad (3.8.13)$$

Following along the same lines, the spin magnetic moment is conventionally expressed as a vector,

$$\boldsymbol{\mu}_s = \frac{\kappa}{2c_0}\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \mathbf{r} \times \rho\overline{\overline{\mathbf{V}}}\ \mathrm{dvol} \quad ; \qquad (3.8.14)$$

but, because the charge density circulates always perpendicular to the radius vector **r** (see Figure 3.8.1), the only components that do not cancel in the integration over all θ and α are the z components. Again, this is better described by writing a spin magnetic dipole moment diadic,

$$M = \frac{\mu_s}{2}(\mathbf{ij} - \mathbf{ji}) \quad , \qquad (3.8.15)$$

where,

$$\mu_s = \frac{\kappa}{2c_0}\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \rho\overline{\overline{V_\alpha}}(r \sin\theta)\ \mathrm{dvol} \quad , \qquad (3.8.16)$$

and ρ is the charge density of Eq.(3.7.2). With the proper substitutions and integrations,

$$\mu_s = \frac{4\pi\kappa K_s \phi_0}{3c_0}\left[1 - \left(1 + \frac{2}{\delta}\right)\varepsilon^{-2/\delta} + \frac{8}{\delta^3}\,T\left(\frac{\delta}{2}\right)\right] , \qquad (3.8.17)$$

and the usual vector magnetic moment is the vector of M, which is M_v or,

$$\boldsymbol{\mu}_s = \mathbf{k}\mu_s \quad . \qquad (3.8.18)$$

Taking the ratio μ_s/σ with the help of Eqs.(3.6.3), (3.6.4) and (3.7.3),

$$\frac{\mu_s}{\sigma} = \pm \frac{e}{m_0 c_0} \frac{\left[1 - \left(1 + \frac{2}{\delta}\right)\varepsilon^{-2/\delta} + \frac{8}{\delta^3} T\left(\frac{\delta}{2}\right)\right]}{\left[1 - \left(1 + \frac{4}{\delta}\right)\varepsilon^{-4/\delta} + \frac{64}{\delta^3} T\left(\frac{\delta}{4}\right)\right]} \quad , \qquad (3.8.19)$$

where the sign is + for the positron and – for the electron. It is a well established fact that the theoretical value of μ_s/σ given by Dirac's equation is $e/m_0 c_0$, and that the measured ratio is slightly larger. The latter is the result of the necessity of making the measurement on an ensemble of particles, and is of no special interest at this point. The actual, or intrinsic μ_s/σ ratio of individual electron/positrons is given by Eq.(3.8.19); and for any δ is slightly *smaller* than $e/m_0 c_0$. Since the effects of the brackets in Eq.(3.8.19) and the ensemble measurements are opposite, the exact value of δ cannot yet be determined; but indications are that $\delta < 0.06$, and the maximum $\overline{\overline{V_\alpha}}$ occurs at a radius less than 2×10^{-15} cm. Assuming that $\delta < 0.06$, then,

$$\sigma = \frac{\pi \kappa K_s \phi_0^2}{3 c_0^2} \qquad (3.8.20)$$

and

$$\mu_s = \frac{4 \pi \kappa K_s \phi_0}{3 c_0} \qquad (3.8.21)$$

differ from the intrinsic values by less than 10^{-13} parts. In the remainder of this work Eqs.(3.8.20), (3.8.21), (3.8.12), and (3.8.17) will all be referred to as intrinsic, unless otherwise specified.

The constant K_s is not determined in this derivation, because it is established during the particle production process. Its magnitude is specified by the complicated relationships set up by the input conditions of the interaction. At the present time, this complicated interaction problem has not been solved. However, the value of κK_s can be obtained by using the experimental value for μ_s,

$$\mu_s = 3.2875524 \times 10^{-20} \quad \text{ergs/hlG} \quad , \qquad (3.8.22)$$

directly in Eq.(3.8.21), which yields,

$$\kappa K_s = 1.2233488 \times 10^{-13} \quad \text{erg-cm}^2/\text{des}^2 - \text{sec} \quad . \qquad (3.8.23)$$

Substituting this value for κK_s in Eq.(3.8.20) produces a value for the electron's angular momentum,

$$\sigma = 5.2728633 \times 10^{-28} \quad \text{erg-sec} \quad . \qquad (3.8.24)$$

L. O. Heflinger has pointed out that, since the outer field is equivalent to a magnetic dipole, combining Eqs.(3.8.9), (3.8.21) and (9.4.7), K_s can be eliminated and $\kappa = 3\phi_d/\phi_0$, or,

$$\kappa = 1.4018715 \times 10^{17} \quad . \qquad (3.8.25)$$

If this value is used for κ , then,

$$K_s = 8.7265399 \times 10^{-31} \quad . \qquad (3.8.26)$$

From Eq.(3.8.9), the maximum $\overline{\overline{V}}_\alpha$ at $\delta = 0.06$ $(\theta = \pi/2)$ is found to be $\overline{\overline{V}}_{\alpha\,max} = 1.9537 \times 10^{-2}$ cm / sec .

It might be thought that the spin would interfere with or distort the outgoing ℓ-waves. That this is not a serious problem can be seen by first recognizing that the exaggerated illustration in Figure 3.2.1 is somewhat misleading. When the actual numbers are examined, the wavelength of the ℓ-waves is found to be very long relative to r_i , as indicated in Figure 3.8.2. This means that the maximum $\overline{\overline{V}}_\alpha$ is essentially at the origin; and at that point, during the time it takes the electron spin to rotate only one second of arc, more than 200 full ℓ – wave cycles leave the electron's concentration region ($r < 10r_i$).

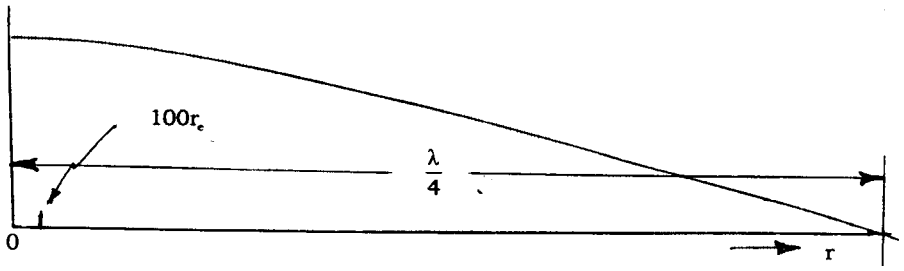

Figure 3.8.2 The ℓ-wave λ is very large relative to r_i.

3.9 **The Electron's Gravitic Field**: From Eqs.(3.3.14), (3.3.15) and (3.5.5), the travelling ℓ-waves are given by,

$$\mathbf{V}_{\cdot t} = \hat{\mathbf{r}} \frac{a}{r} \varepsilon^{-r_i/r} C \quad , \tag{3.9.1}$$

and,

$$\phi_{\cdot t} = \frac{\phi_d a}{c_0 r} \varepsilon^{-r_i/r} \left(C - \frac{c_0}{\omega r} \left(1 + \frac{r_i}{r} \right) S \right) \quad . \tag{3.9.2}$$

To understand how the electron's gravitic field is generated, it is helpful to look at the central region where $r<30r_i$. This is facilitated by expanding the cosine, for example, as,

$$C = \cos \omega t \ \cos \frac{\omega r}{c_0} + \sin \omega t \ \sin \frac{\omega r}{c_0} \quad ,$$

which, in the region $r<30r_i$, reduces to $C \cong \cos \omega t$. The sine becomes $S \cong \sin \omega t$. In the same region,

$$\frac{c_0}{\omega r} \left(1 + \frac{r_i}{r} \right) \gg 1 \quad ,$$

so, the travelling ℓ-waves in the central region of the electron are,

$$\mathbf{V}_{\cdot t} \cong \hat{\mathbf{r}} \frac{a}{r} \varepsilon^{-r_i/r} \cos \omega t \quad , \tag{3.9.3}$$

and,

$$\phi_{\cdot t} \cong - \frac{\phi_d a}{\omega r^2} \varepsilon^{-r_i/r} \left(1 + \frac{r_i}{r} \right) \sin \omega t \quad , \tag{3.9.4}$$

In fact, they represent a pure compression/expansion oscillation, which, being a disturbance, generates standing ℓ-waves as well as the traveling waves.

What the electron's standing ℓ-wave looks like can be determined by solving Eq.(2.12.5) reduced to the static case,

$$\nabla^2 \overline{\overline{\eta}}_s = 0 \quad , \tag{3.9.5}$$

where $\mathbf{V}_{\cdot s}$ and $\overline{\overline{\eta}}_s$ of the standing ℓ-wave are related through Eq.(2.12.3),

$$\overline{\overline{\eta}}_s = \overline{\overline{\mathbf{V}^2_{\cdot s}}} \quad . \tag{3.9.6}$$

In spherical coordinates (r, θ, α), Eq.(3.9.5) can be written,

$$\frac{d^2 \overline{\overline{\eta}}_s}{dr^2} + \frac{2}{r} \frac{d \overline{\overline{\eta}}_s}{dr} = 0 \quad , \tag{3.9.7}$$

which has a solution,

$$\bar{\bar{\eta}}_s = K_h + \frac{K_f}{r} \quad . \tag{3.9.8}$$

Since all incremental fields approach zero as $r \to \infty$, $K_h = 0$, and,

$$\bar{\bar{\eta}}_s = \overline{\overline{\mathbf{V}_{\cdot s}^2}} = \frac{K_f}{r} \quad . \tag{3.9.9}$$

The simplest standing ℓ-wave solution, compatible with Eq.(3.9.3), that satisfies Eq.(3.9.9) is,

$$\mathbf{V}_{\cdot s} = \hat{\mathbf{r}} \sqrt{\frac{K_g}{r}} \cos \omega t \quad , \tag{3.9.10}$$

where $K_g = 2K_f$.

Going back to the traveling ℓ-wave of Eq.(3.9.3), the maximum amplitude of $\mathbf{V}_{\cdot t}$ occurs at $r = r_i$. Here it will be assumed that the standing velocity wave $\mathbf{V}_{\cdot s}$ matches $\mathbf{V}_{\cdot t}$ at that radius, leading to,

$$K_g = \frac{a^2}{r_i \varepsilon^2} \quad , \tag{3.9.11}$$

remembering that ε is the base of natural logarithms ($\psi = \varepsilon^{-1}$ at $r = r_i$). Thus, in Eq.(3.9.10),

$$K_g = 2.43133 \times 10^{-34} \quad \frac{cm^3}{sec^2} \quad . \tag{3.9.12}$$

The standing wave field picture is completed by substituting Eq.(3.9.10) in Eq.(2.12.4) to find,

$$\phi_{\cdot s} = -\frac{3\phi_d}{2\omega} \frac{\sqrt{K_g}}{r^{3/2}} \sin \omega t \quad . \quad r \geq r_e \tag{3.9.13}$$

Now it is possible to show that $\mathbf{V}_{\cdot s}$ and $\phi_{\cdot s}$ establish the electron's gravitic field, but first a brief summary of these standing waves' characteristics will be given that will prove helpful later on. Eqs.(3.9.10) and (3.9.13) indicate that these waves do not propagate; but, instead, the whole field quivers in and out in unison. Since, $\overline{\overline{\phi_{\cdot s} \mathbf{V}_{\cdot s}}} = 0$, no contribution to the electron's bulk density field is made by these standing ℓ-waves. Furthermore, for the electron at rest, Eq.(2.8.13) shows that $\bar{\bar{\mathbf{V}}}_s = 0$. Considering the total velocity at each point as the sum of $\mathbf{V}_t$ and $\mathbf{V}_s$, it is possible to show, with the help

of Eq.(2.8.8), that the only significant contribution to the electron's acceleration field, at distances greater than $100r_i$, is,

$$\overline{\overline{\mathbf{a}_s}} = \overline{\overline{\mathbf{V}_{.s} \bullet \nabla \mathbf{V}_{.s}}} = -\hat{\mathbf{r}} \frac{K_g}{4r^2} \quad . \qquad (3.9.14)$$

All other contributions to the acceleration field are either much smaller than $\overline{\overline{\mathbf{a}_s}}$ or they change sign every 4.853×10^{-10} cm and will not have any effect on an object larger than 10^{-9} cm across. Space will not permit including this straightforward but tedious demonstration.

All particles except photons and neutrinos have this standing wave field, and its importance appears when large numbers of particles are combined into sizable *neutral* objects. Under those conditions, the bulk fields are mostly confined inside the large object, and the sum of all the acceleration fields remains outside. Notice that $\overline{\overline{\mathbf{a}_s}}$ is directed radially *inward* for both negative and positive charged particles.

From Galilei's time onwards, it has been recognized that *the basic characteristic of a spherical gravitic field is that all objects at the same distance from its center accelerate towards its center at the same rate if unimpeded*. This is generally expressed in the form of Newton's law of gravitation,

$$\overline{\overline{\mathbf{a}}} = -\hat{\mathbf{r}} \frac{GM}{4\pi r^2} \quad , \qquad (3.9.15)$$

Where M is the mass of the source body, and G is the gravitational constant. If, for the moment, it is assumed that *the natural state of any object is to move to oppose its time average acceleration with respect to the ether*, then any object in the field described by Eq.(3.9.14) will accelerate towards the center of the field. By comparing Eqs.(3.9.14) and (3.9.15), the constant G will be seen to have the value $\pi K_g / M$. Converting the electron's energy from Eq.(3.6.3) to mass units,

$$m_0 = 9.10939 \times 10^{-28} \text{ gm} \quad , \qquad (3.9.16)$$

and Newton's gravitation constant is found to be,

$$G = \frac{\pi K_g}{m_0} = 8.38503 \times 10^{-7} \ \frac{cm^3}{g - sec^2} \quad . \qquad (3.9.17)$$

Here G differs from the usual value by a factor of 4π used in Eq.(3.9.15) to express the idea that the $4\pi r^2$ in all spherical fields has geometrical significance.

This completes the picture of the electron at rest. Table 3.9.1 lists the related properties found so far. However, the electron has many other interesting characteristics when it is in motion. These will be discussed in the following.

TABLE 3.9.1

CONSTANTS FOR AN ELECTRON AT REST

ϕ_d	8.9875517×10^{20}	descartes	Datum ether density
ϕ_0	-1.92333×10^{3}	descartes	Incremental density (r=0)
r_i	3.522426×10^{-14}	cm	Effective radius
q_e	-1.7026924×10^{-9}	hlc	Charge distortion
E_0	8.18711×10^{-7}	ergs	Energy distortion
m_0	9.10939×10^{-28}	gm	Mass
σ	5.2728633×10^{-28}	erg-sec	Spin angular momentum (intrinsic)
μ_S	3.2875524×10^{-20}	erg/hlG	Spin magnetic moment (intrinsic)
ω_e	7.7634396×10^{20}	rad/sec	Electron ℓ - wave frequency
G	8.38503×10^{-7}	$cm^3/g-sec^2$	Gravitational constant

3.10 **The Constant Velocity Electron**: An electron in motion exhibits several characteristics not evident when it is at rest. Some of these are velocity dependent and some result from acceleration. While the velocity dependent properties are amenable to formal analysis, most of the acceleration effects are mathematically intractable. Nevertheless, even in those cases, considerable insight evolves from the qualitative picture available. In the following constant velocity case, the concepts of kinetic energy, inertia and momentum will be derived, and the physical basis of the de Broglie frequency will be made clear.

The analysis begins with the derivation of the ℓ-waves as a solution of the traveling wave Eq.(2.12.1). This is no easy task, but a hint from the theory of sound suggests one simplification. For example, since it is well known that a moving point source of waves produces wave fronts that are spherical, but with centers strung out along the axis of motion, the sustaining wave of Eq.(2.12.1) will be set

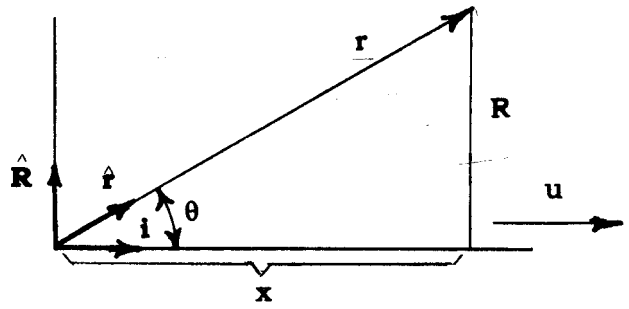

Figure 3.10.1 Cylindrical coordinates for the moving electron.

up in cylindrical coordinates (electron velocity **u** in the x direction, see Figure 3.10.1), so that,

$$\frac{\partial^2 \bar{\bar{\eta}}}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{\bar{\eta}}}{\partial R} + \frac{\partial^2 \bar{\bar{\eta}}}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 \bar{\bar{\eta}}}{\partial t^2} - \frac{1}{\bar{\bar{\eta}}}\left(\left(\frac{\partial \bar{\bar{\eta}}}{\partial R}\right)^2 + \left(\frac{\partial \bar{\bar{\eta}}}{\partial x}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial \bar{\bar{\eta}}}{\partial t}\right)^2\right)$$
$$= -\frac{c_0 \omega}{\phi_d D}\left(\frac{\partial}{\partial R}\left(\overline{\overline{\phi . V_{.R}}}\right) + \frac{\overline{\overline{\phi . V_{.R}}}}{R} + \frac{\partial}{\partial x}\left(\overline{\overline{\phi . V_{.x}}}\right)\right) \quad . \tag{3.10.1}$$

The simplification comes from the fact that, because the electron moves at constant velocity **u** in the x direction,

$$\frac{\partial}{\partial t} = -\,u\frac{\partial}{\partial x} \quad , \tag{3.10.2}$$

so that Eq.(3.10.1) can be reduced to,

$$\frac{\partial^2 \bar{\bar{\eta}}}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{\bar{\eta}}}{\partial R} + \frac{1}{\gamma^2}\frac{\partial^2 \bar{\bar{\eta}}}{\partial x^2} - \frac{1}{\bar{\bar{\eta}}}\left(\left(\frac{\partial \bar{\bar{\eta}}}{\partial R}\right)^2 + \frac{1}{\gamma^2}\left(\frac{\partial \bar{\bar{\eta}}}{\partial x}\right)^2\right)$$
$$= -\frac{c_0 \omega}{\phi_d D}\left(\frac{\partial}{\partial R}\left(\overline{\overline{\phi . V_{.R}}}\right) + \frac{\overline{\overline{\phi . V_{.R}}}}{R} + \frac{\partial}{\partial x}\left(\overline{\overline{\phi . V_{.x}}}\right)\right) \quad . \tag{3.10.3}$$

The familiar motion factor,

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c_0^2}}} \quad , \tag{3.10.4}$$

is a constant, and time has been eliminated from the equation. Still the non-linearity of the equation precludes any direct solution. Fortunately, there are some round about ways to get to the answer, but their discussion must be postponed until a number of other subjects have been developed. So, the solution will be presented here with no proof other than it satisfies the equation by direct substitution.

The solution for $\overset{=}{\eta}$ is,

$$\overset{=}{\eta} = \gamma \frac{a^2 \psi^{*2}}{2r^{*2}} \quad , \tag{3.10.5}$$

and the compatible $\mathbf{V}_\cdot$ and $\phi_\cdot$ are,

$$V_{\cdot x} = \frac{a\psi^* x^*}{r^{*2}} C \quad , \quad V_{\cdot R} = \gamma \frac{a\psi^* R}{r^{*2}} C \quad , \tag{3.10.6}$$

$$\phi_\cdot = \pm \frac{\phi_d a \psi^*}{c_0 r^*} \left(C \mp \gamma \frac{c_0}{\omega_e \mathcal{R}} \left(1 + \frac{R}{\psi^*} \frac{\partial \psi^*}{\partial R} + \frac{x^*}{\gamma \psi^*} \frac{\partial \psi^*}{\partial x} \right) S \right) \quad .$$

With some nomenclature borrowed from the theory of sound[1],

$$C = \cos \frac{\omega_e}{\gamma} \left(t \mp \frac{\mathcal{R}}{c_0} \right) \quad , \quad S = \sin \frac{\omega_e}{\gamma} \left(t \mp \frac{\mathcal{R}}{c_0} \right) \quad , \tag{3.10.7}$$

$$\psi^* = \varepsilon^{-r_i / r^*} \quad , \tag{3.10.8}$$

and,

$$x^* = \gamma(x - ut) \ , \ r^* = \sqrt{x^{*2} + R^2} \ , \ \mathcal{R} = \gamma(r^* \pm \beta x^*) \ , \ \beta = \frac{u}{c_0} \ . \tag{3.10.9}$$

The value of ω_e in these equations is that found for the electron at rest. The upper sign results in the outgoing waves of the electron, whereas the lower sign represents the incoming waves of the positron. Eqs.(3.10.6) are equivalent to Eqs.(3.9.1) and (3.9.2), except for the differences stemming from the replacement of spherical symmetry with cylindrical symmetry, as exhibited by the need for x and R components in the velocity wave. *In the following, only the upper signs, applying to the electron, will be retained*. Because the source is

1. E.U.Condon, H.Odishaw, Handbook of Physics, p.3-117, McGraw-Hill, N.Y. (1958).

in motion, the relationship between $\bar{\bar{\eta}}$ and $\mathbf{V}_\cdot$ becomes,

$$\bar{\bar{\eta}} = \gamma \frac{r^{*2}}{x^{*2} + \gamma^2 R^2} \bar{\bar{\mathbf{V}}}_\cdot^2 \quad . \tag{3.10.10}$$

The $\phi_\cdot$ and $\mathbf{V}_\cdot$ of Eq.(3.10.6) satisfy the continuity equation, Eq.(2.12.4). Their product leads to,

$$\bar{\bar{\phi}}_\cdot \mathbf{V}_\cdot = \frac{\phi_d a^2 \psi^{*2}}{2 c_0 r^{*3}} \left(\mathbf{i} x^* + \hat{\mathbf{R}} \gamma R \right) \quad . \tag{3.10.11}$$

By combining Eqs.(2.15.1), (2.14.2) and (3.10.2), the bridge equation can be written, with the help of Eqs.(3.5.2), (3.5.5) and (3.7.3),

$$\mathbf{i} \frac{1}{\gamma^2} \frac{\partial \bar{\bar{\phi}}}{\partial x} + \hat{\mathbf{R}} \frac{\partial \bar{\bar{\phi}}}{\partial R} = - \frac{e}{4\pi} \frac{\psi^{*2}}{r^{*3}} \left(\mathbf{i} x^* + \hat{\mathbf{R}} \gamma R \right) \quad . \tag{3.10.12}$$

Now, equating coefficients,

$$\frac{\partial \bar{\bar{\phi}}}{\partial x} = - \gamma^2 \frac{e}{4\pi} \frac{\psi^{*2}}{r^{*3}} x^* \quad , \quad \frac{\partial \bar{\bar{\phi}}}{\partial R} = - \gamma \frac{e}{4\pi} \frac{\psi^{*2}}{r^{*3}} R \quad . \tag{3.10.13}$$

Just as in the case of Eq.(3.5.1), these can be integrated to give the constant velocity equivalent of Eq.(3.5.6),

$$\bar{\bar{\phi}} = \phi_0^* \left(1 - \psi^{*2} \right) = \phi_0^* \left(1 - \varepsilon^{-2 r_i / r^*} \right) \quad , \tag{3.10.14}$$

where,

$$\phi_0^* = \gamma \phi_0 = - \gamma \frac{e}{8\pi r_i} = - \gamma \frac{\phi_d a^2 b}{4 c_0 r_i} \quad . \tag{3.10.15}$$

Having laid the groundwork for describing the physical operation of the moving electron/positron, the concepts of charge and energy will be re-examined in that context. For this purpose it is possible to use a slightly simpler form of Eq.(3.10.14), in which the ether density pattern of the constant velocity electron is frozen at one instant of time, say t=0. Then $\bar{\bar{\phi}}$ becomes,

$$\bar{\bar{\phi}} = - \gamma \frac{e}{8\pi r_i} \left(1 - \varepsilon^{-2 r_i / r'} \right) \quad , \tag{3.10.16}$$

where,

$$r' = \left(\gamma^2 x^2 + R^2 \right)^{\frac{1}{2}} = r \left(1 + \zeta \cos^2 \theta \right)^{\frac{1}{2}} \quad , \tag{3.10.17}$$

and,

$$\zeta = \gamma^2 - 1 \quad . \tag{3.10.18}$$

That the two forms of r' are equivalent can be seen from Figure 3.10.1.

The first question of interest is: When the moving charge density is integrated over all space, what is the total charge of the moving electron? Has it changed? From the definition of the dynamic concentration of Eq.(2.20.2), the moving charge density distribution function can be obtained from,

$$\rho = -\left(\nabla^2 \bar{\bar{\phi}} - \frac{u^2}{c_0^2} \frac{\partial^2 \bar{\bar{\phi}}}{\partial x^2} \right) \quad , \tag{3.10.19}$$

which, upon substitution of $\bar{\bar{\phi}}$ from Eq.(3.10.16), results in,

$$\rho = -\gamma \frac{e r_i}{2\pi r'^4} \varepsilon^{-2r_i / r'} \quad . \tag{3.10.20}$$

Just as in the static case, the total charge is found from,

$$q_e = -\int_{space} \frac{\gamma e r_i \varepsilon^{-\frac{2r_e}{r\sqrt{1+\zeta\cos 2\theta}}}}{2\pi r^4 \left(1+\zeta\cos^2\theta\right)^2} \, dvol \quad ,$$

where r' has been replaced by its spherical polar coordinate form as taken from Eq.(3.10.17). This simplifies the integration, which yields,

$$q_e = -e \quad . \tag{3.10.21}$$

From what is known about moving charged particles, this result was expected.

To observe the same process applied to the energy density distribution, the dynamic distortion energy can be found from Eq.(2.19.2). Using $\bar{\bar{\phi}}$ from Eq.(3.10.16), the energy density of the constant velocity electron is,

$$\varepsilon_e = \frac{\gamma^2 e^2}{32\pi^2 r'^4} \varepsilon^{-4r_e / r'} \quad , \tag{3.10.22}$$

and the total moving energy of the particle is,

$$E = \int_{space} \frac{\gamma^2 e^2 \varepsilon^{-\frac{4r_i}{r\sqrt{1+\zeta\cos 2\theta}}}}{32\pi^2 r^4 \left(1+\zeta\cos^{2\theta}\right)^2} \, dvol \quad .$$

The result of the integration is,

$$E = \gamma E_0 \quad , \tag{3.10.23}$$

where E_0 is the electron/positron rest energy of Eq.(3.6.3) and Table 3.9.1.

The energy relationship of Eq.(3.10.23) is well known, and is usually ascribed to so called special relativity. Clearly, no relativistic approach was used in the preceding derivation. All of the physics involved only one observer, and the result is directly attributable to properties of the ether and the structure of the electron/positron fabricated from it. Implicit in the preceding development is the resolution of the infamous 4/3 problem. Its key resides in the proper understanding of and definition of energy and energy density. Rohrlich[1] discussed the problem in detail from the present day accepted viewpoint and described earlier attempts to resolve it. Those earlier investigators managed a mathematical solution that gave the correct form of Eq.(3.10.23). Rohrlich, himself, used a similar approach; i.e. forcing Lorentz invariance and coming up with correction terms for the static definitions of energy and momentum. Although Rohrlich's results are formally the same as Butler's[2], the latter took the correct approach based upon redefining energy density, allowing a much more intuitive interpretation of the mathematical forms. He was prevented from obtaining the correct physical interpretation by using $\mathbf{E}$ and $\mathbf{H}$ to define energy density rather than $\mathbf{A}$ and ϕ.

To see exactly what happens when an electron/positron (e/p) is brought up to some velocity $\mathbf{u}$, first consider the particle at rest. Based on the development in Section 3.5, the contour surfaces of constant $\bar{\bar{\phi}}$ are shown to be spheres, as represented in Figure 3.10.2a. Using the equations of the present section, the corresponding surfaces of constant $\bar{\bar{\phi}}$ for a moving e/p are the oblate spheroids appearing in Figure 3.10.2b. So much emphasis has been placed on the Lorentz "contraction" and the electric field $\mathbf{E}$, in present day texts, that it is almost always overlooked that *the potential* $\bar{\bar{\phi}}$ *does not contract longitudinally to the motion but expands laterally*. This is why the surrounding function or concentration of the particle changes. It is also why the $(\nabla\bar{\bar{\phi}})^2/2$ distortion of the moving particle, integrated over all space, increases. Note that for each

1. F.Rohrlich, Classical Charged Particles, Addison-Wesley Publ. Co., Reading, MA (1965).
2. J.W.Butler, Amer. J. Phys. **36**, 936 (1968); **37**, 1258 (1969).

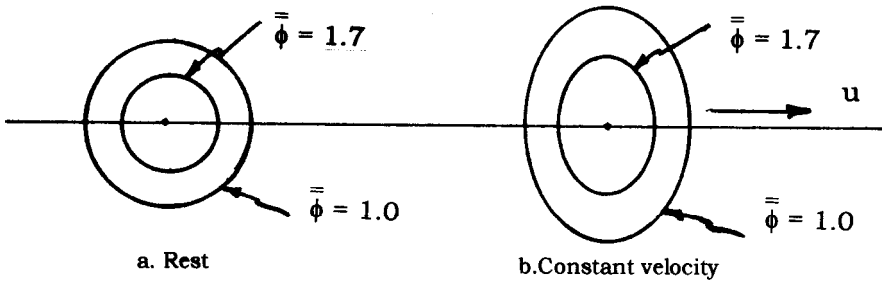

Figure 3.10.2 Lateral expansion of the moving electron/positron, (e/p).

different constant velocity **u**, a specific lateral extension and shape is required. Thus, a specific amount of surrounding function distortion is identified with each velocity, as is a specific amount of distortion energy. Within any $\bar{\bar{\phi}}$ contour, a specific amount of reduced (or additional) ether is also identified with each velocity value. Thus, given a particular size and shape of the field, the associated velocity **u** is determined.

The exact details of interaction of, say, two electrons, will be deferred till later to allow a more logical development; but here, while ignoring certain details, it is possible to form a very useful mental picture. An electron will be acted upon to bring it from rest up to some velocity **u**. However, there are no sticks or stones to move it. There is only ether, and specifically, only ether in the form of another particle. So, as illustrated in Figure 3.10.3, the sequence starts with

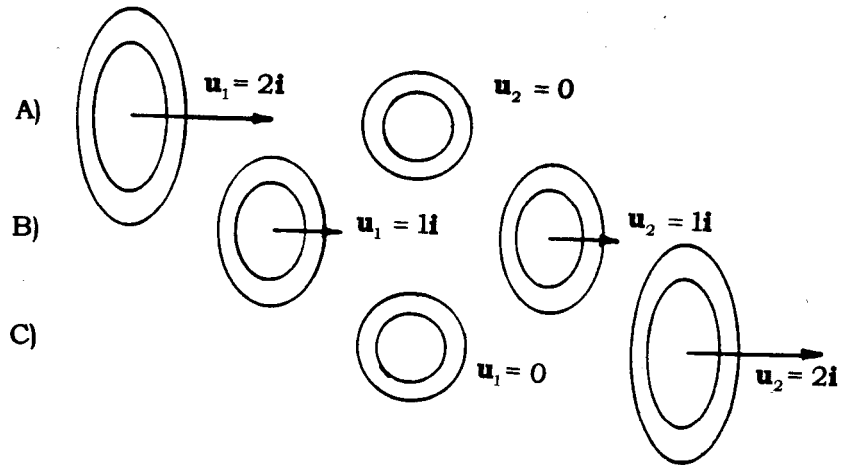

Figure 3.10.3 Energy exchange between two electrons.

one energized electron moving at velocity $\mathbf{u}_1 = \mathbf{i}2$ approaching another electron which is at rest. It must be emphasized that the contours shown are not edges or surfaces in any real sense; they are equipotential or equidensity levels of the ether. There are an infinite number of these imaginary surfaces increasing in size from the electron center to the far reaches of space. Clearly, certain liberties are taken in the simplified picture of Figure 3.10.3. Nevertheless, electron number one is originally carrying excess deformation energy (over its rest value) and number two is at rest. Later, the excess in 1 has caused 2 to move away, at the same time resulting in a transfer of distortion from 1 to 2. Number 1 cannot run slower unless distortion is removed from its field and the latter is allowed to change shape to exactly match the reduced velocity. Number 2 cannot take on the transferred distortion unless it moves and changes shape to exactly match the condition of its moving at its new velocity. As the process of transfer continues, the first electron finally gives up all of the excess distortion and comes to rest. Number 2, meanwhile, has taken on all of the original distortion and is now moving at the velocity originally exhibited by 1. The shape of 2 is now also exactly the same as the original shape of 1. In this example, radiation has been neglected.

Delving further into the operation of an e/p, Eq.(3.10.23) can be written as,

$$E = \frac{E_0}{\sqrt{1 - \dfrac{u^2}{c_0^2}}} \quad ; \qquad (3.10.24)$$

and this can be expanded in series to give,

$$E = E_0 \left(1 + \frac{1}{2}\frac{u^2}{c_0^2} + \ \ldots\ldots\ldots\ldots \right) \quad . \qquad (3.10.25)$$

For small velocities, all higher order terms of the series are negligible; and, making use of Eq.(3.6.4), the excess energy of the particle due to its constant velocity is,

$$E_k \cong \frac{1}{2}\frac{E_0}{c_0^2} u^2 = \frac{1}{2} m_0 u^2 \quad . \qquad (3.10.26)$$

This is called the *kinetic energy* of the moving particle. At higher velocities, the exact form is found by subtracting E_0 from E of Eq.(3.10.23), so that,

$$E_k = E - E_0 = E_0 (\gamma - 1) \quad . \qquad (3.10.27)$$

This brings the discussion to the following point. The electron is a small depletion of ether prevented from sagging by a sustaining wave running away from its geometrical center. There is no rock in the middle, there are no objects in the field. It has two kinds of deformation that are significant in determining its charge and energy. When it moves, it changes shape in a very precise way, increasing its distortion content. The excess distortion, called its kinetic energy, is determined by its shape and velocity. Eq.(3.10.24) *represents the total energy of the moving electron/positron. Contrary to conventional belief, the electron's magnetic field carries no energy*. The full implication of this will be discussed in detail later.

3.11 **Inertia and Momentum**: The quantitative discussion of momentum, etc., will be presented later on. All that is needed here is a brief statement about the physical nature of momentum and inertia. In connection with Figure 3.10.3, the interaction of two electrons was described. Before the #1 electron had approached close enough to #2 to have a significant effect on it, their condition could be described as follows. Number 2, being at rest, was a solution of the field equations, and assuming the boundary conditions did not change, it would sit permanently at the same location forever. Number 1, being in motion at constant velocity, was also a solution of the field equations, and assuming the boundary conditions did not change, it would continue along a straight line at constant velocity forever. These are not mathematical statements, but physical. In both cases, the boundary conditions are $\bar{\phi} \cong 0$, $\bar{\bar{\phi}}_a \cong \phi_d$ far out. For the electron at rest, the reduced $\bar{\bar{\phi}}_a$ near the center has spherical contours, held up by spherical wave fronts, all matched up from $r = 0$ to $r \to \infty$. For the charge in motion, the reduced $\bar{\bar{\phi}}_a$ is oblate in its contours, all moving in a single direction, while curved wave fronts leave the geometrical center along paths all exactly proportioned so that just the right amount of $\bar{\bar{\phi}}_a$ arrives at each point in the field to maintain the shape and overall velocity distribution $\bar{\bar{\mathbf{V}}}$, etc. Otherwise, the "particle" would cease to exist. Only when #1 approached close enough to #2 to lower the ether density from ϕ_d to some $\bar{\bar{\phi}}_a$, on the side of approach to #1, would both see the boundary conditions change and then adjust their representative flow pattern solutions of the field equations. *This is the physical meaning of inertia*. Only when the boundary conditions change will a solution of the field equations be modified. Inertia is obviously not a property of the ether itself, but of the solutions to the field equations; i.e. inertia is a property of particles, not ether.

Momentum, $\mathbf{p} = m\mathbf{u}$, can be understood physically by realizing that it is a combination of the effect of inertia and the fact that it takes time to bring about the changes in velocity of a particle such as the electron. First the boundary conditions must change, usually by bringing another field into the outermost regions of the electron's field. As the electron moves away from or towards the changing region, the excess deformation energy must be carried throughout the electron's field by the modified sustaining waves, which are propagating at the speed of light. Only when the shifted deformation moves in a very prescribed manner and causes the shape of the electron density $\bar{\bar{\phi}}$ to maintain the proper configuration to match the overall instantaneous velocity and motion of the electron field can the electron-external field combination remain as a valid solution of the field equations. Thus, *time* is involved. *It is this time delay that begets the concept of momentum*. Later on, the formal connection between the time variation of the particle deformation and the change in the external boundary conditions will be worked out. Clearly momentum is not a property of the ether but of solutions to the field equations; i.e., momentum is a property of particles, not ether.

Numerical examples reviewing momentum and energy calculations will be presented later, but the equations most often used for this are given here in Table 3.11.1. These equations are always called relativistic in modern texts, but no relativistic condition has entered into their derivation. They come directly from a single observer's solution of the field equations for a moving electron/positron.

TABLE 3.11.1

ENERGY AND MOMENTUM FORMULAS

$$E = E_0 + E_k \qquad E^2 = (pc_0)^2 + E_0^2 \qquad \gamma = 1 + \frac{E_k}{E_o}$$

$$p = mu = \frac{E}{c_0}\sqrt{1 - \frac{1}{\gamma^2}} \qquad u = c_0\sqrt{1 - \frac{1}{\gamma^2}} \qquad u = \frac{pc_0^2}{E}$$

$$p = \frac{E_k}{c_0}\sqrt{1 + 2\frac{E_0}{E_k}}$$

3.12 **Electron Radiation**: Propagating energy in the form of radiation is normally detected at long distance from a changing configuration of charges. It is regarded as originating with those charges, but shaking free in a non-reversible process. It propagates until intercepted. Some of the most puzzling phenomena in conventional physics originate at the interface of the emitting charge and the freely propagating radiation. That is the inevitable result of the lack of knowledge about the charge's internal structure. The present section deals with the mechanism by which extended charges (e.g. electrons) create radiation.

The standard approach to this problem is to first solve Maxwell's equations for the fields radiated from point charges moving in explicit ways. Using this to define the net radiated energy leaving a charge, that energy is associated with the acceleration of the charge. This whole approach is basically unsound because Maxwell's equations, as they are used today, represent only the *weak field*; so that, even though they are adequate to handle the free radiation, they cannot be used for the interface between it and the *extended* electron. As it turns out, many situations exist where electrons accelerate but do not radiate. A simple closed loop of wire carrying a steady current offers resistance to rapidly accelerating electrons through their collisions with the conductor atoms and their sudden decelerations. Certainly the most conspicuous case is that of atoms in their ground state, where electrons orbiting and accelerating towards the nucleus do not radiate. So strong is the belief that Maxwell's equations determine the presence or absence of radiation, that the belief in atomic orbits has been relinquished. Too bad, since they are there. The simple fact is that Maxwell's equations are necessary but not sufficient to indicate whether charges radiate. The full field equations must be solved to see whether the total field has a free or radiated part.

From the ether viewpoint, various stable configurations called particles engage in motions that can result in variations in the deformation, which, although conserved, moves about. Some part of the total deformation is bound in the particles, i.e. is an intrinsic part which, if it were not there, would mean the particle identity was lost. As a particle speeds up, this bound deformation increases; and, conversely, when it slows down, decreases. When increasing, the source of the acquired deformation is another particle or collection of particles that unload deformation energy by first shifting it into the form of an interaction deformation shared between them, after which it is absorbed by the speeded up particle. Radiation occurs during the shifting process. For example, normally a slowly moving particle adjusts its speed to allow the interaction distortion to move throughout the particle in such a way that just the right amount arrives where it is needed to fulfill the requirements of the total field equation solution of the particle moving in the external field.

However, in situations where the speed change is too rapid or the direction changes sharply, i.e. high acceleration occurs, it happens that interaction energy out in the field cannot move in just the right way to keep up with the particle. In such cases, the renegade deformation cannot be reclaimed by either the particle or the external field and it escapes. That lost deformation is the radiation.

What is needed is a simple intuitive way to decide when radiation will occur. The ether provides this. When no bound ether element is changing shape, no radiation can occur. When bound ether elements are changing shape, radiation occurs, with greater radiation resulting from more violent changing. The full import of this concept will gradually appear as more complicated structures, such as atoms and macroscopic field problems, are studied.

3.13 **Turning**: One aspect of the electron's characteristics that has no counterpart in contemporary physics is the result of its shape change with motion. By expanding laterally to the direction of its motion, the electron has an established axis in the direction of its motion; and it could be expected that, in certain situations, that axis could shift direction. In other words, the electron shape could turn. It might then reasonably be expected, based on the discussion in the previous section, that whether or not its incremental ether elements turned, as its path deviated from a straight line, could have a profound effect on its physical operation. At this point, it cannot be emphasized enough that this turning is not the least like the spin examined earlier; and often, when turning is discussed, the spin will be ignored, because the frictionless, massless nature of the ether allows them both to operate without interfering with each other.

Once the electron/positron has a *shape*, due to its motion, the *relative* motion of that shape and the ether that composes the particle represents one of the most significant and controlling properties of that particle, fundamental to important aspects of e/p radiation. Figure 3.13.1 illustrates two alternative modes of path deviation; one

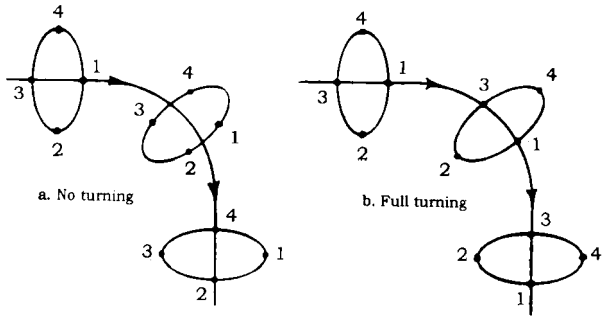

Figure 3.13.1 Path deviation with and without turning.

with no ether turning, and the other with full ether turning. The dots on the equi-density contours identify particular ether elements in the $\overline{\overline{\phi}}$ field of an electron. In the full turning case these elements move with a flow pattern that corresponds to rigid body rotation. In the non-turning case, the flow pattern is more complex, with particular elements in the density pattern approaching one another (2 and 4) and other elements receding from each other (1 and 3). This corresponds to the fact that in the non-ether turning or partial ether turning particle, the $\overline{\overline{\phi}}$ field around each point is deforming, or changing shape; whereas, in the full ether turning particle the $\overline{\overline{\phi}}$ field around each numbered point is not deforming, or changing shape. Thus, in the former case, radiation will take place; whereas, in the latter, it will not. Here, again, the full significance of this phenomenon will gradually appear as more complicated processes are discussed.

3.14 **The de Broglie Frequency**: In this section, another property of the electron that results from its sustaining wave will be discussed. Except for the behavior of atoms, none of the observations of the related phenomena had been made before 1924 when de Broglie first proposed "matter waves" and the current explanations by quantum mechanical approaches are both inaccurate and non-intuitive.

Figure 3.14.1 illustrates the condition of a constant velocity electron as described in Section 3.10. The first half of the figure indicates the outgoing sustaining waves, whereas the second part is a plot of their wave-fronts at any particular time, say t = 0. From

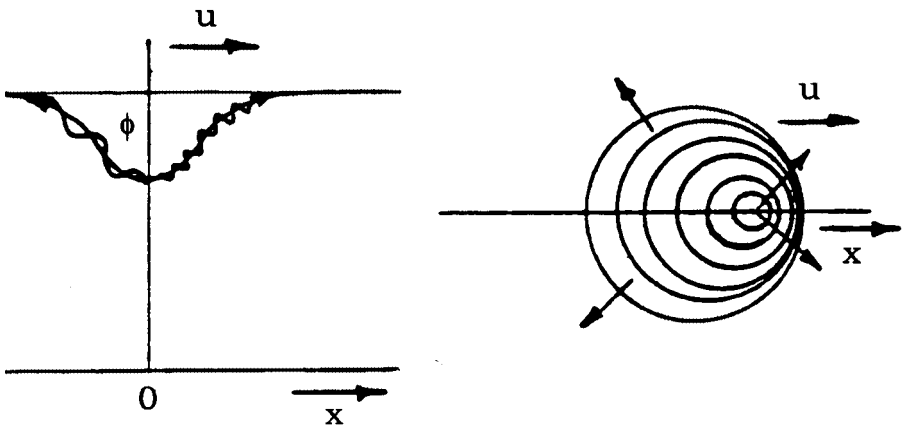

Figure 3.14.1 A constant velocity electron.

Eqs.(3.10.6) and (3.10.7), those fronts are specified by the phase angle,

$$\delta = \frac{2\pi\nu_e\mathcal{R}}{\gamma c_0} \quad ; \tag{3.14.1}$$

and with the aid of Eqs.(3.10.9), $\mathcal{R}$ can be found along the x axis, where,

$$\mathcal{R} = \begin{cases} \dfrac{x}{1-\beta} & , \quad x > 0 \\ \dfrac{|x|}{1+\beta} & , \quad x < 0 \end{cases} \quad . \tag{3.14.2}$$

Substituting these values into the phase δ, the frequency along the x axis is found to be changed to,

$$\begin{aligned} \nu_f &= \frac{\nu_e / \gamma}{1-\beta} \quad , \quad x > 0 \\ \nu_b &= \frac{\nu_e / \gamma}{1+\beta} \quad , \quad x < 0 \end{aligned} \quad . \tag{3.14.3}$$

This change is exactly equivalent to the ordinary doppler shift of a source or sink of sound; and, the electron's being a source of the sustaining waves, the front edge frequency is increased and the back is decreased, which is the opposite of the positron (sink).

For a free electron, the only effect of these changes is the adjusted shape described in Section 3.10. However, when interacting with another particle, the effect produced on the outcome is related to the *difference* between the front and back frequencies. For this reason, the difference frequency assumes a significance of major importance. Combining the frequencies of Eqs.(3.14.3), the difference frequency is,

$$\nu_d = \nu_f - \nu_b = \frac{\nu_e}{\gamma}\left(\frac{1}{1-\beta} - \frac{1}{1+\beta}\right) \quad ,$$

which can be manipulated into the form,

$$\nu_d = 2\gamma\beta\nu_e \quad . \tag{3.14.4}$$

To bring the derivation more in line with the conventional approach, the momentum (see Table 3.11.1) and Eq.(3.14.4) can be used to write,

$$pc_0 = \gamma m_0 u c_0 = \frac{m_o c_0^2}{\nu_e}\frac{\nu_d}{2} \quad , \tag{3.14.5}$$

which shows that the momentum and the difference frequency vary linearly because,

$$h = \frac{m_0 c_0^2}{\nu_e} \qquad (3.14.6)$$

is a constant. When ν_e, m_0 and c_0 from Table 3.9.1 are substituted into Eq.(3.14.6), the value of the constant h is found to be,

$$h = 6.6260759 \times 10^{-27} \quad , \quad \text{erg-sec} \qquad (3.14.7)$$

the well known Planck's constant. With this in mind, Eq.(3.14.5) can be written as the de Broglie difference frequency[1],

$$\nu_d = 2c_0 \frac{p}{h} \quad . \qquad (3.14.8)$$

The customary way of writing this expression is,

$$\lambda = \frac{h}{p} \quad , \quad \text{WRONG} \qquad (3.14.9)$$

and this relationship is called a "quantum mechanical" equation; but it is simply another way of writing Eq.(3.14.8) which comes from the doppler shift of the electron's ℓ-wave. Physically there <u>is</u> a difference *frequency*. However, the Eq.(3.14.9) is probably better not used, because there is <u>no</u> difference wave, so the wavelength λ is nothing more than the inverse of Eq.(3.14.8) expressed in different units. The proper way to invert Eq.(3.14.8) is to write,

$$\Lambda_d = \frac{2c_0}{\nu_d} = \frac{h}{p} \quad . \qquad (3.14.10)$$

Λ_d is **not** the wave length of a mysterious wave that travels along curved paths. It is determined by real $\ell-$waves that propagate radially from the electron's center. It has nothing to do, directly, with the wavelength of any wave. Instead, in this constant linear velocity case, it is simply *the distance the <u>electron</u> travels during* $2/\beta$ *cycles of the difference frequency*.

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1. Although the author has chosen to call ν_d the de Broglie frequency, it should not be confused with the conventional $\nu_{db} = E/h$, a fictitious frequency of a fictitious wave that is thought to travel along with the moving particle; not as a single wave (which leads to very unphysical velocities), but as a wave packet composed of many waves. The electron's radial $\ell-$waves, however, are just as real as t-waves.

CHAPTER 4

THE DATUM

4.1 **Introduction**: In Section 2.3, it was said that a complete description of the ether has two parts, visualizable definitions of its physical characteristics and the formal equations that relate them. The latter are summarized in Table 2.22.1, and most of the visualizations were covered in Chapter 2. However, there are two important features of the ether that require further elaboration before the more complicated forms of matter are developed. These are basic phenomena that can be visualized without considering the properties of constructs, such as the electron/positron, examined in Chapter 3. More to the point, they are processes that occur, in the datum ether, where massive particles have not yet been introduced. One was mentioned briefly in Section 2.13 as the compression/oscillation condition that relates the ether's natural oscillation frequencies to its compression distortion. The other, called datum fluctuations, resembles the "chop" in the ocean of the datum ether. The present chapter will concentrate on these two phenomena.

4.2 **Datum Fluctuations**: Anyone who has seen a glassy lake mirror the scene about the shore understands the deterministic picture of the datum drawn in Section 2.3. The absolute observer was defined as one who, in a region free of energy, sees the datum ether at rest, having zero velocity everywhere. In the case of the glassy lake, any activity in the water about the shore generally destroys the reflected image overall; because a gentle chop ripples across the surface. The same condition applies in the ether, where all the particles far away send out waves that produce a minute, random chop throughout space. *The effect is so small it can be ignored in most calculations of the type used in the previous chapter*. Nevertheless, these small datum fluctuations exert significant effects in situations where constructs are very close to being unstable. In some cases, datum fluctuations can push a construct "over the top" into instability. It is these fluctuations that introduce statistics into the deterministic picture.

4.3 **The Vacuum**: Until very recently, modern physicists regarded the vacuum of space as a *void* in which particles and waves moved about. However, because of the considerable development of high energy particle physics, this void has been given more and more physical

properties; until, now, it is clearly a physical entity.[1] This book is essentially a study of its properties, and the use of "vacuum", "quintessence" or any other term but "ether" to describe it is just an artifice.

Conventionally the fluctuations are viewed as more structured than the ether theory requires, with both the random chop and the forming of whole particles and their subsequent annihilation at each point, temporarily violating energy conservation. In the ether picture, there is distortion moving about in the form of the chop; but, although that distortion can pile up temporarily, at a point, to values as great as those of the particles, no actual particles are formed (with their bulk shapes and radial ℓ-waves all properly deployed) in regions originally free of particles.

In later chapters, only a few simple properties of the datum fluctuations will be required to understand their role in the world of particles and waves. However, before getting to that point, it is useful to highlight the role they have played in the controversy over their fundamental contribution to main-line physics.

4.4 **Statistical Quantum Mechanics**: Every student of physics is keenly aware of the "great revolution" (1905-1926) that split modern physics from classical physics and set Quantum Mechanics over all. Many are also aware of the reluctance of Einstein and others to give up the deterministic basis for the world, along with cause and effect and all the other paradoxically dismissed fundamentals. Generally, the belief predominant today is that Einstein lost and indeterminacy won. However, in view of the preceding chapters, such a belief is not justified. Not only is the world deterministic, but the goal of this work is to present the final ether equations which describe it all deterministically.

This is not to suggest that the outcome of every experiment is exactly predictable, with no statistical element present. The intent is to show that, even when the outcome is only predicted as a set of alternatives with their assigned probabilities, the underlying physics is as deterministic as that of the steam in a boiler, where it is hopeless to follow the motion of all the individual particles. Moreover, the goal of this section is to indicate just how conventional quantum mechanics, with its successes and repressions, fits into main-line physics.

Almost from the beginning of the quantum theory, a controversy has existed; and though it has often been over simplified, it is carried

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1. Physics Through the 1990s, *Elementary Particle Physics*, (National Academy, Washington, D.C. 1986) p.71.

along with several sub-streams. Nevertheless, the whole set divides itself into roughly two parts with their respective proponents. A focal point for the split came from Einstein, Podolsky, and Rosen (EPR) in 1935.[1] The two sides of one key part of the disagreement were pinpointed by Freistadt in a summary paper:[2]

> "(a) The statistical description given by quantum mechanics is the most complete description possible: the behavior of individual sub-microscopic systems is intrinsically indeterminate, erratic, and non-causal; in fact, the very concept of a single precise model must be abandoned in favor of two complementary, mutually interfering pictures. However, the statistical laws of quantum mechanics give a seemingly regular and causal behavior to ensembles of large numbers of sub-microscopic systems.
>
> (b) The statistical description given by quantum mechanics is inadequate: there exists a more fundamental, and, in principle, fully causal description, with respect to which quantum mechanics takes a position analogous to the position held by classical statistical mechanics (of an ensemble of molecules). At the present time, this is not known, but one must continue the search for it, as it is unreasonable to assume that there are physical phenomena not governed by physical laws.
>
> The first point of view was urged most strongly by Bohr and Heisenberg, and the second by Einstein and Planck."

It is clear that not all antagonists, on either side, would state the difference in exactly this way; but it is at least representative. As is commonly found in most controversies in physics, advocates of both viewpoints were correct in some respects and wrong in others. For example, this form of the argument produced the concept of "hidden variables" and the numerous proofs that they did not exist. The preceding portions of the present work show that the search should have been for *variables right out in plain sight*, and they surely do exist. Recently, a variation in the argument has added excitement and confusion. It might be called the Bohm-Bell diversion[3] and is described in a write-up by Mermin.[4] As summarized by Mermin, the

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1. A.Einatein, B.Podolsky & H.Rosen, Pbys.Rev. **47**, 777 (1935).
2. H.Freistadt, Nuovo Cimento, Suppl. **5**, Ser. 10, 1 (1957).
3. D.Bohm, Quantum Theory, p 614, Prentice Hall, N.J. (1951). J.S.Bell, Pbysics, **1**, 195 (1964).
4. N.D.Mermin, Pbysics Today, p 38, April (1985).

EPR paper attacked "the doctrine that physical properties have in general no objective reality independent of the act of observation." This latter position is one taken by most modern physicists. "Einstein didn't like this. He wanted things out there to have properties, whether or not they were measured." Unfortunately, beyond this, the discussion was couched in terms of rejecting "spooky actions at a distance". The experiment of Bohm's, the assertion that Bell's theorem if proven, proves Einstein wrong, and the results of the Orsay experiments[1] just show that the meager background of experimental work on the ether leaves much to be done. Even in the present state of "too much theory and not enough experimental work", *most* of the mysterious effects involved in this controversy will be explained in later chapters. There are so few that cannot now be dealt with, that it can be said, with some confidence, they will be after further work.

The author believes that Einstein was just insisting on cause and effect. Since the spins, polarizations, correlations, etc., involved in all of these experiments are, as in the previous chapters presented here, explainable in terms of deterministic structures in the ether, perturbed by the datum fluctuations, both the observed phenomena (when correctly observed) and Einstein (when correctly understood) are right.[2]

Without entering more deeply into the various aspects of this controversy, it will be informative to examine a related area of considerable activity in the literature during the last seventy years. In many instances a specific quantum effect, e.g. Plank's radiation law, is derived *classically*. The more ambitious find a way to derive Schroedinger's equation classically. Originally, these efforts were just attempts to identify the mysterious quantum mechanical ψ of Schroedinger's equation with some real physical variable. Schroedinger himself, after some spurious attempts, concluded that $\psi\psi^*$ (ψ^* is the complex conjugate of ψ) represented the distributed charge density in, for example, an atom.[3] Almost immediately afterwards, Born suggested that $\psi\psi^*$ represented the probability that an electron could be found in a particular small volume of space.[4] This is the accepted view today. Some confusion still exists about whether this probability applies only over a whole ensemble of

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1. A.Aspect,P.Grangier & G.Roger, Pbys.Rev.Lett. **47**, 460 (1981), **49**,91(1982). A.Aspect,J.Dalibard & G.Roger, Pbys.Rev.Lett. **49**, 1804 (1982).
2. P.Pearle, Amer.J.Pbys. **35**, 742 (1967). E.Nelson, Quantum Fluctuations, Princeton Univ. Press (1985).
3. E.Schroedinger, Ann.pbysik, **81**, 109 (1926).
4. M.Born, Z.Pbysik, **37**, 863; **38**, 803; **40**, 167 (1926).

identical atoms or to each atom alone, examined over some long time.[1] The latter could only be true if the ergodic hypothesis applied to the atom[2]; but it is not at all clear that the electrons trace out all possible positions represented in the ensemble, even though the orbit recessions, mass variations, etc., might make it possible. This question is yet to be resolved.

Even before wave mechanics, Einstein and Stern[3] attempted to derive Planck's radiation law using zero-point energy of an harmonic oscillator (but, as Boyer points out, not zero-point radiation). Nernst first speculated in 1916[4] that there was an electromagnetic zero-point radiation in space. However, the first steps in the direction that appears ultimately to be useful in statistical quantum mechanics were taken in 1926. De Broglie[5] suggested that Schroedinger's equation was related to the particle trajectories perturbed by a quantum mechanical property, and Madelung[6] showed how it was related to a hydrodynamical model. Seven years later, Furth[7] pointed out a formal analogy between Schroedinger's equation and Brownian motion. Finally, in 1944, Liebowitz[8] derived the Schroedinger equation on the basis of a "smoothest" density of perturbed orbit paths, from the Hamilton-Jacobi equation, after defining a *probability of presence* for the orbit. He did not, however, speculate on a cause for the perturbations. For some unexplainable reason, this work of Liebowitz' was totally ignored for twenty years, although parts of it are preferred by the present author as the best approach to the problem. A more commonly accepted position, closely related, was summed up by Rosen[9] in 1945. The decade from 1945 to 1955 was nicely surveyed by Freistadt[10], omitting Liebowitz. During this time, the theory took on a more definite and detailed form. Numerous papers

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1. A.E.Ruark & H.C.Urey, Atoms,Molecules and Ouanta, Vol II, 544, Dover Publications, N.Y. (1964). E:.C.Kemble, The Fundamental Principles of Quantum Mechanics, p 55, Dover Publications, N.Y. (1958).
2. F.Reif, Fundamentals of Statistical and Thermal Physics, p 584, McGraw-Hill Book Co. (1965).
3. A.Einatein & O.Stern, Ann.Pbysik, **40**, 551 (1913). See T.H.Boyer, Pbys. Rev.D, **1**, 2268, footnote 20 (1970).
4. W.Nernst, Verhandl.Deut.Phys.Ges. **18**, 83 (1916).
5. L.de Broglie, Compt.Rend. **183**, 447 (t926); **184**, 273; **185**, 360 (1927).
6. E.Madelung, Z.Pbysik, **40**, 322 (1926).
7. R.Furth, Z.Pbysik, **81**, 143 (1933).
8. B.Liebowitz, Pbys.Rev. **66**, 343 (1944); **68**, 53 (1945); **69**, 131 (1946).
9. N.Rosen, J.Elisha Mitchell, Sci.Soc. **61**, 67 (1945).
10. H.Freistadt, Hmovo Cimento S!uppl. 3, 1 (1957).

appeared[1], which, although not attacking this problem directly, supplied useful ideas, pro and con for later work. However, direct work on the problem was taken up in 1952 by Bohm[2], who extended the ideas of de Broglie and Madelung, proposing that the perturbation was caused by a quantum mechanical *potential* related to the wave function. In that same year, Fenyes[3] suggested that, instead of using a driving force potential, the path variations could be described as a Markov process. Shortly after, Weizel[4] described the random motion in terms of collisions with hypothetical particles he called zerons, an extension of Fenyes' approach; and Bohm and Vigier[5] shifted the mechanism to random variations arising through a sub quantum medium.

Up to this point, the main line of development essentially implied a zero-point fluctuation mechanism closely related to some type of Brownian motion. A second line of approach was taken by Braffort, et. al.[6], by going back to an electromagnetic zero-point fluctuation that was assumed to be the residual of all the motions of all the charges in the universe. This allowed the deduction of a stationary ground level for the harmonic oscillator[7].

The form of the causal theory described by Freistadt[8] in his summary paper was closest to the Bohm-Vigier approach, but from 1960 onward, all of the variations plus some new ones were developed more rigorously and completely. Wesley[9] tried to give up the connection between ψ and the probability. Marshal1[10] (1963) gave a full elaboration of the electromagnetic basis for the fluctuation. Kershaw[11] extended the Markov chain variation, and Comisar[12] (1965) started a new reaction by linearizing the Brownian motion technique. Over this same time span, numerous other closely and not so closely

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1. R.P.Feynman, Rev.Mod.Phys. **20**, 367 (1948). J.Moyal, Proc.Camb.Phil.Soc. **45**, 99; M.S.Bartlett & J.E.Moyal, Proc.Camb.Phil.Soc. **45**, 545 (1949).
2. D.Bohm, Phys.Rev, **85**, 166 (1952); **85**, 180 (1952).
3. I.Fenyes, Z.Physik, **132**, 81 (1952).
4. I.W.Weizel, Z.Physik, **134**, 246 (1953); **135**, 270 (1953); **136**, 582 (19!54).
5. D.Bohm & J.P.Vigier, Phys.Rev. **96**, 208 (1954).
6. P.Braffort, M.Spighel & C.Tzara, Compt.Rend. **239**, 157 (1954).
7. P.Braffort & C.Tzara, Compt.Rend. **239**, 1779 (1954).
8. H.Freistadt, loc.cit.
9. J.P.Wesley, Phys.Rev. **122**, 1932 (1961).
10. T.W.Marshall, Proc.Roy.Soc. **A276**, 475 (1963); Proc.Camb.Phil.Soc. **61**, 537 (1965); Nuovo Cimento, **38**, ? (1965).
11. D.Kerehaw, Phys.Rev. **136**, B1850 (1964).
12. G.G.Comisar, Phys.Rev, **138**, B1332 (1965).

related papers and books appeared[1]. A very competent job of deriving Schroedinger's equation on the basis of Brownian motion was carried out by Nelson[2] (1966), including a compact but well directed summary of the earlier work. He concluded that, at least in the limited area examined, the phenomena originally argued as a reason for quantum mechanics could certainly be explained classically (i.e. deterministically). A year later[3], Lande attempted to abandon deterministics completely and derive quantum mechanics from a set of very basic new statistical principles. Since the latter just attempts to augment the accepted majority opinion on the subject, such work is not of interest here[4].

A major milestone in this area of developing a deterministic description of the world occurred in 1968 in the form of two papers by T.H. Boyer[5]. Inspired by the work of Kershaw[6] and of Nelson[6], he showed a "direct connection between Casimir's[7] zero-point energy calculations and Lifshitz' [8] general theory of retarded dispersion forces between macroscopic objects". The perturbing field was assumed to be randomly fluctuating and *transverse*, so it was essentially that of Nernst[6], Braffort[6], Marshall[6], etc. In the second paper, Boyer applied the same approach to Casimir's[9] model of a charged particle and showed that model to be invalid, mainly because of "the repulsive aspect of retarded dispersion forces which was conjectured by Verway and Overbeck".[10] For the next two decades, although several mild flurries about a classical deterministic alternative to quantum

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1. R.C.Bourret, Nuovo Cimento, **l8**, 347 (1960); Can.J.Phys, **43**, 619 (1965). D.Bohm, Causality & Chance in Modern Physics, Harper & Bros, N.Y (1957); Quantum Theory Radiation and High Energy Physics, Vol 111, p 345, ed D. Bates, Academic Press, N.Y. (1962); R. Schiller, Phys.Rev. **125**, 1100, 1109, 1116 (1962); F.Halbwachs, Nuovo Cimento, **36**, 832 (1965); D.M.Grimes, Proc.I.E.E.E. **53**, 737 (1965); I.E.E.E.Spectrum, **55** (1966); Electromagnetism and Quantum Theory, Academic Press, N.Y. (1969); B.Liebowitz, Nuovo Cimento, **39**, 1201 (1965); H.Surdin, P.Braffort & A. Taroni, Nature, **210**, 405(1966); E.Santamato, Phys.Rev.D, **29**, 216(1984).
2. E.Nelson, Phys.Rev. **150**, 1079 (1966).
3. A.Lande, Physics Today, **55**, Feb. (t967).
4. M.Born & W.Biem, Physics Today, **51**, Aug. (1968).
5. T.H.Boyer, Phys.Rev. **174**, 1631, 1764 (1966).
6. Loc. Cit.
7. H.B.Casimir & D.Polder, Phys.Rev. **73**, 360 (1948).
8. E.M.Lifshitz, Soviet Phys. J.E.T.P. **2**, 73 (1956).
9. H.B.Casimir, Physica **19**, 846 (1956).
10. E.J.W.Verwey & J.Th.G.Overbeck, Theory of Stability of Lyophobic Colloids, p 104, Elsevier Publ.Co. Amsterdam (1948).

mechanics or related topics by others have appeared[1], the main thrust was powered by Boyer. From 1968 to 1988, he produced at least two papers per year, solid convincing papers, on this topic extended into many directions. Although a detailed following of this field is, again, not along the direct line of the present work, the writer cannot resist at least a few comments on Boyer's contribution.

In 1969, after elaborating[2] on the long range Van der Waals potentials of dispersion theory discussed earlier, Boyer produced a superb classical derivation of Planck's law based on the electromagnetic zero-point fluctuations[3]. One of the points raised by him, as well as by earlier workers, relates to the fact that the spectrum of the fluctuations has energy per normal mode given by,

$$\varepsilon = \frac{h\omega}{4\pi} = \frac{h\nu}{2} \quad ; \qquad (4.4.1)$$

which has no limit as ν increases. Nevertheless, the results of calculations in the aforementioned papers produced finite effects. In those papers, a *cutoff function* was used; but, in his next paper[4], Boyer showed that even before that cutoff is effective, practical properties of materials, such as their conductivity, restrict the effects to finite levels. He then proceeded to point out that it is the *vector* (electromagnetic) nature of the fluctuations that cause this result. A scalar field would require a cutoff.

Almost immediately, another paper of moment appeared[5], in which he turned the fluctuation argument around. Instead of postulating the fluctuation as before, Boyer showed that the oscillations are *necessary* if classical thermodynamics and classical statistical mechanics are to form a consistent whole. Then on the basis of their existence, h appears only as "the constant setting the scale of the zero-point electromagnetic radiation spectrum". As if that were not enough, the development continued the reformulation of the concept of *entropy*, eliminating some of the paradoxes of the old classical theory. Finally, this paper corrected a popular concept related to quantum statistics. Standard texts, by emphasis, generally attribute the use of "quantum" statistics, rather than "classical", to the fact that *identical* particles are counted. Boyer pointed out that indistiguishability is really not the essence of quantum statistics,

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1. e.g. M.C.Robinson, Can.J.Phys. **47**, 963 (1969). D.T.Pegg, Nature **222**, 362 (1969).
2. T.H.Boyer, Phys.Rev. **180**, 19 (1969).
3. _____, Phys Rev. **182**, 1374 (1969).
4. _____, Phys.Rev. **183**, 2039 (1969).
5. _____, Phys.Rev. **186**, 1304 (1969).

since it must also be considered in classical cases where applicable.

After a short writeup[1] clarifying the meaning of the third law of thermodynamics in classical physics, aided by zero-point fluctuations, Boyer applied the fluctuation theory to the specific heat of a plane, rigid, electric-dipole rotator having classical interactions with thermal electromagnetic radiation[2]. This was followed by two papers further clarifying statistical counting, Gibbs paradox, and entropy[3]. In 1971, all of this early work of Boyer's was briefly summarized by Theimer[4], who presented some simplified derivations for the purpose of more easily understanding the more detailed analyses of Boyer.

From 1971 to 1988, Boyer published over 30 papers touching on many other ties between classical and quantum physics, not all involving zero-point fluctuations. Some treated the Aharanov-Bohm[5] experiment, others the definition of momentum and energy together with a classical model of the electron[6]. The latter disagreed with Robrlich[7], Butler[8], and also with sections of the present Chapter 3; but even in disagreement, Boyer gave new insight. The remaining publications deal with more detailed penetration of the subjects of the earlier papers and great elaboration of the fluctuation approach. Boyer's work is an inspiration.

From the point of view of this discussion of datum fluctuations, there are two papers of Boyer's that summarize the electromagnetic (transverse wave) picture[9], which he calls "random electrodynamics". This approach assumes that the fluctuations are photons and antenna radiation produced by suns and odd matter scattered about the universe. The fluctuations are often considered to be in equilibrium with matter in a self perpetuating state. With this in mind, since the appearance of Casimir's 1948 paper, the possibility of extracting some of the immense datum fluctuation energy predicted on the basis of Eq.(4.4.1) has been taken seriously by some[10]. Much

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1. T.H.Boyer, Phys.Rev.D, **1**, 1526 (1970).
2. _____, Phys.Rev.D, **1**, 2257 (1970).
3. _____, Amer.J.Phys, **38**, 771, 849 (1970).
4. O.Theimer, Phys.Rev.D, **4**, 1597 (1971).
5. T.H.Boyer, Amer.J.Phys, **40**, 56 (1972); Phys.Rev.A, **9**, 68 (1974).
6. _____, Phys.Rev.D, **25**, 3246, (1982).
7. F.Rohrlicb, Phys.Rev.D, **25**, 3251 (1982).
8. J.W.Butler, Amer.J.Phya. **36**, 936 (1968); **37**, 1258 (1969).
9. T.H.Boyer, Phys.Rev.D, **11**, 790-808, 809-829 (1975).
1O. R.L.Forward, Phys.Rev.B, **30**, 1700 (1984).

of the work after 1980 has been influenced by this possibility[1], and the overall work continues.

A number of the earlier papers, e.g. some of those based on a Markov process or those similar to Liebowitz' ideas, do not specify the source of the Brownian motion. This leaves open the possibility that the fluctuations are not transverse (electromagnetic) but are longitudinal (ℓ-waves). Since the latter carry no energy, and affect particles by *turning* their paths without exchanging energy (just as a magnetic field acts on an electron), it could be that there is no energy in the datum fluctuation field. As will be explained in the later discussion of the gravitic field, energy is a complicated and not completely understood entity; so it is too early to claim that the datum fluctuations carry recoverable energy just on the basis of the Casimir effect. Considering how electromagnetic radiation is formed, it is more likely that the fluctuations are both transverse and longitudinal, with energy carried only by the former.

Little more needs saying here about this important area of advance. The author shall reserve judgment of the exact nature of the fluctuations until further experimental work is done. If those fluctuations are transverse (electromagnetic), then they are photons and antenna radiation and could behave like Weizel's zerons. On the other band, if they are just the buffeting resulting from all of the crisscrossing sustaining waves (longitudinal), then they could produce Brownian motion by turning the electrons. However this puzzle comes out, the result is that quantum mechanics, Schroedinger's equation, etc., are all classical and just statistical mechanics of ensembles[2]. This does not mean that h, for example, is only a constant in statistical mechanics. Clearly, from earlier chapters, h is related to the internal oscillating frequency of the electron; so, it is surely a constant related to a distortion-oscillation property of the ether itself.

4.5 **The Compression/Oscillation Characteristic & Particle Mass**: In Section 2.13, the compression/oscillation characteristic of the datum ether was said to be similar to that of a mass/spring system which obeys the frequency /density relationship of Eq.(2.13.2),

$$\omega = \mathcal{G}(\phi_{\bullet_m}) \quad . \tag{4.5.1}$$

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1. See H.E.Puthoff's papers for references. H.E.Puthoff, Phys.Rev.D, **35**, 3266 (1987); Phys.Rev.A, **35**, 2333 (1985); Phys.Rev.A, **40**, 4857 (1989); Speculations in Science and Technology, **13**, 247 (1990).
2. E.Nelson, Quantum Fluctuations, Princeton U.Press, (1985).

No method of measuring this function directly is known at present, so all estimates of the form of $\mathcal{G}(\phi_{.m})$ must be based on inferences from high energy particle physics and atomic data. In spite of the myriad of particle collision experiments and other related investigations, the present best guess as to the form of $\mathcal{G}(\phi_{.m})$ is the very incomplete curve depicted in Figure 4.5.1. Only part of the lower three horizontal regions are known with any certainty, but even from this meager information, the predilection of the ether for certain frequencies suggests the mechanism by which the particle masses are fixed.

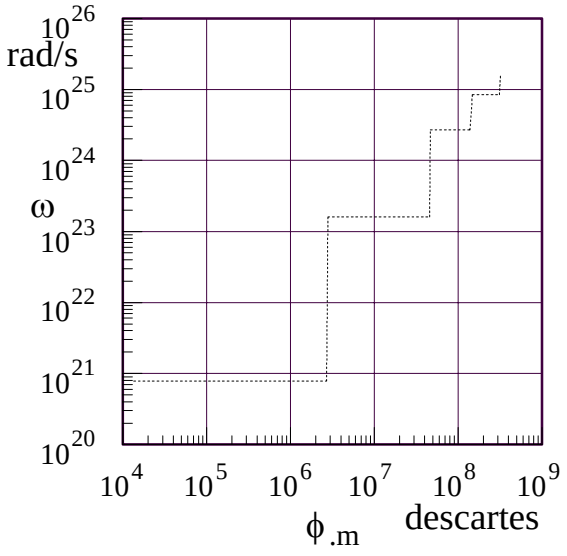

Figure 4.5.1 The ether's compression/oscillation characteristic.

In Figure 4.5.1, the lowest known particle frequency, that of the electron, is marked. It was this "preferred" frequency that was used in Section 3.4 to fix the electron's rest energy using Eqs.(3.3.12),(3.5.2), (3.5.5) and (3.6.3). It doesn't require a great leap of imagination to guess that other solutions of the ℓ-wave equations will lead to more massive particles that correspond to higher oscillation frequencies. What is really needed is some basic experimental technique, perhaps based on the type of equipment used to measure the Casimir effect, that will determine the ether frequency characteristic. At present, this is only a vague hope, so the methods available for a roundabout deduction of the curve will be presented in the next chapter. Meanwhile, almost nothing is known about the great body of datum ether. It represents one of the remaining frontiers of physics.

CHAPTER 5

THE LAYERED PARTICLES

5.1 **Introduction**: For the last 100 years, particle physics has been successful primarily on the experimental side. The rare theoretical success has been the classification scheme called the "Standard Model", and the prediction of the existence of *one* particle. An extensive mathematical literature, e.g. string theory, N dimensions, etc., has done little to advance the knowledge of particle structure or predict particle masses.

The normal mutual support of theory and lab work has failed here, because QM is *non-visualizable* and suppresses *cause and effect*. This has guided experimentalists to the "bigger machine" approach; but particles are very flexible, and at some collision energy level they just come apart and new particles are formed. So, the limit has perhaps been reached in "bigger is better".

The ultimate goal is to change the present approach to particle structure by avoiding the difficulties of a dogged adherence to the use of successive layers of *point particles* to describe matter. Ascribing so many properties to point entities is at least unsatisfying. Of course, with only the rigor of the few presently known solutions of the field equations, the following involves considerable *conjecture*. A good part of it could be wrong and later discarded. Nevertheless, the basic approach, i.e. extended particles rather than point particles, will surely continue to be part of any picture, for the simplicity and intuitive aspects are indispensable. The built in potential for specifying the masses of all the particles demands that attention be paid to this change in viewpoint. For the most part, processes empirically deduced from available measured data will be called upon for the visualization[1]. This is the last great frontier of fundamental physics; and, to complete the picture, an immense effort will be required, a hopeless task for one lone explorer. The job of laying out the whole field of particle structure will require a long effort by many investigators. The following is a small step in applying Main-Line physics to find a more flexible particle classification system and to point the way to a simpler, visualizable analytical picture.

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1. Sources used here include: Phys.Rev.D, Particles and Fields, July (1996) Part I; D.H.Perkins, Introduction to High Energy Physics, 2nd Ed. Addison-Wesley Publ.Co. Mass. (1982); B.C.Duff, Fundamental Particles, Taylor & Francis, London & Philadelphia (1986); F.Halzen & A.D.Martin, Quarks and Leptons, John Wiley & Sons, N.Y. (1984); I.S.Hughes, Elementary Particles, 2nd Ed. Cambridge U. Press (1985); R.N.Cohn&G.Goldhaber, The Experimental Foundations of Particle Physics, Cambridge U. Press (1989).

5.2 **Particle Categories**: Here, the *conventional* categories of particles (e.g. leptons, baryons, etc.) will be abandoned. A new set, based on simple intuitive ether properties will be adopted. Particles are ether configurations that can act as relatively concentrated units for some significant time. If the structure cannot change without some outside influence, the particle is stable. If it *must* redistribute itself into a new form (i.e. "convert"), it is unstable. In the simplest organization of particle categories, no distinction between stable and unstable particles is made; and stability is just another property. However, as discussed later, stability is a complex problem. Based on the available information, there are only two different classes of fundamental particles:

1. Layered particles (layerons) - electric
2. c particles (c-ons) - magnetic

The c particles, photons and neutrinos, travel at the speed of light in free space, *and are quite different in structure from all the other particles*. Neutrinos allow conservation of spin angular momentum in particle interactions. Just how they carry electric energy away from interactions is not yet understood. Photons carry energy away from charged particles, generally orbiting. Some details of the c-on particles' makeup will be presented in Chapter 10.

The layered particles are composed of spherically symmetrical ether density distributions, in some ways similar to the electron, supported by ℓ-waves and stacked in various arrangements of potential $\overline{\overline{\phi}}$ (see Figure 5.2.1). Examples of 1, 2 and 3 layered particles are the positron, pion and proton respectively. In the

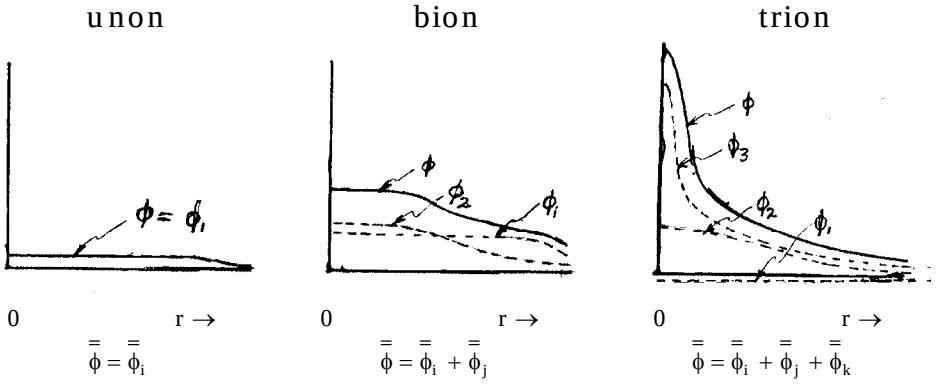

Figure 5.2.1. Layered particles.

following, the layerons will be discussed in detail. The separation of layerons and c-ons helps to emphasize that layerons are bulk distortions with purely *electric* energy (see Secs. 3.10, 9.10 and 15.6).

A proper analysis of the layerons begins with the unons, for which the theory is much advanced. The subsequent description of the bions and trions is far from complete, and leans heavily on the unon analysis. Nevertheless, the *visualization* is carried to the point where, even without the final rigorous answers to many important questions, the overall picture is almost totally understandable and only awaits the formal filling out of the many specific cases.

5.3. **Particle Measurements**: Many of the particle types studied by physicists are man-made, and only a very few are involved in the structure of the world (e, p, n). Table 5.3.1 lists the most important low energy particles and shows their decay products, which help to visualize how the particles are constructed. Numerous other vacuum disturbances, called *resonances*, have some particle-like behavior, but here they are not considered to be particles.

In spite of the fact that most of the data used to describe particle characteristics at present are obtained by high energy collisions of beams and targets, and that this information is indispensable, it is not of paramount importance here where the goals are somewhat different. Instead, the discussion will lean toward particle decay, because only the electron and the proton (and the neutron when part of a nucleus) are stable. All other particles decay in a relatively short time after formation. The only difference between the bombardment and decay conversion processes is in the complexity of the initial conditions. Particle *conversion* involves only *one* pseudo-stable or unstable particle that redistributes its distortion to a lower energy configuration.

In particle *decay*, energy distortion, uninfluenced by any outside presence other than the datum fluctuation, will always redistribute in a downhill direction, i.e. produce only constituents of energies smaller than the original (all adding up to the original energy, of course). This means that any of the particles to be examined will convert to one or more of those listed in Table I at lower rest energies only. These are not the only conversions the listed particles undergo, but those omitted, which may be very numerous in type, are found only a small fraction of the time, less than a few percent, and often as little as, say, 10^{-6} percent.

TABLE 5.3.1

PARTICLE CONVERSION PRODUCTS

		Products	Percent
Unons			
	$e^{\pm} \rightarrow$	stable	
	$\mu^{-} \rightarrow$	$e^{-}\bar{\nu}_e\nu_\mu$	100
	$\tau^{-} \rightarrow$	$\mu^{-}\bar{\nu}_\mu\nu_\tau$	18
		$e^{-}\bar{\nu}_e\nu_\tau$	17
		bions, c-ons	65
Bions			
	π^0	$\gamma\gamma$	98.8
		$\gamma e^{+}e^{-}$	1.2
	π^{-}	$\mu^{-}\bar{\nu}_\mu$	100
	η	$\gamma\gamma$	39.3
		$3\pi^0$	32.1
		$\pi^{+}\pi^{-}\pi^0$	23.2
		$\pi^{+}\pi^{-}\gamma$	4.8
	K^{+}	$\mu^{+}\nu_\mu$	63.5
		$\pi^{+}\pi^0$	21.2
		bions, unons	15.3
All others yield mixes of bions, trions and c-ons			
Trions			
	$p \rightarrow$	stable	
	$n \rightarrow$	$pe^{-}\bar{\nu}_e$	100
	$\Lambda^0 \rightarrow$	$p\pi^{-}$	63.9
		$n\pi^0$	35.8
	$\Sigma^{+} \rightarrow$	$p\pi^0$	51.6
		$n\pi^{+}$	48.3
	$\Sigma^0 \rightarrow$	$\Lambda\gamma$	100
	$\Sigma^{-} \rightarrow$	$n\pi^{-}$	100
All others yield mixes of bions, trions and c-ons			

5.4. **Unons**: The most common single layer particle is the electron/positron, which was discussed in detail in Chapter 3. Referring back to Sections 3.5 – 3.7, the e/p is a single dip (bump) of ether density $\bar{\bar{\phi}}$, held in place by its ℓ-waves, and of a form,

$$\bar{\bar{\phi}} = \phi_0(1 - \psi^2) \quad , \tag{5.4.1}$$

where $\psi = \varepsilon^{-r_i/r}$ is the intrinsic ℓ – wave shape function, and r_i is the inflection radius. This single layer of potential yields single layers of energy and charge density, which indicates that the e/p is a unon with total rest energy and charge,

$$E_0 = 2\pi\phi_0^2 r_i \quad \text{and} \quad q = 8\pi\phi_0 r_i \quad . \tag{5.4.2}$$

It is important to recognize that the solution of Eq.(3.3.12) *for the ℓ-wave shape function was in no way specific to the electron.* Only when the particular frequency ω_e was used, in Eq.(3.5.2), did the central density ϕ_0 of the bulk distribution $\bar{\bar{\phi}}$ identify the particle as the electron. The implication was that, during the formation process (e.g. pair production), the ether had been distorted enough to start the oscillating ℓ-wave with a peak density of at least $\phi_{\bullet m} = 1.3457 \times 10^4$ des. Then, in accordance with the $\omega/\phi_{\bullet m}$ characteristic of Figure 4.5.1, the ℓ-wave oscillation continued at $\omega_e = 7.7634 \times 10^{20}$ rad/sec. This suggests that, if the initial distortion had been much greater, unons of higher frequencies corresponding to the steps in Figure 4.5.1 would have been found, *and they have been.*

The analysis in Chapter 3 applies to all the unons, if ω is properly chosen, so Eq.(3.9.4) for the traveling $\phi_\bullet$ wave in the particle's central region can be used to find $\phi_{\bullet m}$ for the various unons. The density oscillates, so the *peak* amplitude at every radius is reached each time $\sin\omega t = \pm 1$, reducing Eq.(3.9.4) to,

$$\phi_{\bullet p} = \frac{\phi_d a}{\omega r^2} \varepsilon^{-r_i/r} \left(1 + \frac{r_i}{r}\right) \quad , \qquad r < 30 r_i \tag{5.4.3}$$

where r_i designates the *effective* radius (inflection point of the bulk density distribution) of each unon as determined by its ω. Eq.(3.5.5)

establishes r_i as,

$$r_i = \frac{D}{\omega} \quad , \tag{5.4.4}$$

Combining Eqs.(5.4.3) and (5.4.4), it is simple to show that the maximum ℓ-wave density $\phi_{\bullet m}$ occurs at the radius $r_p = C_m D/\omega$, where $C_m = (\sqrt{3}-1)/2$. Thus,

$$\phi_{\bullet m} = \frac{\phi_d a}{\varepsilon^{\frac{1}{C_m}} C_m^{\,2}\, D^2}\left(\frac{1}{C_m}+1\right)\omega \quad ,$$

or,

$$\omega = K_\omega \phi_{\bullet m} \quad , \tag{5.4.5}$$

where,

$$K_\omega = 5.769032\times 10^{16} \quad . \tag{5.4.6}$$

The difference between Eqs.(5.4.5) and (2.13.2), is that $\omega = \mathcal{G}(\phi_{\bullet m})$ describes the compression/oscillation property of the ether itself, whereas $\omega = K_\omega \phi_{\bullet m}$ describes a property of particle structure, i.e. a *solution* of the ℓ-wave equation. *Both conditions must be satisfied*. Eq.(5.4.5) can be plotted over the $\mathcal{G}(\phi_{\bullet m})$ curve of Figure 4.5.1, and a unon should be found wherever it intersects a frequency plateau. Figure 5.4.1 shows this plot.

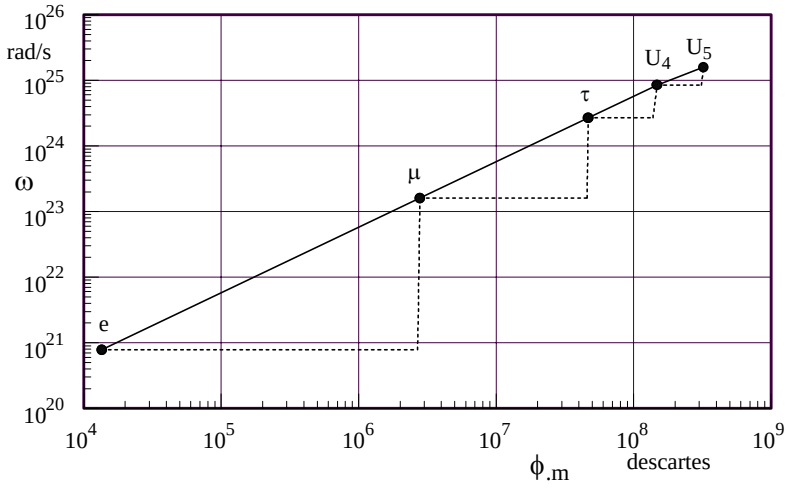

Figure 5.4.1 The unon family.

In 1937, Anderson and Neddermeyer[1] published the first crude data on the muon (μ), but it took 10 years to separate its properties from those of a bion $(\pi^{\pm})$ with roughly the same energy. Whereas the electron is believed to be a stable particle, the μ has a mean life of only 2.1970×10^{-6} sec. It is still possible, however, that all unons are *basically* stable. In fact, since they are described by the same set of equations as the electron, it is likely. If they are basically stable, then it is only the datum fluctuations, triggering them into "conversion" into lower energy particles, that gives the appearance of instability. The amount of distortion compressed into their small volumes then establishes their mean lifetimes.

The next higher energy unon is the τ, which has a shorter mean life of 2.910×10^{-13} sec. This makes it much more difficult to detect, and it was only found by Perl, et al[2] as late as 1975. No other unons have been observed, but some properties of the multi-layer particles suggest that the higher frequency plateaus of Figure 5.4.1 make 4th and 5th unons possible.

5.5 **Measurement of the Unon Family ω's**: At this point, something more should be said about the compression/oscillation curve, and the roundabout method for measuring it. Later on, in Chapter 7, a completely classical derivation of the hydrogen atom will be presented that uses only Newton's laws and some properties of the extended electron that were described in Chapter 3. That derivation can be used to find the equation for the frequencies in the hydrogen spectrum in the form,

$$\nu = \frac{e^4}{8m_0^2c_0^6}\nu_e^3\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \quad ; \tag{5.5.1}$$

or in the more usual form of the inverse wavelength,

$$\bar{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \quad , \tag{5.5.2}$$

where,

$$R_H = \frac{e^4}{8m_0^2\,c_0^7}\nu_e^3 \quad , \tag{5.5.3}$$

1. S.H.Neddermeyer & C.D.Anderson, Phys. Rev., **51**, 884 (1937).
2. M.L.Perl, et al, Phys. Rev. Lett., **35**, 1489 (1975).

and n_f and n_i are the final and initial orbit numbers of the radiating electron respectively. The *measured* values of $\bar{\nu}$ from the many transitions, including microwaves from free hydrogen in outer space, provide a value for R_H, the Rydberg constant, *one of the most accurately measured constants known*. It is found to be,

$$R_H = 1.0973731572 \times 10^5 \text{ cm}^{-1} \quad . \qquad (5.5.4)$$

If this value is used in Eq.(5.5.3), along with the measured values of m_e, e and c_0, then ν_e is,

$$\nu_e = \sqrt[3]{\frac{8 m_0^2 c_0^7}{e^4} R_H} = 1.2355898 \times 10^{20} \text{ cyc/sec} \quad , \qquad (5.5.5)$$

or $\omega_e = 7.7634396 \times 10^{20}$. *This is essentially a measured value for* ω_e.

The same procedure can be followed to find ω_μ, thanks to an important series of experiments done between 1954 and 1957[1] by Stearns, et al. In these trials, various materials were bombarded with μ^- or π^- particles; and the spectra of the μ^- or π^- atoms, in which an electron was replaced by the μ^- or π^-, were measured. By a calculation similar to the one above, ω_μ is found to be very close to the value used in the present work.

Because the basic measurement of $\ell-$wave frequency just described is difficult, particularly for higher energy unons with very short lifetimes, a more practical shortcut is used here. The unon ω's are calculated from their measured bulk rest energies by combining Eqs.(3.5.2), (3.5.5) and (3.6.3) to give,

$$E_0 = \frac{\pi \phi_d^2 a^4 b^2}{8 c_0^2 D} \omega \quad . \qquad (5.5.6)$$

The expression can be simplified by using the derived constant of Eq.(3.14.6), leading to,

$$E_0 = h\nu \quad , \qquad (5.5.7)$$

where,

$$h = \frac{\pi^2 \phi_d^2 a^4 b^2}{4 c_0^2 D} = 6.6260755 \times 10^{-27} \text{erg-s} \quad . \qquad (5.5.8)$$

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1. M.B.Stearns, in Progress in Nuclear Physics, **6**, 108-137 (1957); M.B.Stearns & M.Stearns, Phys. Rev., **105**, 1573-82 (1957).

Eq.(5.5.6) is represented in Figure 5.5.1, showing the bulk energies of the extended unon family of particles as a function of their frequencies.

Unfortunately, determinations of ω by this method require the fore knowledge of each unon's existence, and the measurement of its rest energy or mass. As mentioned in Section 4.5, there is no presently known method for measuring the $\mathcal{G}(\phi_{\cdot m})$ curve directly, so the two top steps shown in Figures 5.4.1 and 5.5.1 are just guesswork inspired by some inferred ideas based on multi-layer particle structure. Even if unons U_4 and U_5 can be formed, their tremendous compaction would lead to such extremely short mean lives that their observation might be out of the question.

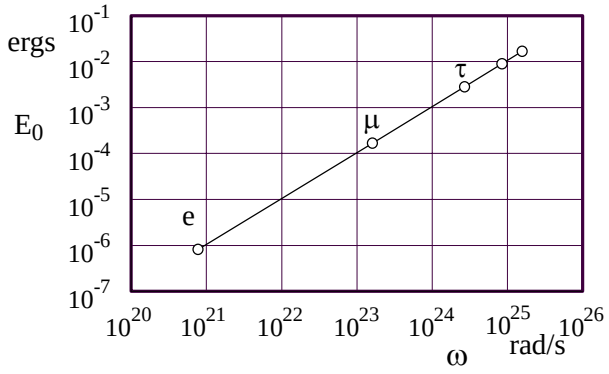


Figure 5.5.1 The unon family.

The Energy Compaction Relationship

Eliminating ϕ_0 between the rest energy E_0 and charge q found in Eqs.(5.4.2) above,

$$E_0 r_i = \frac{q^2}{32\pi} \quad , \quad \text{erg-cm} \qquad (5.5.9)$$

a relationship called the *energy compaction equation*. It indicates that the more energetic unons have smaller radii. For $q = e^{\mp}$, $E_0 r_i = 2.8838 \times 10^{-20}$ erg-cm .

Unon Size and Stability

The unons of interest will be limited to the series of whole charged particles, i.e. e, μ, τ, ..., that can exist alone and be observed for some finite time. Using the compaction relationship derived above, and the *measured* values of E_0 for each of the unons, the calculated values for r_i and ϕ_0 are listed in Table 5.5.1 with each particle's observed mean life.

TABLE 5.5.1

UNONS, THE "PREFERRED" ETHER STATES

	E_0 (ergs)	r_i (cm)	ϕ_0 (hlvolts)	mean life (s)
e	8.1871×10^{-7}	$r_1 = 3.5224\times10^{-14}$	1.9233×10^{3}	Stable
μ	1.6929×10^{-4}	$r_2 = 1.7035\times10^{-16}$	3.9768×10^{5}	2.1970×10^{-6}
τ	2.8472×10^{-3}	$r_3 = 1.0129\times10^{-17}$	6.6886×10^{6}	2.9100×10^{-13}

The interesting features of Table 5.5.1 are that, first, although each of these unons has the same charge $e^{\mp}$ the more energetic particles have larger center potentials; and, their energy being packed into a smaller volume correlates with their being less stable. Second, it appears that the unon sequence is a set of *preferred* states that can exist as "pseudo-stable" particles *because of a fundamental property of the ether.*

5.6 **Preferred Ether States**: The μ and τ are often called "big electrons", because, like the electron, they have only the same two simple characteristics, their center potentials and their inflection radii. In the later discussion of multi-layer particles, it becomes clear that the multiple layers are similar to these three unons. In fact, *the radii of the layers in multi-layer particles are essentially the same as those listed in* Table 5.5.1. In one way this is surprising, but why it is true can be understood better from the following.

The Layer Radius/Frequency Equation

An important relationship between the inflection sphere radius of a unon and the unon's ℓ – wave frequency is obtained by eliminating E_0 from Eqs.(5.5.7) and (5.5.9), with the result,

$$\omega_i r_i = \frac{q^2}{16h} = D \quad , \tag{5.6.1}$$

where $D = e^2/16h = 2.7346139 \times 10^7$ rad-cm/s. Figure (5.6.1) depicts this relationship for the unon family of particles. Eq.(5.6.1) takes the surprise out of the concept of preferred ether states, for although it is difficult to imagine how preferred *radii* could be a basic condition in the ether, it is comfortable to think of preferred ether *frequencies* as basic. So, assuming that Figure 5.6.1 indicates that the ether has preferred *frequency* states, Eq.(5.6.1) shows that this also establishes preferred radii.

Figure 5.6.1 indicates two possible unons, U_4 and U_5, *that have not yet been observed*. In analyzing the more massive, composite particles, it is clear that there are at least two more preferred states, 4 and 5; but their great instability may make their existence possible only inside the composite particles and not observable as unons of higher order. The values shown are *educated guesses*.

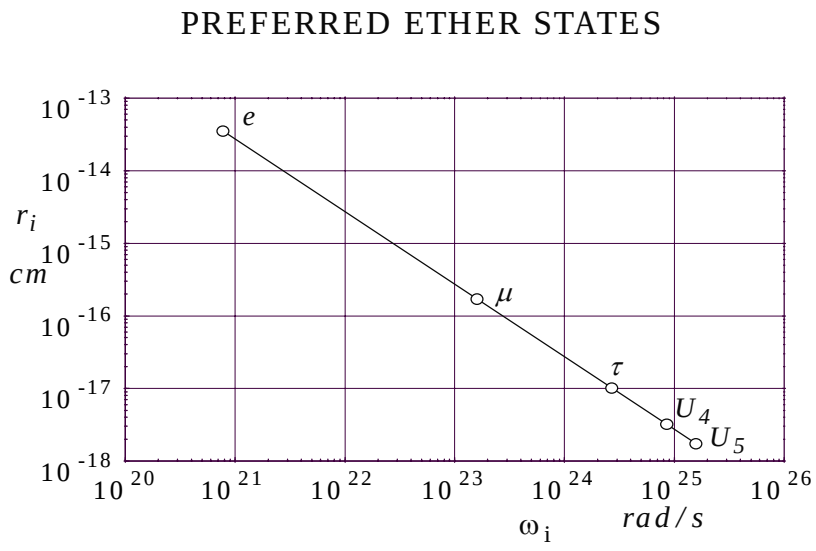


Figure 5.6.1 Layer radius/frequency relationship

Table 5.6.1 lists the important "rest" characteristics of the three known unons.

TABLE 5.6.1

UNONS

Electron e	Muon μ	Tau τ
τ_e = Stable	$\tau_\mu = 2.1970 \times 10^{-6}$ s	$\tau_\tau = 2.910 \times 10^{-13}$
$E_{0e} = 0.51100$ MeV	$E_{0\mu} = 105.66$ MeV	$E_{0\tau} = 1,777.1$ MeV
$r_e = 3.5224 \times 10^{-14}$ cm	$r_\mu = 1.7036 \times 10^{-16}$	$r_\tau = 1.0129 \times 10^{-17}$
$q_e = 1.7027 \times 10^{-9}$ hlC	$q_\mu = q_e = e$	$q_\tau = q_e = e$
$\phi_{0e} = 1923.3$ hlV	$\phi_{0\mu} = 3.9768 \times 10^{5}$	$\phi_{0\tau} = 6.6886 \times 10^{6}$
$\sigma = \frac{h}{4\pi} = 5.2729 \times 10^{-28}$ erg-s for all →		
$\mu_e = 3.2910 \times 10^{-20} \frac{\text{erg}}{\text{hlG}}$	$\mu_\mu = 1.5920 \times 10^{-22}$	$\mu_\tau = 9.4258 \times 10^{-24}$
$m_e = 9.1094 \times 10^{-28}$ g	$m_\mu = 1.8835 \times 10^{-25}$	$m_\tau = 3.1679 \times 10^{-24}$
$\omega_e = 7.7634 \times 10^{20} \frac{\text{rad}}{\text{sec}}$	$\omega_\mu = 1.6052 \times 10^{23}$	$\omega_\tau = 2.6998 \times 10^{24}$

Quarks

From the 1960's on, it has been understood that the more elaborate particles are constructed of objects, now called quarks, that sometimes behave in a manner similar to unons but have fractional charges ±e/3 and ±2e/3. They are thought to be "point" charges like the conventional electron model. *Little is known about the spatial arrangement of these objects inside a composite particle.*

In interactions between quarks and external projectile particles, the *quarks behave as if they were independent entities, but no individual quark has ever been observed outside its housing particle.* This suggests that the composite particles might be made up of very flexible constructs similar to the finite unons described earlier, but having fractional charges, two components for the mesons and three for the baryons. In that case, although the components might freely move for short distances, if one of the components were forced out of a composite particle, *because of its fractional charge, it would not qualify as one of the preferred unon solutions listed in* Table 5.6.1, and so would decay; as would the remaining debris from the original

particle. If this is a correct description of composite particles, then all of the properties of the Standard Model are preserved and yet a greater flexibility results.

The classification scheme described in the following includes all *whole charge* particles, but not photons and neutrinos. Using the Standard Model as a guide, the two "point" quarks that make up mesons and the three "point" quarks that make up baryons are replaced with the finite solutions of Eq.(5.4.1). *To avoid confusing the properties of the "point" quarks with these finite constructs, the term quark will not be used to describe the particle components.*

5.7 **Multi-layer Particles**: At this point the overall particle problem expands intolerably, and a logical, stepwise process of solving it demands an almost endless chain of decisions between possible alternative choices of methods and visualizations. The writer has made certain specific choices, and has carried the process as far as time and resources permit. Although most of the key structure is presented here, there are still volumes of calculations and measurements to be made in verifying and filling out of the structure as developed. Since the same can be said for the conventional "Standard Model", in its present state, the two approaches should be evaluated on the basis of their simplicity and their ability to complete the picture.

A case in point is the "spin crisis" related to proton structure. Conventional theory has great difficulty and becomes dauntingly complex in trying to explain the presence of "strange" quarks in deep-inelastic collision scattering of electrons by protons, with a subsequent confusion about the spins involved.[1] The ether theory presented here, on the other hand, has no problem at all in showing that the great energy brought into the proton by the entering electron can excite the next deeper layer, which gives what appears to be the presence of the "strange" quark.

The spin problem of the Δ^{++}, which forced the addition of "color" to the standard model, is also non existent in the ether theory, because the particle is formed of three *different* concentric layers, so none of the "quarks" is in the same state as any other.

There are three major steps in carrying out the multi-particle structure analysis. The first is to adopt a system that categorizes the various combinations of layers based upon the visualization of the physical distortions in the ether. The second is to develop a more general solution of the ℓ-wave equations of Section 2.12 than was required for the unons. This helps to establish the possible charges

1. B.Schwartzchild, Physics Today, p21, June (1999).
K.Rith & A.Schafter, Scientific American, p58, July (1999).

and energies of the particles, and reduces the number of significant entries in the classifying system. The third is to examine the mountain of experimental data in order to relate the various observed particles with the entities in the categorizing scheme. The classification system and its nomenclature are essentially complete. However, while a more general solution of the ℓ-wave equations is presented, and while it appears to offer most of what is needed for the description of the multi-layer particles, it may not be the most general solution, and some changes may be necessary to make all the categories match up with the voluminous data. The latter task is only begun here.

5.8 **The Multi-layer Classification System**: Whereas the quarks have specific charges assigned, the present scheme first indicates only the number of components a particle has. The basis for the nomenclature system is represented in Figure 5.8.1. The numbered markers represent the relative radii of layers, which are not equally spaced, indicating only the stacking order. The unons, bions and trions are designated by U_i, B_{ij} and T_{ijk} respectively, *where the* i, j *and* k *indicate the layers, and read from the outermost layer inwards.* The possibility that "quadrons" also exist can be accommodated by writing Q_{ijkl}, etc. In all these cases, *the Eq.(5.4.1) constructs take on only the preferred radii listed in Table 5.5.1,* so the subscripts indicate the size and shape of the components.

In this system, the correspondence of the U_i designation to the unon family is, $U_1 \rightarrow e$, $U_2 \rightarrow \mu$ and $U_3 \rightarrow \tau$, etc., each successive particle having a single potential structure with a higher frequency, a smaller radius and a greater energy (see Fig. 5.2.1). The total charge of each is $\pm e$.

When considering the multi-layer particles, several new characteristics appear. For example, *the individual layers do* ***not*** *have different frequencies, each multi-layer particle has just one basic frequency* ω, *and a single* ℓ*-wave establishes all the layers.*

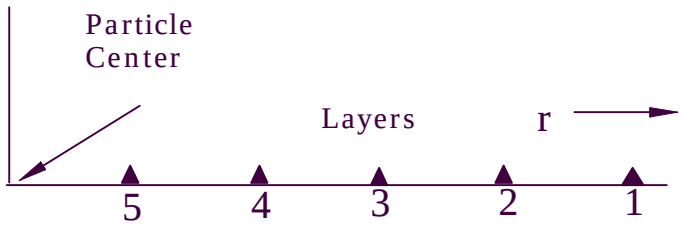


Figure 5.8.1 The preferred ether layer radii.

The frequency ω will **not** be one of the "preferred" layer frequencies. Instead, the single ℓ – wave of a multi-layer particle acts like a driver that *rings* the two or three "preferred" layers constituting that particular particle. For example, the proton will be shown to have the structure T_{123}; i.e. a potential component ϕ_1 with inflection radius r_1, a higher potential component ϕ_2 with smaller inflection radius r_2, and a very high potential component ϕ_3 with a still smaller inflection radius r_3 (see Figure 5.2.1). On the other hand, there could be another trion T_{245}, with components ϕ_2, ϕ_4 and ϕ_5, and inflection radii r_2, r_4 and r_5. Only those "preferred" radii given in Table 5.5.1 and the possible r_4 and r_5, still to be determined accurately, ever appear in the *components* of multi-layer particles; but, each multi-layer particle has just one ℓ – wave frequency ω.

It is convenient to set up the next step in the classification system on the basis of the smoothed out charge density shell ρ rather than the potential ϕ or the energy density shell ε_e, since the total integrated charge of any shell is constant, even when the particle is in motion. The conventional "Standard Model" adopts a very rigid classification scheme that *combines* the layers and charges in a way that is too inflexible. The increased flexibility of the new system comes from the fact that the *charges* of each component have yet to be specified. Now it appears that *all independent, observable particles have total charges that are integral multiples of e*. Because of this empirically determined fact the *total* particle charge distortion is $\pm Ne$, where N is an integer; so, at least in the two layer particles with charge e, *some of the layers must have fractional charge*. The fractional charges have been found to be either $\pm e/3$ or $\pm 2e/3$.

In the present system, *the charge sign and magnitude are indicated, separately from the layers, by superscripts*. For example, the proton is identified as the trion T_{123}^{-++122} where the superscripts indicate that the fractional charges of the components are -e/3, +2e/3, +2e/3. The neutron is tentatively identified as T_{123}^{-+-121}. Thus, $B_{ij}^{\alpha\beta}$ $(i \le j)$ and $T_{ijk}^{\alpha\beta\gamma}$ $(i < j < k)$ represent the complete description of the multi-layer particle categories (except for the spins), where α, β, γ are given values of $\overset{+}{1}$, $\overset{-}{1}$, $\overset{+}{2}$, $\overset{-}{2}$, $\overset{+}{3}$, or $\overset{-}{3}$ for the six possible charge choices. Here, $\overset{+}{1}$ indicates a charge of $+e/3$, $\overset{-}{2}$ a charge of $-2e/3$ and $\overset{+}{3}$ a charge of $+e$.

5.9 **Bion Configurations**: Bions are classified into three groups:

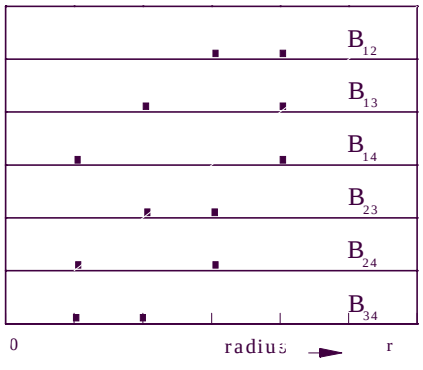
1. Concentric layer bions, B_{ij} } $i < j$

2. Eccentric layer, inside orbiters,

3. Outside orbiters B_{ii}

Group 1 bions have two shells, one inside the other, with a common center. They have a net charge of $e^{\pm}$, whereas the orbiters have two layers of equal and opposite charge that give a net charge of zero. Group 2 bions have two shells, one inside the other, with centers displaced and both orbiting a common center. Group 3 bions form a "system" like positronium, with two separated, equal fractional charge shells, orbiting a common center. Observed *concentric* bions are tentatively identified as $\pi^{\pm}$, $D^{\pm}$, $D_s^{\pm}$, and $B^{\pm}$. *Inside orbiter* bions are probably K^0, D^0, B^0 and B_s^0 and *outside* orbiter bions are most likely $\pi^0, \eta, \eta', \eta_c$ and Υ. The latter (B_{ii}) decay like Positronium and, similarly, produce two photons. *This two photon radiation is their hallmark.*

All the bions are possibly stable, in a fundamental sense (if it were not for the datum fluctuations), but *all bions convert to lower energy forms shortly after their formation.* Because the bions decay rapidly, their correct analysis must address the transient case, which has many mathematical difficulties. Therefore, the measured bion energies are always slightly smaller than the values calculated from the "concentric, static" approximation.



The B_{ij} bion hierarchy

Figure 5.9.1

B_{ij} bions decay mostly into unons and neutrinos. Figure 5.9.1 diagrams the first six forms of the B_{ij} bion hierarchy. For each of these B_{ij} designations, there are *several* possible combinations of charge. For complete generality the charges $\overset{+}{3}$ and $\overset{-}{3}$ were included earlier; but in the following, to relate to the present view of quarks, the $\overset{+}{3}$ and $\overset{-}{3}$ classes

will not be developed. As an example, consider the lowest order bion category $B_{12}^{\alpha\beta}$. Each of the two layers could have one of four different charge distortions, so the total possible types of B_{12} bions is 16. However, this number is reduced considerably because the total particle charge must be zero or an integral multiple of $\pm e$. Therefore, the total number of combinations that could possibly represent real particles is reduced to:

$$B_{12}^{\overset{+}{1}\overset{-}{1}}\ ,\ B_{12}^{\overset{+}{1}\overset{+}{2}}\ ,\ B_{12}^{\overset{+}{2}\overset{+}{1}}\ ,\ B_{12}^{\overset{+}{2}\overset{-}{2}}\ ,$$

and,

$$B_{12}^{\overset{-}{1}\overset{+}{1}}\ ,\ B_{12}^{\overset{-}{1}\overset{-}{2}}\ ,\ B_{12}^{\overset{-}{2}\overset{-}{1}}\ ,\ B_{12}^{\overset{-}{2}\overset{+}{2}}\ .$$

Now, since the second row represents four particles that are exactly like those in the first row, except that their charges are opposite, the second row particles are called the "anti-particles" of those in the first row. Thus, the B_{12} category describes only four different, possible particles (and their anti-particles). Subsequent analysis, using the ℓ-wave equation, can help to decide which, if any, is a real particle and to identify one or all with those observed. The charge density distributions of the tentative $B_{12}^{\overset{+}{1}\overset{+}{2}}$ particle and its anti-particle are depicted in Figure 5.9.2. A similar process can be carried out for all B_{ij} bions. The result is that each bion category can have only four

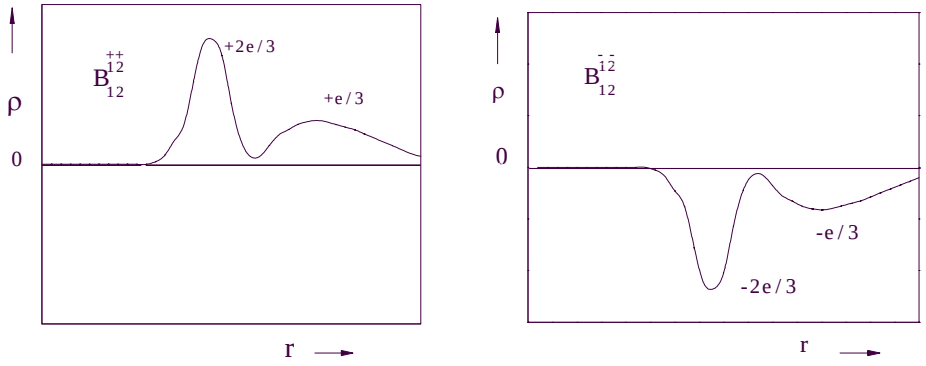


Figure 5.9.2 Examples of possible B_{12} bions

possible particles (and their anti-particles):

$$B_{ij}^{+-11}\ ,\ B_{ij}^{++12}\ ,\ B_{ij}^{++21}\ ,\ B_{ij}^{+-22}\ . \qquad (5.9.1)$$

The first and last of these are inside orbiters, the second and third are concentric bions.

Unlike the unon family, which has a "stable" particle, the electron, at the base of its energy ladder, the bions are all short lived, and even the lowest energy B_{ij} bion is triggered to convert into unons and neutrinos. No more need be said about B_{ij} bions until later, when the solution of the ℓ-wave equation is used to reduce their possible number.

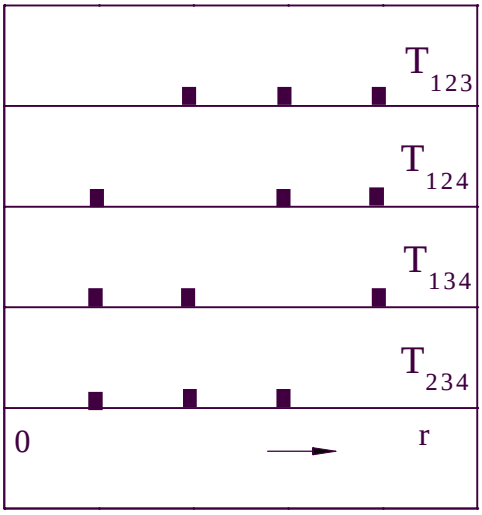
In the most general case of the outside orbiter category, each $B_{ii}^{\alpha\beta}$ yields only three possible types of bions (they are their own anti-particles):

$$B_{ii}^{+-11}\ ,\ B_{ii}^{+-22}\ ,\ B_{ii}^{+-33}\ . \qquad (5.9.2)$$

The last, B_{ii}^{+-33}, is essentially positronium, which is not considered a fundamental particle. Analysis of all orbiters involves more than just solving the ℓ-wave equation, which only gives the structure of the layers themselves. Because orbiters are "systems" over and above the layers, they require an analysis that shares the orbit matching properties of atomic systems, which will be discussed in detail in Chapter 7.

5.10 **Trion Configurations**: The trions come in combinations of *concentric* shells, or *eccentric inside orbiters*. Although work is in progress, the only accurately calculable concentric trion at this time is the stable proton. The great majority of trions appear to be *inside* orbiters, none of which has been finally identified yet due to mathematical intractability. The success of the proton analysis demands that the much more difficult problem of the *orbiter* trions be pursued, particularly that of the neutron.

The same procedure that established the possible bions is applicable to the trions as well. However, in the basic trion hierarchy, there might not be outside orbiters. Thus, for now, the *basic* trion configurations given here involve only concentric cases of $T_{ijk}^{\alpha\beta\gamma}$ where $i<j<k$. The result is that the first few T_{ijk} appear as in Figure 5.10.1. Each category has 3 layers, and each layer can have one of 4 possible



The concentric trion hierarchy.
Figure 5.10.1.

charges, so every category has 64 *possible* particles. Here, again, the requirement for ±Ne (N=0,1,2,...) total particle charge reduces the number of possibles to one out of eight, or 8 (plus their anti-particles). Subsequent analysis can reduce this number.

Of particular interest in this class is the proton, since it is the only "stable" trion known. As before, with the unons and bions, all of these concentric trions might be *basically* stable but susceptible to triggering into energy conversions by the datum fluctuations, which accounts for their short mean lifetimes. The proton, however, is stable, and the lowest sustainable trion form, just as the electron, at the bottom of the unon energy ladder, is stable.

The charge assignments for the 8 tentative trions of each set are:

$$\overset{---}{(1\,1\,1)}\ \overset{--+}{(1\,1\,2)}\ \overset{-+-}{(1\,2\,1)}\ \overset{-++}{(1\,2\,2)}$$

$$\overset{+--}{(2\,1\,1)}\ \overset{+-+}{(2\,1\,2)}\ \overset{++-}{(2\,2\,1)}\ \overset{+++}{(2\,2\,2)}$$

Each of the T_{ijk} categories can have these charge assignments and their negatives. The advantage of this classification system over the standard model is that it frees the charge assignments from the quarks and provides the layer visualization, which is simple to use and to remember.

5.11 **Multi-layer Particle Analysis**: Analysis of *concentric* multi-layer particles runs parallel to the derivation in Chapter 3, and Sections 5.4-5.6 of this chapter. Figure 5.11.1 lists the principal steps involved.

UNON ANALYSIS

1. Assume a simple trial potential.

$$\bar{\bar{\phi}} = \phi_0(1 - \psi_i^2)$$

2. Find a correct ℓ – wave shape function.

$$\psi_i = \varepsilon^{-r_i/r}$$

3. Solve for the correct charge density.

$$\rho = -\nabla^2\bar{\bar{\phi}} = 4\frac{\phi_0 r_i^2}{r^4}\varepsilon^{-2r_i/r}$$

Figure 5.11.1. Unon analysis outline.

MULTI-LAYER ANALYSIS

1. Assume a trial potential.

$$\bar{\bar{\phi}} = \phi_0(1 - \psi^2) \quad = \phi_{01}(1 - \psi_1^2) + \phi_{02}(1 - \psi_2^2) + \phi_{03}(1 - \psi_3^2) + \ldots$$

2. Find each layer's new ℓ – wave shape function.

$$\psi_i = \varepsilon^{-\frac{r_e}{r}\left[1+K_i\ E_2\left(\frac{r}{r_e}\right)\right]}$$

(r_e is the *effective* radius of the whole particle)

3. Find the K_i for each layer.

$$K_i = \left(\frac{r_i}{r_e} - 1\right)\varepsilon^{r_i/r_e} = \left(\frac{\omega}{\omega_i} - 1\right)\varepsilon^{\omega/\omega_i}$$

4. Solve for the correct layer charge densities.

$$\rho_i = -\nabla^2\bar{\bar{\phi}}_i = \frac{q_i r_e}{2\pi}\left(1 + K_i\varepsilon^{-r/r_e}\right)\frac{\psi_i^2}{r^4}$$

Figure 5.11.2. Multi-layer analysis outline.

Although the multi-layer equations are more elaborate, the outline in Figure 5.11.2 shows that roughly the same steps are necessary to determine the *layered* particle's characteristics. In the following, these steps are elaborated upon and the method is applied to *concentric* bions and trions.

Earlier it was said that *all* unons had charge distortion $\pm e$. When multi-layer particles are observed, at least two types are found to have a total charge of $\pm 2e$. This results, during the particle's formation, from the original compression's being large enough to generate three or more layers of charge distortion with a *total charge* greater than e, so that the creation process settles out with a larger amplitude ℓ-wave, as expressed by,

$$\mathbf{V}_\bullet = \hat{\mathbf{r}}\sqrt{N}\,\frac{a}{r}\,\psi C^{\mp} \quad , \quad C^{\mp} = \cos\omega\left(t \mp \frac{r}{c_0}\right) \quad , \qquad (5.11.1)$$

(upper sign outgoing, lower sign in going) where N is an integer that determines the particle's *total charge* $\mp Ne$, and ψ is again a function that approaches unity as $r \to \infty$. Here again, the ether's non-linearity produces stepwise levels of distortion. From Eq.(5.11.1) and the ℓ-wave continuity equation,

$$\phi_\bullet = \pm\sqrt{N}\,\frac{\phi_d a\psi}{c_0 r}\left(C^{\mp} \mp \frac{c_0}{\omega}\left(\frac{1}{\psi}\frac{d\psi}{dr} + \frac{1}{r}\right)S^{\mp}\right) \quad . \qquad (5.11.2)$$

Combining Eqs.(5.11.1) and (5.11.2) gives,

$$\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}} = \pm N\,\frac{\phi_d a^2\psi^2}{2c_0 r^2} \quad \text{and} \quad \nabla\bullet\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}} = \pm N\,\frac{\phi_d a^2}{2c_0 r^2}\,\frac{d\psi^2}{dr} \quad . \qquad (5.11.3)$$

From Eq.(3.7.1),

$$\rho = -b\nabla\bullet\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}} = \mp N\,\frac{\phi_d a^2 b}{2c_0 r^2}\,\frac{d\psi^2}{dr} \quad , \qquad (5.11.4)$$

which can be integrated over all space from $r = 0$ to $r \to \infty$, with the result,

$$q = \mp 2\pi N\,\frac{\phi_d a^2 b}{c_0} = \mp Ne \quad . \qquad (5.11.5)$$

Although a few multi-layer particles form with $N = 2$, most have the same *total charge* found in the unons.

5.12 **Multi-layer Trial Potential**: Although later the proton will be used as an example, it is straightforward to generalize the process for any *concentric* particle. The principal idea is that, as discussed in Section 2.17, *the only physical presence in a particle layer is its potential* $\overline{\overline{\phi}}_i$; so, in the multi-layer particle, *the only physical presence is the* ***sum*** *of the potentials* $\overline{\overline{\phi}}_i$ of the particle's layers (see Figure 5.2.1),

$$\overline{\overline{\phi}} = \overline{\overline{\phi}}_1 + \overline{\overline{\phi}}_2 + \overline{\overline{\phi}}_3 + \ldots\ldots \quad , \tag{5.12.1}$$

Paralleling the unon derivation, the same simple form of trial solution of Maxwell's scalar equation is taken as,

$$\overline{\overline{\phi}} = \phi_0(1 - \psi^2) \quad , \tag{5.12.2}$$

where ψ is the multi-layer shape function. In terms of the individual layer potentials, this becomes,

$$\overline{\overline{\phi}} = \phi_{01}(1 - \psi_1^2) + \phi_{02}(1 - \psi_2^2) + \phi_{03}(1 - \psi_3^2) + \ldots\ldots \quad , \tag{5.12.3}$$

which reduces to,

$$\overline{\overline{\phi}} = \phi_0 - (\phi_{01}\psi_1^2 + \phi_{02}\psi_2^2 + \ldots\ldots) \quad , \tag{5.12.4}$$

where,

$$\phi_0 = \phi_{01} + \phi_{02} + \phi_{03} + \ldots \quad . \tag{5.12.5}$$

The ϕ_{0i}'s can each be positive or negative.

5.13 **Multi-layer Shape Function**: *All layerons have a single ingoing or outgoing* ℓ*-wave with a single characteristic frequency* ω. The thing that distinguishes the multi-layer particles is that they have a composite shape function. Comparing Eqs.(5.12.2) and (5.12.4),

$$\psi^2 = \frac{\phi_{01}}{\phi_0}\psi_1^2 + \frac{\phi_{02}}{\phi_0}\psi_2^2 + \frac{\phi_{03}}{\phi_0}\psi_3^2 + \ldots \quad , \tag{5.13.1}$$

where the ϕ_{0i}/ϕ_0 are fractions related to the amount of charge in each layer of the composite particle. Eq.(5.13.1) defines the *total* multi-layer shape function ψ ; and, because the ψ_i of the layers are

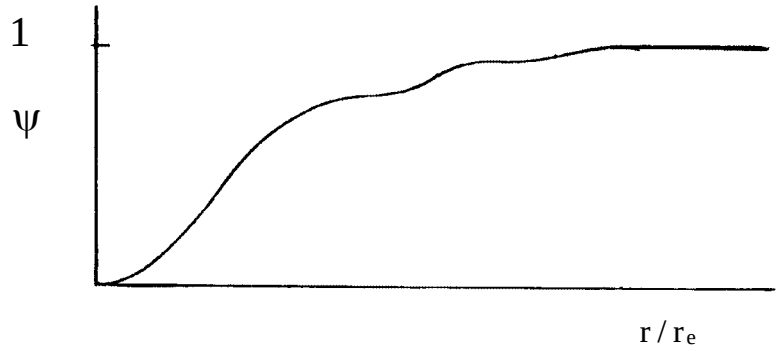


Figure 5.13.1. A composite shape function curve.

not equal, the total shape function has more wiggles than the unon shape function, as shown in Figure 5.13.1.

In summarizing the results of the classification system and the multi-particle analysis, to this point, a conflict arises in the following way. Looking back to Eq.(5.5.7), *that relationship between the particle's rest energy and frequency appears to hold for all concentric layerons*, and it is an essential part of the so called "quantum" properties of matter. Furthermore, the tests of Stearn's, et al, discussed in Section 5.5, which included pionic atoms (electron replaced by a π^- bion), have been repeated, since 1957, with more massive bions. All such tests indicate that the orbit selection implied by the de Broglie frequency, to be discussed in Chapter 7, applies in general. Thus, *all these phenomena require that any particle have one, single ℓ-wave frequency*.

On the other hand, the preceding analysis shows that if the same shape function $\psi = \varepsilon^{-r_i/r}$ used for the unons were used for each of the layers, multiple frequencies would be required, one for each layer. The resolution of this problem comes through finding a new, more general, layer ℓ-wave shape function.

5.14. **The New Layer Shape Function**: The most interesting aspect of multi-layer particles is that *the layers are essentially independent*, though an indispensable part of the whole particle; i.e. *the shape functions, ψ_i, of the individual layers must satisfy the ℓ-wave equation independently*. They can be determined by first differentiating Eq.(5.13.1), leading to,

$$\frac{d\psi^2}{dr} = \frac{\phi_{01}}{\phi_0}\frac{d\psi_1^2}{dr} + \frac{\phi_{02}}{\phi_0}\frac{d\psi_2^2}{dr} + \frac{\phi_{03}}{\phi_0}\frac{d\psi_3^2}{dr} + \ldots \quad . \quad (5.14.1)$$

Combining Eqs.(5.11.3) and (5.14.1), the ℓ-wave equation for the individual layers can be written,

$$\nabla^2 \overset{=}{\eta}_i - \frac{1}{\overset{\equiv}{\eta}_i}\left(\nabla \overset{=}{\eta}_i\right)^2 = -N\frac{\phi_{0i}}{\phi_0}\frac{a^2\omega}{r^2 D}\psi_i\frac{d\psi_i}{dr} \quad , \qquad (5.14.2)$$

where, from Eqs.(5.11.1) and (5.13.1),

$$\overset{=}{\eta}_i = N\frac{\phi_{0i}}{\phi_0}\frac{a^2}{2r^2}\psi_i^2 \quad . \qquad (5.14.3)$$

Substitution of Eq.(5.14.3) in Eq.(5.14.2) leads to,

$$\frac{d^2\psi_i}{dr^2} - \frac{1}{\psi_i}\left(\frac{d\psi_i}{dr}\right)^2 + \left(\frac{2}{r} + \frac{\omega}{D}\right)\frac{d\psi_i}{dr} - \frac{\psi_i}{r^2} = 0 \quad . \qquad (5.14.4)$$

This equation, which applies to the individual layers, is identical to Eq.(3.3.12) which gave the unon's shape function. In the previous unon cases, for simplicity, a very limited solution of Eq.(5.14.4) was used that applies only to unons. At this point, *it is necessary to use a more general solution*.

The new ℓ-wave shape function for each layer takes the form,

$$\psi_i = \varepsilon^{-\frac{r_e}{r}\left[1+K_i\ E_2\left(\frac{r}{r_e}\right)\right]} \quad , \qquad (5.14.5)$$

where $E_2\left(r/r_e\right)$ is the exponential integral of the second kind[1], r_e the effective radius of the whole multi-layer particle and K_i an, as yet, unspecified constant. Figure 5.14.1 depicts the family of ψ_i curves with K_i as the parameter. The $K_i = 0$ curve applies to the unons, and gives the simple structure discussed earlier. However, *in multi-layer particles, each layer has a different value of* K_i; and the radii of the layers are specified by both the particle frequency ω, which is common to all layers in any one particle, and the values of the parameter K_i, which sets the inflection radius of each potential layer. The *shapes* of the ψ_i are determined primarily by the factor $\varepsilon^{-r_e/r} = \varepsilon^{-D/\omega r}$, just as before, but *the purpose of the* K_i *is to allow*

1. Handbook of Mathematical Functions, National Bureau of Standards, AMS 55, p228.

each layer to adjust itself, relative to the total particle frequency ω, *so that the layer frequencies* ω_i, *given by* Eq.(5.6.1), *match the "preferred" layer frequencies.*

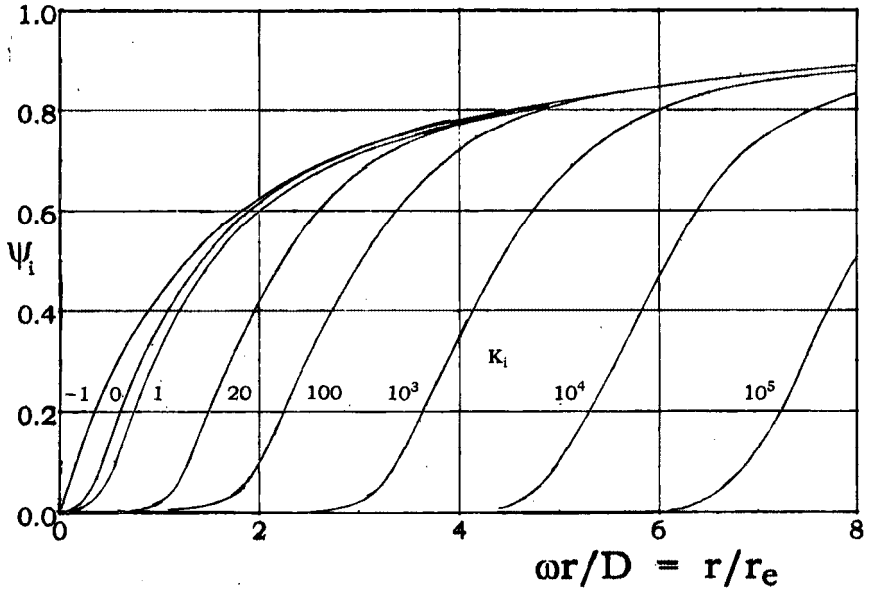


Figure 5.14.1. Layer shape function curves as a function of K_i.

Two forms of the derivative of ψ_i are required in the following,

$$\frac{d\psi_i}{dr} = \frac{D}{\omega r^2}\left(1 + K_i \varepsilon^{-\frac{\omega r}{D}}\right)\psi_i \quad , \tag{5.14.6}$$

and,

$$\frac{d\psi_i^2}{dr} = \frac{2D}{\omega r^2}\left(1 + K_i \varepsilon^{-\frac{\omega r}{D}}\right)\psi_i^2 \quad . \tag{5.14.7}$$

5.15. **Multi-layer Particle Charge Density**: The charge density of a *concentric* multi-layer particle, obtained with the help of Eqs.(5.11.4), (5.11.5) and (5.14.1), is,

$$\rho = \frac{q}{4\pi r^2}\left(\frac{\phi_{01}}{\phi_0}\frac{d\psi_1^2}{dr} + \frac{\phi_{02}}{\phi_0}\frac{d\psi_2^2}{dr} + \frac{\phi_{03}}{\phi_0}\frac{d\psi_3^2}{dr} + \ldots\right) \quad , \tag{5.15.1}$$

which can be integrated over all space from $r = 0$ to $r \to \infty$, with the result,

$$q = q_1 + q_2 + q_3 + \ldots \quad , \tag{5.15.2}$$

where,

$$q_i = \mp \frac{\phi_{0i}}{\phi_0} q \quad . \tag{5.15.3}$$

From Eqs.(5.15.1) and (5.14.7), the charge density of each layer is specified by,

$$\rho_i = \frac{q_i D}{2\pi\omega}\left(1 + K_i \varepsilon^{-\frac{\omega r}{D}}\right)\frac{\psi_i^2}{r^4} \quad . \tag{5.15.4}$$

For larger K_i, the charge shells move out to larger radii and their peak values are lowered. Nevertheless, when Eq.(5.15.4) for the layer charge density is integrated over all space, the *layer charges* q_i *are found to be independent of* K_i, as anticipated by Eqs.(5.11.5) and (5.15.2). It is this fortunate circumstance that permits using the layer charges in the classification scheme. Thus, *K_i is a parameter that can be varied after specifying the charge structure to be examined*; and, although it changes the radii of the charge shells as well as the energy shells, its major effect is on the magnitudes of the layer energies.

5.16 **The Multi-layer Bulk Density Equation**: Starting with Eq.(5.11.3) and the bridge equation,

$$\nabla\bar{\phi} = b\bar{\bar{\phi}}\cdot\bar{\bar{\mathbf{V}}} = \pm\hat{\mathbf{r}} N \frac{\phi_d a^2 b}{2c_0 r^2}\psi^2 \quad , \tag{5.16.1}$$

and,

$$\frac{d\bar{\phi}}{dr} = \pm N \frac{\phi_d a^2 b}{2c_0 r^2}\psi^2 \quad . \tag{5.16.2}$$

Integration produces the bulk density distribution,

$$\bar{\phi} = \bar{\phi}_1 + \bar{\phi}_2 + \bar{\phi}_3 + \ldots \quad , \tag{5.16.3}$$

where,

$$\bar{\phi}_i = S_i - \frac{q_i}{4\pi}\int_0^r \frac{\psi_i^2}{r^2}\, dr \quad , \tag{5.16.4}$$

and S_i is a constant of integration. When $r = 0$, $\bar{\bar{\phi}}_i = \phi_{0i} = S_i$; so,

$$\bar{\bar{\phi}}_i = \phi_{0i} - \frac{q_i \omega}{4\pi D} I_i(r) \quad , \tag{5.16.5}$$

where,

$$I_i(r) = \frac{D}{\omega} \int_0^r \frac{\psi_i^2}{r^2} dr \quad . \tag{5.16.6}$$

As $r \to \infty$, $I_i(r) \to I_i(\infty)$ and $\bar{\bar{\phi}}_i \to 0$. Thus, recognizing that the ℓ – wave analysis leading to Eq.(5.4.4) also determines the effective radius of a whole multi-layer particle to be,

$$r_e = \frac{D}{\omega} \quad , \tag{5.16.7}$$

then,

$$\phi_{0i} = \frac{q_i}{4\pi r_e} I_i(\infty) \quad , \tag{5.16.8}$$

and,

$$\bar{\bar{\phi}}_i = \phi_{0i} \left(1 - \frac{I_i(r)}{I_i(\infty)} \right) \quad . \tag{5.16.9}$$

5.17 **Determination of K_i**: It was stated earlier that each potential layer can be associated with a K_i that determines the inflection radius of that layer. Starting with the gradient of the layer potential in Eq.(15.16.5), differentiating it with respect to r, setting the differential to zero and solving for K_i leads to,

$$K_i = \left(\frac{\omega r_i}{D} - 1 \right) \varepsilon^{\omega r_i / D} \quad , \tag{5.17.1}$$

where r_i is the inflection and maximum energy density radius of the layer. One way to look at Eq.(5.17.1) is to recognize that with Eq.(5.16.7) it can be written in the form,

$$K_i = \left(\frac{r_i}{r_e} - 1 \right) \varepsilon^{r_i / r_e} \quad . \tag{5.17.2}$$

If K_i of a layer is a very large number, that layer is far out from the main energy of the particle. If K_i is near zero the layer is close to the effective radius r_e. For $-1 < K_i < 0$, the layer is smaller than r_e.

An even more useful way to think about Eq.(5.17.1) results from combining Eqs.(5.4.4), (5.17.2) and (5.16.7) in the form,

$$K_i = \left(\frac{\omega}{\omega_i} - 1 \right) \varepsilon^{\omega/\omega_i} \quad , \qquad (5.17.3)$$

where the ω_i are the "preferred" frequencies listed in Table (5.17.1). Eq.(5.17.3) is the most convenient for determining the multi-layer particle structure.

TABLE 5.17.1

"PREFERRED" FREQUENCIES

Layer	ω_i (rad/sec)	r_i (cm)
1	7.76344×10^{20}	3.52243×10^{-14}
2	1.60523×10^{23}	1.70356×10^{-16}
3	2.69981×10^{24}	1.01289×10^{-17}
4	8.508×10^{24}	3.214×10^{-18}
5	1.580×10^{25}	1.731×10^{-18}

5.18 **Multi-layer Particle Energy**: Starting with the incremental ether density of Eq.(5.16.3), the gradient at each point in space is,

$$\nabla\bar{\phi} = \nabla\bar{\phi}_1 + \nabla\bar{\phi}_2 + \nabla\bar{\phi}_3 + \ldots\ldots \quad ; \qquad (5.18.1)$$

and, from Eq.(2.19.1), the total electric energy density of a *concentric* layeron is found to be,

$$\begin{aligned} \varepsilon_e = \tfrac{1}{2}(\nabla\bar{\phi})^2 = {} & \tfrac{1}{2}(\nabla\bar{\phi}_1)^2 + \tfrac{1}{2}(\nabla\bar{\phi}_2)^2 + \ldots. \\ & + \nabla\bar{\phi}_1 \bullet \nabla\bar{\phi}_2 + \nabla\bar{\phi}_1 \bullet \nabla\bar{\phi}_3 + \ldots. \\ & + \nabla\bar{\phi}_2 \bullet \nabla\bar{\phi}_3 + \nabla\bar{\phi}_2 \bullet \nabla\bar{\phi}_4 + \ldots. \\ & + \ldots. \end{aligned} \quad . \qquad (5.18.2)$$

Integrating Eq.(5.18.2) over all space gives the total energy of the particle in the form,

$$\begin{aligned} E_0 &= E_1 + E_2 + E_3 + ... \\ &+ E_{12} + E_{13} + E_{14} + ... \\ &+ E_{23} + E_{24} + E_{25} + ... \\ &+ .... \end{aligned} \quad , \qquad (5.18.3)$$

where the E_i are the layer "self" energies and the E_{ij} are the "interaction" energy deformations, between the layers, stored in the ether during the particle formation process.

The energies, E_i, of the individual layers are found by first differentiating Eq.(5.16.4) to obtain,

$$\frac{d\bar{\phi}_i}{dr} = -\frac{q_i}{4\pi}\ \frac{\psi_i^2}{r^2} \quad , \qquad (5.18.4)$$

and then writing the energy density as,

$$\varepsilon_{ei} = \tfrac{1}{2}\left(\nabla\bar{\phi}_i\right)^2 = \frac{q_i^2}{32\pi^2}\ \frac{\psi_i^4}{r^4} \quad . \qquad (5.18.5)$$

When Eq.(5.18.5) is integrated over all space, the *layer* self energy is found to be,

$$E_i = \frac{q_i^2}{8\pi r_e}\ J_i(\infty) \quad , \qquad (5.18.6)$$

where,

$$J_i(r) = r_e\int_0^r \frac{\psi_i^4}{r^2}\,dr \quad . \qquad (5.18.7)$$

The interaction energies, E_{ij}, can be written,

$$E_{ij} = \frac{q_i q_j}{4\pi r_e}\,J_{ij}(\infty) \quad , \qquad (5.18.8)$$

where,

$$J_{ij}(r) = r_e\int_0^r \frac{\psi_i^2\,\psi_j^2}{r^2}dr \quad . \qquad (5.18.9)$$

In the more complicated cases, involving orbiters, in addition to the self and interaction energies, the orbiting layers are deformed and

include their extra orbital kinetic energies which must be added in to give the total particle energy.

5.19 **Multi-layer Particle Spin and Magnetic Moment**: In Section 3.8, where the electron's spin was taken up, it was described as an ether vortex essentially independent of the density distribution and the ℓ-waves. Its only pertinent characteristic was that the vortex velocity increased with r inside a very small radius ($\delta r_i \leq 0.06 r_i$), and the external velocity gradually decreased as r increased ($\rightarrow 0$ as $r \rightarrow \infty$). It was stated there, that the angular persistence of the frictionless vortex and its external field produced the appearance of angular momentum; but the calculation of the spin using the conventional idea of energy density rotating about an axis was somewhat forced, and the only fundamental connection between the measured spin and the velocity vortex field came through the unknown constant κK_s. In a multi-layer particle, the spin picture is essentially the same for the individual layers.

The calculation for the layer spin starts with Eq.(3.8.11), and differs from the derivation there in the substitution of Eq.(5.18.5) for the energy density in terms of the new ψ_i. The layer spin can then be expressed as,

$$\sigma_i = (\kappa K_s)_i \frac{q_i^2}{12\pi c_0^2}\left[\int_0^{\delta r_i} \frac{\psi_i^4}{(\delta r_i)^3}\, dr + \int_{\delta r_i}^{\infty} \frac{\psi_i^4}{r^3}\, dr\right] \quad , \qquad (5.19.1)$$

where the first integral in the bracket is negligible with respect to the second. A very good approximation can be written in the form,

$$\sigma_i \cong (\kappa K_s)_i \frac{q_i^2}{12\pi c_0^2 r_e^2} L_i(\infty) \quad , \qquad (5.19.2)$$

where,

$$L_i(r) \cong r_e^2 \int_0^r \frac{\psi_i^4}{r^3}\, dr \quad . \qquad (5.19.3)$$

In the same way, the magnetic moment for each layer is found by starting with Eq.(3.8.16) and substituting Eq.(5.15.4) for the layer charge density. The result is

$$\mu_{si} \cong (\kappa K_s)_i \frac{2}{3} \frac{q_i}{r_e c_0} M_i(\infty) \quad , \qquad (5.19.4)$$

where,

$$M_i(r) \cong r_e^2 \int_0^r \left(1 + K_i \varepsilon^{-r/r_e}\right) \frac{\psi_i^2}{r^3}\, dr \quad . \qquad (5.19.5)$$

As discussed in Section 3.8, $\kappa_i = 24\pi c_0^2\, r_e / q_i = 3c_0^2 / \phi_{0i}$. This can be

used with Eq.(5.16.8) to reduce Eq.(5.19.4) to,

$$\mu_{si} \cong K_{si} 8\pi c_0 \left[\frac{M_i(\infty)}{I_i(\infty)} \right] \quad . \tag{5.19.6}$$

For any reasonable, given ω, r_e or K_i, tables of $M_i(\infty)$ and $I_i(\infty)$ reveal that the bracketed quantity has the value,

$$\left[\frac{M_i(\infty)}{I_i(\infty)} \right] = \frac{1}{2} \quad , \tag{5.19.7}$$

to within some small error. Thus, Eq.(5.19.6) becomes,

$$\mu_{si} \cong K_{si} 4\pi c_0 \quad . \tag{5.19.8}$$

Likewise Eq.(5.19.2) can be simplified to the reduced form,

$$\sigma_i \cong K_{si} 4\pi \phi_{0i} \left[\frac{L_i(\infty)}{I_i^2(\infty)} \right] \quad . \tag{5.19.9}$$

Here, however, *the bracketed quantity varies over a wide range*; so, at first sight, Eq.(5.19.8) appears to be the natural route for establishing the value of K_{si}. In Section 3.8, the *measured* value of the electron's magnetic moment μ_s was used to establish the corresponding values of K_s and the spin σ_s. Unfortunately, *the magnetic moments of individual layers of composite particles have not been measured.*

A multi-layer particle consists of several spinning charge layers that can reorient freely and independently of each other. In the ground state of a *concentric* layeron, the magnetic moments will act to align some of the layers, tending to put them into pairs that cancel some or all of the magnetic effects. Care must be exercised, when comparing the measured magnetic moment of a whole particle to that calculated, to consider which layers are paired and which are free to reorient separately.

To avoid this difficulty, the calculation for the layer K_{si}'s is reversed from Section 3.8. In the Standard Model, all "quarks" have spin $1/2$, $\sigma_i = \hbar/2 = 5.272863 \times 10^{-28}$ erg – sec, so *it is assumed here that all the layers also have spin* $1/2$. Therefor, the layer *spins* are used to find the K_{si} values.

In Section 3.8, the electron's spin and magnetic moment were derived and related through its *gyromagnetic ratio*,

$$\frac{\mu_s}{\sigma} \cong \frac{e}{m_0 c_0} \quad . \text{ (electron)} \tag{5.19.10}$$

Because the individual layers in the multi-layer particles have structure similar to the electron's, by combining the above Eqs. (5.18.6), (5.19.2) and (5.19.4), the gyromagnetic ratio of each layer can be expressed in the form,

$$\frac{\mu_{si}}{\sigma_i} = \frac{q_i}{m_{0i}c_0}\left[\frac{J_i(\infty)\, M_i(\infty)}{L_i(\infty)}\right] \quad . \qquad (5.19.11)$$

Upon checking the values of J_i, M_i and L_i for a given K_i, frequency ω or r_e, the bracketed term is found to be *unity* (discounting a small error) for all possible layers, with the result,

$$\frac{\mu_{si}}{\sigma_i} = \frac{q_i}{m_{0i}c_0} \quad . \text{ (layers)} \qquad (5.19.12)$$

In terms of the values that will already be known when this stage of future calculations is reached,

$$\mu_{si} = \frac{q_i c_0}{E_i}\sigma_i \quad , \qquad (5.19.13)$$

and,

$$K_{si} = \frac{\mu_{si}}{4\pi c_0} \quad . \qquad (5.19.14)$$

This is the last step in the formal analysis of multi-layer particles to be presented here. There are other related quantities that can be calculated, but they do not require any further development, since they involve only well known procedures.

5.20 **Multi-layer Particle Calculations**: In the preceding analysis, five integrals that depend upon ψ_i were derived. No closed form solution for any of these is known to the writer. Instead, they must be evaluated by numerical integration. The integrals are designated as, $I_i(\infty)$, $J_i(\infty)$, $J_{ij}(\infty)$, $L_i(\infty)$ and $M_i(\infty)$, and they have been used in both graphical and tabular form. The most fundamental application of the analysis starts by *choosing a particular layer/charge configuration to see what kind of particle it represents*. The analysis ***predicts*** its frequency ω, its rest energy E_0, and beyond that its physical structure, charge distribution, magnetic moment etc. These are then used to determine whether or not such a particle has been observed. In every case, *the basic "existence" test is that Eqs.(5.5.7) and (5.18.3) are satisfied simultaneously*. Several techniques were used to evaluate the integrals, including computer integration with Q Basic, and graphics. When solving problems it is convenient to have

the program in Basic provide, *during the calculation*, the values of all the integrals for a given K_i. In some cases, a table that permanently provides sets of the integrated values for several frequencies over a selected range is more useful.

Tables of $I_i(\infty)$, $J_i(\infty)$, $J_{ij}(\infty)$, $L_i(\infty)$ and $M_i(\infty)$ are presented in Appendix I. In the following the argument, ∞, will not be indicated, and these integrals will just be designated by I_i, J_i, etc. The tabulated values are nowhere near as accurate as particle physicists generally take their work, but the purpose here is to illustrate the techniques rather than to supply final particle characteristics. In making up the tables, certain approximations were used in the extreme ranges. Generally, these have a very small effect on total particle structure. For example, where the values of ω/ω_i are greater than 100, the K_i become cumbersomely large, but it can be shown that the following approximations apply,

$$I_i(\infty) \cong J_i(\infty) \rightarrow \frac{1}{\omega/\omega_i} \quad , \qquad \omega/\omega_i > 100 \qquad (5.20.1)$$

and, with less accuracy,

$$L_i(\infty) \rightarrow \frac{1}{2(\omega/\omega_i)^2} \quad , \qquad \omega/\omega_i > 100 \qquad (5.20.2)$$

and,

$$M_i(\infty) \rightarrow \frac{1}{2\omega/\omega_i} \quad . \qquad \omega/\omega_i > 100 \qquad (5.20.3)$$

In doing the integrals for the interaction constants J_{ij}, it is easy to show that substitution of,

$$K_{ij} = \tfrac{1}{2}\left(K_i + K_j\right) \quad , \qquad (5.20.4)$$

allows using the J_i integral calculations for the J_{ij}, thus eliminating the need for a considerable amount of computation. In that case, an effective ω_{ij} can be defined so that,

$$K_{ij} = \left(\frac{\omega}{\omega_{ij}} - 1\right)\varepsilon^{\omega/\omega_{ij}} \quad , \qquad (5.20.5)$$

and, in the extreme ranges, it can be shown that,

$$\frac{\omega}{\omega_{ij}} \cong \frac{\omega}{\omega_i} - 0.6931 \quad , \quad \omega/\omega_i > 100 \qquad (5.20.6)$$

which allows the approximation of Eq.(5.20.1) to be used for J_{ij}.

5.21. **The Concentric Particle Existence Criterion**: As mentioned above, the existence criterion is based upon the two energy Eqs.(5.5.7) and (5.18.3). In terms of the derived integrals, Eq.(5.18.3) becomes,

$$E_0 = \frac{1}{8\pi r_e}\Big[q_1^2 J_1 + q_2^2 J_2 + \ldots + 2q_1q_2J_{12} + 2q_1q_2J_{23} + \ldots + 2q_2q_3J_{23} + \ldots\ldots + 2q_3q_4J_{34} + \ldots\ldots\Big] \quad , \qquad (5.21.1)$$

and Eq.(5.5.7) can be written as,

$$E_0 = h\nu = \hbar\omega \quad . \qquad (5.21.2)$$

Eq.(5.18.7) indicates that the J_i values depend upon ω through ψ_i (see Figure 5.14.1), so *there are 2 independent equations for* E_0 *as a function of* ω.

Considerable ease in the calculations derives from a slight change of variable. By defining a quantity,

$$J = \frac{8\pi D}{e^2\omega}E_0 = \left[\frac{q_1^{\,2}}{e^2}J_1 + \frac{q_2^{\,2}}{e^2}J_2 + \ldots\ldots + 2\frac{q_1q_2}{e^2}J_{12} + 2\frac{q_1q_3}{e^2}J_{13}\ldots\ldots + 2\frac{q_2q_3}{e^2}J_{23} + \ldots\ldots\right] \quad , \qquad (5.21.3)$$

Eq.(5.21.2) becomes,

$$J = \frac{4hD}{e^2} = 0.25 \quad . \qquad (5.21.4)$$

For a given layer/charge configuration, Eqs.(5.21.3) and (5.21.4) can be plotted as functions of ω, as illustrated in Figure 5.21.1; and the *predicted* particle frequency ω is found at their intersection. The particle's rest energy E_0 follows from Eq.(5.21.2).

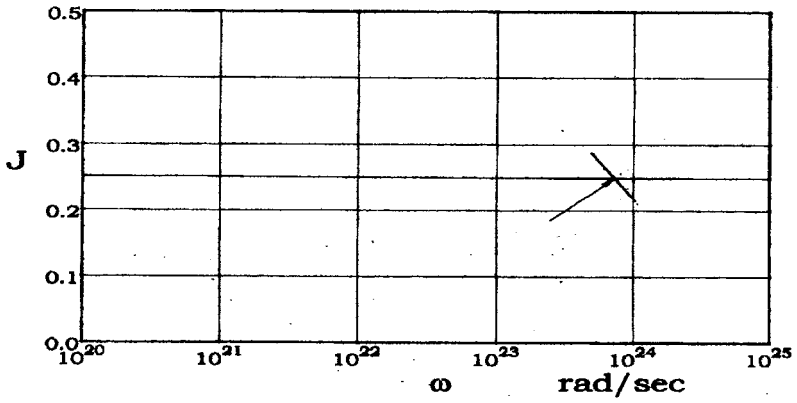


Figure 5.21.1. Eqs.(5.5.7) and (5.18.3) solved for ω.

No measured data taken from a particle being investigated are used in this calculation. The only *measured* data involved in setting up the system are the unon frequencies and radii of Table 5.17.1 required to establish the "preferred" vacuum frequencies. The only integrals involved in the existence criterion are the J_i's and the J_{ij}'s.

5.22. **Predicted Concentric Particles**: It is now possible to plot the various concentric bion and trion predictions along the $J = 0.25$ line of Figure 5.21.1. First, however, the unons can be retrieved as a check on the system. As an example, choose the $U_3^{\bar{3}}$ layer/charge configuration. Eq.(5.21.3) becomes $J = J_3$ and Eq.(5.21.4) is $J = 0.25$; so, from Appendix I, if the J_3's for several ω's are plotted, as in figure 5.21.1, the crossover point is found to be at $\omega = 2.70 \times 10^{24}$. The more exact value, found from the non- graphical, Q Basic program calculation, is $\omega = 2.6998 \times 10^{24}$, indicating that $U_3^{\bar{3}}$ is the τ unon. This is not surprising, since that ω was used to establish the 3rd layer preferred frequency.

Concentric Bions

The existence criterion for bion layer/charge forms is,

$$J = \frac{q_i^2}{e^2} J_i + \frac{q_j^2}{e^2} J_j + 2\frac{q_i q_j}{e^2} J_{ij} = 0.25 \quad .$$

A typical example is the B_{23}^{++12}, which leads to,

$$J = \frac{1}{9} J_2 + \frac{4}{9} J_3 + \frac{4}{9} J_{23} \quad .$$

The graphical solution is done with a programmable hand calculator to add the three terms. Starting anywhere in Appendix I, an ω is chosen and the corresponding J_2, J_3 and J_{23} terms are added. If the above sum is larger than 0.25, a larger ω is chosen. If the above sum is smaller than 0.25, a smaller ω is chosen. About 4 points, in a fairly small range of ω with 2 above and 2 below 0.25, are usually adequate. When plotted on a graph like Figure 5.21.1, the 0.25 crossover point gives the proper ω. In the case of B_{23}^{++12}, $\omega = 2.820 \times 10^{23}$. The more accurate value obtained from the non-graphical, program calculation is $\omega = 2.8217 \times 10^{23}$.

The analysis has been used to plot the intersecting curves for all concentric bions up to, and including, the fifth layer. Figure 5.22.1 shows the result. In trying to assess which observed particles might

correspond to these solutions, it must be remembered that all bions are "unstable", i.e., they convert into lower energy particles. Later on, in calculating the proton properties, the fact that it is a truly stable particle that does not convert in its free state allows the intersection values and the observed E_0, ω to be essentially exact. This is not true for the concentric bions, so the results require some interpretation. For example, their mean life is about 10^{-8} to 10^{-12} seconds. If, as has been suggested earlier, they are basically stable, but are triggered

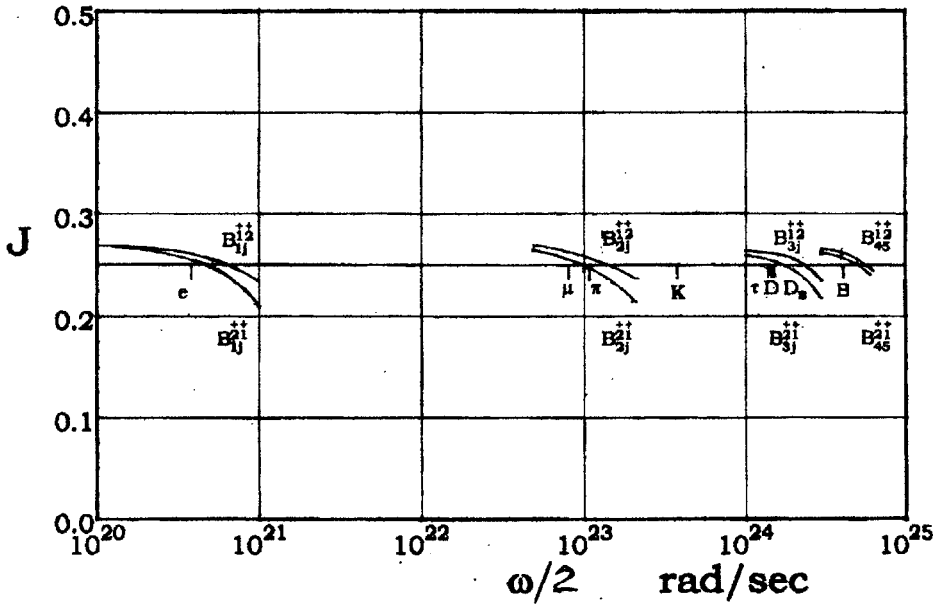


Figure 5.22.1. The concentric bion solutions.

by the datum fluctuations, the mean lives should be dependent on both the fluctuations themselves and the energy of the particles' layers. However, no solutions of the time dependent ℓ-wave equations have been carried out. Furthermore, the cutoff frequency for the datum fluctuation spectrum is not known. Thus a certain amount of guesswork is involved in the following, and *the observed particles should not be expected to exactly match the intersection points. Notice that Figure 12 is plotted versus* $\omega \backslash 2$.

Looking at Figure 5.22.1, a few significant observations can be made. For one, *a particle's outer layer establishes the major grouping pattern.* For example, $B^{++}_{21}{}_{14}$ is so close in energy to $B^{++}_{21}{}_{12}$ that the

curves are almost identical. Likewise, B^{++}_{23} and B^{++}_{24} are essentially equal to each other. Also, Figure 5.22.1 shows that *the solutions are grouped in pairs* distinguished only by a reversal of the charge magnitudes of the layers, i.e., B^{++}_{ij} and B^{++}_{ij}. Their energies are fairly closely matched, with B^{++}_{ij} being roughly between 10% and 50% higher. Thus, *there are only eight significantly different <u>concentric</u> bion solutions to be observed*.

The concept involved in "conversion" is that, if B^{++}_{25} forms, it quickly shrinks so that layer 5 becomes layer 4 or layer 3, and during this conversion its energy distortion is changing, along with its frequency, by constant interaction with the datum fluctuations, which can allow for small increases and decreases of energy. When this process is observed in an accelerator experiment, what is finally seen might be the π^+. However, the π^+ also has a very short life, with perhaps the $^{++}_{12}$ charge distortion shifting to $^{++}_{21}$ and finally combining into one layer to form U_2, the μ^+. The μ^+ is again unstable, and so the conversion goes down the E_0/ω ladder to the positron, which is in equilibrium with the datum fluctuations.

Figure 5.21.1 also reveals other pertinent bion properties. For example, the B^{++}_{45} / B^{++}_{45} pair apparently decay so quickly that what is observed is the bion conventionally designated B^+. Perhaps if the detectors were changed, the U_4 could also be observed.

Again, the B^{++}_{3j} / B^{++}_{3j} pair appear to be observed as either, D_s^+, D^+, or U_3 which is the τ^+ unon. The fact that the frequencies corresponding to U_4 and U_5 were estimated without being observed could explain why the predicted bions intersect so far above the related observed particles. Lower ω_i values would shift them downward.

There is an anomaly here; because, while in most cases, the conversion products are grouped fairly closely to the bion pair they originate from, the $K^{\pm}$ seems too far below the $B^{\alpha\beta}_{3j}$ to be part of that group. No explanation is known for the strange location of the $K^{\pm}$ on the E_0/ω ladder.

Finally, the $B^{++}_{i\,j}$ (12) / $B^{++}_{i\,j}$ (21) pair probably convert into the positron, but it is surprising that they have not been observed. Perhaps they have, but are so close to being a positron that it has been assumed that is what they are. It is clear that much more work remains in this process of identifying and classifying the various bions. Table 5.22.1 lists the concentric bion energy ladder.

TABLE 5.22.1

CONCENTRIC BION GROUND STATES

$\omega \left(\frac{\text{rad}}{\text{sec}}\right)$	Concentric Bion	Observed Particle	E_0 (ergs)	E_0 (MeV)
1.168×10^{25}	B^{12}_{45}		1.232×10^{-2}	7,690
9.680×10^{24}	B^{21}_{45}		1.021	6,373
8.508		(U^+_4)	8.972×10^{-3}	5,600
8.020		B^+	8.458	5,279
4.180	B^{12}_{3j}		4.514	2,817
3.200	B^{21}_{3j}		3.375	2,107
2.870		D^+_s	3.154	1,969
2.840		D^+	2.995	1,869
2.700		τ^+	2.847	1,777
7.500×10^{23}		K^+	7.910×10^{-4}	493.7
2.820	B^{12}_{2j}		2.974	185.6
2.120		π^+	2.236	139.6
1.988	B^{21}_{2j}		2.044	127.6
1.605		μ^+	1.693	105.7
1.370×10^{21}	B^{12}_{1j}		1.445×10^{-6}	0.901
9.360×10^{20}	B^{21}_{1j}		9.871×10^{-7}	0.616
7.764		e^+	8.187	0.511

Concentric Trions

The existence criterion for trion layer/charge forms is,

$$J = \frac{q_i^2}{e^2} J_i + \frac{q_j^2}{e^2} J_j + \frac{q_k^2}{e^2} J_k + 2\frac{q_i q_j}{e^2} J_{ij} + 2\frac{q_i q_k}{e^2} J_{ik} + 2\frac{q_j q_k}{e^2} J_{jk} = 0.25 \quad .$$

A typical example is the T_{123}^{-++122}, which leads to,

$$J = \frac{1}{9} J_1 + \frac{4}{9} J_2 + \frac{4}{9} J_3 - \frac{4}{9} J_{12} - \frac{4}{9} J_{13} + \frac{8}{9} J_{23} \quad .$$

From the tables in Appendix I, the J_i's and J_{ij}'s for the range of ω from 8×10^{23} to 2×10^{24} are repeated in Table 5.22.2. The above sum is found for each ω, and plotted in Figure 5.22.2. The crossover point is at $\omega = 1.40\times10^{24}$ rad/s. The accurate non-graphical, program value is $\omega = 1.4051\times10^{24}$.

TABLE 5.22.2

ω	J_1	J_2	J_3	J_{12} & J_{13}	J_{23}	J
8×10^{23}	9.704×10^{-4}	0.1417	0.2691	9.711×10^{-4}	0.1517	0.317
1×10^{24}	7.763×10^{-4}	0.1228	0.2646	7.768×10^{-4}	0.1309	0.288
1.5×10^{24}	5.176×10^{-4}	0.0913	0.2636	5.177×10^{-4}	0.0963	0.243
2×10^{24}	3.882×10^{-4}	0.0723	0.2584	3.883×10^{-4}	0.0756	0.214
Coefficient	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$-\frac{4}{9}$	$\frac{8}{9}$	

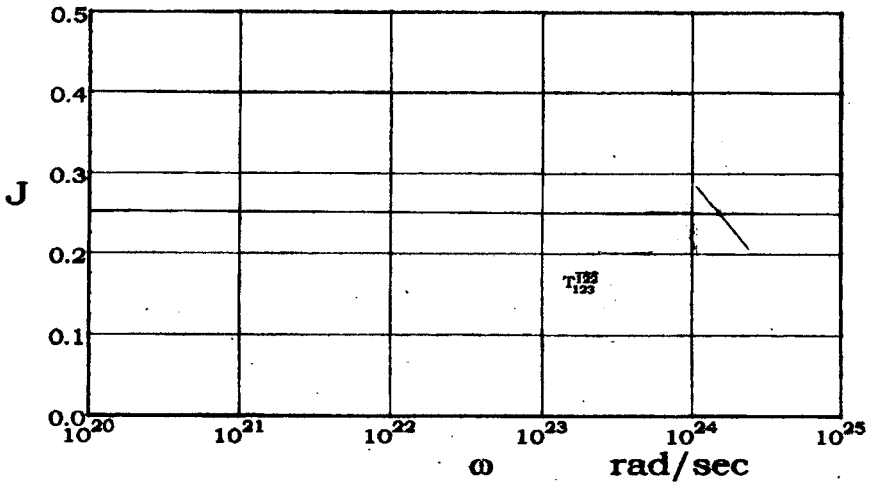


Figure 5.22.1. Predicted T_{123}^{-++122} Frequency.

Thus, the ω predicted for the $T_{123}^{\bar{1}22}$ is just 1.43% lower than the *measured* proton frequency, $\omega = 1.42548 \times 10^{24}$, so the $T_{123}^{\bar{1}22}$ may be tentatively identified as the proton.

When the existence criterion is applied to the remaining trion layer/charge forms depicted in Figure 5.10.1, the total energy of Eq.(5.18.3) is almost always too low to cross over the $J = 0.25$ level. This indicates that most other trions are orbiters requiring the added orbital kinetic energy to make up the total.

5.23 **The Self Consistent Adjustment**: The close agreement between the predicted and measured proton frequencies encourages confidence in the method, and invites effort to find the source of the error. Since the system, to this point, just uses three given numbers (the e, μ and τ frequencies) to determine a fourth (the p frequency), if properly set up, it should be *self-consistent* and without error. One of the many possible sources of error could be the specification of the unon preferred frequencies. Of the four particles, the e and p are *stable* (the only stable particles known), and their rest energies can be measured with great accuracy. On the other hand, the μ and τ are very short-lived and their frequencies quickly shift lower during their decay, making their rest energies difficult to measure accurately. *With a small adjustment of the preferred frequencies of layers two and three the system becomes self-consistent.*

The reality is that *both might need correcting*, but a simple calculation indicates that to correct the 1.4% error, the second layer ω need only change a comparable amount, whereas the third layer ω, because of its location, must change about 40%. It is unreasonable to suppose that the *measured* τ energy is anywhere near that low, so only the second layer ω will be adjusted. The *corrected* preferred frequencies are listed in Table 5.23.1. *Future concentric bion and orbiter trion work might show both need changing.*

The corrected ω_2 is increased by 2.5%, which has only a small effect on the bion predictions expressed in Figure 5.22.1 and Table 5.22.1. For example, the ω for B_{23}^{+-21} increases only 1.7%, which does not affect the relationships in Figure 5.22.1 significantly.

TABLE 5.23.1

SELF-CONSISTENT PREFERRED FREQUENCIES

Layer	ω_i (rad/sec)	r_i (cm)
1	7.76344×10^{20}	3.52243×10^{-14}
2	1.62973×10^{23}	1.67796×10^{-16}
3	2.69981×10^{24}	1.01289×10^{-17}
4	8.508×10^{24}	3.214×10^{-18}
5	1.580×10^{25}	1.731×10^{-18}

5.24 **Self Consistent Proton Structure**: The overall analysis of proton structure can now be presented in *self-consistent* form. It follows a procedure somewhat different from the one used in the *prediction* calculation. Once the prediction shows that the layer/charge combination indicates a particular observed particle, as many characteristics of that particle are tested as possible.

The *measured* E_0 for the proton is 938.27203 MeV or 1.5032818×10^{-3} ergs. To satisfy Eq.(5.21.2), it must have a frequency $\omega = 1.4254856\times10^{23}$ rad/sec. If it is to be a real particle, at that frequency it must also satisfy Eq.(5.18.3) for the three layers 1,2,3, so that,

$$E_0 = E_1 + E_2 + E_3 + E_{12} + E_{13} + E_{23} \quad . \qquad (5.24.1)$$

With ω_i's from Table 5.23.1:

$$\frac{\omega}{\omega_1} = 1.8361527\times10^{3} \quad , \quad \frac{\omega}{\omega_2} = 8.7467592 \quad , \quad \frac{\omega}{\omega_3} = 0.52801199$$

Invoking Eq.(5.17.3):

K_1 = out of range , K_{12} = out of range

$K_2 = 4.8729207\times10^{4}$, K_{13} = out of range

$K_3 = -0.80028312$, $K_{23} = 2.4364203\times10^{4}$

Using the accurate, non-graphical program and the approximations in Eqs.(5.20.1) to (5.20.6):

$$J_1 = 5.44617\times10^{-4} \quad , \quad J_{12} = J_{13} = 5.4482266\times10^{-4}$$
$$J_2 = 0.09611 \quad , \quad J_{23} = 0.10153$$
$$J_3 = 0.26428$$

Eq.(5.16.7) gives $r_e = 1.9183736\times10^{-17}$, so the energies of Eqs.(5.18.6) and (5.18.8) are:

$$E_1 = 3.63871\times10^{-7} \quad , \quad E_{12} = E_{13} = -1.45604\times10^{-6}$$
$$E_2 = 2.56853\times10^{-4} \quad , \quad E_{23} = 5.42677\times10^{-4}$$
$$E_3 = 7.06299\times10^{-4}$$

Then, Eq.(5.24.1) leads to $E_0 = 1.50328\times10^{-3}$ ergs, to the accuracy limit of the accurate program.

Next, the program indicates that:

$$I_1 = 5.44617\times10^{-4} \ , \ L_1 = 1.48304\times10^{-7} \ , \ M_1 = 2.72309\times10^{-4}$$
$$I_2 = 0.11489 \ , \ L_2 = 5.42896\times10^{-3} \ , \ M_2 = 5.74709\times10^{-2}$$
$$I_3 = 0.60451 \ , \ L_3 = 7.92868\times10^{-2} \ , \ M_3 = 0.30228$$

These are used with Eqs.(5.16.8), (5.19.13) and (5.19.14) to calculate the layer center potentials ϕ_{0i}, magnetic moments μ_{si} and spin constants K_{si}:

$$\phi_{01} = -1.28222\times10^{3} \ , \ \phi_{02} = 5.40984\times10^{5} \ , \ \phi_{03} = 2.84646\times10^{6}$$
$$\mu_{s1} = 2.46567\times10^{-20} \ , \ \mu_{s2} = 6.98598\times10^{-23} \ , \ \mu_{s3} = 2.54053\times10^{-23}$$
$$K_{s1} = 6.54492\times10^{-32} \ , \ K_{s2} = 1.85437\times10^{-34} \ , \ K_{s3} = 6.74362\times10^{-35}$$

The next step is to try to visualize how these three layers with their spins and magnetic moments arrange themselves. The total proton spin is 1/2 so clearly two of the layers align themselves to cancel their spins. In all likelihood, the two *inner* layers (2,3) interact to do just that, at the same time *reducing the net magnetic field they compose*. They are the layers of greatest energy, whereas the outer layer is very much like an electron with 1/3 the charge, and energy very much smaller than that of the two inner layers.

In measuring the proton's magnetic moment, the outer layer will precess independently at a high frequency commensurate with that usually observed for electrons. This has probably been ignored in the past. If the two inner layers were to lock together at 180^{o}, the proton's measured magnetic moment would be the difference between μ_{s2} and μ_{s3}, or 4.44545×10^{-23} ergs/hlG, a value 11.2% lower than the observed $\mu_{sp} = 5.0056 \times 10^{-23}$ ergs/hlG. However, any misalignment results in a *larger* magnetic moment; so, if layer 3 is normally precessing in the magnetic field of layer 2, when no external field is applied, then that, coupled with a possible random disturbance from the datum fluctuations, could account for the higher measured value. The misalignment angle can be calculated from the shell structure.

5.25. **Orbiters**: Analysis of the orbiters, is considerably more difficult; particularly that of the *inside* orbiters, since they have *overlapping* shells on which is superimposed a dynamic orbiting motion. *The proper analysis would involve solving for the full field, transient case.* It is clear that to do this for all the possible ground state configurations and then identify the observed particles accordingly is a task that could take many man-years.

Outside Orbiters

In Chapter 7 it is shown that, although even there the *full field* solution of the two particle hydrogen structure is the basic approach, it is possible to use a "separatist" approximation, i.e. a two body, planetary calculation that gives excellent results. At first glance, the *outside* bion orbiters, for example $B_{i\,i}^{+-\alpha\alpha}$, suggest the hydrogen atom and positronium as models; but it is not obvious that the same analytical approach can be used. Close proximity of the two layers adds many complications. Nevertheless, the fact that positronium lends itself to similar macroscopic analysis is encouraging, but only preliminary attempts to analyze the *outside* orbiter bions have been made. Table 5.25.1 lists their *measured* properties.

Inside Orbiters

Unfortunately, the case for the inside orbiters, whose *measurements* are listed in Table 5.25.2, is quite different; since the $B_{ij}^{+-\alpha\alpha}$ shells will surely overlap and require a full field solution. At this time, nothing

more is known about the structure of these particles and it probably will be a long time before such transient solutions are available.

TABLE 5.25.1

OUTSIDE ORBITER BIONS

Observed Particle	E_0 (ergs)	ω_ℓ (rad/sec)	E_0 (MeV)
π^0	2.1626×10^{-4}	2.0506×10^{23}	1.3498×10^{2}
η	8.7711×10^{-4}	8.3172×10^{23}	5.4745×10^{2}
η'	1.5345×10^{-3}	1.4551×10^{24}	9.5777×10^{2}
η_c	4.7742×10^{-3}	4.5272×10^{24}	2.9798×10^{3}
Υ	1.5157×10^{-2}	1.4373×10^{25}	9.4604×10^{3}

TABLE 5.25.2

INSIDE ORBITER BIONS

Observed Particle	E_0 (ergs)	ω (rad/sec)	E_0 (MeV)
K^0 $\begin{cases}K_S^0\\K_L^0\end{cases}$	7.9736×10^{-4}	7.5610×10^{23}	4.9767×10^{2}
D^0	2.9873×10^{-3}	2.8328×10^{24}	1.8645×10^{3}
B^0	8.4582×10^{-3}	8.0204×10^{24}	5.2792×10^{3}
B_S^0	8.6026×10^{-3}	8.1574×10^{24}	5.3693×10^{3}

Trion Orbiters

Except for the clear cut case of the stable proton, the trion analysis is in an even more primitive state than that of the orbiter bions; because only a few of the trions appear to be simple, concentric layerons. Even in their ground states, most of the trions are probably orbiters. Only the briefest sketch of the trion family identification will be attempted here.

The approach taken is to *first assume that all trions are simple, concentric particles*. The calculations described previously are carried out for each one. The earlier analysis of the proton, $T_{123}^{\bar{1}22++}$, is typical of the procedure. When this is done for all the configurations represented in Figure 5.10.1, extended to include the fifth layer, *only those structures with numerical values listed in Table 5.25.3 yield J values large enough to cross the 0.25 line*. This indicates that the concentric structure has energy (frequency) too low to represent a physical particle solution. On the other hand, all configurations *below* the dashed line in Table 5.25.3 have energies *much greater* than any observed trion, so they are probably too compacted to do more than just explode and convert. In the $(\bar{1}22^{-++})$ column, the proton and its two siblings are essentially the same particle, with 124 and 125 both converting almost instantaneously to the proton by shifting the inner layer out to the three position. In the $(\dot{2}1\dot{2}^{+-+})$ column, the 345 configuration is down 3% from Λ_c^+ and 10% from Σ_c^+. Similarly, in the $(\dot{2}2\dot{2}^{+++})$ column, the energies (frequencies) indicated are about 20% low for Δ^{++} and 60% low for Σ_c^{++} (no other particles with charge 2e have been observed). Although the $T_{345}^{\bar{1}\bar{1}\bar{1}-}$ structure has *more* energy than Δ^-, Σ^-, Ξ^- or Ω^-, and could appear to be any one of these with less energy during the decay process, this does not seem to be a good

TABLE 5.25.3

CONCENTRIC TRION FREQUENCIES

$T_{ijk}^{\alpha\beta\gamma}$	$\bar{1}\bar{1}\bar{1}^{---}$	$\dot{2}\bar{1}\bar{1}^{+--}$	$\bar{1}\dot{2}\bar{1}^{-+-}$	$\bar{1}\bar{1}\dot{2}^{--+}$	$\dot{2}\dot{2}\bar{1}^{++-}$	$\dot{2}\bar{1}\dot{2}^{+-+}$	$\bar{1}22^{-++}$	$\dot{2}\dot{2}\dot{2}^{+++}$
123			n?				p	1.47×10^{24}
124							1.47×10^{24}	1.47×10^{24}
125							1.46×10^{24}	1.48×10^{24}
134							1.88×10^{25}	1.90×10^{25}
135								
145								
234							1.67×10^{25}	2.10×10^{25}
235							2.14×10^{25}	
245								
345	4.48×10^{24}	3.38×10^{24}				3.38×10^{24}	2.20×10^{25}	8.40×10^{25}

approach. Similar reasoning applies to the T^{+--}_{345} configuration and $\Delta^0, \Lambda^0, \Sigma^0$ or Ξ^0. Except for these last two unexplained cases, all observed trions, including those ($\overset{+\,+\,+}{222}$) listed, appear to be orbiters; since *the only way to bring their energies up to the observed values is to include their orbiting energies*. This has not yet been worked out for all the reasons given earlier in discussing the orbiter bions.

There are roughly 16 trions observed in the ground state (spin $\frac{1}{2}$) and 10 in the spin $\frac{3}{2}$ state that should be associated with the sixty or so low energy categories in Table 5.25.3. By far the most important of these are the proton and neutron. There is little chance of a full field, transient analysis of the neutron in the near future; but, based on a "guess" that the so called "strong" interaction acts similarly for both protons and neutrons because each of their two outer layers have a similar form, the neutron is indicated in Table 5.25.3 as being in the ($\overset{-\,+\,-}{121}$) column. This is not absolutely necessary; since it might only be the *outer* layer of each trion that determines the "strong" interaction. In that case, the neutron might match a different configuration in Table 5.25.3.

One problem that arose in the quark model was the three parallel quark spins in a particle such as the Δ^{++}, which was assigned three identical u quarks, a clear violation of the Pauli exclusion principle. To resolve this problem, **the property of quark color was adopted**. *The exclusion principle is a manifestation of the single solution principle, i.e. no two identical solutions of the field equations can exist in the same place at the same time*. Referring back to Figure 5.2.1, three layers with parallel spin clearly do not represent identical solutions of the field equations, so **no need for color arises.**

5.26 **Resonances**: Most of the spin zero bions and the spin ½ trions are conventionally called "stable" particles (with half lives $\tau > 10^{-19}$ sec), even though the proton (anti proton) is the only particle among them with observed stability. In addition, many other bions and trions with higher energies and spins are seen. They are called "resonances", and represent extremely short lived transition states ($\tau < 10^{-20}$ sec) where the interactions are so violent that there is even less time to settle briefly into a pseudo-stable form. Little can be said about these resonances, at the present state of knowledge, except that some are probably inside orbiters, where $n_\psi > 1$, and involve increased angular momentum. Others, like the "mesons" with spin 2, are most likely not bions but complex "systems" of several layers

paired in various ways. While all these resonances are interesting as aids to understanding the nature of particles, they have little to do with the structure of matter in the form of nuclei and atoms, etc. Thus, lack of detailed knowledge of them will not detract from the picture being constructed here.

5.27 **Change and Interaction Processes**: There are two processes by which structures such as particles, nuclei or atoms decay or change, *conversion* and *separation*. Although they overlap somewhat, conversion is essentially a process in which a single identifiable structure changes into a different identifiable structure, with the addition or subtraction of energy, another particle or both. Separation is the breaking apart of an identifiable structure composed of particles, into two or more identifiable structures or particles, usually with a release of energy. There are numerous overlap cases, particularly in high energy collision experiments.

Separation is generally not found in particle decay; because, as mentioned in the discussion on quarks in Section 5.6, neither layers removed from multi-layer particles nor the remaining layers are solutions of the field equations. On the other hand, separation accounts for the majority of changes in atomic nuclei, as discussed in Chapter 6.

Of the conversion processes, only a few occur in nuclear and atomic changes, and nuclear conversion, i.e. neutron or proton conversion, will be considered in Chapter 6.

At present, certain conventions are observed in describing *interaction* processes. For example, bion-bion, bion-trion, and trion-trion interactions are designated "strong". Slow interactions that often have unons and c-ons as by products are designated "weak". It is likely that these designations will be replaced by others more specific when the various deterministic processes are better understood. Keep in mind that, as indicated in Chapter 12, *the concept of "exchange forces" is not accepted as valid*, but the computational methods, Feynman diagrams, etc., are probably valid as a form of bookkeeping to keep track of the energy and charge distortions during redistribution.

Because all layerons are essentially bubble like entities, with all of the attendant softness and flexibility, prone to oscillate, and susceptible to breaking up, it is not surprising *that when the ether is sufficiently stirred up by a cataclysmic collision of particles or even just a very rapid decay*, a number of smaller new particles can be formed. In fact, if enough energy is available, any or all of the various kinds of particles can form, and here this kind of formation process will be called "splatter". In splatter, the original "formless" cloud of energy is broken up into several distortion packets of various sizes and

irregular shapes. These tend to ooze out and are gradually formed by ℓ-waves that determine the particular particles being *created*.

5.28 **Proton Structure Experiments**: Experimentally, some idea of the overall dimensions of *atomic nuclei* can be found from electron-nucleus scattering, which provides one of the main sources of data on nucleus size and charge distribution[1]. The technique involves measuring the scattering cross-section, and from that, determining the form factor of the interaction. Under certain conditions of momentum transfer, the form factor is the Fourier transform of the nucleus charge distribution. Thus, the rough structure of the nucleus, its mean square radius, its general shape, etc., is found. During the 1960's, the same method was used in electron-nucleon (the term used to describe neutrons or protons in the atomic nucleus) scattering experiments, with somewhat surprising results. Beginning with Hofstadter, et al[2], numerous data were gathered, and a generally accepted position on proton structure was established by 1967 [3]. It took the form of four form factors, two for the proton and two for the neutron. The data appeared to fit <u>*one*</u> form for the proton electric and magnetic moment factors and the neutron magnetic moment factor. The neutron electric form factor is approximately zero, but not exactly, because of its distributed charge density. The <u>*one*</u> form factor had the shape known as the *dipole* curve, and it implied a charge distribution,

$$\rho_d = \rho_0 \varepsilon^{-\alpha r} \quad , \quad \alpha = 4.27 \text{ f}^{-1} \quad , \quad \rho_0 = 5.274 \times 10^{-9} \text{ hlc/f}^3 \quad . \quad (5.28.1)$$

This was a surprising result, representing an exponentially decreasing charge density as r increases, with no other structure. Its rms radius was $\sqrt{12}$ / E = 0.80 fermi, which was not unreasonable; but, if it were regarded as an actual charge distribution, it led to totally non-physical results. For example, using ρ_d from Eq.(5.28.1) as the

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1. H.Uberall, <u>Electron Scattering From Complex Nuclei</u>, Academic Press, N.Y. (1971). G.R.Satchler, <u>Introduction to Nuclear Reactions,</u> John Wiley & Sons, N.Y. (1980).
2. R.Hofstadter, et al, Phys.Rev.Lett. **6**, 290 (1961); <u>Electron Scattering and Nuclear and Nucleon Structure,</u> W.A.Benjamin, N.Y. (1963).
3. G.Weber, <u>Int. Symp. on.Electron and Photon Interactions at High Energies</u>, p. 59, Stanford, California (1967). D.H.Perkins, <u>Introduction to High Energy Physics</u>, 2nd Ed. p. 282, Addison-Wesley Publ.Co. Reading, Mass. (1982). F.Halzen & A.D.Martin, <u>Quarks. and Leptons</u>, p. 172 ff, John Wiley & Sons, N.Y. (1984).

source in Poisson's equation, the first integration becomes,

$$\frac{d\bar{\bar{\phi}}}{dr} = \frac{2\rho_0}{\alpha^3}\left(\frac{\alpha^2}{2} + \frac{\alpha}{r} + \frac{1}{r^2}\right)\varepsilon^{-\alpha r} \quad . \qquad (5.28.2)$$

From this, the electric energy density $\varepsilon = (\nabla\bar{\bar{\phi}})^2 / 2$ can be integrated over all space and the resulting proton energy is infinite.

On the other hand, if ρ_d is thought of as a smoothed out average due to the fact that *the electron is a very large probe* with which to measure the much smaller inner nucleon structure, then the distribution of Eq.(5.28.1) is understandable. However, even using the ether theory, this smoothed out form is not easily calculated, because the immense energy of the probing electron brings enough distortion to the central region to actually change the shell structure of the nucleon during the measurement.

Recent Developments

In the 1980's the *experimenters* reached the energy level of the bombarding electrons that causes splattering and the creation of strange quarks. Since the Standard Model is too restrictive, and the quantum mechanical analysis has an unpicture of the structure and behavior of the particles quite different from the one presented in this book, the *theoreticians* invented a seething sea of quarks, forming and disappearing, to account for the presence of the strange quarks. They added the concept of color to alleviate the effect of the restrictive nature of the quark picture. The analysis has grown more and more elaborate, and the results less and less reasonable.

Meanwhile, the experimentalists have improved the data bank by adding several variations of the earlier experiments. One of the better advances has been to use beams of polarized electrons. Each of the new variations has improved the data in both accuracy and range.

In the last year or so, the *conventional analysis* has led to calculations that show the quarks as carrying only a few percent of the proton mass with most of the mass now assigned to the "gluons" and other spirits in the seething sea of the non-ether.

In clear contrast to this mathematical monument, the simple solution of Maxwell's equations and the deterministic calculations presented in this Chapter 5, show **all** of the mass to be in the electromagnetic energy density of the layers and interaction fields.

5.29. **The Neutron**: Little has been said about the neutron except for two brief mentions in Section 5.25. The fact that it is probably an orbiter makes its analysis very difficult. However, experimentally, its

properties are much like the proton. The *free* neutron has a mean life of about 15 minutes. Being slightly more energetic than the proton, it then converts into an electron, a proton and an electron anti-neutrino. Inside the nucleus it combines with a proton or another neutron and manages to remain stable in most of the nuclear arrangements. In that state, the neutron and the proton are the building blocks of matter, as represented in the atomic nucleus, which is the subject of the next chapter. Almost nothing more need be said about any of the other bions or trions from this point on.

CHAPTER 6

THE NUCLEUS

6.1 **Introduction**: By now, it cannot have escaped the reader that although an intuitively pleasing, visualizable picture of nature's operation has been provided by the ether, the availability of only a few solutions of the field equations denies the kind of detailed picture that must be obtained. Nevertheless, even the picture so far presented represents a substantial advance. Having put space, time, coordinates and field equations in a non-paradoxical perspective, the physical operation of particles is understandable on a simple cause and effect basis. Likewise, the deeper physical natures of charge and energy are apparent. Moreover, the inner workings of the electron as an extended entity are clear, illuminating the essence of particle momentum and kinetic energy. Ultimately, all of this leads to the understanding of a deterministic set of "systems" using the ordinary laws of mechanics. It also leads to a statistical variation in observed results as indication of the presence of datum fluctuations in the ether. Finally, in Chapter 7, all of the results of the statistical quantum mechanics, conventionally thought to be the ultimate basis of physics, will be seen to be available as a complementary adjunct to the deterministic description of *ensemble* phenomena, just as statistical mechanics was in earlier times. Thus, any result previously found through the Schroedinger and Dirac equations, with some changes that will be apparent later on, can be taken as a correct *ensemble* characteristic in the ether picture, with the expectation that the more direct deterministic analysis of the individual entities involved in each detailed action in an experiment will yield even further understanding of the physics not possible with the ensemble analysis. Completing the deterministic picture is the immediate and ultimate goal.

The next higher level of complexity of "systems" is encountered in atomic nuclei. Since 1910, it has been known that matter is composed of atoms that are essentially miniature "solar" systems with a massive nucleus at the center orbited by one or more electrons. In Section 2.22 it was stated that solids, liquids, gasses and plasmas could be mostly ignored in the quest for the basic structure of matter, being too voluminous and too far removed from the fundamentals. In some respects, the same condition applies also to the nucleus. Whereas the study of nuclear physics has been predominantly one of trying to discover the motions, positions, affinities, and excitations of the nucleons, including the various energy states, shell structures, etc., these contribute primarily towards understanding of processes for large scale energy production and control, both man made and in nature. In this chapter, the more fundamental side will receive all the

attention; so the nuclear *lowest energy rule* and the nature of the conventionally designated "strong" and "weak" interactions are investigated.

6.2 **The Hydrogen Nucleus**: The simplest nucleus is that of the hydrogen atom, which has a single proton orbited by a single electron. As explained in Chapter 5, the proton is a stable, concentric trion, with a well defined structure. There is little more to be said about this simple nucleon, and the hydrogen atom is analyzed in Chapter 7 using ordinary mechanics.

6.3 **Deuterium**: The next simplest case is found in the deuterium atom, which consists of a nucleus made up of one proton and one neutron, bound together, around which orbits a single electron. Deuterium is a stable isotope of hydrogen, that immediately brings out the problems yet to be solved. In Chapter 5 it was indicated that the detailed structure of the neutron is not known. This stymies any attempt to specify the way in which the proton and neutron are held together in the nucleus. It is known that the binding process is essentially the same for p-p, p-n and n-n, if the coulomb repulsion of the protons for each other is neglected. Electron bombardment of much larger nuclei indicates that the average nucleon spacing is about 2 fermi; but if the neutron were the same size as the proton, it would make more sense that the spacing might be closer to 0.7 f. In Chapter 5 it was implied by the trion calculations that the neutron could be an orbiter, so that if the orbit radius was about 1.7f, *the spacing could be accounted for by the electromagnetic interaction*. Here it is assumed that something similar to this is the correct picture.

At present, the usual designation for nucleon attraction is the "strong" interaction. It is described as an "exchange force" with a *virtual* π^o or $\pi^{\pm}$ being passed back and forth between two nucleons. This picture is also conventionally used for the electromagnetic interaction with a *virtual* photon replacing the bions as intermediary. Later, in Chapter 12, the nature of the electromagnetic interaction will be uncovered, and it definitely does not involve a photon exchange mechanism. It will be shown there, that all ether "forces" are similar, and are basically electromagnetic. So, not only will the idea of a "force" with a *virtual* particle intermediary be abandoned here, but the presence of π bions in the residue of collider experiments is easily understood as simple particle *creation* and not evidence of their momentary presence in the deuteron.

The point should be made that the so called "strong" interaction is not actually very strong. The deuteron provides the best evidence, being held together with a binding energy of 2.224 MeV. This can be compared with the "binding energy" of a positron-electron

combination 2 fermi apart, held by a purely coulomb force, that can be separated by supplying work equal to 0.72 MeV. So, the nucleon binding energy is only 3 times as much as the electrostatic binding energy. In fact, *it isn't the strength of the nuclear force that is noteworthy, but the constancy of the nucleon spacing* of about 2 fermi. To understand the basic adhesion, the sum of the interaction energies of all shells in both trions will at first become more negative as the two nucleons approach each other, so that the negative interaction energy reduces their total energy and they are attracted. Pushing them even closer results in a smaller negative interaction energy, because of their large positive layers, and they are repulsed. Thus, they naturally seek a separation distance.

It was once hoped that studying the structure of the deuteron would lead to *detailed* knowledge of the potential describing the "nucleon interaction". This hope has not yet been realized. The physical facts that must be reconciled to any theory of the deuteron ground state are few. First, its binding energy is $E_b = 2.224$ MeV. Second, the angular momentum quantum number is 1. Third, the nuclear parity is even. Fourth, its magnetic moment Is +0.8574 nuclear magnetons instead of the simple sum of the proton and neutron moments (0.8794); and fifth, its electric quadrupole moment is $Q = +2.7 \times 10^{-27} \text{cm}^2$. Statistical quantum mechanics has provided a first approximation form. The only bound state is $^3S_1$. This first approximation neglects the difference between the sum of the magnetic moments of the two nucleons. It also cannot include the minute quadrupole moment, since it is a spherically symmetric approximation. The square well solution of Schroedinger's equation gives an approximation to the potential well depth magnitude of about 38 MeV. Since the binding energy is only 2.224 MeV, it is a weak bond, and no further bound states are found. A second approximation is generally carried out by adding in a small percentage of the $^3D_1$ state through the modification of the potential to include spin-dependent terms.[1] The probability that the deuteron is in the 3D state is about 4 percent. Several different potentials can be made to yield the correct quadrupole moment; so this approach does not produce a clue to the true potential.

Ruminating on the deuteron structure just described, the combination of a very small amount of $^3D_1$ to account for the magnetic moment and quadrupole moment was chosen on the basis of point particles with spin, energy, and charge. However, it is well known that both the neutron and proton are *extended* distributions, with more complex structures. In the deuteron, and any other nucleus,

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I. L.R.B.Elton, Introductory Nuclear Theory, p 83 ff, Interscience Publ. Inc. N.Y. (1959).

the nucleons are close enough that those complex structures overlap. Thus, the small quadrupole moment and the 2.5 percent change in magnetic moment could be the result of a configuration where each nucleon's charge distribution is minutely distorted along the axis between them. This makes it easier to see intuitively how the quadrupole moment can be positive, since the charge is elongated, whereas in the conventional form it is hard to visualize how a prolate charge deformation results from an orbiting motion.

The failure of conventional theory to establish the basic nucleon interaction extends into the analysis of scattering interactions involving other deuteron conditions. Ether theory considerations indicate that these conventional approaches are probably wrong, and that any success they may exhibit is coincidental. The full field, transient solution of the neutron could resolve this adhesion question.

6.4 **Helium**: Two other simple, stable nuclei bring out certain characteristics of nuclei in general. The first is the isotope of helium with 1 neutron and 2 protons as its nucleus. *In all other stable nuclei, the number of protons never exceeds the number of neutrons*.

The second, the helium nucleus, has 2 neutrons and 2 protons. When stripped of its two orbiting electrons, the nucleus with +2e charge is termed an α particle; and it represents a closed energy level, since only 2 neutrons with opposing spins and 2 protons with opposing spins are allowed in one energy configuration because of the single solution rule (Pauli's exclusion principle).

The preceding few smaller nuclei give a clear enough picture of the *fundamental* problem that remains. The rest of this chapter is a short summary of present conventional visualization of general nuclear structure.

6.5 **Nuclear Structure**: A fairly complete description of the present level of understanding of the structure of the nucleus is available in a number of good text books on the subject.[1] Therefore, only a quick sketch is needed to bring the discussion to the level desired. Since 1932 it has been understood that, except for the simplest form of hydrogen, nuclei are composed of two kinds of nucleons, protons and neutrons, clustered like a handful of marbles. Those nuclei occurring in nature tend to have almost equal numbers of each until the cluster has about 20 nucleons, after which the neutrons outnumber the protons in a gradually increasing ratio. Artificially produced nuclei deviate only nominally from this. Attempts to visualize details of the

1. R.B.Leighton, Principles of Modern Physics, McGraw-Hill Book Co. N.Y. (1959).
L.R.B.Elton, Introductory Nuclear Theory, Interscience Publishers, N.Y. (1959).
A,.Beiser, Perspectives of Modern Physics, McGraw-Hill Book Co. N.Y. (1969).
R. Eisberg & R.Resnick, Quantum Physics, John Wiley & Sons, N.Y. (1974).

structure and motions in these clusters have been numerous, but only limited success, in any rigorous way, has been attained. So far, attempts to describe the "strong" interaction with a potential have been thwarted by the small number of available simple cases and their lack of many distinguishing characteristics. Nucleon spins also influence this "strong" interaction, so a simple potential is not the complete answer.[1]

In spite of these difficulties, a surprisingly good picture of the structure of nuclei is available today. The backbone of this understanding is obtained by simply accepting the existence of the strong interaction, and then, using the energy relationship of Eq.(3.6.4) and the conventional approach to energy and force, considering the stability of a given cluster of nucleons. The effective radius of a neutron or proton is about 1 fermi (10^{-13} cm); and, except for the smallest nuclei, the clusters are approximately spherical. The close packing results in a fairly constant density of nucleons per unit volume, so the spherical clusters have a radius of about,

$$R \cong 1.07\ A^{1/3} \quad , \qquad (6.5.1)$$

where A is the mass number or total number of nucleons. Ignoring, for the moment, the exact mechanism of attraction and repulsion of the individual nucleons, because the range of the strong force is essentially the same as the nucleon radius ($\cong$ 1 fermi), each nucleon is affected only by the 12 nucleons (or fewer in smaller clusters) that surround it. That is, they act just as packed marbles would act; those inside the cluster being surrounded by 12 others and those near the cluster's outer boundary surface interacting with about half that number. As a result, those nucleons inside the cluster feel no net strong force, and are free to move around unobstructed, except for proton charge effects, as long as they do not get closer to or farther away from their nearest neighbors.

These clusters are by no means static, but rather are in relatively violent motion internally. This can be crudely but effectively visualized by thinking of a handful of greased marbles enclosed in a rubber balloon. If the surface is manipulated, the marbles can move about freely along various twisted paths. In the nucleus, the motion is frictionless and much less inhibited. The picture here is quite similar to that of conduction electrons moving in a metal lattice except that here the snaking, turning paths are not centered on fixed points as in the lattice. Instead, the nucleons are twisting and turning about adjacent nucleons. In addition to the motion resulting from its own kinetic energy, each nucleon is also subjected to the random

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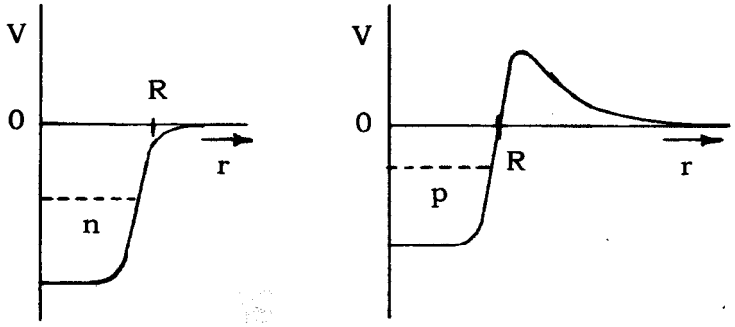
1. J.J.de Swart,et al, Few Body X, **1**, 299c, Ed.B.Zeitnitz, North-Holland, (1984).

fluctuations of the datum ether, which accounts for the statistical aspects of nuclear operation. Whereas the nucleon orbits are basically closed, the perturbations caused by the datum fluctuation can induce a nucleon to attempt to breach the outer boundary. Only then will the outbound nucleon feel a significant force.

Consider an outbound neutron for example. At the center of the nucleus there is no net force acting on it. As the potential escapee approaches the average boundary surface, the number of adjacent nucleons diminishes until, when it is centered on that boundary, there are just 6 or fewer within range of the strong interaction, and all of these are located on the inboard side of the neutron leaving. Therefore, as the neutron traverses a distance equal to its own diameter it goes from a condition of no net force acting on It to maximum retardation and back to zero force. If it has enough kinetic energy to do the required work against this force, it will be a free neutron. Figure 6.5.1a displays a typical potential energy curve for a neutron and a nucleus.

The condition for proton escape is moderately different. For, although a proton located near the center of the nucleus feels no net force, just as the neutron felt none, if that proton moves towards the outside, even before reaching the surface it begins to feel the effect of the coulomb repulsion due to all the other protons scattered evenly throughout the nucleus. The presence of that coulomb repulsion changes the potential curve for a proton to that shown In Figure 6.5.1b. Most notable is the barrier potential just outside R. To escape, a proton must have a kinetic energy large enough to overcome that barrier. Often, the emission of an alpha particle (with a similar kind of potential curve), of kinetic energy lower than the barrier over which it escapes, is observed. Conventionally this is explained as an example of "tunneling", a rather mysterious process involving wave



a. Neutron and nucleus b. Proton and nucleus

Figure 6.5.1 Potential energy curves.

functions. From the ether viewpoint, the explanation is such simpler. The potential curves of Figure 6.5.1 are understood to be nominal, and in actuality are continuously modified by the addition and subtraction of the ether's datum fluctuations, which increase and decrease the potential at any point randomly as a function of time. Thus, after a wait that depends upon the nominal barrier height and the coincidence of a negative fluctuation in V at the same point at which a potential escapee arrives at the same time, the effective barrier is lower than the particle's kinetic energy and the escape is accomplished.

In a similar fashion, any nucleus can be thought of as broken up into two or more arbitrary parts, each with its own kinetic energy, and there is then the possibility that the bonding between those parts could be broken. In principle, deciding what combinations of nucleons are stable and which ones will break apart with no more external influence than the datum fluctuations might appear to be possible only if the nature of the strong interaction were known so that a formal calculation of the orbit and energies of the possible configurations can be made. *In practice, however, the structural stability of nuclei can be explained quite well without that knowledge.*

6.6 **Nuclear Binding Energy**: The internal or rest energy of a nucleus or an atom can be measured by passing it through a mass spectrometer. As a result of the present state of the art, atomic energies are measured with a precision better than 1 part in 10^6. Fortunately, it is the energy <u>differences</u> between nuclei that are important, since the absolute values are not as well determined. For example, if a particular nucleus is made up of Z protons and N neutrons, the difference between the sum of their individually determined energies and the measured energy of the nuclear combination can be found accurately. In the following, the energy values will be given in MeV units, as discussed in Appendix G. Conventionally, it is <u>atomic</u> energies that are measured and tabulated, and the nomenclature used to designate a particular atom is ${}^{N}_{Z}X^{A}$, where X is the chemical element symbol (e.g., H, He, Li, etc.) and,

$$\begin{array}{ll} N = \text{number of neutrons} & A = \text{number of nucleons} \\ Z = \text{number of protons} & A = Z + N \end{array} \qquad (6.6.1)$$

In the following, only <u>nuclear</u> energies are used (the electron energies associated with Z are deleted), and a ground state nuclear

combination will be designated only by $\binom{N}{Z}$. For example, the proton and neutron rest energies are,

$$E\binom{0}{1} = E_p = 938.27 \quad \text{MeV} \quad , \quad E\binom{1}{0} = E_n = 939.57 \quad \text{MeV} \quad .$$

Often these energies are given in atomic mass units which are related to energy through,

$$1\ u = 1\ amu = 1.66957 \times 10^{-24} g = 931.502\ \text{Mev}/c_0^2$$

Now, going back to the idea of measuring the difference between the energies of, say, the boron nucleus, $E\binom{5}{5} = 9324.51$ MeV,[1] and its parts, $5E_p + 5E_n = 9389.26$ MeV; that *difference*, known as the *binding energy*, is $E_b = 9389.26 - 9324.51 = 64.75$ MeV. The meaning is clear. In order to separate those nucleons far enough apart so that they no longer have any significant influence on each other, an outside source of energy equal to E_b must be provided. In the absence of that extra energy, the boron nucleus has at least a chance of staying together. If the binding energy were a negative quantity, such a nucleus could not be assembled. In general, the binding energy of any nucleus is given by,

$$E_b = E_p Z + E_n N - E\binom{N}{Z} \quad , \qquad (6.6.2)$$

where E_p, E_n and $E\binom{N}{Z}$ are all *measured* values as defined above.

At this point it is possible to reconcile the measured binding energy of the nuclides (any single Z,N combination) with the physical picture presented in Section 6.5. To start off, almost exactly the same rationale can be exploited that is commonly applied to a drop of liquid. When a gas is cooled until it is transformed into an essentially incompressible fluid, during the transition from one phase to the other, a certain amount of energy per-molecule-converted has been removed. Called the heat of condensation, it clearly represents a binding energy of the molecules in the liquid. Where only a small amount of liquid is formed, its surface tension pulls it into spherical shape. In that case, the total binding energy of the resulting drop is proportional to the total number of molecules in it, reduced by an amount that accounts for the fact that the surface molecules are not as strongly bound, since they have fewer neighbors. Moreover, if the

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1. Nuclear data used in this chapter taken from: Nuclear Wallet Cards, Ed. by V.S.Shirley, & C.M.Lederer, Isotopes Project, Lawrence Berkeley Laboratory for U.S. Nuclear Data Network, January (1970).

liquid happened to be electrically charged, the binding energy would again be somewhat reduced because of the mutual repulsion. So far, all of these characteristics are duplicates of those described earlier as nuclear properties, and the equation,

$$E_b \cong \underbrace{a_1(Z+N)}_{\text{volume}} - \underbrace{a_2(Z+N)^{2/3}}_{\text{surface}} - \underbrace{a_3 \frac{Z(Z-1)}{(Z+N)^{1/3}}}_{\text{coulomb}} \quad , \qquad (6.6.3)$$

should give a reasonably good estimate of the nuclear binding energy. In fact, with the proper coefficients, it does. The liquid drop approximation comes close, but two further terms are usually appended to the equation to account for spin and de Broglie frequency properties of protons and neutrons not exibited by molecules in a liquid. These should be found by solving the Hamilton-Jacobi equations of motion of the nucleons inside the potential well, but the difficulty of the many body problem precludes this approach. An excellent approximation can be made, however, by using the Schroedinger equation approach, with a square well with rounded corners as the radial potential profile. Combined with the assumption that spin-orbit coupling is LS for small clusters, jj for large clusters and a mixed form for intermediate sizes, this "shell" approximation gives a set of states of motion that accounts for many of the observed nuclear characteristics. For example, since the protons see a well shape different from that seen by the neutrons, two quite separate sets of co-existent orbits are predicted. It should be stressed that these orbits are not necessarily planar or even confined to concentric shells, in spite of the grouping of energy levels that results, but could be long snakes, short snakes, and all varieties of 3 dimensional snake-like paths, only matching the de Broglie difference frequency to maintain the individual nucleon identities. The potential appearing in Schroedinger's equation, being essentially an on-off type of boundary, serves only to confine the orbits. Inside, the nucleons are freely moving, independent of each other, except to act in rapidly exchanged pairs as centers for directional shifts with full turning (frictionless), so that the motion is a special kind of controlled, free particle activity. There is no conflict between the visualization of these orbits and the liquid drop energies discussed above.

If it were possible to solve the deterministic ether field equations for a large nucleus, the fact that those equations can only allow one of each possible solution, i.e. the exclusion principle applies, ensures that at most only two nucleons with opposite spins can occupy any of the allowed stable closed paths. Furthermore, because the potential for the protons is shallower than that for the neutrons, the resulting two different sets of allowed orbits will have two neutrons or two protons, not one of each. Moreover, there will be fewer allowed proton levels than neutron levels, especially for the larger nuclei.

All of these characteristics are supported by the statistical quantum mechanics solution. Not only are the "magic numbers", i.e. the energy "shells" with 2, 8, 20, 28, 50, 82, 126, nucleons explained, but also the observed patterns of nuclear electric quadrupole moments, even-even and odd-odd stability levels, and finally the total nuclear angular momenta.

To find the two terms to supplement Eq.(6.6.3), an approximation within the shell approximation is useful. Since the two sets of particles are moving along frictionless paths with full turning, the only thing happening that resembles a collision is the perturbation due to the datum fluctuation, during which the particles themselves do not exchange energy with each other. They are, therefore, independent and have very long (endless) mean free paths. In fact the whole ensemble can be quite accurately described as constituting a Fermi gas enclosed in a spherical space. Here again, the standard mechanical solution is foiled by the many body aspect of the problem, but the statistical solution is well known.[1] It indicates that the nuclides with Z = N are more stable than their unbalanced isobars (nuclides with equal A) because the latter have nucleons of higher kinetic energy as expressed by a term proportional to $(Z-N)^2/(Z+N)$.

The final term injects an adjustment for the fact that, since the single solution (exclusion) principle applies, the like nucleons are paired in each state, so even-even nuclides having filled states, are more stable then mixed, and mixed are more stable than odd-odd. Thus, a term of the form,

$$\delta\left(\begin{smallmatrix}N\\Z\end{smallmatrix}\right) = \begin{cases} +1 \\ 0 \quad \dfrac{a_5}{\sqrt{Z+N}} \\ -1 \end{cases} \quad , \quad \text{if} \quad \left(\begin{smallmatrix}N\\Z\end{smallmatrix}\right) \begin{cases} \text{both even} \\ \text{mixed} \\ \text{both odd} \end{cases} \quad , \qquad (6.6.4)$$

is added to Eq.(6.6.3). It should be noted that the ether approach is essentially reversed from the conventional nuclear viewpoint; in that, the free motion of the nucleons is not ascribed to the single solution (exclusion) principle, but to the type of particles and their full turning. This, not the exclusion principle, permits using the shell and fermi gas approximations.

The final form of the binding energy equation Is found by combining all of the above terms, so that,

$$\begin{matrix} \text{volume} & \text{surface} & \text{coulomb} & \text{asymmetry} & \text{odd-even} \end{matrix}$$

$$E_b \cong a_1(Z+N) - a_2(Z+N)^{2/3} - a_3\frac{Z(Z-1)}{(Z+N)^{1/3}} - a_4\frac{(Z-N)^2}{Z+N} + \delta\left(\begin{smallmatrix}N\\Z\end{smallmatrix}\right) \quad . \qquad (6.6.5)$$

1. L.R.B.Elton, Loc.Cit. p 107.

Most of the coefficients have been derived on theoretical grounds, albeit crudely. However, the best results are obtained by fitting the equation to the measured data for the nuclide energies. The semi-empirical equation with the best fit has the coefficients (in MeV):

$$a_1 = 15.7517 \quad , \quad a_2 = 17.8010 \quad , \quad a_3 = 0.7107$$
$$a_4 = 23.6951 \quad , \quad a_5 = 11.18 \qquad . \qquad (6.6.6)$$

Except for the smallest values of A, Eq.(6.6.5) with coefficients (6.6.6) gives the correct binding energy to within a few percent (most often better than 1%). Even for smaller nuclides, the results are sometimes surprisingly good. For example, the measured binding energy of the boron nuclide was found to be $E_b = 64.75$ MeV. Using Eq.(6.6.5), $E_{b(calc)} = 64.73$ MeV, and the error is negligible. This indicates that the physical picture of nuclear structure given previously is very close to the fact. Since, as discussed in Chapters 4 and 7, even the datum fluctuations are deterministic, the nucleus has been described in a totally deterministic format, and no need for anything beyond that is anticipated. Further support for this position is obtained from extended consideration of nuclear stability.

6.7 **Nuclear Stability**: The stability of a system of particles is fixed by a very fundamental property of the ether. Starting with Section 2.6, where it was indicated that the ether does not maintain static configurations of $\underline{\underline{\phi}}$, and following on through Section 2.19, where energy was defined as a gradient squared distortion, a picture has been presented that relates to the basic property that the ether moves to reduce that gradient squared distortion to a uniform level where possible; i.e. where the sustaining waves do not prevent it. In Chapter 12, this is shown to be the basis of electrostatic field interaction. More complicated cases abound. For example, in Section 2.11, pair production and annihilation were described. Once separated, there is a tendency for the two charge distortion regions to pull together and combine to form two photons. The latter are velocity distortion regions that move apart at velocity c_0, leaving the volume originally occupied with no distortion ($\phi_a = \phi_d$). Again, in Chapter 7, in describing excited atomic states and the dropping back to the ground state, although it is rationalized as a separatist phenomenon related to electron radiation as the result of path perturbation and turning, etc., it is also pointed out that ultimately those unstable orbits, the stable ground state, and the exclusion phenomenon, are all just characteristics of a single solution of the ether equations for the whole atom taken as a single system with certain movable distortion

concentrations. Thus, if the ether equations available here, are the final ones they could be expected to show not only the conservation of gradient squared distortion, but also the rule of spreading that distortion out to achieve a lowest level. Not much more need be said here about this concept, except to point out that it also appears as the backbone of nuclear stability. Each nucleus or nuclide represents a single complex solution of the ether field equations; and whether or not it can remain in a stable-state depends on whether it is losing distortion energy by radiation or diversion into kinetic energy. Unstable nuclides will change, by separation into parts or by conversion of the nucleons themselves into other particles, until the sustaining waves can maintain the remaining components in stable form. Before this is discussed further, it is useful to look at a general representation of all nuclei.

One way to represent the possible nuclides is to construct a grid such as that In Figure 6.7.1. The vertical lines represent constant total nucleons, and all combinations along any A line are called isobars. Lines sloping *downward* to the right represent nuclei with a fixed number of protons, or isotopes. Lines sloping *upward* to the right indicate combinations with a fixed number of neutrons, or isotones. Since moving along either, an additional neutron or proton is added at each junction, all possible combinations of Z and N have a place on the chart, which extends infinitely. Fortunately, only about 285 stable or long lived nuclei are found in nature. Almost 2400 observable artificial ones have been produced, but all others are so unstable that they come apart too quickly to be observed.

In the chart of Figure 6.7.1, all nuclides along the abscissa have equal numbers of protons and neutrons. Those above the abscissa have an excess of protons, and only one $\binom{1}{2}$ is stable, all others breaking down relatively quickly. Those far above the abscissa have

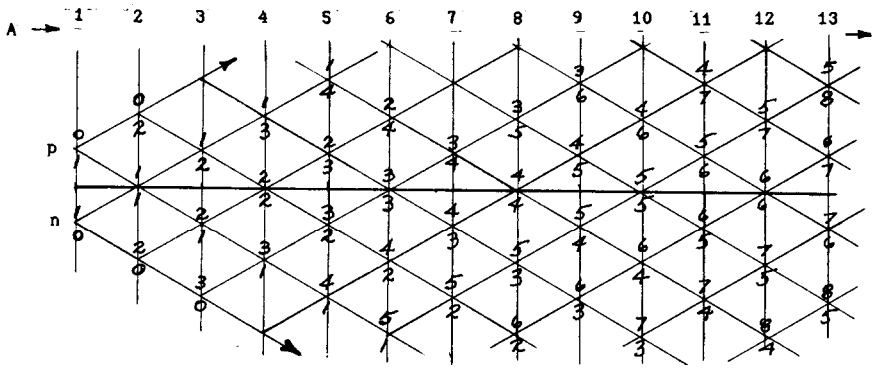


Figure 6.7.1 Generalized grid of possible nuclides.

negative binding energy, due to the proton repulsion, and cannot be assembled. Most of those closer in break down by the nucleon conversion process mentioned before, but some separate into two or more smaller nuclides. Nuclides below the abscissa have an excess of neutrons, but, for some values of A, several isobars can be stable. *Most of those that are unstable break down by nucleon conversion, but some separate into parts.*

Figure 6.7.2 graphs a short portion of the whole grid that includes all of the smaller nuclides whose energies can be measured directly or be determined by direct inference. All but a few are artificially produced, and all have positive binding energies. The numbers shown between nuclides represent the binding energy of the last nucleon added. Since most of those nuclides shown are unstable, it is important to be able to say which are and which aren't. Because separation instability is generally a *faster* process than nucleon conversion instability, the former will be described first.

Separation

In discussing nuclide separation instability, perhaps even more useful than the binding energy is a related quantity called the separation energy E_s. It represents the external energy required to remove any fractional part of a nucleus, i.e. to divide the nucleus into two separated parts. Formally it can be expressed as,

$$E_s = E\binom{N_s}{Z_s} + E\binom{N-N_s}{Z-Z_s} - \binom{N}{Z} \quad , \tag{6.7.1}$$

here the three RHS quantities are *measured*. If the binding energies of the parts are known, E_s can be found from,

$$E_s = E_b\binom{N}{Z} - \left(E_b\binom{N_s}{Z_s} + E_b\binom{N-N_s}{Z-Z_s}\right) \quad . \tag{6.7.2}$$

Again using the boron nucleus as an example, the energy required to separate one proton from it is,

$$E_s = E_b\binom{5}{5} - \left(E_b\binom{0}{1} + E_b\binom{5}{4}\right) \quad . \tag{6.7.3}$$

In the sense used here, an individual neutron or proton involves only one nucleon and therefore has zero binding energy, i.e.,

$$E_b\binom{0}{1} = E_b\binom{1}{0} = 0 \quad . \tag{6.7.4}$$

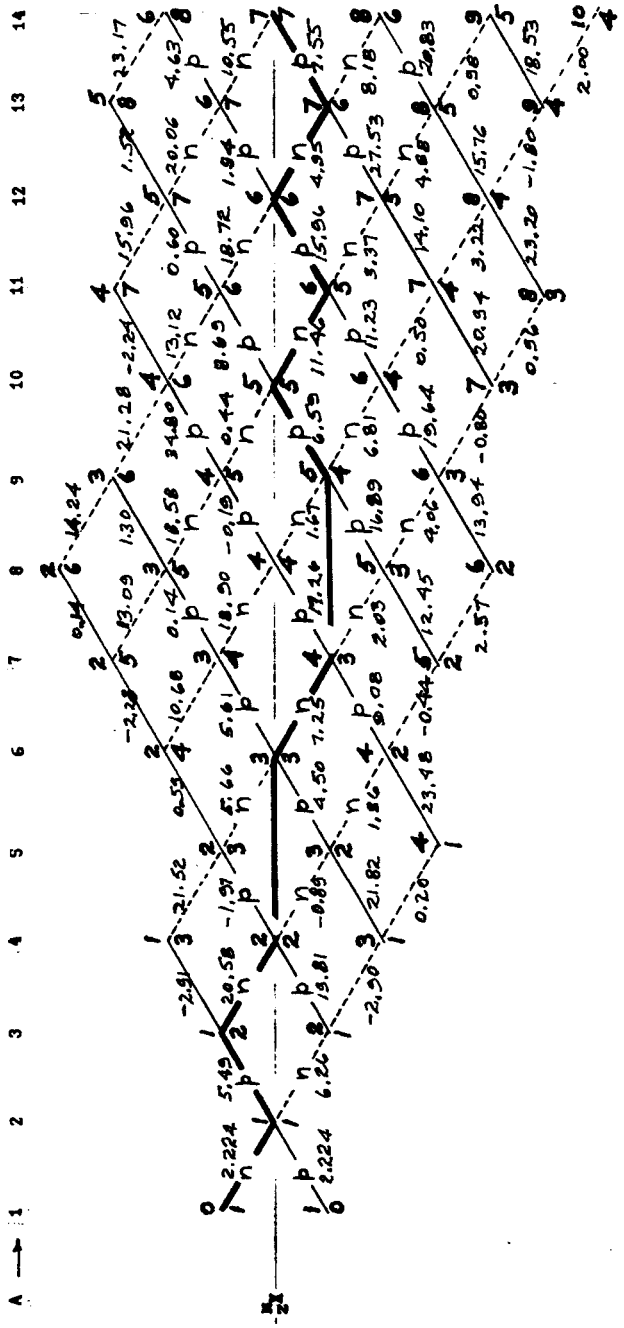


Figure 6.7.2 Last nucleon separation energies.

Substituting the actual energy values, the measured binding energy of the beryllium nucleus $E_b\binom{5}{4}$ is 58.16 MeV, and Eq.(6.7.3) gives $E_s = 6.59$ MeV. Because a single nucleon is being removed, the separation energy is the same as the last-nucleon binding energy shown in Figure 6.7.2. A positive separation energy indicates that a single proton will *not spontaneously* leave the boron nucleus, particularly since there is, in addition, the proton barrier discussed earlier. The above separation energy does not guarantee that after some significant time has elapsed the "tunneling" type of proton emission will not occur. That must be checked in a separate calculation.

Now, looking back at the chart in Figure 6.7.2, a few specific nuclides will be examined to illustrate some of the points made earlier. First, consider the neutron $\binom{1}{0}$ and the proton $\binom{0}{1}$. As single particles, their binding energies are zero in the separation scheme. Under other considerations they can exhibit both instability and convertibility, but only as a basic property of the particle itself and not like the separation of nucleons by doing external work on them. The latter is the nature of the separation instability under discussion. The first separation stable nuclide formed by combined nucleons is the deuteron $\binom{1}{1}$. From Figure 6.7.2, its binding energy is 2.224 MeV. Not shown are its isobars, the dineutron $\binom{2}{0}$, and the diproton $\binom{0}{2}$, which are unstable because of the spin dependence of the strong interaction mentioned earlier.

Adding a proton or a neutron to $\binom{1}{1}$ produces helium isotope $\binom{1}{2}$ or hydrogen isotope tritium $\binom{2}{1}$ with respective separation energies 5.49 and 6.26 MeV. These ensure that a nucleon will not spontaneously leave, reducing them to deuterons. A separate calculation shows that $\binom{1}{2}$ does not exhibit "tunneling", so both $\binom{1}{2}$ and $\binom{2}{1}$ are separation stable.

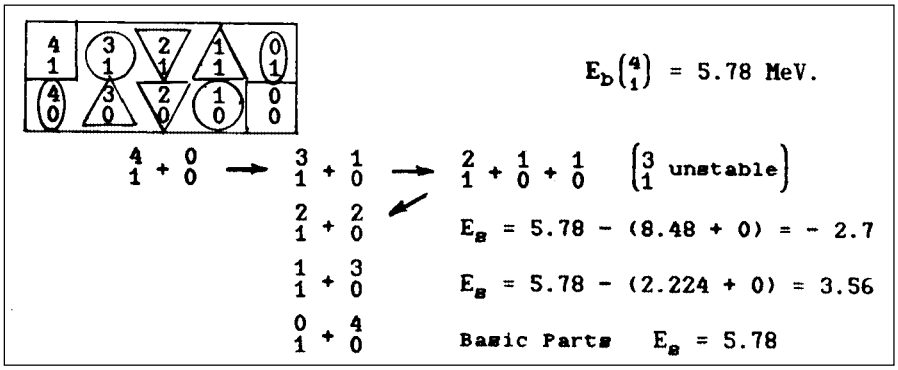
Going on to A = 4 in the chart, combining $\binom{1}{2}$ and a neutron or $\binom{2}{1}$ and a proton produces $\binom{2}{2}$, the α particle or helium nucleus. The last-nucleon binding energies ensure complete separation stability against n or p decay. The isobars $\binom{3}{1}$ and $\binom{1}{3}$, having negative last-nucleon separation energies, decay spontaneously, and do not appear on a chart of separation stable nuclides. For small nuclides, it is convenient to determine the binding energy of the whole nuclide by just adding all of the last-nucleon binding energies up to

that nuclide. For example, E_b for $\binom{1}{1}$ is 2.224, and E_b for $\binom{2}{2}$ is 2.224 + 5.49 + 20.58 or 28.29 MeV. A final check on the separation stability of $\binom{2}{2}$ considers the possibility that it might decay into two deuterons $\binom{1}{1}+\binom{1}{1}$. Using Eq.(6.7.2),

$$E_s = 28.29 - 2 \times 2.224 = 23.64 \text{ MeV}.$$

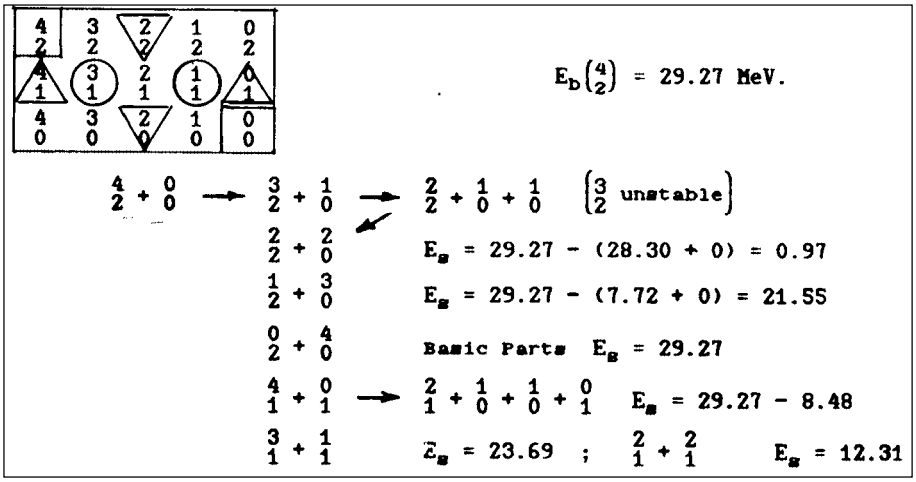
Clearly this separation does not occur, so $\binom{2}{2}$ is separation stable.

The next set of isobars in Figure 6.7.2, A = 5, has *no stable member*. Nuclides $\binom{3}{2}$ and $\binom{2}{3}$ both have negative last-nucleon separation energies as indicated, so they decay to $\binom{2}{2}$. Only $\binom{4}{1}$ must be checked in more detail. As A increases, it becomes practical to be more systematic in looking at the decay products. In the case of $\binom{4}{1}$ they can be represented as follows:

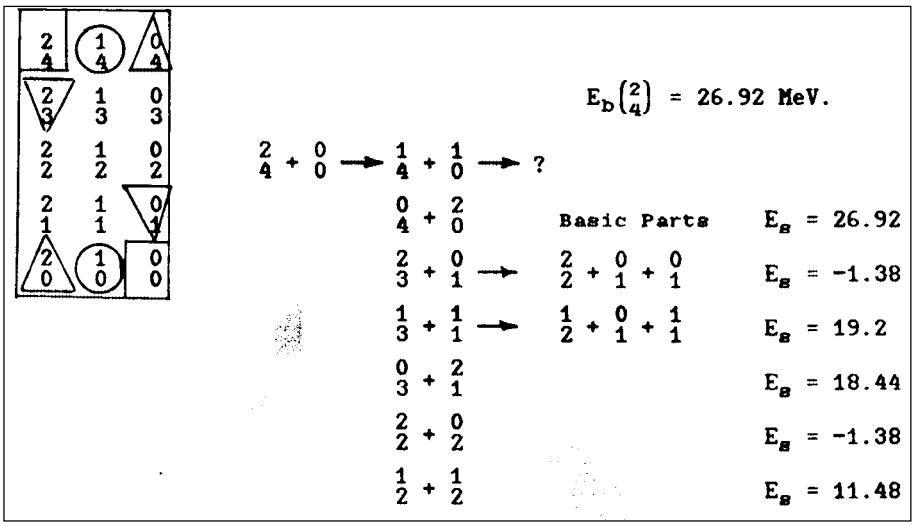


The negative separation energy indicates that $\binom{4}{1}$ decays into $\binom{2}{1}$ and 2 neutrons, and is therefore *not* separation stable.

For A = 6, both $\binom{4}{2}$ and $\binom{3}{3}$ are <u>separation</u> stable, but $\binom{2}{4}$ is not. This can be seen as follows:



Since no negative E_s appears, $\binom{4}{2}$ is <u>separation</u> stable. A similar exercise shows $\binom{3}{3}$ to be stable also. On the other hand,



showing that $\binom{2}{4}$ is <u>separation</u> *unstable*.

When this process is continued for all values of A, a reduced graph of all of the separation stable nuclides can be constructed. Figure 6.7.3 sets forth the separation stable nuclides for the smaller A values. Out of the original set of 48 shown in Figure 6.7.2 for values of A up to and including A = 14, only 34 are separation stable; and the other 14 undergo a very rapid decay by separation. It is well known that only a small number of those nuclides graphed in Figure 6.7.3 are actually stable. The second mechanism of breakdown, which applies to those that remain, will now be considered.

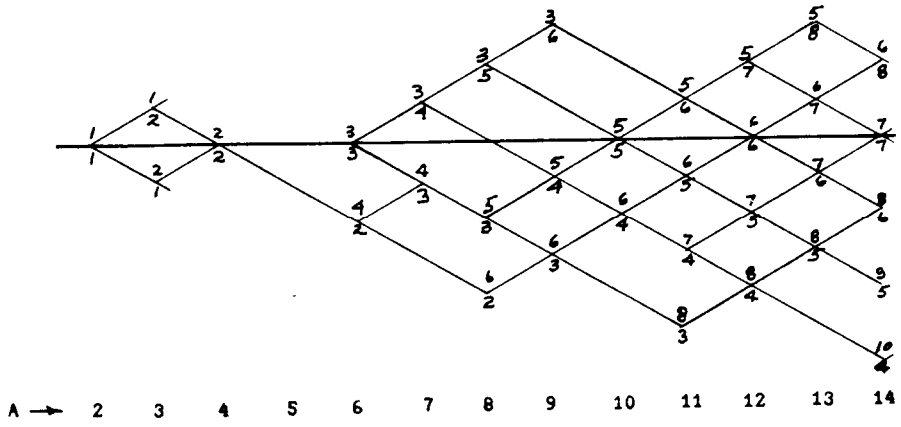


Figure 6.7.3 Separation-stable nuclides.

Nucleon Conversion

Conversion in this case is a fundamental particle property of the nucleons. Here, why it occurs and the results of its occurrence will be reviewed. Because it is a relatively slow process when compared with separation, the latter will always occur first in cases where both processes are possible. Thus, it is only necessary to consider those nuclides depicted in a separation stable grid such as that in Figure 6.7.3. Of the two types of nucleons, neutrons and protons, the former convert by a less complex process, and for that reason will be examined first.

The neutron is only a stable particle under very restricted conditions. A free neutron decays with a mean life of only 15 minutes. Only inside the nucleus, in a closed orbit, is it stable. Nevertheless, it should not be thought that the neutron has particle parts like nuclides have nucleons. Instead, it must be visualized as a single complicated 3 dimensional density pattern where $\bar{\bar{\phi}}$ and $(\nabla\bar{\bar{\phi}})^2$

are only semi-stable out in the open. As the distribution of $\bar{\bar{\phi}}$ slowly redistributes itself, three independent ether configurations evolve out of the original configuration. Here, it is sufficient to set up a simple method for determining when the neutrons inside a nucleus will break down. It will be based on the same energy principle as separation decay.

In the neutron conversion process, it can be considered to have a binding energy which it gives up when it decays. Along the same line as the development of Eq.(6.6.2), the neutron binding energy Is defined to be,

$$E_b(n) = E_p + E_e - E_n \quad . \tag{6.7.5}$$

Substituting the values of E_p, E_e and E_n from Section 6.6,

$$E_b(n) = -0.782 \text{ MeV} \quad , \tag{6.7.6}$$

indicating the neutron's instability. Now, the neutron conversion energy of the nuclide $\binom{N}{Z}$ can be defined as,

$$E_c = E\binom{N-1}{Z+1} + E_e - E\binom{N}{Z} \quad ; \tag{6.7.7}$$

which means that in nuclides with an excess of neutrons, it is possible (although not necessary) for one neutron to decay into a proton, resulting in a nuclide with one less neutron and one more proton, plus an emitted electron that leaves the nucleus. In the early study of radioactive materials, this was called β decay, the electron being the β particle. Simplification of Eq.(6.7.7) can be had by using Eq.(6.6.2) to convert to nuclide binding energies, with the result that,

$$E_c = E_b\binom{N}{Z} - \left(E_b\binom{N-1}{Z+1} + E_n - E_p - E_e\right) \quad .$$

With the aid of Eqs.(6.7.5) and (6..7. 6), this becomes,

$$E_c = E_b\binom{N}{Z} - \left(E_b\binom{N-1}{Z+1} + 0.782\right) \text{ MeV} \quad . \text{ (Neutron conversion)} \tag{6.7.8}$$

Understanding of proton conversion is wholly dependent on the total system energy conversion picture. External to a nucleus, a proton is a truly stable particle, being a relatively simple solution of the ether equations. Nevertheless, its structure ($\bar{\bar{\phi}}$ and $(\nabla\bar{\bar{\phi}})^2$

distribution) is similar to, though not exactly the same as, the neutron. The difference controls the stability vs instability of the two. Inside a nucleus, even one with excess neutrons, it is possible that the overall energy configuration can readjust and redistribute itself so that one more neutron and one less proton remain, with the creation and ejection of a positron. The proton conversion energy of the nuclide $\binom{N}{Z}$ is defined as,

$$E_c = E\binom{N+1}{Z-1} + E_{\overset{+}{e}} - E\binom{N}{Z} \quad ; \qquad (6.7.9)$$

where $\overset{+}{e}$ is used to designate a positron. Again, using Eq.(6.6.2),

$$E_c = E_b\binom{N}{Z} - \left(E_b\binom{N+1}{Z-1} + E_p - E_n - E_{\overset{+}{e}}\right) \quad ;$$

which, with the aid of Eqs.(6.7.5), (6.7.6) and Section 6.6 becomes,

$$E_c = E_b\binom{N}{Z} - \left(E_b\binom{N+1}{Z-1} - 1.804\right) \text{ MeV} \quad . \text{ (Proton conversion)} \qquad (6.7.10)$$

Another form of proton conversion occurs in nuclei within atoms when an inner orbit electron undergoes datum perturbation great enough to cause it to pass close enough to the nucleus to be captured. In that case, the conversion represents a total ether solution adjustment of the atom, not just of the nucleus. The electron capture conversion energy of the nuclide $\binom{N}{Z}$ is defined as,

$$E_c = E\binom{N+1}{Z-1} - \left(E\binom{N}{Z} + E_e\right) \quad , \qquad (6.7.11)$$

i.e. the combined energies of the original nuclide and the orbital electron will convert into a nuclide with one less proton and one more neutron and the annihilation of the electron. By the same steps as used above, this becomes,

$$E_c = E_b\binom{N}{Z} - \left(E_b\binom{N+1}{Z-1} - 0.782\right) \text{ MeV} \quad . \text{ (Electron capture)} \qquad (6.7.12)$$

The three Eqs.(6.7.8), (6.7.10) and (6.7.12) can now be used to establish the final stability chart.

Referring back to the chart of separation stable nuclides in Figure 6.7.3, each of the A = 3 isobars $\binom{2}{1}$ and $\binom{1}{2}$ must be examined for the possibility of conversion. Starting with $\binom{2}{1}$, which has an excess of

neutrons, Eq.(6.7.8) gives,

$$E_c = 8.48 - (7.72 + 0.78) = -0.02 \text{ MeV} \quad ;$$

so $\binom{2}{1}$ *converts* by electron emission to $\binom{1}{2}$ which is both separation and conversion stable. Likewise, $\binom{4}{2}$ converts to $\binom{3}{3}$, which is also stable. A somewhat different condition attends $\binom{4}{3}$, however. There Eq.(6.7.8) gives,

$$E_c = 39.24 - (37.6 + 0.78) = +0.86 \text{ MeV} \quad .$$

so $\binom{4}{3}$ does not convert to $\binom{3}{4}$, and is both separation and conversion stable. The excess of protons in $\binom{3}{4}$ might permit proton conversion, and according to Eq.(6.7.10),

$$E_c = 37.60 - (39.24 - 1.80) = +0.16 \text{ MeV} \quad .$$

Thus, $\binom{3}{4}$ does *not* undergo positron emission decay; but electron capture is still possible. This is checked using Eq.(6.7.12), to give,

$$E_c = 37.60 - (39.24 - 0.78) = -0.86 \text{ MeV} \quad ,$$

indicating that, after a considerable time, electron capture-proton conversion will occur. The result is that $\binom{3}{4}$ decays to $\binom{4}{3}$.

The last case of direct interest here is composed of the A = 8 isobars, $\binom{6}{2}$, $\binom{5}{3}$ and $\binom{3}{5}$. Using Eq.(6.7.8) as before, it is straightforward to show that $\binom{6}{2}$ decays into $\binom{5}{3}$ and $\binom{5}{3}$ into $\binom{4}{4}$ by electron emission (β^- decay). Also, using Eq.(6.7.10), $\binom{3}{5}$ decays into $\binom{4}{4}$ by positron emission (β^+ decay). This brings out an interesting point. In the case of $\binom{3}{4}$ there was not enough excess energy in the nucleus to induce the proton conversion, and not until the extra energy of the captured electron was absorbed from the total atom structure did proton conversion (e_k decay) occur. In the case of $\binom{3}{5}$, both positron emission and electron capture are possible in principle because both Eqs.(6.7.10) and (6.7.12) yield negative conversion energies. In fact, from the equations, any case where β^+ decay can occur also has a

favorable energy balance for electron capture. But In practice, where β^+ decay is possible, it will always occur first, since it is a simple adjustment of the nuclide energy. Electron capture requires a long time during which the electron must come into the vicinity of the proton, and this is often a matter of days or even years.

Continuing the process of examining larger and larger nuclides finally yields a much smaller total number of *stable* nuclides (i.e. both separation and conversion stable). A portion of that stable set is presented in Figure 6.7.4

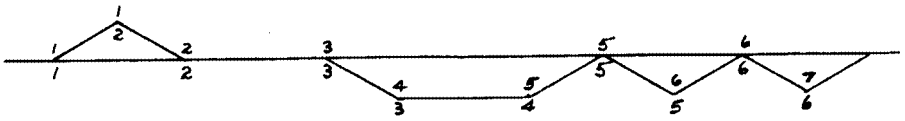


Figure 6.7.4 The stable nuclides for small A.

Going back to the original grid in Figure 6.7.1, of the 116 possible nuclides up to and including A = 14, 68 are unknown either because they have negative binding energies or decay by one of the two processes so quickly that they cannot be observed. The remaining 48, shown in Figure 6.7.2, are either stable (11), or decay by separation (14) or by conversion (23). Throughout the whole extended grid of Figure 6.7.1, the simple deterministic picture of nuclear structure and stability agrees with observation with no exception. Of course, there are many more nuclear phenomena that have not been discussed, e.g., α radioactivity, excited levels in the nucleus, γ radiation decay, nuclear interactions, etc. But, these are outside the area of interest as mentioned before.

Clearly, the processes of separation and conversion when described conventionally are just exhibitions of an "observed" rule of progression of energy reduction in systems. The advantage of the ether approach is in its inherent ability to derive the energy reduction rule from the basic field equations, and although this has not yet been completed, it serves as a powerful motivation for solving the field equations, and as a goal with no real opposition to attainment except perhaps unwarranted discouragement. The terminology "strong" and "weak" interactions customarily applied to separation and conversion are confusing in an ether theory where the strong and weak fields refer to the inner and outer regions of particles' structures. Because the concepts of the strong and weak *fields* are so necessary to the ether theory, in the following the two nuclear processes will be designated the "nucleon Interaction" and the "nucleon conversion"; terms that are, as can be seen, closer to the true physical description of the phenomena than the conventional terms.

CHAPTER 7

THE ATOM

7.1 **Introduction**: The Wave Mechanics explanation of the atom's interior has so proliferated that it has deeply ingrained in the modern physicist's mind its hopeless inability to describe that interior. *A firm belief that no deterministic description is possible can be considered the basis of "Modern" physics.* Yet on what grounds? What is the physical evidence? Take the hydrogen atom. An electron, with 98 percent of its energy concentrated inside a sphere of radius 2OO r_e and 99 percent of its charge inside the same sphere, approaches a proton that is essentially the same size as the electron. If the electron is captured, the innermost stable orbit it can occupy is a circle of radius $1.5024 \times 10^5 r_e$. Thus, *the distance between the two particles is greater than 750 times the sphere of significant influence of either one.*

If the electron is *not* captured, it sails past the proton on an hyperbolic orbit, and no one has ever suggested that any serious change in the electron or proton occurs. In the circular and elliptic orbits of capture, the great distance between the particles and the relatively slow motion of the electron argue that just as little change takes place. Then, why the firm belief that the interior cannot be described deterministically? First, such a description had not been available up until 1989.[1] Second, certain mistaken beliefs related to the application of Maxwell's equations led to the *prediction* of paradoxical non-observed radiation phemomena. Third, a physical reason for the existence of the de Broglie frequency has only been forthcoming recently.[1] Beyond that, the fact that the means for measuring the electron's position in the atom without perturbing its motion are not available has been used as a final reason for outlawing a deterministic atom. None of this has any validity.

Considering the proton-electron separation in the atom, no reason can be suggested why their motion should be any different than outside the atom. In earlier chapters, the control of the ℓ-wave equations has been made clear; and the criterion for when an electron will or will not radiate was presented. The ether description of the extended electron of Chapter 3 has explained the meaning of the de Broglie frequency. Finally, the ability to measure electron motion in the atomic interior directly is irrelevant to its being deterministic. The perturbation in measurement is a *fact*, not the mysterious religion it has become. Considerable evidence has been collected to show that

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1. R.H.Dishington, Physics, Beak Publications, Pacific Palisades, CA (1989).

the particles that make up single atoms retain their simple, individual identities and behave in a straightforward manner.[1]

Earlier, almost all of the electron's properties were presented. This was necessary because it was lack of knowledge of the electron and the related particle structure that allowed certain myths to divert atomic physics towards *statistical ensemble* methods and away from the Main-Line, deterministic picture needed to fill out physics. What is called " classical" physics is capable of describing and *explaining, on the basis of cause and effect*, both atomic and particle physics (and the paradoxes of modern physics) right down to the metaphysical base. There is only one physics, and this and subsequent chapters will attempt to make that clear.

It is not *impossible* to describe what is going on inside atoms. It can be done using only Maxwell's potential equations and Newton's laws. In fact, Quantum Physics, in the present author's opinion, does not constitute a physical theory, at the same level, as the Theory of Electromagnetism and Newton's laws does.

This chapter will present a planetary type description, of electrons orbiting nuclei, that shows how close Bohr and Sommerfeld came to the correct picture, in spite of the rudimentary awareness of particle properties and structure in the early 1900's. They were unable to see the extended nature of the orbiting electrons, and did not realize that the electron, in turning as it orbited, contributed to the total angular momentum. Thus, they failed to match the orbital angular momentum to the value predicted by Newton's laws. *The standard QM analysis of the atom makes the same error*; and, since Shroedinger's equation *cannot* give the correct answer unless the exactly correct mechanical picture is used to enter the energy of the system, QM carries some of the errors along. Using the extended electron described earlier, this will be corrected.

The turning of the orbiting electrons in the ground state ensures that no radiation takes place and that the ground state is stable. Quantization of the orbits is established by the electron's real, doppler shifted longitudinal wave difference frequency. In the next few sections, the structure and operation of the deterministic atom will be described using only ordinary Newtonian mechanics.

7.2 **The Hydrogen Atom**: The simplest atom consists of a single electron orbiting around a single proton. Quantum mechanics gives a proper evaluation of certain aspects of mechanical systems *when the correct description of the mechanics is known*. In the case of the

1. T.Erber et al, "Resonance Fluorescence and Quantum Jumps in Single Atoms", Annals of Physics, **190**, pg 254, March (1989). H.Dehmelt, "Experiments on the Structure of an Elementary Particle", Science, **247**, pg 539, (1989).

hydrogen atom, past lack of knowledge of the *electron* has caused QM to be used erroneously. The radiated frequencies and total angular momenta predicted are correct; but, because a minute electron angular momentum (due to its extension) has been omitted, the orbital periods and *orbital* angular momenta usually given are incorrect. The progression of the approach from the Bohr-Somerfeld model to the wave mechanics un-picture is so well known that nothing will be said about the history except to point out that the reluctance of textbook authors to give up the visualizable properties of the former for the, until now, more quantitative, non-visualizable latter is significant.

While, in principle, only a complete solution of the total field equations can give an exact picture of this miniature "planetary" system, the mechanics of Chapter 12 combined with the properties of the *extended* electron expounded in Chapter 3, keeping in mind the approximate nature of the "force" concept, gives a highly accurate description of atomic operation. If the "turning" angular momentum is added to the QM analysis, the latter also gives the same results for the *ensemble*.[1]

7.3 **Orbit Analysis**: Standard planetary analysis begins by considering the motion of a satellite of mass m_0 moving in a central field (see Figure 7.3.1), e.g. the Moon orbiting a fixed Earth. The presentation here is a modification of Goldstein's treatment of the Kepler problem in astronomy.[2] Customarily, the analysis is carried out with the orbit plane in three dimensions; but, for complete *visualization* of details, here the orbit plane is described with two dimensions. To keep the discussion simple, *even in the atomic case, mass variations will be ignored* ($\gamma = 1$). This will have no important effect on either the picture or the principles presented.

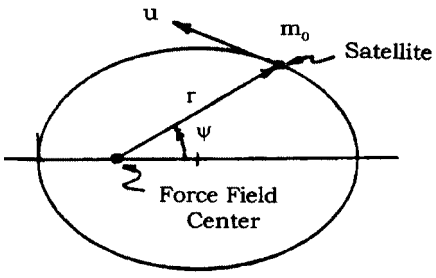

Figure 7.3.1 A planar orbit

1. R.H.Dishington, Physics, pg 340, Beak Publications, Pacific Palisades, CA (1989). ....., Fundamental Questions in Quantum Physics, Ed. Franco Selleri, Hadronic Journal Supplement, **8**, #4, p 395-414,December (1993).
2. H.Goldetein, Classical Mechanics, p. 229, Addison-Wesley Press, Inc. Mass. (1950). See also: R.Becker, Electromagnetic Fields and Interactions, Vol 2, p. 56, Dover Publications, N.Y. (1982); G.F.Lothian, Electrons in Atoms, pgs. 65, 74, Butterworths, London (1963); A.Ruark and H.Urey, Atoms, Molecules, and Quanta, Vol 1, Chs 4 and 5, Dover Publications, N.Y. (1964).

The satellite's energy is defined as $E = T + V$. $V = -k / 4\pi r$ is its potential energy, and its kinetic energy is (HL units),

$$T = \frac{1}{2m_0}\left(p_r^2 + \frac{p^2}{\eta^2 r^2} \right) \quad , \tag{7.3.1}$$

where p_r is its radial momentum and p its *total* angular momentum. To be defined later, η is unity in the ordinary planetary case. In the simplified Moon-Earth system, the orbital angular momentum is,

$$\boxed{p_\psi = m_0 r^2 \dot{\psi}} \quad , \tag{7.3.2}$$

and this is usually entered for p in Eq. (7.3.1). However, *the Moon always presents the same face to the Earth*, rotating one turn about its own axis for each complete orbit. In the atom, the electron *must* turn that way in all possible orbits (see Sections 3.13 and 12.9), with a turning angular momentum (not the spin),

$$p_t = K_t p_\psi \quad . \tag{7.3.3}$$

Thus, for a close parallel to the atomic case the planetary example must be visualized with a similar constraint, with the total angular momentum written as,

$$p = p_\psi + p_t = (1 + K_t) p_\psi \quad . \tag{7.3.4}$$

Next, Newton's second law is used to write the radial force equation,

$$F_r = m_0 \left(\ddot{r} - r(1 + K_t)\dot{\psi}^2 \right) \quad , \tag{7.3.5}$$

and the angular momentum equation,

$$\frac{dp}{dt} = 0 \quad , \quad p = k_\psi \quad \text{(constant)} \quad . \tag{7.3.6}$$

In the general case, for E<0 and attractive force $F_r = -k/4\pi r^2$, a rather long and convoluted derivation[1] leads to closed elliptical orbits ($K_t = 0$) or almost elliptical orbits (precessing, $K_t < 0$; recessing, $K_t > 0$),

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1. A.Ruark and H.Urey, Atoms, Molecules and Quanta, Vol. 1, p. 133, Dover Publications, N.Y.(1964). B.Shore and D.Menzel, Principles of Atomic Spectra, p. 45, J.Wiley and sons, N.Y.(1968).

given by,

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos(\eta\psi)} \quad , \quad \eta = \sqrt{1+K_t} \tag{7.3.7}$$

with the parameters (HL units),

$E = -\frac{k}{8\pi a}$ Constant energy E for each possible orbit.

$p = k_\psi = \eta\sqrt{\frac{m_0 k}{4\pi}(1-\varepsilon^2)a}$ Constant total angular momentum for each possible orbit.

$a = \frac{r_{min} + r_{max}}{2}$ Similar to the semi-major axis of the elliptical case.

$b = \sqrt{r_{min} r_{max}}$ Corresponds to the semi-minor axis.

$\varepsilon = \sqrt{1-\left(\frac{b}{a}\right)^2}$ Orbit eccentricity parameter.

In terms of these parameters, the radial momentum is given as a function of r by,

$$p_r = \sqrt{2m_0 E + \frac{2m_0 k}{4\pi r} - \frac{k_\psi^2}{\eta^2 r^2}} \quad . \tag{7.3.8}$$

Without the *turning* constraint, $K_t = 0$ and $\eta = 1$, reducing these equations to the usual textbook elliptical case. All of this is well known[1], along with the fact that *any choice of semi-major axis a and eccentricity* ε *in the astronomical case (with* $k = Gm_0M$*) will produce a physically realizable orbit* as long as the satellite is far enough away from the force field central mass. Because the energy E is not a function of the eccentricity, any specific choice of a applies to a whole family of pseudo-ellipses and their corresponding total angular momenta p, the largest of which matches the circular orbit with radius a and $\varepsilon = 0$ (see Figure 7.7.1).

In the Moon-Earth case, the center to center distance is about 60 times the Earth's radius. Since the electron-proton separation is at

1. H.Goldstein, Classical Mechanics, Addison-Wesley Press, Inc. Mass. (1950). R.H.Dishington, Physics, Ch.l2, Beak Publications, Pacific Palisades, CA (1989).

least 750 time their significant regions of influence, and these particles exhibit inertia, momentum and all other common properties of ordinary matter, it should be clear that *no obstacle to their executing planetary type orbiting motion exists in any general way*. Therefore, to apply the preceding equations to the hydrogen atom, it is only necessary to set $k = e^2$ and supply a rationale for choosing semi-major axis a, ε and K_t. However, it is at this point that the atomic case begins to differ significantly from the astronomical.

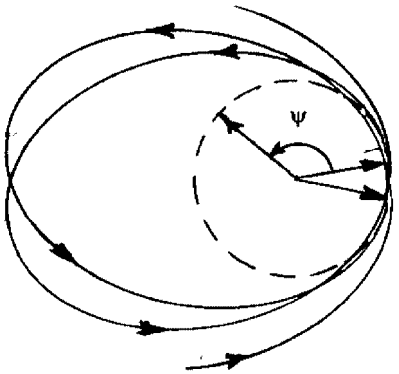

Figure 7.3.2
The r_{min} period is less than $\psi = 2\pi$.

For example, in the purely elliptical case, the period of the orbit represents one complete circuit of the ellipse repeated over and over again. In the atomic case, the orbits are not closed but *recess*, as shown in Figure 7.3.2. Here, the cycle is considered to go from one r_{min} to the next r_{min}, *which shifts orientation* in the cases where $K_t \neq 0$. Because of this shift, each pseudo-ellipse cycle is completed when ψ has swept out only $2\pi/\eta$ radians.

A great deal of knowledge about atoms comes from their radiation spectra. From the time of Bohr and Sommerfeld, it has been clear that atoms exist in stable or pseudo-stable states; and only when an electron shifts from one orbit to another does radiation occur. Because this was never tied to a cause and effect explanation, but only to the mysterious "quantum", the de Broglie "wave" and "h", the orbits and the visualization were ultimately lost. *Here, the cause and effect chain is traced directly to the properties of the electron and its ℓ-waves*, and the method for finding the semi-major axis values of a that give the observed selected orbits is presented.

7.4 **The de Broglie Difference Frequency and Planck's Constant**: In Section 3.14, it was demonstrated, using only Newton's laws, that Planck's constant h is a *derived* constant that relates the electron's momentum to the Doppler difference frequency of its front and back longitudinal waves. The following shows the way the difference frequency and the derived constant h enter the orbit analysis.

Refer back to Section 3.14. There it was shown that when an electron moves along a path at velocity u, its radially outward moving ℓ – waves are Doppler shifted, resulting in a difference frequency

between the front and back waves of,

$$\nu_d = 2\gamma\beta\nu_e \quad , \tag{7.4.1}$$

where, $\beta = u/c_0$ and ν_e is the electron's intrinsic $\ell-$wave *rest* frequency (1.2355898×10^{20} cyc/s, as determined by the Rydberg constant). Eq.(7.4.1) and the *linear* momentum p_L were used to write,

$$p_L c_0 = \gamma m_0 u c_0 = h\frac{\nu_d}{2} \quad , \tag{7.4.2}$$

where h is the *derived* Planck's constant,

$$h = \frac{m_0 c_0^2}{\nu_e} = 6.6260755\times10^{-27} \quad . \tag{7.4.3}$$

However, *for the atomic orbit*, the derivation of Section 3.14 must be modified, as follows, to account for the separation of the *orbital* and *turning* angular momenta.

Using Eq.(7.3.2), the electron's <u>orbital</u> *linear* momentum is,

$$p_L = \frac{p_\psi}{a} = \gamma m_0 u \quad . \tag{7.4.4}$$

Combining Eqs.(7.4.1) and (7.4.4),

$$p_L c_0 = h\frac{\nu_d}{2} \quad , \tag{7.4.5}$$

where again h is the *derived* constant of Eq.(7.4.3). Eq.(7.4.5), when transposed, gives the de Broglie difference frequency[1],

$$\nu_d = 2c_0\frac{p_L}{h} \quad . \tag{7.4.6}$$

However, it is the *total* angular momentum,

$$p = p_\psi + p_t = \eta^2 p_\psi \quad , \tag{7.4.7}$$

that sets the electron's velocity and the difference frequency. Defining the *total* linear momentum, including the "turning", as,

$$p_{Lt} = \frac{p}{a} = \eta^2 p_L = \eta^2\gamma m_0 u \quad , \tag{7.4.8}$$

Eq.(7.4.5) becomes,

$$p_{Lt} c_0 = \eta^2\frac{h\nu_d}{2} \quad . \tag{7.4.9}$$

---

1. Although the author has chosen to call ν_d the de Broglie frequency, it should not be confused with $\nu_{db} = E/h$, a fictitious frequency of a fictitious wave.

As discussed in Section 3.14, *although there is a difference frequency, there is no difference wave*. For an atomic orbit, the proper inversion of Eq.(7.4.9) is,

$$\Lambda_d = \frac{2c_0}{\eta^2 \nu_d} = \frac{h}{p_{Lt}} \quad . \tag{7.4.10}$$

where Λ_d is **not** the wave length of a mysterious wave that travels along curved paths. It is determined by real, longitudinal waves that emanate and propagate radially from the electron's center. Λ_d has nothing to do directly with the wavelength of any wave. Instead, in this orbital case, it is simply *the distance the electron travels during* $2/\eta^2\beta$ *cycles of the difference frequency* ν_d.

7.5 **The Steady-State Orbiting Electron Field**: To better understand the nature of Λ_d in the atom, the total field of an orbiting electron will be visualized. One of the most significant effects of the electron Doppler *difference* frequency occurs when the electron moves periodically in a closed path in a central electrostatic field. In that situation the conditions for the *total* field solution are quite different from those of the free electron. In the *circular* orbit case, for example, only when the effective circumference of the orbit, $L = 2\pi a/\eta$, is related to the *difference* frequency by,

$$\nu_d = n\frac{2}{\eta^2}\frac{c_0}{L} \quad , \tag{7.5.1}$$

where a is the orbit radius, and $n = 1, 2, 3, ...$, can a steady state field solution exist. This can be explained as follows.

An attempt will be made to visualize the overall field surrounding the orbiting electron near the orbit and at great distances from the center. It will be shown that *Eq.(7.5.1) is the criterion necessary to maintain the combined solution in steady state*.

In the hydrogen ground state, for an example, if the electron's $\ell-$waves are drawn to scale, because u/c_0 is small, it is impractical to show enough waves to see their centers displaced to match past positions of the particle as it moves along the orbit. A better idea about the minute but significant effects taking place can be obtained by *artificially exaggerating the velocity* u. Then, the effect in space is seen to be a shifting of the positions of maximum and minimum bunching of the $\ell-$waves, as illustrated in Figure 7.5.1. Full turning of the electron's field is implicit.

The same exaggerated orbit velocity and, in addition, *artificially reduced wave propagation velocity* allows a plot of the *outer* regions of the field. Figure 7.5.2 shows every 2,348th wave front, and the bunching and extending of those fronts can be seen to spiral

outward so that the spacing between successive bunching or extending is,

$$D = c_0 T_{orb} \quad . \qquad (7.5.2)$$

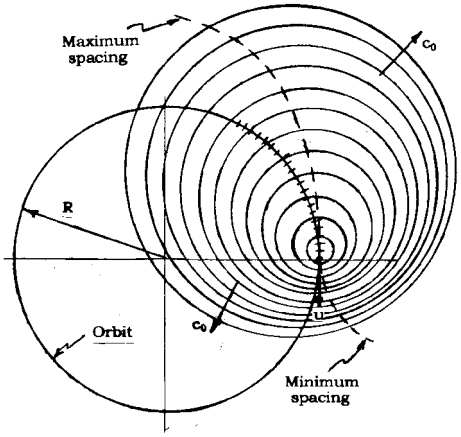

Figure 7.5.1.
Orbiting electron ℓ-waves

By turning the figure in the direction of the electron's motion, the outward motion of the spirals can be seen as a good representation of the total field equation solution for the hydrogen ground state. To be a <u>steady state</u> solution, *the spiral must occupy exactly the same position relative to the orbit, as the electron returns to the same orbit position*, taking into account the orbit recession. There is no problem in the outer field, as long as the correct phasing occurs along the orbit.

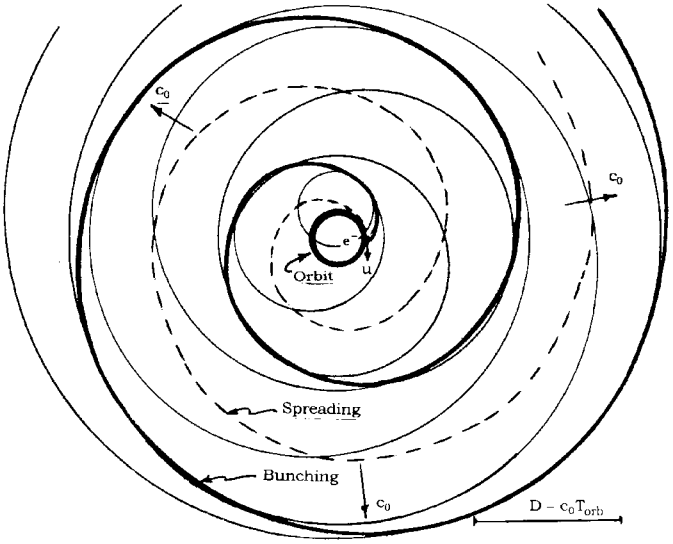

Figure 7.5.2
Outer field diverging waves, exaggerated ground state orbit.

In Figures 7.5.1 and 7.5.2, the two spirals (bunching and spreading) can be seen to approach the orbit tangentially and to join at the electron center. As the electron orbits, the same picture is repeated at each point. Again this can be seen by simply rotating the plots to simulate the electron's motion. Clearly, depending upon the velocity u and the *effective* orbit circumference L, the phase position of the wave fronts at the electron center may or may not be exactly the same as the electron completes its round trip and returns to the recessed starting point. But, if those phases are not identical, then the outer field will be *changing* and the total field equation solution will *not* be steady state.

To find the criterion for a return to the same phase condition, think of a point on the orbit just as the electron is passing. First, a series of front waves, moving in the direction of the electron's motion, cause an oscillation at the point of frequency ν_f. Then a series of back waves, leaving in a direction opposite to the electron's motion, cause an oscillation at the point of frequency ν_b. *The same phenomenon occurs at each point of the orbit circumference,* differing only in the time when the electron passes. Only if the number of front wave cycles $N_f = \eta^2 \nu_f L / 2c_0$, in the distance equal to the *effective* circumference L, and the number of back wave cycles $N_b = \eta^2 \nu_b L / 2c_0$, in that same distance, are both *integers* can the phases and electron oscillation match at each point. Thus *the condition for a steady-state field in this circular orbit* is that the difference between N_f and N_b is also an integer,

$$n = N_f - N_b = \eta^2(\nu_f - \nu_b)\frac{L}{2c_0} = \eta^2 \nu_d \frac{L}{2c_0} \quad , \qquad (7.5.3)$$

which is just Eq.(7.5.1) rearranged.

Thus, those *circular* paths specified by Eqs.(7.5.1) or (7.5.3) are stable, and the elliptical orbits with the same semi-major axis a are pseudo-stable. The profound effect this has on atomic structure was first pointed out by L. de Broglie, through Eq.(3.14.9), although he was inspired by a shrewd guess based on symmetry rather than an understanding of the electron's structure. Unfortunately, the de Broglie wavelength has been emphasized, and a mysterious wave of a much different frequency than the actual frequencies, ν_f and ν_b, directly involved has been used to describe "matter waves" of quantum mechanics. Clearly, the mathematical nature of such waves that can travel along an orbit that is believed not to exist presents a problem to anyone interested in physics. The preceding picture gives a much more realistic description of the phenomena in three dimensions.

7.6 **The Semi-Major Axis** a: It is fortunate that the solution for the orbital energy E is degenerate for all of the non-circular paths that have the same semi-major axis a, because in all those paths the difference frequency and electron velocity vary, and those cases are not truly steady-state. The criterion for finding the allowed a values can be obtained from the *circular* orbit.

Earlier it was shown that Newton's laws were adequate in the astronomical problem. They provided the solution of Eq.(7.3.7), which includes the parameter equation for the *circular* orbit case ($\varepsilon = 0$),

$$p = k_{\psi} = \eta\sqrt{\frac{m_0 k}{4\pi}\,a} \quad . \tag{7.6.1}$$

In the astronomical solution, no further restriction is placed on either the constant total angular momentum p or the orbit radius a. The arbitrary choice of either one determines the other through Eq.(7.6.1). It is at this point the atomic case deviates most, for the choices of p and a are not arbitrary in the atom, *since the electron itself imposes another condition on* p. Consequently, p is determined and a follows directly from Eq.(7.6.1). Thus, p and a are fixed and not arbitrary.

The criterion for finding the allowed total angular momentum and semi-axis a results from the combination of Eqs.(7.4.10) and (7.5.1),

$$L = \frac{2\pi a}{\eta} = n\Lambda_d \quad . \quad n = 1, 2, 3, \ldots . \tag{7.6.2}$$

Here, n is the familiar principal "quantum number", *obtained using only Maxwell's equations and Newton's laws*. Only those orbits specified by Eq.(7.6.2) are stable or pseudo-stable.

Now, to find the alternative equation for p, combine Eqs.(7.4.10) and (7.6.2) to read,

$$p = ap_{Lt} = \frac{\eta n h}{2\pi} \quad . \tag{7.6.3}$$

Equating Eqs.(7.6.1) and (7.6.3) yields,

$$a = \frac{n^2 h^2}{\pi m_0 e^2} \quad . \quad n = 1, 2, 3, \ldots \tag{7.6.4}$$

These values of a, used with the energy parameter in the list following Eq.(7.3.7), specify all the allowable total angular momenta, orbit energies and sizes. They do not establish the *shapes* of the elliptical orbits. That will be taken up next.

7.7 **Orbit Shape and Eccentricity**: The selection rule for *circular* orbits can be rewritten, with the help of Eq.(7.6.3) above, in the form,

$$p_{Lt}L = nh \quad , \quad n = 1, 2, 3... \qquad (7.7.1)$$

where L is the length of path and p_{Lt} is the *total* linear momentum of the electron along the path. If the orbit is *elliptical* the electron's distortion with changing velocity precludes that orbit's stability. Nevertheless, the rate of radiation is not that great, so a kind of pseudo-stability exists. If the radiation is neglected for the moment, it is clear that the steady state field about the orbit is not the simple cyclic spiral discussed above. However, even when the path is elliptical, it is possible to visualize the outer field spiraling outward in non-circular form, always matching the electron's difference frequency as it speeds up and slows down along the path of the orbit. If the match at each point is instantaneously correct, then the proper pseudo-steady state criterion suggested by Eq.(7.7.1) is,

$$\int_{r_{min1}}^{r_{min2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = nh \quad , \quad n = 1, 2, 3... \qquad (7.7.2)$$

where $\mathbf{p}_{Lt}$ is the total linear momentum of the electron,

$$\mathbf{p}_{Lt} = m_0 \dot{r}\hat{\mathbf{r}} + m_0(1 + K_t)r\dot{\psi}\hat{\boldsymbol{\psi}} = p_r\hat{\mathbf{r}} + \frac{p}{r}\hat{\boldsymbol{\psi}} \quad ,$$

and $d\mathbf{s}$ is the differential displacement of the electron along the elliptical path. The integration is carried out over the unclosed section of the ellipse corresponding to the cycle or repetition period from r_{min} to r_{min} and angle of $2\pi/\eta$. In terms of the components, Eq.(7.7.2) becomes,

$$\int_{r_{min1}}^{r_{min2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = \int p_r dr + \int p d\psi = nh \quad . \qquad (7.7.3)$$

The components integrated give,

$$\int p d\psi = p \int_0^{2\pi/\eta} d\psi = \frac{2\pi}{\eta} p \quad , \qquad (7.7.4)$$

and,

$$\int p_r dr = \int_0^{2\pi/\eta} \frac{p\varepsilon^2 \sin^2(\eta\psi)}{(1 + \varepsilon\cos(\eta\psi))^2} d\psi \quad . \qquad (7.7.5)$$

To complete the analysis, it is necessary to ask which orbits are circular and which are elliptical. It can be seen from Eq.(7.6.4), that the smallest orbit is specified by $n = 1$, and succeeding values of n give larger and larger paths; but their *shapes* are not indicated by n.

In fact, it is easy to visualize a set of orbits for which n would be the same, i.e. the total number of difference frequency beats would be the same around the loop, and yet the shape of the paths could be quite different because of the *changing* de Broglie difference frequency. Some clarification comes from a consideration of Eq.(7.7.3), in which, for the same n, the two components of momentum could make different contributions. For example, if the orbit was circular, p_r would be zero and the total momentum contribution would be the angular momentum p. In an orbit of non-circular shape, the radial momentum would not be zero, and the angular momentum ($p = constant$) would be smaller. This would give an elliptical type path with a semi-*minor* axis less than a.

Once the allowed orbit sizes have been found through Eqs.(7.6.4) and (7.7.3), the shapes of the various orbits, due to differences in the de Broglie frequency, can be found from the components of Eq.(7.7.3). It should be emphasized that *it is the whole three dimensional field that must be orbit compatible*. In other words, not only is the total momentum along the orbit path matched to the circumference, but all components of the field must also repeat starting with the new r_{min}. This can only happen if the components obey integer relationships such as,

$$\int p d\psi = p \int_0^{2\pi/\eta} d\psi = n_\psi h \quad , \quad \int p_r dr_r = n_r h \quad . \tag{7.7.6}$$

Finally, from Eqs.(7.3.7), (7.7.3) and (7.7.6),

$$n = n_r + n_\psi \quad , \tag{7.7.7}$$

and,

$$\sqrt{1-\varepsilon^2} = \frac{n_\psi}{n} = \frac{\text{min or axis}}{\text{major axis}} \quad ; \tag{7.7.8}$$

where the axes here are loosely equivalent to those of an ellipse. Figure 7.7.1 sketches the first few allowed orbits. In order to determine them, it was necessary to specify K_t, which is not a universal constant, but *a different constant for each distinct orbit.* In making the sketches in Figure 7.7.1, K_t was assumed to be,

$$K_t = \frac{1}{n_\psi} \quad ; \tag{7.7.9}$$

but the reasons for the choice will only be discussed later on.

Ultimately, K_t should come as a result of solutions of the total field equations. At present, it is only agreement with experiment that confirms the choice of Eq.(7.7.9).

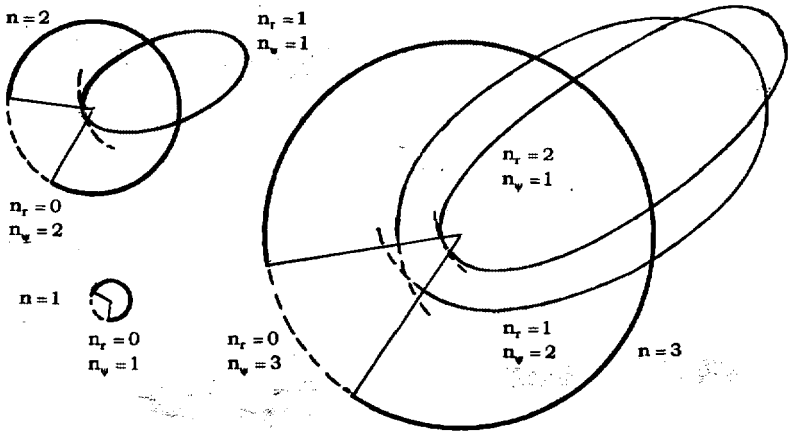

Figure 7.7.1. Recessing atomic orbits.

7.8 **Odds and Ends**: Before concluding the specific discussion of hydrogen and broadening the atomic perspective, a few salient points as well as the values and ranges of the various n_i are in order. Just as in the Bohr atom, the principal quantum number can range from 1 to ∞. *Since strong physical reasons have been given to show that circular orbits are basically the most stable*, the lowest value of n_r is zero, or conversely, the maximum value of n_ψ is n. On the other hand, the *orbits corresponding to linear oscillation of the electron through the nucleus, have not been shown* in Figure 7.7.1, since it is clear that a large amount of energy is needed to force an electron to approach very close to a proton, and experiments have shown that the proton breaks up and other particles (s quarks) are formed. Also, if the linear orbit were allowed, the electron's rate of radiation would be extreme, and the lifetime of that burst of radiation would be so short as to not qualify this case as even pseudo-stable. In fact, that radiation would present a broad spectrum rather than a line. The upshot of these arguments is that the quantum number ranges are,

$$n = 1, 2, 3 \ldots \infty \quad ; \quad n_\psi = 1, \ldots\ldots, n \quad ; \quad n_r = 0, \ldots\ldots, (n-1) \quad . \qquad (7.8.1)$$

Another important item is the omission of the electron's mass variability from the derivation. A proper derivation must include it. It was part of Sommerfeld's model, although too much was left out of that analysis to do more than indicate that the effects were present. Whereas the turning momentum causes the orbit to *recess* a considerable amount, the mass variation has the opposite effect, causing a minute precession. Nevertheless, the effect of the mass variation enters into the energy in a different way and so leads to a measurable effect. Finally, a word should be said in connection with the "Correspondence Principle" used by Bohr, et al. Here *there is no need for it.* In the early days, because the Thomson atom, with its linearly oscillating electrons provided discrete radiation to be identified with the observed spectral lines, the model makers were looking for a match between the mechanical oscillation frequencies and the radiation. In the Bohr model, this only occurred for the outermost orbits, and was interpreted as an asymptotic approach to the "classical" picture. Here, with hindsight, it is known that the lines are radiation emitted during a transition of the electron from one orbit to another. In a typical shift down from one orbit to the next, the electron starts in orbit 1 in a configuration that is stable with respect to de Broglie match and velocity, so that only the persistent buffeting of the zero point fluctuations and its own speed changes perturb the electron's motion. Gradually, the motion reaches a deviation that results in enough electron radiation to prevent recovery of the exact orbit and a slow spiral inwards commences. Soon, the de Broglie match is badly broken and the electron shape oscillates with velocity to produce faster inward spiraling and greater radiation per cycle. Finally, as it approaches closer to the inner orbit, the radiation lessens, the spiraling is slower, although the new velocity is greater, and the electron slowly settles into orbit 2.

It would be surprising if the radiation had the frequency of one of these two orbits. Rather it is logical to suspect that it should be a line of some width, centered perhaps close to the average frequency between the two. In fact, the original Bohr model predicts just such a thing. The presently corrected analysis does so as well. Table 7..8.1 lists the parameters of the first five circular orbits of hydrogen. The radii are essentially those predicted by Bohr, *but the angular velocities of the electron in orbit are found from the* Eqs.(7.6.4), (7.3.4) and (7.3.2). The average ω_{mn} are calculated and the measured values are also given. These latter are the lowest terms of the Lyman, Balmer, Paschen and Brackett series respectively. It is clear that the calculated average ω_{mn} is always less than 8% different from that measured in the line spectrum. Except for the innermost transition, the measured frequency is always higher than the calculated average,

which suggests that the electron spends more time or oscillates more vigorously nearer the inner orbit. None of these details pose a problem to the intuition; so the operation of the hydrogen atom is "classical" right down to its innermost orbit (even by the Bohr Theory), making the "correspondence" principle unnecessary.

TABLE 7.8.1

THE HYDROGEN ATOM

n	R	$\omega_n = \dot{\psi}$	$\omega_{mn} = \frac{\omega_m + \omega_n}{2}$	ω_{mn} (measured)
	(cm)	(sec^{-1})	(sec^{-1})	(sec^{-1})
1	5.2918×10^{-9}	2.9232×10^{16}	1.6726×10^{16}	1.5495×10^{16}
2	2.1178×10^{-8}	4.2193×10^{15}	2.7727×10^{15}	2.8702×10^{15}
3	4.7651×10^{-8}	1.3260×10^{15}	9.5188×10^{14}	1.0046×10^{15}
4	8.4714×10^{-8}	5.7776×10^{14}	4.3983×10^{14}	4.6496×10^{14}
5	1.3237×10^{-7}	3.0191×10^{14}		

It is important to realize that *none of the orbiting rates given in Table 7.8.1 match those given by the standard solution of Shroedinger's equation.* For example, the value $2.9232\times10^{16}\,rad/s$ for the ground state orbit represents the actual, Newtonian orbiting rate found from Eq.(7.3.2). In QM, the total angular momentum p is called the *orbital* angular momentum, and the existence of the electron's angular momentum is not recognized although it is unconsciously included. If the proper mechanical format for the energy, separating the orbital and electron angular momenta, is used as the energy in Schroedinger's equation; then, also taking into consideration the present discussion of the linear orbit vs. the circular, the QM result agrees with Table 7.8.1.

The details of the last few paragraphs should not obscure the fact that the numbers in Table 7.8.1, for example, are not in any way final. They were obtained from the foregoing equations, ignoring such subtleties as the reduced mass resulting from proton motion. Correct procedure would also consider the turning energy of the proton. Table 7.8.1 overlooks this detail in the calculation of $\dot{\psi}$ and ω_{mn} (average).

7.9 **Orbital Angular Momentum**: The main difference between the preceding development and the Bohr-Sommerfeld atom is in the interpretation of the angular momentum and its effect on the type of orbits, etc. The orbital angular momentum is taken here to be p_ψ of Eq.(7.3.2) which corresponds to the usual mechanical planetary concept of orbital momentum. In the B-S model, there was no other angular momentum. In the present case, however, there is included the additional angular momentum of the electron's *turning*. According to Eqs.(7.3.3) and (7.7.9), the added turning angular momentum is,

$$p_t = \frac{1}{n_\psi} p_\psi \quad . \tag{7.9.1}$$

The total angular momentum is then,

$$p = p_\psi \left(1 + \frac{1}{n_\psi} \right) \quad . \tag{7.9.2}$$

By uniting Eqs.(7.7.4), (7.7.6) and (7.9.2),

$$p_\psi = \frac{1}{\eta} \frac{n_\psi h}{2\pi} = \frac{n_\psi}{\sqrt{1 + \frac{1}{n_\psi}}} \frac{h}{2\pi} \quad ; \tag{7.9.3}$$

so that when Eqs.(7.9.2) and (7.9.3) are combined, the total angular momentum is seen to be,

$$p = \sqrt{n_\psi (n_\psi + 1)} \ \frac{h}{2\pi} \quad , \tag{7.9.4}$$

a very familiar form. Since its first appearance in the early days of wave-mechanics, it has entered into every situation where angular momentum is present; and has remained disturbing to student and thinking physicist alike, with its forced acceptance, unexplained, take it or leave it. It is now clear that it represents the inevitable form taken when orbital and *turning* angular momenta are combined. Now, since electrons and all other fundamental particles in which distortion energy binds a $\overline{\overline{\phi}}$ distribution, when moving along curved paths in electric fields, have full turning, the form of the angular momentum will always be that of Eq.(7.9.4). It corresponds to the orbital kinetic energy plus the turning energy,

$$T_{\text{total ang. mom.}} = T_\psi + T_t = T_\psi \left(1 + \frac{1}{n_\psi} \right) \quad . \tag{7.9.5}$$

It was this condition that led to the choice of $1/n_\psi$ for K_t.

7.10. **Three Dimensional Hydrogen Atom**: In the above solution for the hydrogen atom, only two degrees of freedom were considered; whereas, in more complex atoms with two or more electrons, a third degree of freedom is involved in establishing the relative positions of two or more orbit planes. Fortunately, the three dimensional analysis of a hydrogen atom is only slightly more complicated than that given earlier. Since only a few steps are different, the analysis will be sketched here briefly, using the coordinates (r,θ,α) diagrammed in Figure 7.10.1.

First, the kinetic and potential energies are,

$$T = \frac{m_0}{2}\left(\dot{r}^2 + \eta^2 r^2\dot{\theta}^2 + \eta^2 r^2 \sin^2\theta\,\dot{\alpha}^2\right) \quad , \quad V = -\frac{e^2}{4\pi r} \quad . \qquad (7.10.1)$$

The related momenta become,

$$p_\alpha = \eta^2 m_0 r^2 \sin^2\theta\,\dot{\alpha} = k_\alpha \quad , \qquad (7.10.2)$$

$$p_\theta = \eta^2 m_0 r^2\dot{\theta} \quad , \quad \text{not constant} \quad , \quad p_r = m_0\dot{r} \quad , \quad \text{not constant} \quad .$$

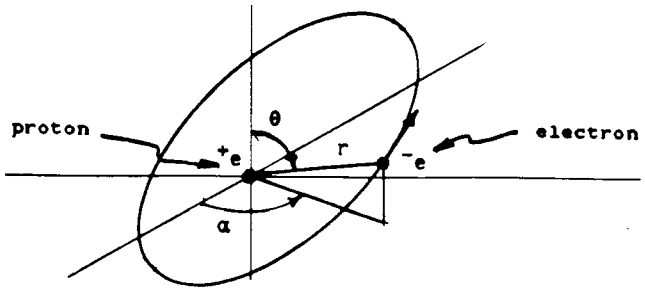

Figure 7.10.1 A three dimensional orbit.

Angle α is not in the plane of the orbit, so $p_\alpha = k_\alpha$ represents the z axis component of the *constant* total angular momentum. In the plane of the orbit, the angle will designated as ψ as before, and the total angular momentum p is a constant related to the two components p_α and p_θ by,

$$p = \sqrt{p_\theta^2 + \frac{k_\alpha^2}{\sin^2\theta}} = k_\psi \quad . \qquad (7.10.3)$$

So all of the results, related to p and a, found earlier for the planar orbit are the same as before. Only the *components* of p present a change from the earlier case.

The p_θ component is the new element in this analysis, and it must satisfy,

$$\int_{\theta_1}^{\theta_2} p_\theta d\theta = \int_{\theta_1}^{\theta_2} \sqrt{k_\psi^{\,2} - \frac{k_\alpha^{\,2}}{\sin^2\theta}}\; d\theta \quad , \qquad (7.10.4)$$

which follows from Eq.(7.10.3). Here, again, the angle θ does not go full circle, but rocks between θ_{min} and θ_{max}. A rather convoluted derivation yields,[1]

$$\int_{\theta_1}^{\theta_2} p_\theta d\theta = k_\psi \int_0^{2\pi/\eta} d\psi - k_\alpha \int_0^{2\pi/\eta} d\alpha = (k_\psi - k_\alpha) 2\pi/\eta \quad , \qquad (7.10.5)$$

since α goes through the same number of radians that ψ does.

Paralleling the earlier planar derivation, the constant total angular momentum p of Eq.(7.10.3) shows that *the principal quantum number n acts in the same way*, so the orbit criterion Eq.(7.7.2) can be repeated here,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = nh \quad , \quad (n = 1, 2, 3, \ldots) \qquad (7.10.6)$$

where,

$$\mathbf{p}_{Lt} = m_0 \dot{r}\, \hat{\mathbf{r}} + m_0 \eta^2 r \dot{\theta}\, \hat{\theta} + m_0 \eta^2 r \sin\theta\, \dot{\alpha}\, \hat{\alpha} \quad ,$$

$$\mathbf{p}_{Lt} = p_r \hat{\mathbf{r}} + \frac{p_\theta}{r} \hat{\theta} + \frac{p_\alpha}{r \sin\theta} \hat{\alpha} \quad .$$

In terms of the components, Eq.(46) becomes,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = \int p_r dr + \int p d\psi = \int p_r dr + \int p_\theta d\theta + \int p_\alpha d\alpha = nh \quad . \qquad (7.10.7)$$

Again, because the total field equation solution is *cyclic in three dimensions*, each of the components of the motion must return to the cycle start condition to be steady-state; so,

$$\int p_r dr = n_r h \quad , \quad \int p_\theta d\theta = n_\theta h \quad , \quad \int p_\alpha d\alpha = n_\alpha h \quad . \qquad (7.10.8)$$

Combining Eqs.(7.10.7) and (7.10.8),

$$n = n_r + n_\psi = n_r + n_\theta + n_\alpha \quad , \qquad (7.10.9)$$

1.H.Goldstein, Classical Mechanics, p 301, Addison-Wesley Press, Inc. (1950).

where the quantum number ranges are,

$$n = 1, 2, 3, \ldots, \infty \quad , \quad n_\psi = 1, 2, 3, \ldots, n \quad , \quad n_\theta = 1, 2, 3, \ldots, (2n_\psi - 1)$$
$$n_\alpha = -(n_\psi - 1), -(n_\psi - 2), \ldots, 0, \ldots, +(n_\psi - 1) \quad . \tag{7.10.10}$$

The limits have been determined purely by physical reasoning which is somewhat more obvious if a more conventional, completely equivalent set of quantum numbers is used. These are,*

Principial q. n.	$n=1,2,3,\ldots,\infty$	(radius a, etc.)
Angular Mom. q. n.	$\ell = 1, 2, 3, \ldots, n$	(orbit shape*)
Magnetic q. n.	$m_\ell = -(\ell - 1), -(\ell - 2), \ldots, 0, \ldots, +(\ell - 1)$	(orbit orientation)

(7.10.11)

where,

$$\ell = n_\psi = n_\theta + n_\alpha \quad , \quad m_\ell = n_\alpha \quad , \tag{7.1012}$$

and the orbit shape is set by,

$$\sqrt{1 - \varepsilon^2} = \frac{n_\theta + n_\alpha}{n} = \frac{\ell}{n} \quad . \tag{7.10.13}$$

It is customary to call the second quantum number (conventionally $\ell - 1$, as ℓ is used here) the orbital quantum number, but more realistically ℓ *represents the total angular momentum* of the electron, both orbital and turning (not spin), through the relationship in Eq.(7.9.4) repeated here,

$$p = \sqrt{\ell(\ell + 1)}\,\frac{h}{2\pi} \quad . \tag{7.10.14}$$

This can be expressed as,

$$p = \ell\sqrt{1 + \frac{1}{\ell}}\,\frac{h}{2\pi} = \eta\frac{\ell h}{2\pi} \quad . \tag{7.10.15}$$

The *orbital* angular momentum is,

$$p_\psi = \frac{1}{\eta}\frac{\ell h}{2\pi} \quad . \tag{7.10.16}$$

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* WARNING: ℓ is <u>not</u> the conventional orbital quantum number, but is increased by 1.

Physically, the circular orbit where $\ell = n$ is certainly a reality, and the $\ell = 0$ case, as discussed earlier, is not regarded as a pseudo-stable state, so the range of ℓ is from 1 to n.

The third, or magnetic quantum number is not significant in any single *free* hydrogen atom because the one electron can take any of the orbits allowed by n and ℓ with any orientation in space; except that, because of the proton's magnetic moment, there is a tendency for the orbit to be perpendicular to the magnetic field lines with the magnetic moment of the electron orbit oriented in the direction opposite the proton's magnetic moment. Physically, this is a very small effect that is normally overshadowed by other conditions. In certain experiments where hydrogen atoms are exposed to an external magnetic or electric field, the external field provides an axis relative to which the various orbits designated by m_ℓ can be shown to be occupied by the hydrogen atom's single electron.

7.11 **The *Free* Hydrogen Atom**: This section will present the structure of the three innermost sets of orbits and their total momenta (including spin) and magnetic moments for a hydrogen atom **free** of any *external* electric or magnetic fields. This is something *that no present textbook can do* because of the non visualizability of the QM ensemble approach. Whenever modern textbooks try to describe the various orbital states, they are forced to immerse the atom in some form of external field. The ensuing complication completely obscures the simplicity of a **free** atom. Here the task is to *visualize a single, deterministic atom in a field free region*.

So far, nothing has been said of the atom's *magnetostatic* field, although it plays a significant role in both the atom's structure and in its radiation process. The magnetostatic field of a single, circular filament of current is often used as a starting point for the description. A detailed discussion can be found in Sections 9.4, 9.6 and 9.7. All that must be said at this point is that the orbiting electron in each orbit creates a vortex magnetic field, roughly dipole in form. When the atom radiates a photon, an electron is falling from one orbit to another, generally smaller. Since the smaller orbit has a smaller magnetic field, the difference in magnetic moment is carried away by the emitted photon along with the difference in angular momentum.

As discussed earlier, n is the usual quantum number that fixes orbit *size* a and energy E,

$$a = \frac{n^2 h^2}{\pi m_0 e^2} \quad (A) \qquad , \qquad E = -\frac{e^2}{8\pi a} \quad (B) \qquad . \quad (7.11.1)$$

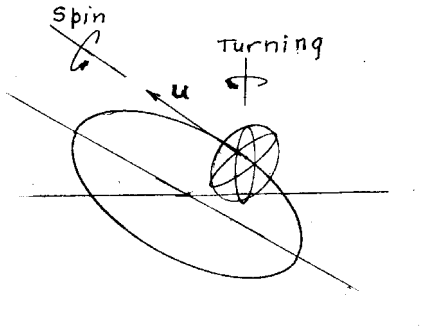

Figure 7.11.1
Orbiting electron turning and spin

Orbit *shape* is set by n_ψ. The electron is spread out *laterally*, because of its kinetic energy, and has an oblate spheroid shape. Also, in the electric field of the proton, it "turns" (in addition to the spin) so its shape axis is along the orbit. The *spin* aligns itself with the electron *shape* axis. Figure 7.11.1 shows the electron, moving in orbit at velocity $\mathbf{u}$, turning and spinning.

Total vector angular momentum

The basic *orbital* angular momentum is,

$$p_\psi = m_0 a^2 \omega \quad , \tag{7.11.2}$$

as predicted by Newton's laws. However, because the electron's shape "turns", in the plane of the orbit, one turn per electron period, this "turning" angular momentum adds to p_ψ to give a total angular momentum,

$$p = p_\psi + p_t = (1 + K_t)\, p_\psi \quad . \tag{7.11.3}$$

When atoms are *quiescent*, the turning factor K_t is related to the orbit shape through,

$$K_t = \frac{1}{n_\psi} \quad . \tag{7.11.4}$$

Combining Eqs.(7.11.2), (7.11.3) and (7.11.4), the total angular momentum, *before adding spin* is (see Section 7.9),

$$p = \sqrt{n_\psi (n_\psi + 1)}\, \frac{h}{2\pi} \quad . \tag{7.11.5}$$

At this point, before discussing the total vector angular momentum, $\mathbf{J}$, and in order to be close to the conventional notation, the substitutions $S = \sigma = \tfrac{1}{2}\hbar$ ($\hbar = h/2\pi$) and $L = p$ can be made. It is customary, in *ensemble* quantum physics, to use the same *form* that appears in Eq.(7.11.5) for S and J; i.e. $\sqrt{s(s+1)}\hbar$ and $\sqrt{j(j+1)}\hbar$. However, from the present point of view, *there is no physical justification for this*, because S and J do not have any extra turning

components. Therefor, the vector spin and total vector angular momentum become,

$$\mathbf{S} = \hat{\mathbf{S}}S = \hat{\mathbf{S}}\tfrac{1}{2}\hbar \quad , \quad \mathbf{J} = \hat{\mathbf{J}}J \quad ; \qquad (7.11.6)$$

and there are no magnetically induced precessions of these vectors involved in the *free* atom.

It is clear from Figure 10 that *the spin vector and the orbit vector are always perpendicular to each other*; so, from Eqs.(7.11.5) and (7.11.6), the total vector angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ has a magnitude,

$$J = \sqrt{L^2 + S^2} = \sqrt{n_\psi(n_\psi + 1) + \tfrac{1}{4}}\ \hbar \quad . \qquad (7.11.7)$$

This very simple equation for J will not be found in the literature, since it depends on the free atom analysis not available to modern physics.

Eqs.(7.11.1) and (7.11.5) give the energy and angular momentum for each orbit. Referring again to Figure 7.7.1, Table 7.8.1 lists some of the values related to those orbits. The radii are given in terms of the *Bohr radius*,

$$a_0 = 5.2918 \times 10^{-9} \quad , \quad \text{cm} \qquad (7.11.8)$$

a convenient unit. The atomic vortex is observed as the atom's magnetic moment,

$$\mu = n_\psi \mu_B \quad , \qquad (7.11.9)$$

where μ_B is the Bohr magneton. Values of μ for the inner orbits of hydrogen are listed in Table 7.11.1.

TABLE 7.11.1

n	n_ψ	a	ω	p_ψ	L = p	J	μ
		cm	s^{-1}	g-cm/s	g-cm/s	g-cm/s	$\frac{\text{ergs}}{\text{hlG}}$
1	1	a_0	2.9233×10^{16}	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
2	1	$4a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	4.2194×10^{15}	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
3	1	$9a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	varies	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
	3	"	1.3260×10^{15}	$2.5981\hbar$	$3.4641\hbar$	$\frac{7}{2}\hbar$	$3\mu_B$

7.12 **Multi-electron Atoms**: It is often stated, inaccurately, that the deterministic ("classical") approach to the atom "failed" when multiple-electron atoms were considered. What failed was the mathematical approach to the three-body problem, aided and abetted by the calculational success of wave-mechanics and the obfuscation of its attendant philosophy. Certainly, based on the present work, there is no obvious reason to suspect that a completely deterministic calculation of any atom cannot be made. Given, the task is horrendous. But, even the earlier Investigations of Bohr, Kimble, Van Vleck, Born and Einstein into the He atom and of Pauli and others into H_2^+ , although they did not succeed well enough to prevent the discarding of such approaches by almost all physicists, managed to contribute enough to keep some embers burning. Now, at last, stimulated by half a century of improved mathematical techniques and computer developments, some support for the deterministic approach appears to be building. Not necessarily because the recent investigators believe in that underlying truth; but because at least they have taken the proven position that the "semi-classical quantization" techniques give considerably simpler approximate solutions with as much accuracy as the corresponding approximate solutions of quantum mechanics. The added benefit, of course, is the ever present physical picture allowing complete visualization.

Take, for example, the analysis of the He atom by Leopold, Percival, and Tworkowski.[1] In comparison with the calculations of first order perturbation theory in quantum mechanics, the results were quite good, with promise of improvement with further work. Even closer agreement was obtained by Strand and Reinhardt[2] in their study of H_2^+ , where the differences from the exact results were less than a fraction of a percent. In fact, a significant number of papers of this general class has been published in the last few years. Not in the same vein, but closely related, are the numerous articles on collision processes, starting, perhaps in any significant way, with Gryzinski.[3] In a literally monumental series of papers, he managed to trigger an avalanche[4,5] of investigations into classical (i.e., deterministic) collisions. First, between particles; but, with enlarged scope, finally

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1. J.G.Leopold, I.C.Percival & A.S.Tworkowski, J.Phys.B: Atom.Molec.Phys. **13**, 1025 (1980).
2. M.P.Strand & W.P.Reinhardt, J.Chem.Phys. **70**, 3812 (1979).
3. M.Gryzinski, Phys.Rev. **115**, 374 (1959), **138**, A305, A3332 & A336 (1965); Phys.Rev.Lett. **14**, 1059 (1965), **24**, 45 (1970).
4. Only a few for example: R.C.Stabler, Phys.Rev. **133**, A1268 (1964). A.E.Kingston, Phyn.Rev. **133**, A1529 & A1537 (1964). R.H.McFarland, Phys. Rev. **139**, A40 (1965). E.Gerjuoy, Phys.Rev. **148**, 54 (1966). K.Omidvar, Phys.Rev. **177**, 212 (1969). J.D.Garcia, Phys.Rev. **177**, 223 (1969).
5. Summarized by M.R.H.Rudge, Rev.Mod.Phys. **40**, 564 (1968).

between atoms, di-atoms, etc., as represented by such groups as W.H. Miller, et. al.[1] Again, along a somewhat different line, a semi-classical radiation theory has appeared; typified by the work of E.T. Jaynes and F.W. Cummings.[2]

Taken together, all of these publications plus the development here militate for the acceptance of the position that, internally, all atoms are deterministic and calculable by ordinary planetary mechanics. Every indication is that these methods and techniques will be refined and improved; so that with time, aided by computers, the labor of working out the atomic structures, the periodic table, etc., will be completed. Completed, incidentally by entirely "classical" procedures. Some may argue that the presence of h and ν_d, etc., is not "classical". Certainly the picture evolved in the preceding chapters and sections is "classical" in that it is based on cause and effect; simple extension of mechanical principles; no mysterious space-time and no mystic regions, shapes, processes, rituals or tabus. It involves no paradoxes, and no non intuitive phenomena. The point has been reached where the artificial division between "classical" and "modern" physics should be abandoned. The mere presence of a constant, h, in much of the formalism only indicates that the ether has properties that can be related to a few constants like c_0 and h. The latter is derivative, just as c_0 is, and the *fundamental* constants are those given in Table 2.10.1. Moreover, in the works cited above, which should be examined for an imense number of references not quoted here, many so called quantum effects have been calculated deterministically. Nothing but hard work to be done and common sense to be used stands in the way of all such phenomena being explained in a similar way.

The author shall resist the temptation to develop the whole atomic scene. In the present case, the most serious changes that should be made related to atomic structure are first, an acceptance of a clear structural picture of the atomic machines, and second, a process of clearing up the loose language used in that field, which often reverses the role of cause and effect. The picture sketched in Section 7.10 can be expressed most effectively in the over-simplified graphic form of Figure 7.12.1. Here the meanings of the three quantum numbers n, ℓ and m_ℓ can be better understood. The true picture is not presentable in this static form because of the effects of the orbit

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1.W.H.Miller, J.Chem.Phys. **53**, 1949 (1970), **81**, 3573 (1984); Plus more than twenty papers with collaborators.

2. E.T.Jaynes & F.W.Cummings, Proc.I.E.E.E:. **51**, 89 (1963). M.D.Crisp & E.T.Jaynes Phys.Rev. **179**, 1253 (1969). C.R.Stroud,Jr. & E.T.Jaynes, Phys.Rev.A, **1**, 106 (1970). E.T.Jaynes, Phys.Rev.A, **2**, 260 (1970). R.K.Nesbet, Phys.Rev.Lett. **27**, 553 (1971).

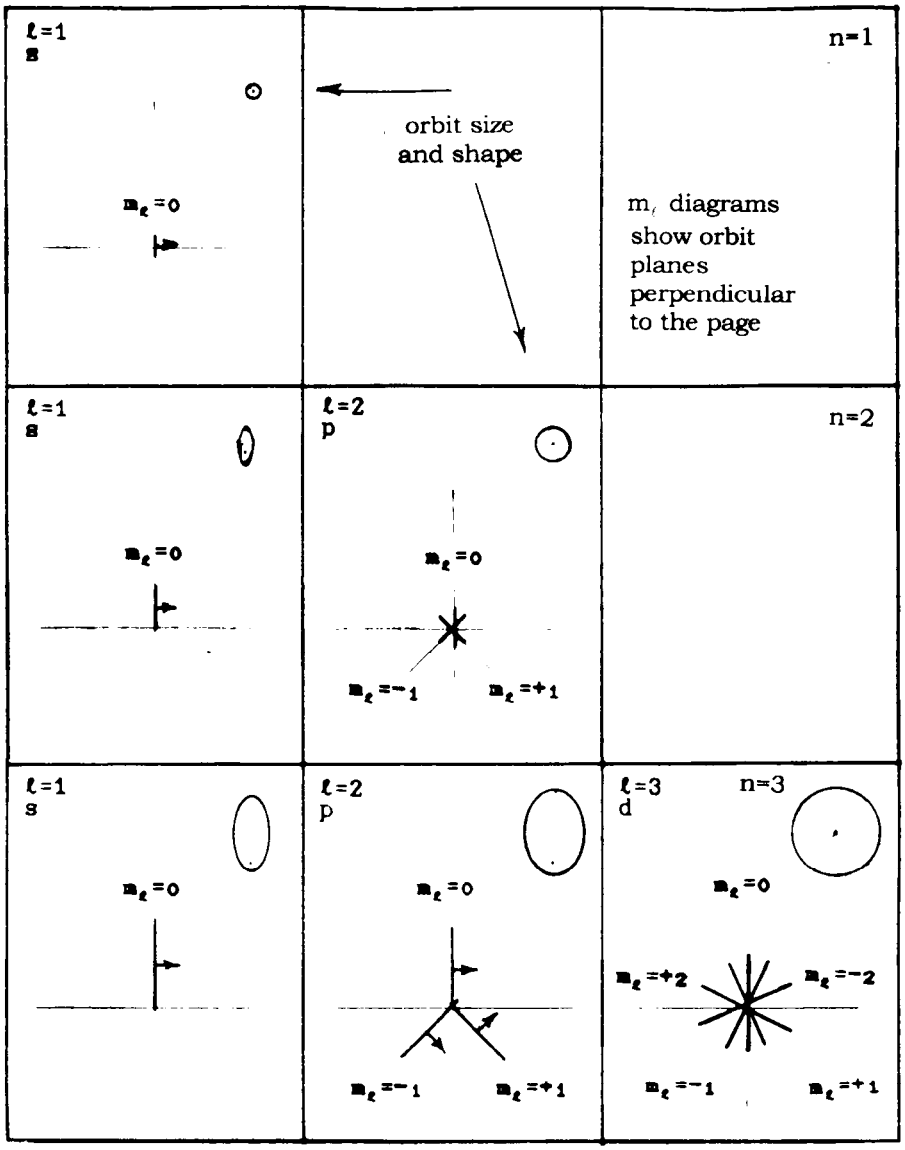

Figure 7.12.1 Allowed atomic orbits through n=3.

recessions, mass variability, spin, etc. Nevertheless, Figure 7.12.1 goes far in making the atom visualizable.

The simplest multiple electron atoms are those with many electrons in closed inner shells and one valence electron. Subject to certain well known assumptions that permit the closed shells and the

nucleus (of many protons and neutrons) to be lumped together and represented by a modified potential, the analysis is very similar to that given for the hydrogen atom. The principal difference is that the allowed orbits are all closer in to the center, but the various values of n, ℓ and m_ℓ are all the same. That is, *no new allowed orbits are created as the larger atoms are built up*. This is still true in atoms with two or more valence electrons.

Figure 7.12.1 represents two conditions at once. First, as nucleons are added to the nucleus and electrons are added externally to match, starting with the innermost orbit n = 1, ℓ = 1, m_ℓ = 0, and filling each successive layer, all of the orbits are filled in order until the next step is n = 3, ℓ = 3. Beyond that point a number of cases of electrons filling orbits out of order are observed, but the general trend Is to follow close to the simple order represented in Figure 7.12.1. Second, all of the empty orbits are available to electrons excited from an inner orbit, and these are responsible for the observed spectra generated as excited electrons drop back into vacated lower energy orbits. The relationship between the orbit structure represented in Figure 7.12.1 and the periodic table, valence, and all the other important chemical and physical properties of atoms are very well presented in any book on modern physics.[1]

In connection with the first condition, i.e. the filling of inner orbits, an example such as nitrogen will be helpful. Both its n=1, ℓ=1 and its n = 2, ℓ = 1 orbits are full (2 electrons each), and its n=2, ℓ = 2 orbits m_ℓ = - 1, 0 and + 1 have one electron each. It is reasonable to understand the behavior of these three electrons in terms of their mutual electrostatic repulsion; i.e. they keep as far away from each other as possible, while still following orbits allowed by their sustaining waves. Probably, instead of being perpendicular to the plane of the paper, as shown, they are arranged more like a propeller, i.e. twisted out of the plane. Since these three orbits are the only ones allowed in that n and ℓ level, the question can be asked, "what does the next electron do?" "Why does it not go into one of those orbits with one of the other electrons?" The answer is, of course, that it does. In fact, as is well known, every orbit shown in Figure 7.12.1 takes two electrons to fill it. Certainly the next obvious question is, "why two and not three?" To answer this question, one further physical condition must be included in the picture of the atom; the electron spin. The quantum numbers n, ℓ, m_ℓ and the orbits they represent, illustrated in Figure 7.12.1, were derived using a simple planetary analysis of an assumed artificial separatist force model of the atom,

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1. G.F.Lothian, Electrons in Atoms, Butterworths, London (1963).
   A.Beiser, Perspectives of Modern Physics, McGraw-Hill, N.Y. (1969).

tracing the center of motion of the electron. The fact that during the electron's creation it had imparted to it a spin angular momentum in order to conserve the system momentum was simply ignored in the electron solution of Section 3.10 and in Section 7.10. Nevertheless, the results of this separatist model analysis are astoundingly accurate; and yet there is much more involved. The true picture, as stated many times in previous sections, is actually that of many little bumps and hollows of ether density all gyrating under the control of a strict set of rules (the ether field equations), policed by their subtle sustaining waves. How many particles are allowed in any region, how they move, etc., are things that are totally decided by the field equations. This includes each electron's spin, which is retained by it until the whole particle field is annihilated, if ever that occurs. So, it is in the total three dimensional field equation solution of the atom that the fact no two electrons can ever orbit the same path with parallel spins is established. Such a condition would be equivalent to allowing two distinct solutions (one or two orbiting electrons) of the field equations with exactly the same boundary conditions. Therefore, calling the spin quantum number m_s ($\pm \frac{1}{2}$), the spin is either right handed or left handed with its axis along the orbit, and no more than two electrons per orbit are allowed. A third would violate the observed rule that no two electrons with exactly the same four quantum numbers n, ℓ, m_ℓ, m_s can exist in any one atom. Conventionally, this is known as Pauli's exclusion principle; but as such is merely an observation. In the ether theory, it is a derivable fact.

7.13 **The Ether Datum Fluctuation**: Based on the preceding discussion, it will be assumed that the motion of particles, i.e. bumps and depletions of ether with their ℓ-waves, can be visualized as deterministic. Nevertheless, because of a residual fluctuation always present in the datum ether, an electron, for example, will move along any path with a continuously varying erratic perturbation of that path. From the separatist force point of view, the particle appears to take on and give off small amounts of energy (distortion) as it vacillates. Photons bombarding the particle could account for the electromagnetic (transverse) fluctuation of Nernst, Braffort, Marshall, and Boyer. On the other hand, if the datum fluctuations were merely scalar variations in density, the sum of numerous longitudinal waves going in all directions, the result would still appear like ordinary Brownian notion. No further speculation along these lines can be made now.

7.14 **Line Widths of Deterministic Atoms**: Earlier, correction of loose language was mentioned as one of the hoped for gains from a deterministic atomic theory. This loose language often appears as part of the description of line widths and rates of return from unstable to stable states. Numerous examples will be obvious to the reader when he attempts to relate the deterministic picture being constructed here with the exposition in almost any text on quantum mechanics. Only one small aspect of the overall confusion will be discussed here as an illustration.

Workers who are accustomed to think in terms of bandwidth-time products commonly used by engineers are often misled in atomic studies by the apparent failure of some of the simple rules about line widths and pulse lengths. For example, a pulse of microwave radiation generated by some electrical circuit has a bandwidth that is inversely proportional to the pulse length transmitted. However, when the fluorescence line width of say a neodymium:YAG crystal (pumped to excite its $^4F_{3/2}$ level) is measured, that line width bears no obvious relationship to the pulse length (life-time) of the fluorescence that results. A mixture of classical physics and conventional quantum mechanics is needed to properly explain this. The usual explanation is always incomplete, and sometimes erroneous. To correctly describe the complementary way that classical physics and quantum statistical mechanics combine to explain the phenomena, it is best to start with a discussion of a *free* atom spectrum.

If a single *free* atom is excited by photon impact with one of its electrons so that the electron takes up an orbit farther out, a simple succession of events follows. After reaching the outer region, the excited electron orbits along its planetary path, with energy constant except for the small buffeting of the datum fluctuations, and, in the non-circular orbits, the oscillations due to electron expansion/contraction. As diagrammed in Figure 7.14.1, after a *delay* period determined by the orbit and the fluctuations, the buffeting has so perturbed the orbit that the exchange of energy between the electron and the datum cannot be stabilized; and the

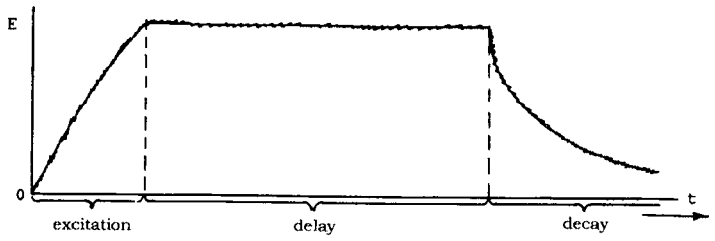

Figure 7.14.1 Excitation, delay and radiation in a single free atom.

electron begins its inward spiral and radiates. During a *decay* period, its energy is depleted by the radiation and it finally settles into its old orbit.

The radiation sequence during the *decay* period can be described completely by photon production, and the associated radiation linewidth is that of the spectrum of a single photon. When a single atom is *not free*, i.e. when it is immersed in a gas or a solid host, if it is excited, the photon generation process during its decay is often shortened and the spectrum of its generated photon broadened by the now increased buffeting of the field surrounding it. All of the *decay* processes that specifically broaden the individual photon spectrum are known as photon broadening mechanisms. Where a whole ensemble of excited, identical atoms form a gas or is immersed in a solid host, the ensemble spectrum formed by any photon broadening mechanism results from the *superposition of all of the essentially identical photons*, each with a spectrum as broad as that of the ensemble.

A second, important characteristic of the photon radiated by a free atom during its *decay* period, is its center frequency. As a result of the datum fluctuations, there is a shift of the energy difference between the initial and final orbits; for, although the fluctuations cancel out during the electron's descent from one orbit to the other, the instantaneous starting and ending energies affect the photon's center-frequency (average orbital frequency) through the total field equation solution of the atom and the datum field. Since the datum field is only known statistically, only an ensemble picture of the photon center-frequency shift can be calculated. When the ensemble comprises a gas or is immersed in a solid host, the center shifting of photons is often increased by the greater buffeting of the surrounding field, giving a more broadened observed spectrum. All similar decay processes are known as center shift broadening mechanisms. Some writers identify photon and center shift broadening as homogeneous and inhomogeneous respectively, but the correspondence is not always exact. Both broaden the spectrum; but the former has all photons essentially identically broadened with identical center frequencies; whereas the latter has unbroadened photons center shifted.

It was Lorentz[1] who pointed out that certain characteristics of radiation from an atomic transition can be described accurately by an equivalent oscillating electron. The ether oscillations of an actual orbiting electron undergoing the transition from one state to another produce a photon that, in *some* respects, is equivalent to one produced by a simply oscillating electron initially displaced from a

1. H.A.Lorentz, The Theory of Electrons, 2nd Ed. p 53,259, Dover Publications Inc. N.Y. (1952).

center of equilibrium. Commonly, the *decay* time and spectral line shape due to photon broadening are calculated from an equation representing a damped oscillation. The following is an abbreviation of a nice development by Stone.[1] In the standard damped oscillator equation for a *free* atom,

$$\frac{d^2s}{dt^2} + \omega_p^2\, s = \frac{F_r}{m_0} \quad , \tag{7.14.1}$$

s represents the displacement of the electron from its equilibrium position, ω_p is the undamped radiation frequency, and F_r / m_0, is the damping term. The damping is caused by the electron's own field, as it radiates, and is found from (see chapter 11),

$$F_r = \frac{e^2}{6\pi c_0^3}\frac{d^3s}{dt^3} \quad . \tag{7.14.2}$$

The final equation comes from the combination of Eqs.(7.14.1) and (7.14.2),

$$\ddot{s} + \omega_p^2\, s - \frac{e^2}{6\pi m_0 c_0^3}\dddot{s} = 0 \quad . \tag{7.14.3}$$

In somewhat oversimplified form (see Stone) the solution is a damped sinusoid,

$$s = s_0 \varepsilon^{-\alpha t} \cos \omega_p' t \quad , \tag{7.14.4}$$

where,

$$\omega_p' = \sqrt{\omega_p^2 - \alpha^2} \cong \omega_p \quad , \tag{7.14.5}$$

and,

$$\alpha = \frac{\pi e^2}{3 m_0 c_0^3}\nu_p^2 = 1.23 \times 10^{-22}\nu_p^2 \quad . \tag{7.14.6}$$

Therefore, the energy radiated is also an exponentially decreasing quantity,

$$E = E_0 \varepsilon^{-2\alpha t} = E_0 \varepsilon^{-t/\tau_p} \quad , \tag{7.14.7}$$

where the photon time constant τ_p is related to α by,

$$\tau_p = \frac{1}{2\alpha} = \frac{4.04 \times 10^{21}}{\nu_p^2} \quad . \tag{7.14.8}$$

The time constant τ_p is often called the "Classical time constant"; but here it will be called the *free* atom photon decay time, being identified

1. J.M.Stone, Radiation and Optics, p 247ff, McGraw-Hill Book Co. N.Y (1963).

with that portion of Figure 7.14.1 during which the particular electron radiates, i.e. creates a photon.

Equation (7.14.4) also represents the *space* distribution of the photon wave that propagates away from the radiating atom, assuming that the photon velocity is constant throughout. Although no *conventional* model of a photon exists, it is possible to utilize some of the measurements on photons to give a rough picture of a representative form. A number of experiments[1] seem to show that a photon is a long needle-like wave train. At $\lambda = 1.06\mu$, for example, the photon emitted by a *free* neodymium atom appears to be 1526 cm long to the point where $c_0 t = c_0 \tau_p$, its diameter is less than 5×10^{-5} cm, and is more likely of the order of 10^{-8} cm. From its beginning to the point $c_0 t = c_0 \tau_p$ it has 1.43×10^7 wave cycles. Although this model might only be roughly like the actual photon, it is close enough to be used in any analysis of photon interactions. A Fourier analysis shows that such a wave train has what is called the Lorentz distribution for its spectrum,

$$\frac{I}{I_p} = \frac{1}{1 + 4\left(\dfrac{\nu - \nu_p}{\Delta\nu_p}\right)^2} \quad , \tag{7.14.9}$$

where here again ν_p is the *undamped* or center frequency, and I_p is the center frequency intensity of the observed single photon radiation. In this case, $\Delta\nu_p$ represents the half power band width, and is related to the damping constant and *decay* time constant by,

$$\Delta\nu_p = \frac{\alpha}{\pi} = \frac{1}{2\pi\tau_p} \quad , \tag{7.14.10}$$

Eqs.(7.14.1) through (7.14.10) apply to a *free* atom, where the only damping is the electron's own field. An atom in a gas or a solid host is subjected to a continuous bombardment by the surrounding lattice atoms or the gas molecules. The result can be a larger instantaneous distortion of the electron's field with a correspondingly increased instantaneous intensity of energy released per oscillation and a shortening of its *decay* time. This produces a shorter photon with more ether distortion in its individual waves. Since the electron, during the photon generating process, was exposed to this excess random buffeting, the photon has essentially a phase-modulated wave structure; and its bandwidth is greater. This preserves

1. A.E.Ruark & H.C.Urey, Atoms. Molecules and Quanta, Vol I, p 81 ff, Dover Publications, Inc. N.Y. (1963).

the bandwidth-time relationship of Eq.(7.14.10), and the spectral shape of Eq.(7.14.9).[1]

Unlike the photon broadened spectrum, just described in a completely deterministic way, the center-shift broadening must be calculated statistically. The source of the statistical variation for a *free* atom is the datum fluctuation, and the formalism used is quantum statistical mechanics and Schroedinger's equation. It should be emphasized that both the Shroedinger and the Dirac form of quantum mechanics yield the same *ensemble* results as the atomic theory of the previous sections *if the correct mechanics is inserted* (electron turning). For *ensemble* calculations, the QM approach is more practical. Conventionally, the interaction of matter and radiation is formalized by considering an ensemble of *free* atoms as a separate system; then, considering a box-like space full of quantized radiation of all modes as a second, separate system; and finally, bringing them together via an interaction energy (small perturbation).[2] This first order theory still yields sharp spectral lines, however, *unless the fact that the population is decreasing exponentially during the* delay *period is* injected *into the formalism*.[3] Such a step provides time for the fluctuations to influence the energy levels of the initial and final orbits. Intuitively, the excited atom population should decrease exponentially; and experimentally this is borne out. More will be said about this when the *delay* itself is discussed. Meanwhile, the effect of the radiation on the sharp energy levels of the original free atom is represented in Figure 7.14.2. The line widths of the various transitions are determined by the spread of each of the energy levels, which depends upon all of the possible transitions to levels below the level evaluated. The original sharp levels are now varied so that the probability that any atom in a particular excited state has an energy E is given by the Lorentzian distribution,[4]

$$p\{E\} = \frac{2}{\pi \Delta E} \frac{1}{1 + 4\left(\frac{E - E_0}{\Delta E}\right)^2} \quad , \qquad (7.14.11)$$

where E_0 is the nominal energy level of the state and ΔE is the "width" of the level. In Eq.(7.14.11), a slight shift of the level has been neglected. If an atom changes from state (2) to state (1), i.e. from energy E_2 to energy E_1 in the specific case shown in Figure 7.14.2,

1. A.E.Siegman, An Introduction to Lasers and Masers, p 103, McGraw-Hill Book Co. (1971).
2. W.Heitler, The Quantus Theory of Radiation, 2nd Ed. p 102, Oxford U. Press (1947).
3. V.Weiskopf & E.Wigner, Zeits.f.Physik. **63**, 54; **65**, 18 (1930).
4. H.E.White, Introduction to Atomic Spectra, p 422, McGraw-Hill Book Co. N.Y.(1934)

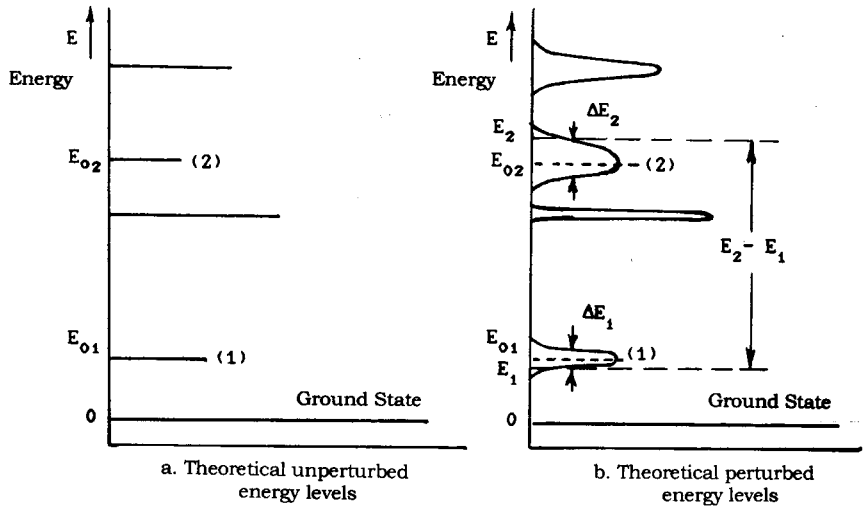

Figure 7.14.2 Center-shift broadening from datum fluctuations.

the photon produced in the transition will have a center frequency,

$$\nu_{21} = \frac{E_2 - E_1}{h} \quad , \tag{7.14.12}$$

generally different from the nominal center frequency. For the total ensemble, the center frequencies of different atoms' transitions can vary over a range governed by the widths of both levels, so that the line width of the center shift broadened spectrum is,

$$\Delta\nu_c = \frac{\Delta E_2 + \Delta E_1}{h} \quad . \tag{7.14.13}$$

The overall result of this center shifting is that any ensemble of free atoms will produce a radiation spectrum (neglecting the radiation reaction) that has a Lorentzian distribution,

$$\frac{I}{I_0} = \frac{1}{1 + 4\left(\frac{\nu - \nu_0}{\Delta\nu_c}\right)^2} \quad . \tag{7.14.14}$$

Notice that the *form* of Eq.(7.14.14) is exactly the same as that of Eq.(7.14.9). Except for that, *they are completely different*, since one represents the spectrum of the individual photon as it is generated and the other represents the whole *ensemble* of all the center-shifted frequencies of all the photons generated by electrons making that transition. It is repeatedly claimed in books on quantum mechanics that Eq.(7.14.14) is the quantum mechanical "equivalent" of Eq.(7.11.9). Nothing could be more untrue, as has been indicated.

The conventional derivation uses transverse electromagnetic waves to describe the radiation in the various modes in the box. While this is correct for a photon field in which the atomic ensemble is immersed, the question of the form of the datum fluctuations should be left open.

If the ensemble is immersed in a low pressure gas (or composes one), where the time between collisions is long, the photon is generated between collisions and is not broadened. However, the simple translational motion of each radiating atom converts the center frequency and translates the whole spectrum of its one photon by the doppler shift. The ensemble therefore exhibits a broadened spectrum, gaussian in shape, whereas the individual photons are radiating according to Eqs.(7.14.1) through (7.14.10). Similar shift effects occur in solids as the result of fields imposed by crystalline strain, lattice defects, and inhomogeneities. Except for the datum fluctuation generated center-shift, all of the other center-shift mechanisms generate gaussian distributions, as the result of the central limit theorem applied to the whole ensemble. Most gaseous and solid host cases involve both photon and center-shift broadening simultaneously. Thus, $\Delta\nu_p$ and $\Delta\nu_c$, which are quite different in cause and magnitude, must be combined, and the resultant distribution is called a Voigt profile,[1] neither a Lorentzian nor a gaussian shape.

Returning now to the time delay before the photon forming process begins; the *delay* time τ_d is unrelated to the spectrum and *decay* time, except that the mechanism that determines both is the buffeting of the datum fluctuations and also the thermal vibrations where present; the *delay* time being how long, typically, the electron in orbit must sustain the buffeting before correlating its motion with the transition configuration, and the *decay* time being how many bumps the electron sustains and how many orbits it makes during the transition. The *delay* time can be visualized better by neglecting the *decay* times, as depicted in Figure 7.14.3. Using the time dependent Schroedinger equation applied to the case of an ensemble of excited ions immersed in a fluctuating field, it can be shown that for any particular ion, the delay time is determined statistically, i.e. there is a small probability,

$$p = A_{21}dt \quad , \tag{7.14.15}$$

that its excited electron will begin radiating and decaying from state 2 to state 1 during any short time interval dt. The quantity A_{21} is known as the Einstein spontaneous emission coefficient. Eq.(7.14.15) has validity only in the limit of very short times, and p really expresses the

1. D.W. Posener, Austral.J.Phys. **12**, 194 (1959).

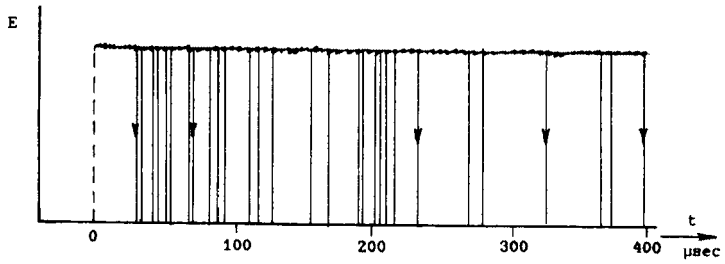

Figure 7.14.3 Delay time distribution of decreasing excited population.

idea that the fraction of ions in state 2 that drop out during dt is,

$$-\frac{dN_2}{N_2} = p = A_{21}dt \quad , \tag{7.14.16}$$

where N_2 represents the total excited ion population. Thus, A_{21} is the rate of decrease of population per unit population,

$$A_{21} = -\frac{1}{N_2}\frac{dN_2}{dt} \quad , \tag{7.14.17}$$

and is a constant for any given system. Clearly A_{21} has nothing to do with the *decay* time or the radiation coming from any individual ion. Eq.(7.14.16) can be solved for the population of the excited state as a function of time. The result is,

$$N_2(t) = N_2(0)\varepsilon^{-A_{21}t} \quad , \tag{7.14.18}$$

which indicates that the population decreases exponentially with a fluorescence time constant,

$$\tau_d = \frac{1}{A_{21}} \quad . \tag{7.14.19}$$

This is exactly what was expected since it was *introduced* into the quantum statistical mechanics calculation that allowed the center-shift broadening of Eq.(7.14.14).

As a practical example of the times and bandwidths, the Nd:YAG rod mentioned at the beginning of this section has very representative numbers. When excited to fluorescence, the observed line width ($\lambda = 1.064 \times 10^{-4}$ cm) at room temperature is $\Delta\nu_f = 5.66\,\text{Å} = 1.50 \times 10^{11}$ cps, and the time constant of the fluorescence decay curve is $\tau_p = 230\ \mu s$. Clearly, the latter has no relationship to

$$\tau_f = \frac{1}{2\pi\Delta\nu_f} = 1.06 \times 10^{-12}\ \text{sec} \quad . \tag{7.14.20}$$

At the fluorescence wave length, $\lambda = 1.064 \times 10^{-4}$ cm , the free-atom, undamped center frequency is $\nu_p = 2.82 \times 10^{14}$ cps. From Eq.(7.14.8), $\tau_p = 5.09 \times 10^{-8}$ sec; and, using Eq.(7.14.10), the free atom photon *decay* linewidth is $\Delta\nu_p = 3.13 \times 10^6$ cps. Based on the datum fluctuation, the free atom center-shift linewidth $\Delta\nu_c$ of Eq.(7.14.13) is of the same order of magnitude. Thus, the free atom combined linewidth $\Delta\nu_p + \Delta\nu_c$ is of the order of 10^7 cps.

When the ions are immersed in a YAG rod, the lattice vibrations at room temperature cause the combined linewidth to broaden to the value given above ($\Delta\nu_f = 1.5 \times 10^{11}$ cps). Reducing the temperature to near zero ^oK shrinks the linewidth to a fraction of an Angstrom, showing a small residual crystal strain, clearly indicating that most of the line broadening is thermal in nature. Separating $\Delta\nu_f$ into its components $\Delta\nu_p$ and $\Delta\nu_c$ is not possible here for several reasons. One is that the line shapes are both Lorentzian, obviating Voigt profile techniques. Another is that, in spite of the fact that one is a center-shifted broadening, the rapid thermal fluctuations ($\cong 10^{12}$ per second) cause every ion in the ensemble to pass through every center frequency represented in the broadened spectrum many times per second so that only the most sophisticated kind of hole burning experiment could provide any distinction. Thus, the time in Eq.(7.14.20) represents the thermal variation for both processes taken together.

Turning now to the *delay* time, $\tau_d = 230\ \mu$sec , the transitions that produce the spectral line of interest are all within the 4f shell. A burst of photons passing through the host rod impacts many of the Nd^{3+} valence electrons in the $4f^3$ orbits of numerous members of the doping ensemble. These electrons are physically transported into pseudo-stable orbits outside the 4f shell from which they rapidly pass (through radiationless transitions, stimulated by rapid manipulation of the site fields around the ions due to lattice vibrations), and gradually they settle Into the $4f^3$ orbits again, but with spin/orbit states that are more energetic than the ground state. These more energetic states are metastable, i.e. on the average it takes a long time before the lattice buffeting and the datum fluctuations cause the spins and orbits to change to return an electron to the ground state. The passage of the population from the metastable state to the ground state, observed throughout the whole host, produces a fluorescence radiation decreasing exponentially as the remaining excited population decreases. The time constant of this process is very long

because, in the free atom, transitions within the 4f shell are forbidden (i.e. $\tau_d \rightarrow \infty$). When the ensemble is immersed in the host rod, the electric fields surrounding the ions break the symmetries and shorten the lifetime. In the pumped states outside the 4f shell the excited electrons are strongly affected by the lattice vibrations, resulting in rapid ($\tau_d \cong 10^{-9}$ sec) radiationless transitions back to the excited 4f states. But, these latter are shielded from the lattice by the outer 5s and 5p shells; so, although the transitions within 4f back to the ground state are permitted, the fluorescence time constant is still a relatively long one, $\tau_d = 230\ \mu$sec.

By now it should be clear that statements indicating that Newton's laws and Maxwell's equations do not apply inside the atom are simply not true. Newton's laws and classical mechanics apply without any modification. Maxwell's equations, when recognized as the weak field equations they are, also apply where they should reasonably be expected to. The need to make small changes in Maxwell's equations for application to the strong field regions inside the particles is a normal classical condition that leads to the visualizable planetary form of atom given here.

CHAPTER 8

RODS, CLOCKS AND PLUMB BOBS

8.1 **Introduction**: In Section 1.4 the broad concept of physics was said to be a combination of *knowing* and measuring. By *knowing* is meant the *understanding*, through cause and effect explanations, of the physics closely related to each new experiment. Generally the new experiment, when *explained*, adds to the understanding. The relative importance of measurement and "knowing" are brought out here by the fact that not until this 8th chapter was there any need to discuss measurement in detail. The intuition and "knowledge" common to every day experience was sufficient to carry the work this far. Now, measurement must be discussed because of the arrived at awareness of the non-rigid, variable nature of layered particles out of which all measuring devices are made.

In modern textbooks, the discussion of measurement usually begins and ends with a description of Einstein's theory of Special Relativity; but even today, the Earth's surface is peppered with literally hundreds of *dissidents*, some knowledgeable, some not, who are confused by the non-intuitive explanation of the observations and calculations SR is used for. Although it is clear to almost any practicing physicist that the Lorentz transformation is an essential tool in any quantitative study, it is equally clear that, for 104 years the explanation of the meaning of the observations and calculations continues to be disturbing to many.[1]

Actually, Einstein's original paper didn't try to explain the results, but just showed that Maxwell's equations required only a few restrictions to give the Lorentz transformation equations. The confusion arises from Minkowski's *space-time* explanation (see Section 1.9). In looking at the results of the Lorentz transformation, if the question was asked, "How can the measuring rod of each of two observers be shorter than the other's?"; the explanation was, "space-time works in mysterious ways". This was the same as saying, "The spirits drank it" to explain evaporation. Cause and effect were abandoned and replaced with paradox. There was no excuse for it.

The present chapter is an attempt to reconcile the successful formalism of the Lorentz transformation with an intuitive, non-paradoxical explanation of the physics based on cause and effect.

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1. See the discussion of A. Ungar, Am. J. Phys. **56**, 814 (1998) and R. W. Brehme, 811. These also have important references.

8.2 **Observers and Labs**: Most experiments are conducted in laboratories that are rooms with benches and equipment (eg. rods and clocks) used to measure, among other things, length and time intervals. These rods and clocks, as well as *the investigators, themselves*, are made of particles that are quite flexible. Lorentz and others have shown that these particles *change shape when accelerated with respect to the ether*; although, *at any constant speed, they maintain the particular distorted shape that corresponds to that speed* (see Section 3.10).

Even more important is the fact that a lab worker *can know immediately if he is accelerating with respect to the ether*, because he senses the acceleration as his body particles are changing shape. Although rods and clocks are the measuring instruments most familiar to everyone, because of their common every day use, a third, and perhaps more important measuring device, is the *plumb bob*. To understand its importance, visualize two observers whose coordinate systems are platforms upon which they rest, fixed. In Figure 8.2.1, the essentials are represented. The platforms are out in free space where negligible gravitic action is present. One platform is moving at constant velocity u relative to the ether, the other is accelerating relative to the ether at the constant rate a = K (the cause of the acceleration is not shown). The particles making up the plumb bob and string in the constant velocity system are not changing in any way, but merely translating through the ether. Thus, wherever they happen to be in that system, they stay, and the string is generally just loosely distributed between the holder and the bob. Not so in the accelerating system. Here, each particle is being "forced" to change

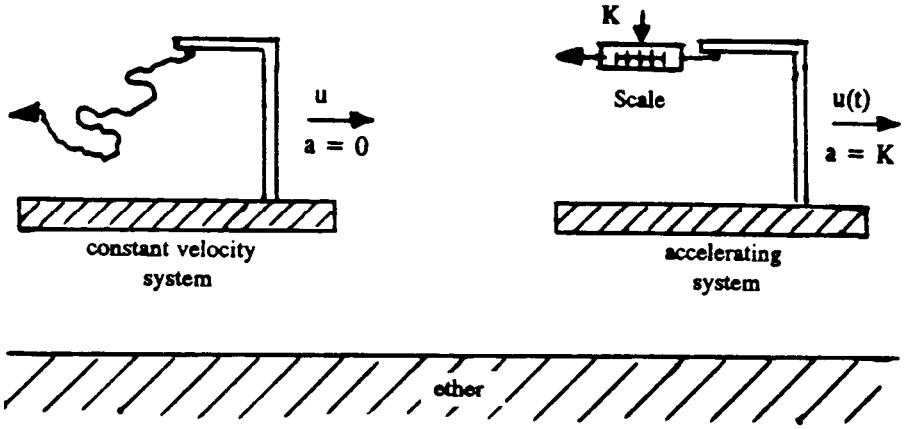

Figure 8.2.1 Observers with plumb bobs.

shape and gain energy, so that it opposes the change; and thus the string is stretched taut and the bob points in the direction opposite the platform acceleration. In fact, if a spring scale is inserted between the holder and the string as shown in Figure 8.2.1, the *magnitude* of that acceleration can be measured.

People who understand neither physics nor relativity have been heard to say, "all motion is relative." This kind of statement is confusing. It is true that the kinematic relationships between the two observers of Figure 8.2.1 are equivalent; that is, geometrically the accelerating system "sees" the constant velocity system accelerate away from it in the opposite direction. But, both of them "know" that only the one is Accelerating (with a capital A); because Acceleration is absolute. It produces an interaction, between each particle's configuration and the ether, that has *physical* effects. This always comes as a shock to those who believe "all motion is relative".

The presence of the ether is brought home dramatically to anyone living in a modern society, because one's body is a plumb bob. It is an extremely sensitive detector of Acceleration. The act of stepping on the gas or of braking suddenly in a car thrusts the ether's powerful presence into one's consciousness; whereas the smooth ride at constant speed on a flat freeway makes it obvious that constant velocity particles sail through the ether unchanging. The sad thing about the last 114 years of ether exile is that it was caused by the rejection of the ether as superfluous in the constant velocity systems, totally ignoring the immensely more important Accelerating systems. Special relativity is to all observers as Newton's first law is to all of dynamics, almost superfluous. Yet the ether was rejected for *all* cases on the basis of its subtle, negligible appearing role in those few restricted areas. Every day, one's own body tests and re-tests the ether's omnipotent presence.

To sum up the basic situation, each observer who is Accelerating anywhere relative to the ether is made personally aware of that Acceleration by his own body and senses. Moreover, he can make an absolute quantitative *measurement* of that Acceleration by the use of a plumb bob and spring scale (or other equivalent Accelerometer). All observers agree on just who is Accelerating and how much, in spite of the geometrical symmetry in the kinematics of the motion. When in doubt use a plumb bob. A major part of the world's operation involves dynamics, i.e. "forces" and Accelerations.

On the other hand, a constant velocity object is a dull and uninteresting thing on the grand scale of things. Its innards are unchanging, and it does not "interact" with the ether, being the simplest kind of solution of that medium's rules and regulations. A set of particles all moving along together form an observer who does not sense his motion, and cannot tell whether or not he is at rest

relative to the ether. Furthermore, a plumb bob and spring scale will not indicate his motion or the ether's presence.

It was stated in Chapter 2 that an *absolute* observer is one with respect to whom the ether is everywhere at rest in a field free lab. That is the condition of a *third* lab observer at rest relative to the ether shown in Figure 8.2.1. Since no constant velocity lab (non-accelerating) observer has any indication of his velocity relative to the ether, it might seem reasonable to assume that none of those observers, with a range of velocities, could identify whether or not they were the *absolute* observer with u = 0. Based on the Principle of Identical Environments (see Section 1.4), it would seem just as reasonable for any one of them to *assume he was the absolute observer*, thus making the calculations for various experiments much easier. When this is true will become clear in the following.

8.3 **Rod Contractions**: Previous chapters have yielded a picture of the layerons as very flexible constructs in the ether that change shape and internal distortion as they change velocity. This raises the question, "If the particles change as they move, how does this affect the rods and clocks?" As first described by Lorentz, a practical measuring rod consists of a number of particles laid end to end, but always in neutral pairs or groups in the form of atoms. These already have their inner dimensions established by their balance of motions and deformations. Conventionally, their "forces" are balanced. Likewise, the "forces" between the atoms are balanced to determine the length of the rod. Similarly, a real clock is composed of many layerons arranged in various ways. The fact that there is no "matter", other than these very flexible bulk layeron constructs in the ether, accounts for all the changes in the rods and clocks when they move relative to the main body of the medium.

First consider a rod constructed of atoms. Having established a symbol for its length L, the next task is to find what happens to its length when it moves at constant velocity with respect to the ether. Here *it will be assumed*, as in the earlier chapters, *that the absolute observer* who fashioned the rod *is at rest relative to the ether*. It will further be assumed that the ether is not deformed in a large region outside the rod. The implications of these assumptions, and objections that an observer cannot know whether or not he is at rest relative to the ether, will later be discussed in detail; and such objections will be shown to be pointless.

When a rod is at rest, its internal "forces" are determined by the negative gradient of $\bar{\bar{\phi}}$; but when the rod is in motion with constant

velocity, the negative dynamic gradient,

$$\mathbf{E} = -\left(\nabla\overline{\overline{\phi}} + \frac{1}{c_0^2}\frac{\partial(\overline{\overline{\phi_a \mathbf{V}}})}{\partial t} \right) \quad , \tag{8.3.1}$$

conventionally called the *electric field intensity*, determines the "force". This quantity was first introduced in Eq.(2.14.2). Earlier it was shown that $\overline{\overline{\phi}}$ in layerons *expands laterally* to their direction of motion, yet, if $\mathbf{E}$ is calculated from Eq.(8.3.1) it is found to *contract longitudinally*. So Lorentz was able to demonstrate[1] (using a point charge field) that when the rod moves at constant velocity, the dynamic gradient (field intensity, $\mathbf{E}$) contours contract in the direction of motion by $1/\gamma$. Here, the full equations for $\overline{\overline{\phi}}$ will be used to get the same result.[2]

Using the positron equivalent of Eq.(3.10.16), with Eq.(8.3.1),

$$\begin{aligned} \mathbf{E}_0 &= \frac{e}{4\pi r_0^{\,2}}\varepsilon^{-2r_e/r_0}\hat{\mathbf{r}} \quad , \quad \text{At rest} \\ \mathbf{E} &= \gamma\frac{er}{4\pi r'^3}\varepsilon^{-2r_e/r'}\hat{\mathbf{r}} \quad , \quad \text{Constant velocity} \end{aligned} \tag{8.3.2}$$

where,

$$r' = \sqrt{\gamma^2 x^2 + R^2} \qquad , \qquad \gamma = \frac{1}{\sqrt{1 - \dfrac{u^2}{c_0^2}}} \tag{8.3.3}$$

and the positron is traveling in the x direction at velocity u. Here, r_0 is an arbitrary contour radius corresponding to the field intensity $\mathbf{E}_0$. To find the shape of the r_0 contour in motion, equate Eqs.(8.3.2). The contraction along the x axis is obtained by setting $R = 0$ and solving for the new x,

$$x = \frac{r_0}{\gamma} \quad , \tag{8.3.4}$$

which is the same result obtained by Lorentz using his "point charge" electron model.

On the basis of this contraction of the particle $\mathbf{E}$ field, Lorentz argued correctly that the total rod length L, when a rod *moves* parallel

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1. H.A.Lorentz, Proc.Acad.Sci.Amsterdam, **6**, 809 (19O4). Reprinted in The Principle of Relativity, Dover Publications Inc. Also see Lectures on Theoretical Physics, Vol 3, Macmillan Co. Ltd., London (1931).
2. R.H.Dishington, Physics, Beak Publictions, Pacific Palisades, CA (1989). ....., Advances in Fundamental Physics, p. 187, M. Barone & F. Selleri Eds., Hadronic Press, Palm Harbor, FL (1995).

to its length, would contract to,

$$L_m = \frac{L}{\gamma} \quad . \tag{8.3.5}$$

Being made of particles that are solutions of the same field equations as the electron/positron, *all material objects contract in the direction of their motion with respect to the ether*. This is not a mathematical or philosophical contraction, but a real physical shortening of the rod moving through the ether. Cause and effect are acting through the behavior of the particles. The reality of this shortening is obscured in present day texts by the introduction of a set of *symmetrical* equations, thus leading to much confusion. Those symmetrical transformation equations cannot and should not be introduced at this stage of the discussion because they involve a set of arbitrary specific acts that can only be justified after further development of the approach used here. By holding off that part of the procedure until later, the source of the usual confusion will be obvious and that confusion will vanish. *It should be quite clear that only one observer is so far involved*.

8.4 **Clock Slowing**: Clocks moving relative to the ether run slower than clocks at rest in it. This is true for all types of clocks, mainly because clocks are made of particles just as rods are. Because there are many kinds of time measuring devices, it is not as easy to prove the general statement as it was for rods; but if each type is examined, it always turns out to be true. Most texts describe simple photon clocks, where a photon is sent out to a mirror and back, detected, and another photon sent, etc. There, the time interval of the moving clock is increased because the photon path length is longer.

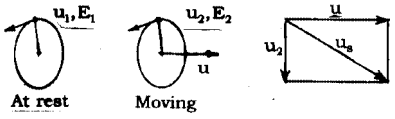

Figure 8.4.1 Circling mass clock.

Another elementary form of clock involves a mass circling on the end of a string or fine wire of negligible mass, as exhibited in Figure 8.4.1. When the clock is at rest, the energy of the circling mass is,

$$E_1 = \frac{E_0}{\sqrt{1 - \frac{u_1^2}{c_0^2}}} \quad ; \tag{8.4.1}$$

where the relationship to its rest energy E_0 is taken from Eq.(3.10.24). Now the clock is moved at constant velocity u in a direction perpendicular to the orbit (this is not necessary, but simplifies the discussion). It is assumed that the motion has been inaugurated without unduly disturbing the orbiting mass, so that its

momentum perpendicular to the clock's direction of motion is conserved. The mass now executes a spiral motion through the ether, and its new total energy is,

$$E_2 = \frac{E_0}{\sqrt{1 - \frac{u_s^2}{c_0^2}}} \quad , \tag{8.4.2}$$

where, $u_s^2 = u_2^2 + u^2$ as seen in Figure 8.4.1. Using the conservation of momentum,

$$\frac{E_2}{c_0^2} u_2 = \frac{E_1}{c_0^2} u_1 ; \tag{8.4.3}$$

which can be combined with Eqs.(8.4.1) and (8.4.2) to yield,

$$\frac{u_2}{\sqrt{1 - \frac{u_s^2}{c_0^2}}} = \frac{u_1}{\sqrt{1 - \frac{u_1^2}{c_0^2}}} \quad . \tag{8.4.4}$$

Squaririg both sides, cross multiplying and cancelling like terms, Eq.(8.4.4) becomes,

$$u_2 = u_1 \sqrt{1 - \frac{u^2}{c_0^2}} = \frac{u_1}{\gamma} \quad . \tag{8.4.5}$$

This result reintroduced into Eq.(8.4.3) shows that,

$$E_2 = \gamma E_1 \quad . \tag{8.4.6}$$

Thus, the orbital speed of the mass particle *decreases*, to compensate for its energy increase due to the motion at velocity u, and just enough to preserve its momentum, which has no reason to change. At rest with respect to the ether, the period of the clock is,

$$T = \frac{2\pi r_1}{u_1} \quad ; \tag{8.4.7}$$

while, in motion with respect to the ether, its period is,

$$T_m = \frac{2\pi r_2}{u_2} \quad . \tag{8.4.8}$$

So, the period in motion is related to the period at rest by ($r_1 = r_2$),

$$T_m = \frac{r_2}{r_1} \frac{u_1}{u_2} T = \gamma T \quad ; \tag{8.4.9}$$

i.e. the clock runs slower. Again, this is not an hypothetical change, but a true slowing of its rate; because it moves relative to the ether and its changes in deformation and speed produce the effect. All clocks, no matter how complex, behave in the same way. Spring clocks, for example, involve length contractions, energy changes, velocity changes and numerous interactions of these, yet they follow Eq.(8.4.9). An easier clock to analyze is an atom clock, where the preceding mass on a string device works in almost the same way, except that the radius of the electron orbit in the atom is held by only an electrostatic gradient when the clock is at rest and by a combined dynamic gradient reduced by a magnetic force caused by the mutual motions of the orbiting electron and the charged nucleus along lateral parallel paths when the clock moves. So far, no one has ever found a clock that did not follow Eq.(8.4.9) when moving through the ether at constant velocity.

It should be clear that in using Eqs.(8.3.5) and (8.4.9) only one observer is involved, and the cause and effect relationship is basic. *Ether properties determine particle properties,* since particles are just stable configurations of ether. *Particle properties determine rod and clock characteristics*. It was once well known that all of the results of special relativity can be derived directly from the ether caused contractions in a completely intuitive way using cause and effect. On the other hand, special relativity as presently propounded has no cause and effect basis. It is presented on a "take it or leave it", "that's the way it is" basis by invoking the mysterious "space-time" as its foundation. No sensible physicist could choose the latter over the former.

8.5. **Laboratory Classification**: The rest of this chapter will concentrate on how much can be known quantitatively in an experiment, on explaining some of the techniques used to abet measurement and on certain odd effects that result from flexible particles. It will be assumed that the qualitative actions of plumb bobs and Accelerating observers are already intuitively understood. Moreover, the status of so called inertial systems, i.e. the set of constant velocity observers whose plumb bobs are relaxed, is also assumed to be the result of intuitively understood extended particle properties. Note that as far as real "inertial systems" are concerned, almost none exist, because of the ever present gravitic fields. Only tests run perpendicular to gravitic fields simulate the inertial system concept. Perhaps the only true inertial system in the universe is that which has its boundaries so far out in space that all known stars and galaxies, etc., are way inside some imaginary box. Then, the observer whose motion is matched to that of the average position of all the matter and energy inside the box would be an inertial observer. He

could not tell if the overall datum ether density ϕ_d was moving relative to him at some constant velocity. This is actually the only problem dealt with by special relativity except in those cases that neglect gravitic fields as an approximation.

Some confusion appears in discussions of rod and clock measurements because of the heavy emphasis on *transformations* between moving observer systems. Actually, all physics can be discovered and explained in just one laboratory, and the so called paradoxes of transformation theory have very simple explanations.

The first step is to recognize that some labs are much better than others in which to do experiments. *Labs are classified according to how the datum ether is flowing through them.* There are three classes of labs:

INERTIAL	1. Primary - Datum ether at rest
	2. Secondary - Datum ether at one constant velocity
NON INERTIAL	3. Accelerating - Datum ether varying in velocity

In choosing a lab for the purpose of discovering the laws of physics, *one class can be eliminated at the outset*. The non inertial labs, including rotating and linearly accelerated labs like that in Figure 8.2.1, require very complicated equations to describe even simple experiments. So, most experiments are done in inertial labs.

An *inertial* laboratory is one in which a *lab worker standing still in the lab feels no acceleration*. If the datum ether is at rest everywhere in an inertial lab, that lab is designated as a *primary* inertial lab. In this type of lab a lab worker is called an *absolute* observer; and the laws of physics, as seen by an absolute observer, take the simplest form. A lab built on the platform of a rotating merry-go-round is definitely **not** a primary inertial lab, and the laws of physics in that lab are very complicated. Disbelievers should try to play catch in that rotating lab.

8.6. **Primary Inertial Labs**: In setting up *any* lab, coordinates must be installed and clocks set. The coordinate system is *attached* to the walls, ceiling and floor, and relates a unique set of three numbers to each point in the lab. Since the ether is *at rest* in primary labs, the point location is unambiguous.

In primary labs, clock setting is also simple. One can imagine, although it is not practical, having a tiny clock *at each point* in the lab

space, wall to wall and floor to ceiling. When doing experiments, in any and all labs, the observer in each lab always assumes that *all the clocks in the lab have been set to read exactly the same time at any given instant.* He thinks that he has used a clock set procedure that ensures this. In secondary labs this is most often not true, but in primary labs *it can always be true if the clocks are properly set.*

The *simplest* procedure, for primary labs, is *light* set. This involves one master clock at rest in the ether, which is at rest in the lab. Light signals are sent from the master clock to all the other lab clocks. There are two versions of light set, one way and two way. In one way, when the signals arrive, the clocks are set to the master clock's time by correcting for the signal time delay, based on the constant velocity of light c_0 in the ether, and the known distance to each point. In two way, each clock sends back a pulse to the master, and the outlying clocks are set to half the round trip time.

A third more difficult method uses a particular form of *contact* set by carrying a clock from the master clock to all the others at a known constant speed and making the proper corrections.(see Eq.(8.4.9)).

All of these methods produce the same true *clock synchronization* of any primary inertial lab's clocks.

8.7 **Contact Clock Set and the One Way Velocity of Light**: The most *fundamental* way to synchronize two identical clocks is to use the most elementary form of "contact set". Surely, when two clocks are at the same point they can be synchronized by "contact", and will remain synchronized as long as they remain together. Now, in a primary inertial lab, let one of these be fixed relative to the ether, and the other moving past it at constant velocity. In this example, the ether is assumed to have the datum density ϕ_d everywhere outside the clocks. As they pass, they are set together, say at $t = 0$. Since the moving clock runs slower, from that initial moment of *contact* on, the moving clock and the fixed clock are geared together by Eq.(8.4.9), and so the absolute or ether observer knows the readings of both clocks exactly, as long as their environment does not change.

A slightly more elaborate "experiment" can be set up as follows. As shown in Figure 8.7.1, the absolute observer uses only one *fixed* clock A, the master clock. He has two other identical clocks, B and C attached to the ends of a rod, previously arranged so that their center to center distance is γL when at rest relative to the ether and A. *During the experiment*, the rod and clocks B and C are moving through the ether in the x direction at constant velocity u. The ether observer (absolute observer) is the only one involved in the experiment, and is the one manipulating the clocks A, B and C.

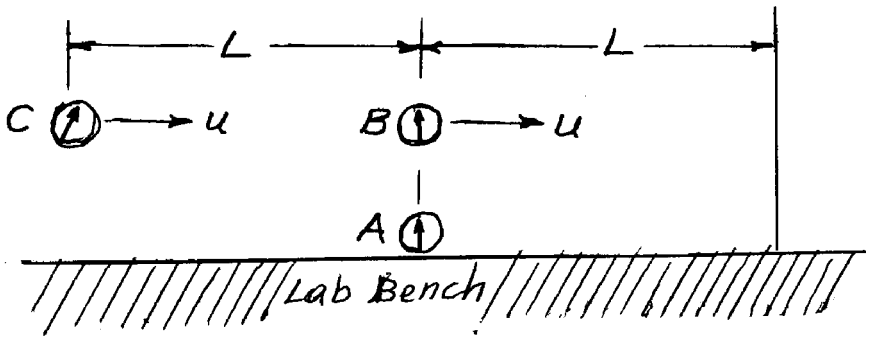

Figure 8.7.1. Contact synchronizing two moving clocks.

The experiment proceeds as follows:

At $t = 0$, B is *contact* set to A's $t = 0$

At $t_{1A} = \frac{L}{u}$, B arrives at L and C arrives at A

B reads $t_{1B} = \frac{L}{\gamma u}$, $\gamma = \frac{1}{\sqrt{1 - u^2 / c_0^2}}$

A *contact* sets C to $t_{1C} = t_{1B} = \frac{L}{\gamma u}$

B and C are now *truly* synchronized *to each other* and remain so as long as they continue at velocity u.

The absolute observer has previously chosen clocks B and C that can send out single light pulses towards each other on demand. He had done an experiment, when the rod and clocks B and C were at rest, and had observed that pulses sent out at the same time arrived at the opposite clock after identical intervals. He now continues the experiment to determine the *one way* pulse arrival times as measured by B and C in motion. Because they are moving, his intuition tells him that B and C will intercept their respective pulses at *different* times, since the pulses travel at the constant velocity c_0 in the ether.

The results of the experiment go as follows:

$$\text{At} \quad t_{1A} = \frac{L}{u}, \qquad t_{1B} = t_{1C} = \frac{L}{\gamma u}$$

B and C emit pulses

While the light from B is going to C, C keeps moving; so the lab observer sees the pulse going a distance $L - x$ to get to C, arriving at:

$$t_{2A} = \frac{L - x}{c_0} + t_{1A}$$

But, $$x = u(t_{2A} - t_{1A})$$

so:

$$t_{2A} - t_{1A} = \frac{L}{c_0 + u} \quad \text{and} \quad t_{2B} - t_{1B} = t_{2C} - t_{1C} = \frac{L}{\gamma(c_0 + u)}$$

Conversely, while the pulse from C is going to B, B is moving away from it and a similar calculation reveals that:

$$t_{3A} - t_{1A} = \frac{L}{c_0 - u} \quad \text{and} \quad t_{3C} - t_{1C} = t_{3B} - t_{1B} = \frac{L}{\gamma(c_0 - u)}$$

Now, suppose that there is a second observer riding along on the *moving* platform (rod), using the clocks B and C and the rod length γL. If he calculates the apparent propagation velocities of the photons received by B and C, he gets,

$$c_B = \frac{\gamma L}{\Delta T_B} = \gamma^2(c_0 + u) \quad , \quad c_C = \frac{\gamma L}{\Delta T_C} = \gamma^2(c_0 - u) \quad . \qquad (8.7.1)$$

Indeed, he has measured the *one way* velocity of light and <u>it is different in both directions</u>, *a first order effect*.

This is no philosophical trick experiment, but a real one, although it has not yet been performed as described. If it is, no matter who does it or where, the result will always come out as stated above. Here, the question must be asked, "does anything about this experiment offend the intuition?" The answer musts be "no". for, both the ether observer and the one moving with the platform accept contact synchronization as the most fundamental; and, through cause and effect, they also accept the rod and clock rate changes. Both

expect the velocity of propagation to appear different in the two directions, because both know that the pulses propagate at the velocity c_0 relative to the ether, and the platform is moving with respect to the ether. *If the experiment is repeated with the platform moving in the opposite direction, the pulse from* B to C *will take longer.*

A simplified form of the same experiment is illustrated in Figure 8.7.2. The roles of A and B are the same as before, but C is replaced by a fixed detector. Only one photon, going from B to C, is involved, and the travel time of that photon will be the same as in the other case. Unfortunately, even this version may not be simple enough. Unless someone thinks of a much better variation, this experiment might not be possible, because of the great distances and short times required.

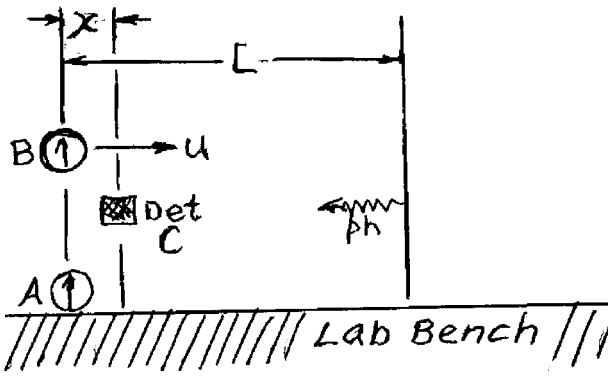

Figure 8.7.2 .
Simplified one way velocity of light experiment.

Nevertheless, there are other indications that the outcome is stated correctly. For example, Sagnac[1] mounted four mirrors on a rotating platform and sent two light beams in opposite directions around the loop they formed. The interference fringes showed that their propagation velocities matched the amounts predicted by Eqs.(8.7.1). Later larger loops were used, and Michelson[1] built a huge outdoor circuit of mirrors and evacuated pipes and accurately measured the Earths' rotation rate by the same technique. It is true that these rotation system tests are not done in a primary, or even a secondary inertial lab environment; but they still present nearly the same simple, unchanging flow of ether as seen by the photons. The steady state, radial ether acceleration is cancelled out in the opposing massless photon paths.

Several important conclusions can be drawn from this experiment. For example, a distinction must be made between the phrase "the laws (equations) of physics" and the term "experiments". If the procedure described in Figure 8.7.1 were repeated, with two ether clocks and one moving clock, and the roles of the two sets of clocks

1. H. P. Robertson, T.W. Noonan, Relativity and Cosmology, pg.38, Sagnac, pg. 40, Michelson, W. B. Saunders Company, Philadelphia (1968).

exactly reversed, the two moving ether clocks would see exactly the same thing observed by the ether observer in the original experiment. Thus, the experiment supports the idea that identical *experiments* in *all* inertial labs yield identical results (see Section 1.4). It contradicts the statement that the laws (equations) of physics are the same in *all* inertial systems; because, in the particular ether theory being described here, the fundamental equations (e.g. Maxwell's) are assumed, from the start, to apply in primary ether labs, whereas the inertial lab represented by the moving clocks and rod in the experiment would require a *modified* set of equations to accommodate the two different propagation velocities observed.

8.8 **Actual Ether Experiments**: There are two classes of ether experiments that have been conducted since around 1887:

1. Measurements to determine
   a. Two way velocity of light
   b. Clock slowing
   c. Rod contraction

2. Measurements to detect constant velocity motion relative to the datum

All accurate *measurements of the velocity of light* have been *two way* measurements. The presently accepted value is regarded as well established. Until an experiment equivalent to the one described in Section 8.7 is achieved, the one way velocity must be deduced.

Clock slowing was first measured directly as late as 1938.[1] Even then it was a very difficult measurement involving the frequency shift of radiating hydrogen atoms moving in a beam. The time dilation had to be separated from a much larger doppler shift component. Later, the lifetimes of decaying unons and bions both in cosmic rays and finally in large accelerators gave convincing support to the fact[2]. A reasonable interpretation of the Hafele-Keating experiment also concurs[3].

It is true that *rod contraction* has never been measured *directly*, but that hardly takes away from the fact of its existence. Certainly the notorious Michelson-Morley experiment led both Fitzgerald and Lorentz to accept the contraction as the most reasonable explanation

1. H.E. Ives, G.E. Stilwell, J.Opt.Soc.Amer. **28**, 215 (1938); **31**, 369 (1941).
2. B. Rossi, D.B. Hall, Phys.Rev. **59**,223 (1941). Durbin, Loar & Havens, Phys.Rev. **88**, 179 (1952). J. Bailey et al, Nature, **268**, 301 (1977).
3. J. Hafele, R. Keating, Science, **177**, 166 (1972).

of the results. Many examples of so called relativistic experiments depend upon rod shortening as well as the two preceding characteristics in such a way that it would require a fantastic coincidence for all of them to come out as observed. Moreover, Selleri's theorem[1] elegantly demonstrates that *if any two of the three conditions above are found to hold, the third must also be in effect.*

Referring to Item 2 above, it is well known that *no experiment has yet determined the constant velocity of the ether relative to any inertial system*. From the following examples, it should be clear that this will always be the case. On the other hand, everyone has performed an experiment that shows the acceleration of the ether.

8.9 **Cause and Effect Cure of the "Paradoxes"**: The analysis of almost any experiment involving moving observers and using the Lorentz transformation leads to "paradoxical" results when "explained" on the basis of space-time. When the ether is used to describe the same proceedings, no paradox is found. One example of this is the popular twin paradox.

The classical twin paradox has one twin stay at home and the other travel a long way away, turn around and come back, only to find that the stay-at-home has aged more than the traveler. There are two versions of this experiment, one involving *accelerations* and the other involving only *constant velocities*. Solution of the former involves several approximations, but the velocity case is straightforward, so it will be presented here. For simplicity, three clocks will be used in place of "twins". Figure 8.9.1 diagrams the essentials. A is a fixed clock, and B and C are moving in opposite directions at the same velocity u (this is not necessary but simplifies the discussion). At the start, they have the positions shown and no attempt has been made to synchronize any of them. As C passes A, however, C is *contact* synchronized to A, the ether clock. When C and B pass each other, B is *contact* synchronized to C; and this takes place at a distance L from A. When B passes A, their times are compared, and it is found that A reads a later time than B.

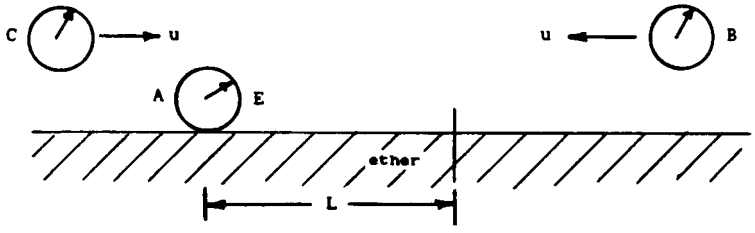

Figure 8.9.1 Constant velocity twin experiment.

1. F. Selleri, Apeiron, **4**, 100 (1997).

There is no paradox here, just the facts. If A reads $\Delta T = L/u$ when C passes B, C according to Eq. (8.3.9) will read $\Delta T/\gamma$. B is contact set to $\Delta T/\gamma$, and after another $\Delta T = L/u$, as read by A, B passes A. At that time, A reads $2\Delta T = 2L/u$ and B reads $2\Delta T/\gamma$, according to Eq. (8.3.9), as a result of its motion through the ether. Therefore, B is "younger" than A by the ratio $1/\gamma = \sqrt{1 - u^2/c_0^2}$. Here, again, this is no mathematical or apparent change, but a real effect that present day clocks are on the verge of showing comfortably. Chapter 7 showed that atoms are little solar-system like arrangements, essentially clocks; and, *if* age is actually related to how many times the electrons orbit, living twins (or triplets) would truly age differently.

All the arguments in the literature notwithstanding, there is no question whatever of this clock effect's not taking place. It has only been introduced here to provide another insight into the ether and its properties. True, from the relativistic point of view, it may be just as hard to swallow as any of the "which contraction is real" answers, but the cause and effect picture rendered possible by the ether makes it easy to understand.

It is the asymmetry that provides and removes the paradox. In the velocity experiment, the *single* clock is the older and the two clock set that reverses is the younger. In the acceleration experiment, the sensing of Acceleration (with a capital A) provides the asymmetry. If the stay-at-home never feels Acceleration, and the traveler does, the stay-at-home is older. *If both were to part and then each feel the same Acceleration sequence ,their ages would be equal upon reuniting.*

Beyond merely clarifying the "paradox", this experiment offers an excellent way to illustrate how the contractions prevent inertial system observers from detecting their own absolute motion relative to the ether. To do this, the above experiment will be described in a more formal way. Because, in the second part of the experiment, the observer A will be an inertial observer with no knowledge of his velocity relative to the ether, all measurements are taken as contact readings or counting marks on one scale or another without recourse to any *light set* type clock synchronizations. This means that A does not make any velocity measurements. These are all observed by B and C and later by the ether observer E. Referring to Figure 8.9.2, A, who is the ether observer (unknown to him), has a long rod that extends in both directions. It has evenly spaced marks all along its length. The experiment starts with C and B farther apart than shown, and each moving towards the other at a previously agreed upon speed u'' which they measure by counting the marks on A's rod, as they pass, and their own time. When C,A contact occurs, A and C start timing. When C,B contact occurs, C time is $\Delta T_1''$ and B is set. At the

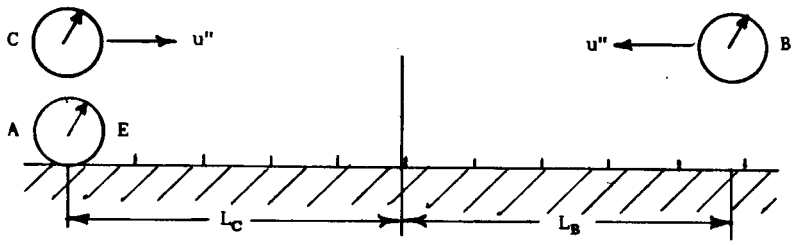

Figure 8.9.2 Detailed constant velocity twin experiment.

instant of B,A contact, B reads $\Delta T'' = \Delta T_1'' + \Delta T_2''$ and A reads $2\Delta T'$. B is slower than A by the ratio,

$$R = \frac{\Delta T_1'' + \Delta T_2''}{2\Delta T'} = \frac{\Delta T''}{2\Delta T'} \quad . \tag{8.9.1}$$

Since A is at rest in the ether, the moving clocks C and B run slower than the ether clock by,

$$\frac{\Delta T''}{2\Delta T'} = \sqrt{1 - \frac{u'^2}{c_0^2}} = \frac{1}{\gamma'} \quad . \tag{8,9.2}$$

But, u' is not measured in this experiment; so, to get this ratio in terms of the measured velocity u'', notice that,

$$u' = \frac{L_c}{\Delta T'} \quad , \quad u'' = \frac{2L_c}{\Delta T''} \quad \text{and} \quad R = \frac{\Delta T''}{2\Delta T'} = \frac{u'}{u''} \quad .$$

Now, using the identity,

$$(\gamma')^2 = 1 + (\gamma')^2 \frac{(u')^2}{c_0^2}, \tag{8.9.3}$$

and $R = 1/\gamma'$ (combining Eqs.(8.9.1) and (8.9.2)),

$$R = \frac{1}{\gamma'} = \frac{1}{\sqrt{1 + \dfrac{(u'')^2}{c_0^2}}} \quad ; \tag{8.9.4}$$

which gives the ratio of times in terms of the measured velocity. Except for the conversion to the form of Eq.(8.9.4), this result is no different from that derived earlier; but the details of the process are now significantly more specific.

In the second part of the experiment, let A be an inertial observer who is *not at rest in the ether*; but who, of course, is unaware of that fact. If A, B, and C repeat the experiment, the result will be exactly the same. It must be, or A would be able to know of his motion relative to the ether. Following Figure 8.9.3, B and C set their velocities as before to the value $u'' = u''_b = u''_c$. However, because A is moving at velocity V, *their true velocities relative to the ether are not equal*, but instead, as will be shown, are related to u'' by,

$$u'' = u''_c = \gamma\,\gamma_c(u_c - V) \quad , \quad u'' = u''_b = \gamma\,\gamma_b(u_b + V) \tag{8.9.5}$$

where,

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{V^2}{c_0^2}}} \quad , \quad \gamma_b = \frac{1}{\sqrt{1 - \dfrac{u_b^2}{c_0^2}}} \quad , \quad \gamma_c = \frac{1}{\sqrt{1 - \dfrac{u_c^2}{c_0^2}}} \quad . \tag{8.9.6}$$

If L_c is the point of C,B contact, the corresponding time C reads is $\Delta T_1''$ and E reads,

$$\Delta T_1 = \gamma_c \Delta T_1'' \quad . \tag{8.9.7}$$

By that time A has moved,

$$L_{A1} = V\Delta T_1 \quad , \tag{8.9.8}$$

and the distance to the C,B point from A is $L_1 = L_c - L_{A1}$. Thus, C and B see that point as,

$$L_1' = \gamma(L_c - L_{A1}) = \gamma L_1 \quad . \tag{8.9.9}$$

At the time of B,A contact, A has moved to $L_{A1} + L_{A2}$ and B has moved

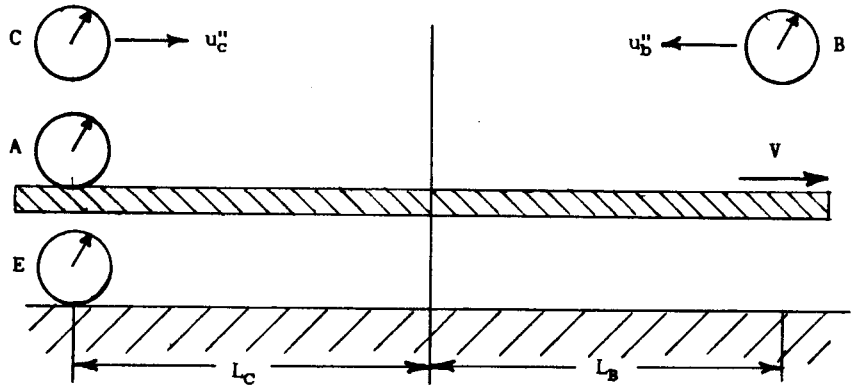

Figure 8.9.3 Non ether observer twin experiment.

from L_c to $L_{A1} + L_{A2}$, in time $\Delta T''$ (as read by B). At that time, E reads $\Delta T_1 + \Delta T_2$, where,

$$\Delta T_2 = \gamma_b \Delta T_2'' \quad \text{and} \quad L_{A2} = V\Delta T_2 \quad . \tag{8.9.10}$$

E sees C moving at velocity,

$$u_c = \frac{L_c}{\Delta T_1} = \frac{L_c}{\gamma_c \Delta T_1''} \quad , \tag{8.9.11}$$

whereas C interprets this as,

$$u_c'' = \frac{L_1'}{\Delta T_1''} = \gamma \frac{(L_c - L_{A1})}{\Delta T_1''} \quad . \tag{8.9.12}$$

Combining Eqs.(8.9.7), (8.9.8), (8.9.11) and (8.9.12) gives the relation in Eq.(8.9.5). E sees B moving at velocity,

$$u_b = \frac{L_c - L_{A1} - L_{A2}}{\Delta T_2} = \frac{L_c - L_{A1} - L_{A2}}{\gamma_b \Delta T_2''} \quad , \tag{8.9.13}$$

whereas B Interprets this as,

$$u_b'' = \frac{L_1'}{\Delta T_2''} = \gamma \frac{(L_c - L_{A1})}{\Delta T_2''} \quad . \tag{8.9.14}$$

Combining Eqs.(8.9.10), (8.9.13) and (8.9.14) gives the second relation in Eq.(8.9.5). From the preceding equations, it follows that,

$$\Delta T_1'' = \Delta T_2'' \quad , \quad \frac{\Delta T_1}{\gamma_c} = \frac{\Delta T_2}{\gamma_b} \quad , \quad \text{and} \quad \gamma_c(u_c - V) = \gamma_b(u_b + V) \quad . \tag{8.9.15}$$

At the end of the experiment, A reads,

$$\Delta T_1' + \Delta T_2' = \frac{\Delta T_1}{\gamma} + \frac{\Delta T_2}{\gamma} \quad . \tag{8.9.16}$$

At that same time, B reads,

$$\Delta T'' = \Delta T_1'' + \Delta T_2'' = \frac{\Delta T_1}{\gamma_c} + \frac{\Delta T_2}{\gamma_b} \quad . \tag{8.9.17}$$

So, B is slower than A by the ratio,

$$R = \frac{\Delta T''}{\Delta T_1' + \Delta T_2'} = \frac{2\gamma}{\gamma_c + \gamma_b} \quad . \tag{8.9.18}$$

As a check, if $V = 0$, $\gamma = 1$, $\gamma_c = \gamma_b = \gamma'$, so that, $R = 1/\gamma'$ as before.

Now, the γ, γ_b and γ_c of Eqs.(8.9.18) and (8.9.6) are given in terms of E measurements, not B and C. To convert to their values, the same approach is used that was used earlier. Assume for a moment that B and C also counted marks on E. Then,

$$u''_{ec} = \frac{L_c}{\Delta T''_1} = \gamma_c u_c \quad , \quad u''_{eb} = \gamma_b u_b \quad , \tag{8.9.19}$$

$$R_{e1} = \frac{1}{\gamma_c} = \frac{u_c}{u''_{ec}} \quad \text{and} \quad R_{e2} = \frac{1}{\gamma_b} = \frac{u_b}{u''_{eb}} \quad .$$

Therefore,

$$R_{e1} = \frac{1}{\sqrt{1 + \frac{(u''_{ec})^2}{c_0^2}}} \quad \text{and} \quad R_{e2} = \frac{1}{\sqrt{1 + \frac{(u''_{eb})^2}{c_0^2}}} \quad . \tag{8.9.20}$$

Substituting Eqs.(8.9.20) into Eq.(8.9.18),

$$R = \frac{2\gamma}{\sqrt{1 + \frac{(u''_{ec})^2}{c_0^2}} + \sqrt{1 + \frac{(u''_{eb})^2}{c_0^2}}} \quad . \tag{8.9.21}$$

Now, with the aid of Eqs.(8.9.19) and (8.9.5),

$$u''_{ec} = \frac{u''}{\gamma} + \gamma_c V \quad , \quad \text{and} \quad u''_{eb} = \frac{u''}{\gamma} - \gamma_b V \quad ; \tag{8.9.22}$$

which, combined with Eq.(8.9.21) and a very round about and tedious manipulation, finally gives,

$$R = \frac{1}{\sqrt{1 + \frac{(u'')^2}{c_0^2}}} \quad . \tag{8.9.23}$$

This is exactly the same as the result from the original experiment. Thus, A cannot tell that he is not the ether observer.

At this point it must be emphasized that there is nothing mysterious about this. On the contrary, it was expected as the inevitable result of the kind of contractions the particles composing rods and clocks undergo because of their nature. All of the previous examples, and many others[1] indicate that regardless of which inertial system observer sets up an experiment, the result is always the same,

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1. R. H. Dishington, PHYSICS, Beak Publications, Pacific Palisades, CA (1989).
_____, PHYSICS 2001, loc. cit. (2001).

so that *no inertial system observer can detect his uniform motion with respect to the ether.* Yet in Section 8.6, using contact setting of two clocks, the one way velocity of light was shown to be *different* in opposite directions, and both were different from c_0. Therefore, it would seem that here was a case where the ether motion could be determined. In fact, it can't. If the same experiment were set up with the roles of the two sets of clocks exactly reversed, the two moving clocks would see exactly the same thing observed by the ether observer in the earlier example; whereas the present ether observer would see that it all came about because of the rod and clock changes, but that the moving observer couldn't tell the difference.

Without knowing about the rod and clock changes, the investigators working between 1865 and 1910 made over 50 different kinds of experiments[1], some quite exotic, to detect the ether's motion relative to an inertial system. It was, for obvious reasons, never found. All of these experiments were staunch evidence for the existence of the ether; otherwise, how could such a variety of complex cancellations be explained in terms of cause and effect? Although the mass of evidence as well as the logic of the extension of the properties of particles to the measuring rods and clocks makes it almost certain that ether motion relative to inertial systems cannot be detected, no general proof, starting with the length contraction and the slowing of clocks, has been given so far in this narrative. Nevertheless, to retain cause and effect as the basis of physics, here *it is accepted as a fact.*

8.10. **Secondary Inertial Labs**: Strong arguments have been advanced here to preserve visualization, cause and effect, intuition and determinism, using an ether, while at the same time avoiding metaphysics at the physical level. In this chapter, however, certain experiments have been described that have caused physicists no end of soul searching and mysticism in resolving the results obtained. For example, though the laws (equations) of physics (see Chapter 2) are in simplest form in primary inertial labs, apparently no one is able to say which inertial labs are primary. It has been demonstrated here, that in secondary labs with *truly synchronized clocks*, the laws of physics take on complicated forms that must include the different propagation times caused by the constant velocity of propagation relative to the ether. The way out of this dilemma is narrow, but intuitively satisfying.

It is likely that all inertial lab experiments ever done have been done in secondary labs; and the essential difference between those

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1. H.A.Lorentz, Lectures on Theoretical Physics, Vol 1, Macmillan & Co. Ltd. London (1927). E.Whittaker, A History of the Theories of Aether and Electricity, Thomas Nelson & Sons, Ltd. London, Vol I (1951), Vol II (1953).

labs and the primaries is that in the primaries *all light pulses propagate in any direction at the same velocity* c_0. For this reason, there are several ways to set *primary* lab clocks so that they are all *truly synchronized*. As depicted in Figure 8.10.1, in secondary labs with *truly synchronized* clocks, each *pair* of points in the space has *two* propagation velocities associated with it due to the constant ether flow through the lab (not shown in the Figure). Light pulses going in opposite directions between the points have different speeds (see Section 8.7). Thus, using a clock set procedure that would *truly synchronize* the secondary lab's clocks would be a very bad choice. The equations of physics in that lab would be ferociously complicated.

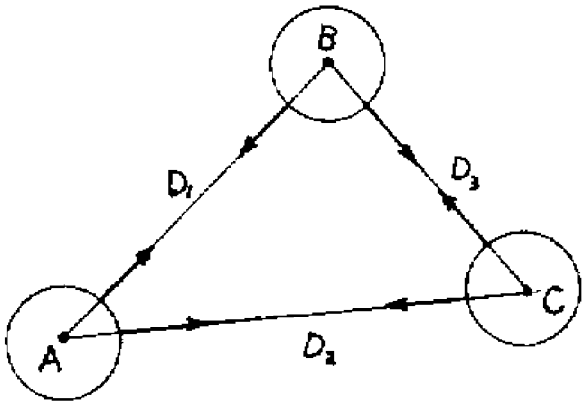

Figure 8.10.1 Three points in a secondary lab.

The proper approach to secondary lab clock set is to find a way to <u>*de-synchronize*</u> the clocks so that both directions of propagation between two points have the same *calculated* velocity, using the incorrect clock time. *This is done by first giving up the contact clock set used in Section 8.7. One-way light set from a master clock also fails.* Special Relativity actually gets the desired result, but its methods are so abstract and unmotivated that many of its users and advocates are unaware that the clocks are deliberately *de-synchronized*.

The correct method for clock setting in secondary inertial labs is found as follows. Referring to Figure 8.10.1, it is seldom necessary to actually set the time at every point in the 3d lab space. Often, only a few clocks are placed about the lab, and the significant propagation is along just those few paths. As an example, in Figure 8.10.1, only three paths and three clocks are shown. The latter are represented by the three black dots at the centers of the spherical wave fronts shown

propagating away from the point clocks, which are at any three points in the lab. The constant velocity ether flow through the lab is not shown, because it cannot be known to the lab workers. As derived in Section 8.7, given *truly synchronized* clocks, there were *two* actual propagation velocities caused by the *constant* ether velocity in the secondary lab. However, in that example, the ether flow was along the line between the two clocks being set. In the general case of Figure 8.10.1, if the single, constant velocity ether flow happened, by huge coincidence, to be along one of the paths, then it would surely cut across the other paths.

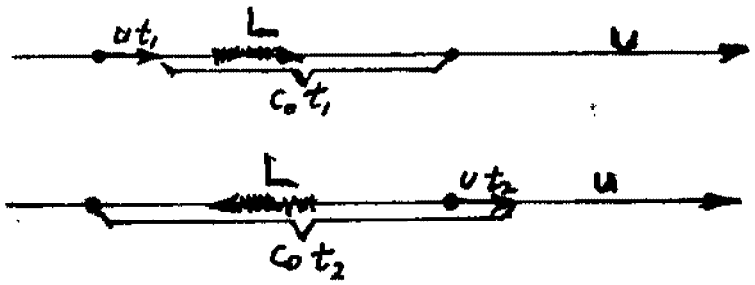

Figure 8.10.2
Ether flow u parallel to L.

Figures 8.10.2 and 8.10.3, illustrate two very *special* flow configurations between any pair of clocks, and Figure 8.10.4 covers all others. The first, Figure 8.10.2, represents the example of Section 8.7, where it was shown that the *absolute observer* sees the out-back propagation times as,

$$t_1 = \frac{L}{c_0 + u} \quad , \quad t_2 = \frac{L}{c_0 - u} \quad ,$$

where u now represents the secondary lab's ether flow to the right above, and the clocks and L are fixed. Clock slowing makes the lab's times,

$$t_{\ell 1} = \frac{L}{\gamma(c_0 + u)} \quad , \quad t_{\ell 2} = \frac{L}{\gamma(c_0 - u)} \quad .$$

If the rod's length in a primary lab is γL , then rod shortening causes the lab observer to measure the propagation velocities,

$$c_{\ell 1} = \frac{\gamma L}{t_{\ell 1}} = \gamma^2(c_0 + u) \quad , \quad c_{\ell 2} = \frac{\gamma L}{t_{\ell 2}} = \gamma^2(c_0 - u) \quad . \quad (8.10.1)$$

It should be pointed out that, in Eqs.(8.10.1), the second order γ^2 factor accounts for the rod shortening and clock slowing, and the first order factor supplies the effect caused by the constant velocity of propagation c_0 relative to the moving datum ether.

In this particular secondary lab, the proper way to set the clocks is to force the one way velocity of propagation to be the same, and equal to c_0. This is done by setting,

$$t_\ell = \tfrac{1}{2}(t_{\ell 1} + t_{\ell 2}) \quad . \tag{8.10.2}$$

The result is,

$$t_\ell = \frac{1}{2}\left[\frac{L}{\gamma(c_0 + u)} + \frac{L}{\gamma(c_0 - u)}\right] = \frac{L}{\gamma}\left[\frac{c_0}{c_0^2 - u^2}\right] = \frac{\gamma L}{c_0} \quad ,$$

so the lab worker measures the propagation velocity $c_\ell = \gamma L / t_\ell = c_0$ *both ways*.

If, by another rare coincidence, the datum flow through the lab was at right angles to the line between two clocks, the timing (as seen by an absolute observer) would be as illustrated in Figure 8.10.3. The

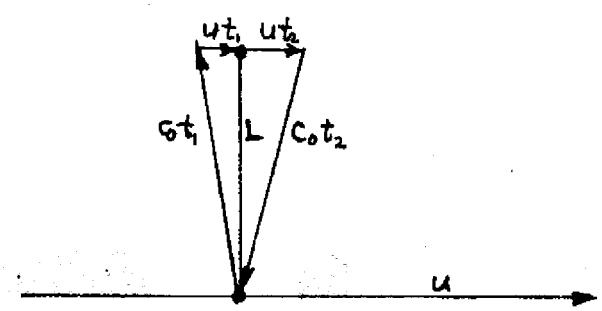

Figure 8.10.3
Ether flow Perpendicular to L.

ether is flowing to the right in the lab and the absolute observer finds the light pulse transit times to be,

$$t_1 = \frac{L}{\sqrt{c_0^2 - u^2}} = \gamma\frac{L}{c_0} \quad , \quad t_2 = \frac{L}{\sqrt{c_0^2 - u^2}} = \gamma\frac{L}{c_0} \quad .$$

In this case, there is no rod shortening, because the rod length is at right angles to the ether velocity u, but clock slowing makes the lab's times,

$$t_{\ell 1} = \frac{L}{c_0} \quad , \quad t_{\ell 2} = \frac{L}{c_0} \quad .$$

Substituting the lab times in the two way clock set Eq.(8.10.2), the result is,

$$t_\ell = \tfrac{1}{2}\left(\frac{L}{c_0} + \frac{L}{c_0}\right) = \frac{L}{c_0} \quad ,$$

so the lab worker measures the propagation velocity $c_\ell = L/t_\ell = c_0$ *both ways.*

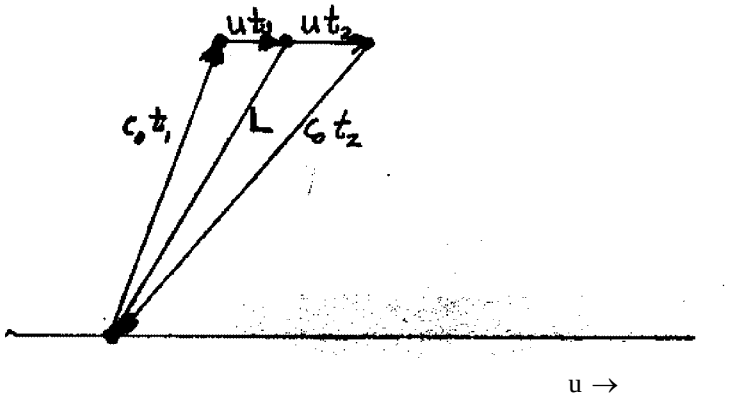

Figure 8.10.4
Ether flow in any direction relative to L.

The general case depicted in Figure 8.10.4 requires a more elaborate derivation that need not be repeated here, because the derivation of the Lorentz transformation gives the same result later on. Einstein's Special Relativity postulate about the constancy of the velocity of light is nothing more than a statement that two way light set, using Eq.(8.10.2), *de-synchronizes* the clocks and, *effectively converts any secondary inertial lab into the equivalent of a primary inertial lab, with calculated propagation velocity* c_0 *in all directions.*

8.11 **The Ether Observer**: Conventionally, the inability of inertial system observers to detect their motion relative to the ether has been used to argue against the ether's existence; but *a far more useful conclusion can be drawn.*[1] Any reader who has come this far who still does not appreciate the power of the ether interpretation over the space-time approach can perhaps find guidance from the following shift in viewpoint. Past writing has indicated that the result of all experiments on the ether and the implications of Relativity (from here on, this word refers only to the first postulate) are that the inertial observer cannot tell whether or not he is the ether observer. A more effective way to look at the results is to realize that they also mean

1. R.H.Dishington, Physics, Beak Publ. , Pacific Palisades, CA (1989).

that any inertial observer can *assume* he is the ether observer, and the results he observes are exactly what the ether observer would see doing the same *experiment*. Now, since the phenomena discussed earlier, in all chapters including this one, are intuitively obvious in terms of an ether observer, that is how the physics should always be explained. Assume the ether observer is doing the experiment. All other inertial observers will see the same thing. This is true *without* using the two way light set procedure of special relativity. It is also true using the light set procedure, or any other reasonable procedure.

8.12 **The Lorentz Transformation Without Space-Time**: What is the future role of the Lorentz transformation and of special relativity? Space-time offers nothing to the physicist. It consists of two postulates; the basic Relativity principle, which is just a limited form of the Principle of Identical Environments, and correct; and the constancy of the velocity of light, generally incorrect. *The second postulate only holds true for the arbitrary setting of clocks by two way light set.* For two-clock contact setting (see Section 8.7) the one way velocity of light is different in opposite directions, so *the second postulate is wrong as presently stated.* Thus, Einstein's form of special relativity must be used with great care and an appreciation of its non-fundamental nature, especially as it is erroneously used to try to explain intuitively what is being observed, is indispensable. All that will be retained is the basic Relativity principle; perhaps it should be called Law of Relativity (or the Law of Identical Environments).

Conversely, the Lorentz transformation has a utilitarian value. This is because two way light set is generally the simplest process available. When it is used, and rods are read by moving observers carrying them along, then the Lorentz transformation does give the proper method for *calculating* the readings of the moving clocks and rods in terms of those of the assumed ether observer, and vice versa. In order to explain away the last vestiges of paradox or confusion associated with the Lorentz transformation, it will be derived here in its usual form, but with a slightly different emphasis, inasmuch as the ether will be tacitly assumed as the basis of its validity.

8.13. **Lorentz Transformation and Simultaneous Synchronization**: Before carrying out the derivation, a crucial difference between the two-clock contact and light-set experiments must be indicated. Referring back to Section 8.7, notice that *the methods used by the two sets of clocks for synchronization were not the same.* That is, during the experiment, *the experiences of the two inertial observers were not symmetrical.* There is nothing about the Law of Relativity that requires it. The only reciprocal requirement imposed by Relativity is that if the whole procedure is repeated with the roles of the clock pairs

reversed, then the results must be indistinguishable from the first experiment.

A well known derivation of the Lorentz transformation is described by Robertson.[1] In it, the claim is made that the Lorentz transformation follows from two postulates:

1. Relativity

2. The Fitzgerald-Lorentz matter contraction.

Unfortunately, *this is not quite true*. It has been shown in earlier sections that the contraction coupled with the clock slowing, *cause* the Law of Relativity to be true, *even in systems where the Lorentz transformation and the constancy of the velocity of light are not directly applicable*. Actually, Robertson's derivation is a much more restrictive process than those specified by the Law of Relativity and the contraction. It invokes the implicit assumption that both systems are "de-synchronized" in the same way at the same time. In other words, that everything happening to the two inertial observers is mutually symmetrical, simultaneously. *This is a much more restrictive condition than the Lorentz contraction and Relativity impose*. It is this very tight restriction that is responsible for much of the confused intuition. Once the implications of this implicit restriction are understood, much of the mystery vanishes.

Continuing with Robertson's derivation, Figure 8.13.1 displays the usual pair of observer systems in relative motion, at constant velocity V as seen by the ether observer S. A rod at rest in S', with one end at the origin and the other at (x',y',z'), will appear to S to have its ends at

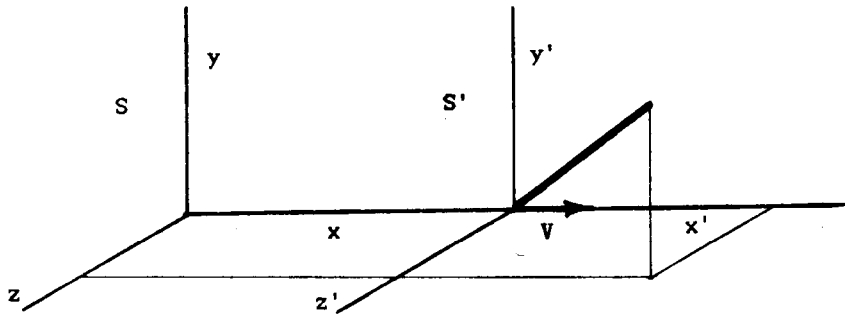

Figure 8.13.1 Inertial systems in relative motion.

1. H.P.Robertson & T.N.Noonan, Relativity and Cosmology, p 43, W.B. Saunders Company, Philadelphia, PA (1968).

(Vt, 0, 0) and (x,y,z) respectively. The matter contraction with motion through the ether requires that,

$$x - Vt = \frac{1}{\gamma} x' \quad , \quad y = y' \quad , \quad z = z' \quad , \qquad (8.13.1)$$

where,

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c_0^2}}} \quad . \qquad (8.13.2)$$

Next, Robertson writes,

$$x' + Vt' = \frac{1}{\gamma} x \quad , \quad y' = y \quad , \quad z' = z \quad , \qquad (8.13.3)$$

on the basis of the Relativity Law; i.e. inertial observers performing the same experiment see the same results. This is acceptable from the ether viewpoint because it is known that the time slowing of clocks and the contraction work together to ensure that result.

At this point Robertson says that Eqs.(8.13.1) and (8.13.3) may be solved for the Lorentz transformation:

$$x' = \gamma(x - Vt) \ , \ y' = y \ , \ z' = z \ , \ t' = \gamma\left(t - \frac{V}{c_0^2} x\right) \ , \qquad (8.13.4)$$

and,

$$x = \gamma(x' + Vt') \ , \ y = y' \ , \ z = z' \ , \ t = \gamma\left(t' + \frac{V}{c_0^2} z'\right) \ . \qquad (8.13.5)$$

This is the step that imposes the <u>simultaneous</u> transformation not required by Relativity or the Fitzgerald-Lorentz matter contraction. As Robertson points out, *this is equivalent to the adoption of the postulate of the constancy of the velocity of propagation*. Earlier that postulate was shown not to be true in general, but to result from the very arbitrary and restrictive two way method of setting the clocks. That is what results when the Lorentz transformation is used.

In spite of the severely restrictive character of the Lorentz transformation, with its two way light-set clocks and its constant <u>calculated</u> velocity of light in all inertial systems, its utility in setting up experiments and in certain types of problem solving make its use logical and practical. *It should never be used to try to explain the physics*. The simple ether picture is better for that purpose, basic and correct. Almost all of the intuitional difficulties of special relativity came from the unreal philosophical decoration of "space-time" and

from the simultaneously symmetrical imposition of the Lorentz transformation.

8.14 **Implications of the Lorentz Transformation**: Going back to Figure 8.13.1 and Eqs.(8.13.4), which are used by S to find out what S' measures; if S wants to know of the origin of S', he sets $x' = 0$ into the first equation and finds it satisfied by $x/t = V$. If S' wants to know of the origin of S, he uses the inverse Eq.(8.13.5), setting $x = 0$ and finding it satisfied by $x'/t' = -V$. *S and S' both measure their speeds as equal, with opposite velocities*. By simply reversing the sign and making the primes unprimed and vice versa, S' and S will be interchanged, with the original S' system now the ether observer equivalent and the original S system now the moving observer equivalent. With this arbitrary clock set arrangement, the two are completely symmetrical, simultaneously. It is in this connection that the greatest philosophical or metaphysical confusion enters in the conventional approach. The ether eliminates this.

To the question, "which rod is really shorter or which clock truly slower?", common conventional answers are that question has no meaning [1], irrelevant[2], the space-time manifold[3]. Since 1960, it has become popular to just omit such questions, as though intuition is in the way, and should be ignored. In fact, the question is perfectly valid. So is the answer. *In every case, the rod or clock moving fastest relative to the ether, is the shortest or slowest.* The fact that it is not possible to determine how the ether is flowing external to any inertial system does not invalidate that answer. The previous sections have explained clearly and intuitively why the ether flow cannot be measured by an inertial observer. The intuitive difficulty was artificially introduced by an arbitrary choice of the simultaneously symmetric transformations. The Lorentz transformation is used as a general formalism, but should not be used to obtain an intuitive grasp of any given experiment, a number of at first sight strange results can be deduced through its use. There are numerous expositions of these effects available.

Section 8.15 summarizes the principal gains in understanding resulting from the use of the ether as a motivation for the many choices made in making measurements.

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1. A.Sommerfeld, Electrodynamice, p 227, Academic Press, N.Y.N.Y. (1952).
2. R.B.Lindesy & H.Margenau, Foundations of Physics, p 340, John Wiley & Sons, N.Y. (1936).
3. I.S.Sokolnikoff, Tensor Analysis, p 267, John Wiley & Sons, N.Y. (1951).

8.15 **Inertial Lab Measurement Summary**:

The Principle of Identical Environments (Relativity Principle) holds between *all* inertial labs:

> Identical experiments in **any** two inertial labs yield identical results.

Primary Inertial labs:

*No inertial observer can determine that he is in a primary inertial lab; but, (before he sets his c locks) any inertial observer can assume he **is** in a primary inertial lab, and the* experimental *results he observes (now including his choice of clock setting) will be exactly what a primary observer would see doing the **same*** experiment *(with the same clock setting).*

Secondary Inertial Labs:

There are two types of secondary inertial labs of particular interest: Those with truly contact *synchronized c locks and those with two way light set de-synchronized c locks.*

TRULY SYNCHRONIZED CLOCKS	TWO WAY LIGHT SET CLOCKS
1. The secondary lab clocks are set with difficulty	1. The secondary lab clocks are easily set
2. The clocks are truly synchronized *in the most fundamental way*	2. The clocks are *deliberately de-synchronized* to ensure 3. below
3. The principle of the constancy of light propagation velocity is **not** generally true	3. The one way calculated velocity of light is always the same, c_0
4. The *equations (laws)* of physics do **not** generally have the same form as in the inertial lab used to set the clocks	4. The equations (laws) of physics are exactly the **same** in all two way light set inertial labs
5. The Lorentz transformation does **not** generally apply between the truly synchronized lab and the inertial lab used to set the clocks	5. The Lorentz transformation applies between any two way light set inertial labs

CHAPTER 9

MAGNETICS

9.1 **Introduction**: This chapter is a turning point in the world description. In the first eight chapters, the structure of material objects was developed to a point where only well known and understood ensemble descriptions were needed to extend and complete the picture. *It is noteworthy that the whole development of the "steady state" forms of matter involved only electric energy*. At this point, however, many peripheral forms (e.g. antenna radiation, photons, neutrinos, etc.), that were mentioned previously but not seriously studied, require magnetic fields for their description. Just as earlier the physical natures of charge and electric energy density were presented, the basic natures of a magnetic field and its energy density are given here. After that, various complexities that magnetic fields are capable of are looked into in greater detail.

9.2 **The Magnetic Field**, $\overline{\overline{\phi_a \mathbf{V}}}$: The conventional definition of a magnetic field ($\mathbf{B}$) is given in terms of the very restrictive concept of "force". Here a magnetic field is said to exist wherever the time average ether flow vector $\overline{\overline{\phi_a \mathbf{V}}}$ is not zero. *Thus*, $\overline{\overline{\phi_a \mathbf{V}}}$ *is the physical embodiment of the magnetic field*. It is related to the conventional magnetic vector potential through,

$$\mathbf{A} = \frac{\overline{\overline{\phi_a \mathbf{V}}}}{c_0} \quad . \qquad \text{(des)} \qquad (9.2.1)$$

In terms of the incremental form of Eq.(2.15.1),

$$\mathbf{A} = \frac{\bar{\bar{\phi}}\,\mathbf{u}}{c_0} \quad . \qquad (9.2.2)$$

Recognizing $\overline{\overline{\phi_a \mathbf{V}}}$ as the physical essence of magnetism is important, because *there are well known cases where significant physical effects are produced in regions where the conventional* $\mathbf{B}$ *field is zero everywhere*; but $\overline{\overline{\phi_a \mathbf{V}}}$ causes these effects. The conventional $\mathbf{B}$ field is a measure of the ether flow "vorticity",

$$\mathbf{B} = \frac{1}{c_0} \nabla \times \overline{\overline{\phi_a \mathbf{V}}} = \frac{1}{c_0} \nabla \times \bar{\bar{\phi}}\,\mathbf{u} \quad . \qquad (9.2.3)$$

Because $\phi_d >>> \bar{\bar{\phi}} + \phi_.$ in any practical case,

$$\mathbf{A} \cong \frac{\phi_d}{c_0}\ \bar{\bar{\mathbf{V}}} \quad , \tag{9.2.4}$$

and,

$$\bar{\bar{\phi}}_a \mathbf{V} \cong \phi_d \bar{\bar{\mathbf{V}}} \quad . \tag{9.2.5}$$

9.3 **Magnetostatics**: Magnetostatic fields are the simplest of all ether conditions other than no field at all. They are just those regions of space where stationary circulation exists in the ether flow pattern. These circulation flows can exist whether or not there is an incremental ether density, but the term *magnetostatics* refers to situations where $\bar{\bar{\phi}} = 0$. The conditions for the existence of a magnetostatic field can be summarized by:

$$\bar{\bar{\mathbf{V}}} \neq 0 \ , \ \bar{\bar{\phi}} = \phi_. = \mathbf{V}_. = 0 \ , \ \phi_a = \phi_d$$
$$\frac{\partial \bar{\bar{\phi}}}{\partial t} = \frac{\partial \phi_.}{\partial t} = \frac{\partial \bar{\bar{\mathbf{V}}}}{\partial t} = \frac{\partial \mathbf{V}_.}{\partial t} = 0$$
All higher time partials are zero.

Conditions For Magnetostatics (9.3.1)

When these conditions are substituted into Eqs.(2.16.3) and (2.21.1), the equations reduce to,

$$\nabla^2 \bar{\bar{\mathbf{V}}} = 0 \quad , \quad \nabla \bullet \bar{\bar{\mathbf{V}}} = 0 \quad . \tag{9.3.2}$$

Two examples will now be given to illustrate a well known method for solving Eqs.(9.3.2), and both will prove useful to further develop the concepts of ether flow and particle action. The first is the flow field between two parallel plane sheets of current. As seen in Figure 9.3.1, the flow of electrons is in two large parallel sheets of area great compared with the distance between them. In a central region, where edge effects can be neglected, the ether flow is assumed to be parallel to the x axis, so that $\bar{\bar{V}}_y = \bar{\bar{V}}_z = 0$. On this basis, Eqs.(9.3.2) become,

$$\frac{\partial^2 \bar{\bar{V}}_x}{\partial y^2} = 0 \quad , \quad \frac{\partial \bar{\bar{V}}_x}{\partial x} = 0 \quad . \tag{9.3.3}$$

Integrating the first of these twice leads to,

$$\bar{\bar{V}}_x = Ky + K_1 \quad . \tag{9.3.4}$$

Clearly, Eq.(9.3.4) satisfies the continuity equation in Eqs.(9.3.2) and (9.3.3) exactly. To match the physical conditions depicted in Figure 9.3.1, the constant K_1 can be chosen to be zero; meaning that an observer at the y = 0 plane half way between the plates sees the ether

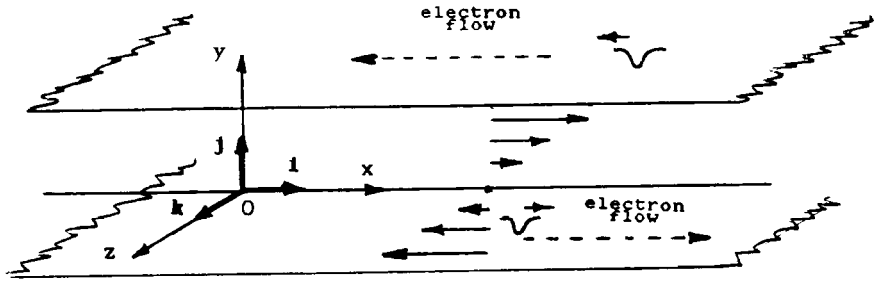


Figure 9.3.1 Ether flow between two current sheets.

at rest at any point on that plane, but exhibiting a vorticity. Above or below the horizontal plane through the origin of the rectangular coordinate system, the ether can be seen to have what is called laminar flow. In vector form,

$$\overline{\overline{\mathbf{V}}} = \mathbf{i}\,\mathrm{K}y \quad . \tag{9.3.5}$$

By applying Table D.6.1, the rate of rotation tensor for this field is found to be,

$$\overline{\overline{\Omega}} = \frac{1}{2}\frac{\partial \overline{\overline{V}}_x}{\partial y}(\mathbf{ji} - \mathbf{ij}) \quad ; \tag{9.3.6}$$

which together with Eq.(E.6.11) shows that the vorticity vector is related to the flow by,

$$2\overline{\overline{\mathbf{w}}} = \overline{\overline{\Omega}}_v = \nabla \times \overline{\overline{\mathbf{V}}} = -\mathbf{k}\frac{\partial \overline{\overline{V}}_x}{\partial y} \quad . \tag{9.3.7}$$

From Eqs.(9.3.4) and (9.3.7) the constant K can now be identified with the vorticity through (see Eq.C.4.5),

$$\mathrm{K} = -2\overline{\overline{w}}_z = -2\overline{\overline{\Omega}}_m \quad ; \tag{9.3.8}$$

which allows Eq.(9.3.5) to be rewritten,

$$\overline{\overline{\mathbf{V}}} = -\mathbf{i}2\overline{\overline{w}}_z y = -\mathbf{i}2\overline{\overline{\Omega}}_m y \quad . \tag{9.3.9}$$

As a matter of curiosity, the *acceleration* field can be found from Eq.(2.8.8), with the result,

$$\overline{\overline{\mathbf{a}}} = \overline{\overline{\mathbf{V}}} \cdot \nabla \overline{\overline{\mathbf{V}}} = -2\overline{\overline{w}}_z y \frac{\partial \overline{\overline{\mathbf{V}}}}{\partial z} = 0 \quad . \tag{9.3.10}$$

No gravitation-like acceleration appears in this field, and neutral objects at rest in the laminar flow will not be moved. Outside the sheets the velocity field is zero (this is a pathologically non-physical example).

The correspondence between the familiar magnetostatic equation,

$$\mathbf{B} = \nabla \times \mathbf{A} \quad , \tag{9.3.11}$$

and Eq.(9.3.7) can be made more explicit using Eq.(9.2.4), with the result,

$$\mathbf{B} = \frac{\phi_d}{c_0} \nabla \times \overline{\overline{\mathbf{V}}} = 2 \frac{\phi_d}{c_0} \overline{\overline{\mathbf{w}}} \quad . \tag{9.3.12}$$

Thus, using the standard method for finding the magnetostatic field from its current sources, and aided by Eq.(9.3.12), a vorticity integral can be written,

$$\oint \overline{\overline{\mathbf{w}}} \cdot d\mathbf{r} = \frac{\sqrt{\pi} c_0}{10 \phi_d} \, I \, (\text{amps}) \quad . \tag{9.3.13}$$

This *mixed unit* form gives the vorticity in radians per second, if dr is in centimeters. Because the current here is defined as *opposite* to the electron flow, its direction is correctly related to the direction of integration of $\overline{\overline{w}}$ by the usual right hand rule. For numerical results it is necessary to specify ϕ_d in Heaviside-Lorentz units. With this substitution, Eq.(9.3.13) becomes,

$$\oint \overline{\overline{\mathbf{w}}} \cdot d\mathbf{r} = 5.9123 \times 10^{-12} \, I \, (\text{amps}) \quad . \tag{9.3.14}$$

Going back to the example of Figure 9.3.1, and assuming a rather impractically high current of say 1000 amperes per centimeter of current sheet width, the vorticity found by integrating Eq.(9.3.14) around a loop enclosing one centimeter of the current sheet is $\overline{\overline{w}} = 5.91 \times 10^{-9}$ radians per second, or about 33.7 years per rotation. At first hand this might lead to the expectation that negligible effect on a test electron at rest between the sheets would be observed. Later, it will be shown that the situation is more complex, since the electron might deform, spin up etc.; and most magnetic effects result from very small ether velocities exaggerated by its great density. For the present, based on experience, it will be assumed that an electron at rest between the sheets will feel no stimulus. If, on the other hand the electron moves, then the well known Lorentz "force" will act. The mysteries of the latter effect will be dispelled in Chapter 12.

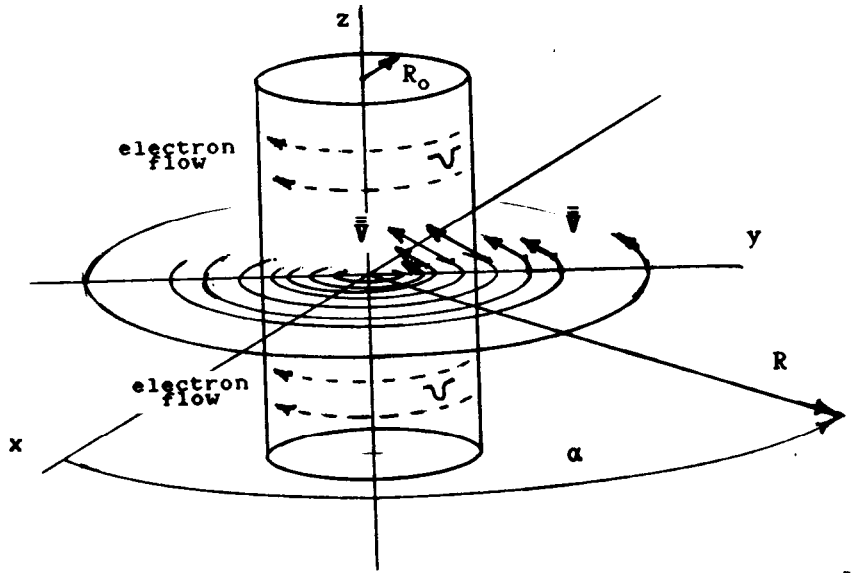


Figure 9.3.2 Ether flow in a long solenoid.

A second magnetostatic field of consequence is that of a long solenoid. Taking the flow of electrons to be in a cylindrical sheet, as illustrated in Figure 9.3.2, the ether flow circulates in the opposite direction. For a coil with diameter very much smaller than its length, the field near the middle of the coil, lengthwise, is essentially cylindrical; and Eqs.(9.3.2) can be written in cylindrical coordinates,

$$\frac{d^2\overline{\overline{V}}_\alpha}{dR^2} + \frac{1}{R}\frac{d\overline{\overline{V}}_\alpha}{dR} - \frac{\overline{\overline{V}}_\alpha}{R^2} = 0 \quad , \quad \frac{d\overline{\overline{V}}_\alpha}{d\alpha} = 0 \quad , \qquad (9.3.15)$$

where it has been assumed that $\overline{\overline{V}}_R = \overline{\overline{V}}_z = 0$. The first order equation in Eq.(9.3.15) implies that the flow is in the form of a cylindrical circulation. The magnitude of the velocity field can be found from the second order equation,

$$\overline{\overline{V}}_\alpha = K_2 R + \frac{K_3}{R} \quad . \qquad (9.3.16)$$

Physical considerations permit establishment of the values for K_2 and K_3. For example, inside the solenoid, at R = 0, $\overline{\overline{V}}_\alpha = 0$ so $K_3 = 0$; but, outside, as $R \to \infty$, $\overline{\overline{V}}_\alpha \to 0$ and $K_2 = 0$. Both inside and

outside, at $R = R_0$, $\overline{\overline{V}}_\alpha = \overline{\overline{V}}_0$; so the final form of the velocity field is,

$$\overline{\overline{\mathbf{V}}} = \hat{\boldsymbol{\alpha}}\frac{\overline{\overline{V}}_0}{R_0}R \quad , \text{ inside} \quad , \quad \overline{\overline{\mathbf{V}}} = \hat{\boldsymbol{\alpha}}\frac{R_0\overline{\overline{V}}_0}{R} \quad , \text{ outside} \quad . \qquad (9.3.17)$$

Thus, inside the ether turns as a rigid body and outside the adjacent layers of ether slip to allow the velocity at the center and at great distance from the center to be zero. The vorticity found from Eq.(9.3.17) is,

$$\overline{\overline{\mathbf{w}}} = \frac{1}{2}\nabla\times\overline{\overline{\mathbf{V}}} = \begin{cases} \mathbf{k}\dfrac{\overline{\overline{V}}_0}{R_0} & , \text{ inside} \\ 0 & ; \text{ outside} \end{cases} \qquad (9.3.18)$$

which brings the discussion to a very critical point. Notice that the *vorticity outside the solenoid is zero.* Nevertheless, the circulation $\boldsymbol{\Gamma} = \oint \overline{\overline{\mathbf{V}}}\bullet\, d\mathbf{r}$ is not, since the velocity field is constant around any circular path at a fixed radius from the coil axis. Whether or not there is vorticity, the circulating flow can exert physical effects. It was pointed out earlier that the magnetic field **B** corresponds to the "flow vorticity" of the velocity field. In Maxwell's E&M, **B** is regarded as physically significant and **A** as a mathematical tool. For some time now, the feeling that **A** might be more physical has slowly developed because of its role in quantum electrodynamics. Nevertheless, no one appears ready to give up **B** as more basic. Clearly, from the preceding examples, **A**, representing the ether velocity $\overline{\overline{\mathbf{V}}}$, is the only physically real variable in the magnetic field; and **B**, defined in terms of force on a unit pole is as empty as any other definition of "force" (see Chapter 12). It is true that **B** is representative of the vorticity in the ether flow, as in Eq.(9.3.12); but the artificial direction of the curl *vector* has little or nothing to do with the directions in which the physical actions are taking place.

Eq.(9.3.18) invokes another significant consideration. The source of stationary circulation fields can be moving charged particles, most often electrons, continuing along closed paths. Inside the closed paths, near the center, the circulating flow field has rigid body vorticity. However, farther from the center and outside the paths, it is commonly found that, although there is always circulation, there is less ordered vorticity of low magnitude or no vorticity at all. Here, the non-vorticity circulating fields are just as real as those with vorticity; and *the Lorentz force exerted on charged particles will not be used as a criterion for the existence of a magnetostatic field.* Any solution of Eqs.(9.3.2) will be regarded as a magnetostatic field.

Continuing with the example of the long solenoid, an inevitable development appears when Eq.(2.8.8) Is used to determine the acceleration field about the solenoid,

$$\overline{\overline{\mathbf{a}}} = \overline{\overline{\mathbf{V}}}\cdot\nabla\overline{\overline{\mathbf{V}}} = \begin{cases} -\hat{\mathbf{R}}\left(\dfrac{\overline{\overline{V}}_0}{R_0}\right)^2 R \quad , & \text{inside} \\ -\hat{\mathbf{R}}\dfrac{\overline{\overline{V}}_0 R_0}{R^3} \quad . & \text{outside} \end{cases} \tag{9.3.19}$$

The flow exhibits a centripetal acceleration, and it might be thought that any small neutral object should feel the primary inertial system (the ether) accelerate towards the central axis of the solenoid. Here the situation is more complicated, and this question will be examined in detail in Chapter 12. Nevertheless, it is of some interest to check on the magnitudes of $\overline{\overline{\mathbf{V}}}$ and $\overline{\overline{\mathbf{a}}}$ in an extreme case. For example, let the radius of the solenoid be 1 cm and the current be 100 amperes per cm. Then, the vorticity inside the solenoid is $\overline{\overline{w}}_z = 5.9123\times10^{-10}$ rad/sec (see Eq.(9.3.14)), and the velocity just inside the electron sheet is $\overline{\overline{V}}_0 = \overline{\overline{w}}_z R_0 = 5.9123\times10^{-10}$ cm/sec. Again, this small velocity is misleading because, as later developments show, this velocity coupled with the great density of the ether can produce significant motions. In this example, the acceleration just inside the electron sheet is $\overline{\overline{a}}_0 = 3.5\times10^{-17}$, again a very small number, but still with possible significance.

Take another solenoid, for example, wrapped with 20 gage copper wire (Diam. = 8.12×10^{-2} cm) and 12.32 turns/cm. The coil diameter is 1 cm and its cross sectional area is 0.785 cm^2. From Eq.(9.3.14), a 1cm length of solenoid, conducting a current of 12.32 amp/cm (Mks), produces a vorticity $\overline{\overline{w}} = 7.28\times10^{-11}$ rad/sec. In this case, the ether inside the coil is rotating as a rigid body, so its velocity at the wire is 3.64×10^{-11} cm/sec. By assuming 8.44×10^{22} atoms per cm^3 in the wire, and one free electron per atom, the total number of electrons in 1cm of the coil is 1.692×10^{22}; so the average velocity of electrons in the wire is found to be 0.0143 cm/sec. To find the ether velocity related to the motion of the individual electrons, a crude estimate of the value at the center of each electron can be made by recognizing that the net ether carried by the incremental density ϕ_0, at the electron center moving at u = 0.0143 cm/sec (the electron velocity), is equal to the actual density $\phi_d + \phi_0$ moving at $\overline{\overline{V}}_e$. Roughly then, $\overline{\overline{V}}_e = \phi_0 u / \phi_d$; and, using the value of ϕ_0 from Eq.(3.5.2),

$\overline{\overline{V}}_e = 3.06\times10^{-20}$ cm/sec. Thus, the ether vortex velocity at the coil is 1.19×10^{9} times that related to the electrons' motion.

9.4 **A single Current Loop**: The magnetostatic field of a single, circular filament of current is of particular interest. Figure 9.4.1 illustrates some of the coordinates used to describe the field, the structure of which is well documented in the literature. *The form of the equation presented here, however, is simpler than the one commonly used.* It was derived by Heflinger[1]; and, expressed in terms of the ether flow, is written,

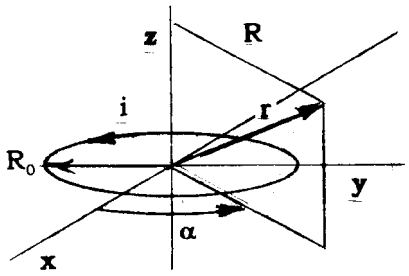


Figure 9.4.1 Loop coordinates.

$$\overline{\overline{\phi_a \mathbf{V}}} = \hat{\alpha}\frac{i}{\pi}\sqrt{\frac{R_0}{kR}}\left[K(k)-E(k)\right] \quad , \tag{9.4.1}$$

where i is the loop current (hl amp) and K(k) and E(k) are the standard, complete elliptic integrals. However, *the parameter k is different from that used in several widely known references*, and is defined as,

$$k = \zeta\left(1-\sqrt{1-1/\zeta^2}\right) \quad , \tag{9.4.2}$$

where,

$$\zeta = \frac{1+(R/R_0)^2+(z/R_0)^2}{2R/R_0} \quad . \tag{9.4.3}$$

The "velocity" approximation of Eq.(9.2.5) converts Eq.(9.4.1) into,

$$\phi_d \overline{\overline{V}}_\alpha = \frac{i}{\pi}\sqrt{\frac{R_0}{kR}}\left[K(k)-E(k)\right] \quad . \tag{9.4.4}$$

This flow field is cylindrical, with no R or z component; and in the plane of the loop ($z = 0$) bears some resemblance to the solenoid field described previously. Figure 9.4.2 portrays the velocity distribution for two values of z, zero and $0.5R_0$. Since the current in the filament is actually propelling the vortex, in the $z = 0$ plane the maximum of

1. L.O. Heflinger, private communication.

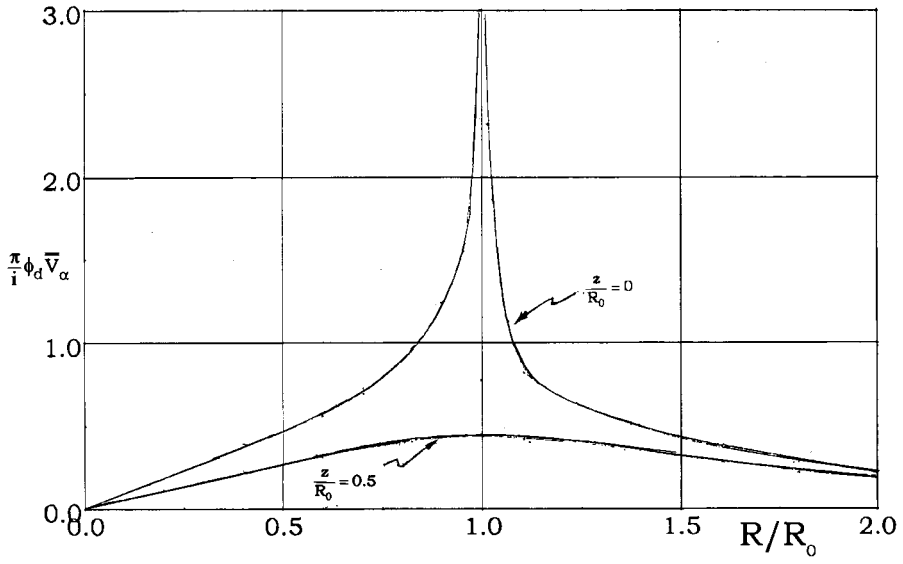


Figure 9.4.2 Velocity field of a single current loop.

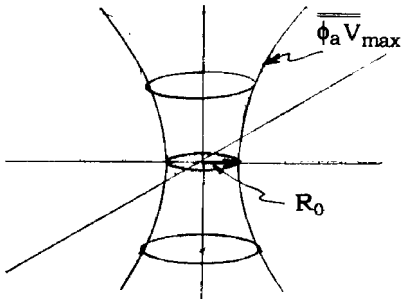
$\overline{\overline{\phi_a \mathbf{V}}}_\alpha$ occurs at the radius R_0 of the filament; and the flow tapers off outside R_0. However, the field differs from that of the solenoid because there is no enhancement, by neighboring current loops, to multiply the vortex effect. Therefore, the field is weaker, the rigid body turning in the central region does not extend very far out, and the maximum velocity occurs at greater R values for larger values of z. The sheet representing $\overline{\overline{\phi_a V}}_{max}$ is drawn in Figure 9.4.3

Figure 9.4.1 shows that, $r = \sqrt{R^2 + z^2} = R / \sin\theta$, so where $r >> R_0$, Eq.(9.4.4)can be approximated by,

$$\phi_d \overline{\overline{V}}_\alpha \cong \frac{i}{4\pi} A \frac{R}{r^3} \quad , \qquad (9.4.5)$$

where, A is the area πR_0^2 enclosed by the loop of current. This relationship indicates that the flow field far from the loop has the characteristics of a magnetic dipole of strength,

$$\mu = \frac{i}{c_0} A \quad , \qquad (9.4.6)$$



Maximum flow surface

Figure 9.4.3

leading to the expression,

$$\phi_d \overline{\overline{V}}_\alpha \cong \frac{c_0}{4\pi}\mu\frac{R}{r^3} \quad . \tag{9.4.7}$$

In vector form,

$$\phi_d \overline{\overline{\mathbf{V}}} \cong \frac{c_0}{4\pi}\frac{\boldsymbol{\mu}\times\mathbf{r}}{r^3} \quad . \tag{9.4.8}$$

Some relationships between the units are listed in Table 9.4.1.

TABLE 9.4.1

q	$\rightarrow$	hl coul	$\rightarrow$	des•cm		
i	$\rightarrow$	hl amp	$\rightarrow$	hl coul/sec	$\rightarrow$	des•cm/sec
μ	$\rightarrow$	erg/hlG	$\rightarrow$	hl coul•cm	$\rightarrow$	des•cm^2
$\overline{\overline{\overline{\overline{\phi_a \mathbf{V}}}}}$	$\rightarrow$	des•cm/sec				

As an example, consider a single loop of the smaller solenoid of Section 9.3. The loop radius is $R_0 = 0.5$ cm, and the current is $i = 1.0627\times10^{10}$ hl amp (1 amp Mks). The loop area is 0.7854 cm^2. At the point $R = 2R_0$, $z = 0.5R_0$, $r = 1.0308$ cm; so Eq.(9.4.5) gives $\phi_d \overline{\overline{V}}_\alpha = 6.0641\times10^8$ des•cm/sec. This is about 6% lower than the exact value illustrated in Figure 9.4.2. From Eq.(9.4.6), the magnetic dipole moment of this loop is found to be $\mu = 0.2784$erg/hlG.

Finally, comparing the vortex velocity $\overline{\overline{V}}_\alpha$ at the edge of the wire ($R = 0.92R_0$) with the internal flow $\overline{\overline{V}}_e$ of the relatively slow moving electrons, $\overline{\overline{V}}_\alpha \cong 5.415\times10^{-21}$ cm/sec and $\overline{\overline{V}}_e = 0.0143\phi_0/\phi_d$ or 3.060×10^{-20} cm/sec. Unlike the long solenoid, which has a vortex much larger than the electron flow, here the vortex at the loop is actually one fifth as strong as the electron flow.

9.5 **The Vortex Drive Mechanism**: Up to this point, little has been said about the detailed mechanism by which vortices are generated by circulating charged particles. In the case of the single current loop described in Section 9.4, the picture of the electrons driving the central vortex was at least intuitively satisfying. Yet, even in that case, the intuition required the aid of a formal solution involving the

moving electrons as sources of the field. Such a formal approach starts with the *macroscopic* form of Equation (2.21.2),

$$\nabla^2 \overline{\overline{\phi_a \mathbf{V}}} - \frac{1}{c_0^2} \frac{\partial^2 \overline{\overline{\phi_a \mathbf{V}}}}{\partial t^2} = - \mathbf{J} \quad , \tag{9.5.1}$$

where $\mathbf{J}$ is the current density,

$$\mathbf{J} = \rho \mathbf{u} \quad , \tag{9.5.2}$$

and ρ is the charge density of *whole charged particles* (usually electrons) creating the flow field. In this case, the positions and motions of the source charges are known and given.

For stationary fields, where the source current is steady, Eq.(9.5.1) reduces to,

$$\nabla^2 \overline{\overline{\phi_a \mathbf{V}}} = - \mathbf{J} \quad , \tag{9.5.3}$$

which has the solution,

$$\overline{\overline{\phi_a \mathbf{V}}} = \frac{1}{4\pi} \int \frac{\mathbf{J}}{\mathbb{R}} \, dvol \quad . \tag{9.5.4}$$

Here the field is found at each fixed point $\mathbf{r}$ by integrating over all space that includes source current $\mathbf{J}$ using the variable vector $\mathbf{r}'$, so that,

$$\mathbb{R} = \left| \mathbf{r}' - \mathbf{r} \right| \quad . \tag{9.5.5}$$

In the case of the single current loop, Eq.(9.5.4) yielded Eq.(9.4.1), and it was not difficult to visualize the connection between the source electrons and the vortex.

However, there are configurations where the generating mechanism is not as obvious, and first impressions can be very misleading. The example set forth in the next section brings out some of the reasons the well known relationships presented earlier cannot be applied arbitrarily to situations which look very similar but have subtle differences.

9.6 **The Atomic Vortex**: In Chapter 10, the structure of the photon is investigated, and most of the clues leading to its visualization come from the process of its generation and the machine that produces it. The latter, of course, is the atom; and one most influential component of atoms, related to photon generation, has not yet been described. It is a magnetic field. Generally, not much is said about magnetic effects in atoms, and often these are so small they cannot be

observed.[1] However, this particular magnetic field is well known to produce several significant effects.

Consider the hydrogen atom ground state, for example. Its single orbiting electron is traveling in a closed loop, and *conventional analysis assumes that it behaves as a single loop of current* of magnitude,

$$i = \frac{e}{T} = \frac{e\omega}{2\pi} \quad . \tag{9.6.1}$$

The values of ω and e from Tables 7.6.1 and 5.4.1 give a magnitude for the equivalent current of $i = 7.9216 \times 10^6$ hl amp. At this point one might be tempted to use Eqs.(9.4.1) through (9.4.6) to determine the flow and the magnetic moment; but, because of the extended nature of the electron, this leads to a considerable error. Clearly, the actual mechanism of generating the vortex is sensitive to the *difference* between the orbiting electron and a current carrying loop. However, it is clear that the current 7.9216×10^6 hl amp is the correct amount of charge being carried around the orbit per second, so that it is the true current. It is only the Eqs.(9.4.1) through (9.4.6) that do not represent the true vortex generated.

To obtain the correct form of the vortex, the charge distribution of the extended electron of Chapter 3 must be used in Eq.(9.5.4) to describe the moving charge source. The integration of this complicated field has not yet been accomplished. Nevertheless, the effect of the extended turning charge density appears to allow an *effective* current to be defined that can be used in Eqs.(9.4.1) through (9.4.6) to give the correct field. That effective current is,

$$i_{eff} = \eta i \quad , \tag{9.6.2}$$

where η is the "turning" factor defined in Eq.(7.3.7). For the hydrogen ground state, $\eta = \sqrt{2}$, so the effective current is $i_{eff} = 1.1203 \times 10^7$ hl amp. When this value is used in Eq.(9.4.6), for example, the magnetic moment of the hydrogen ground state is found to be,

$$\mu_B = 3.287553 \times 10^{-20} \text{ ergs / hlG} \quad , \tag{9.6.3}$$

commonly known as the Bohr magneton. Eqs.(9.6.2) and (9.4.6) together give the correct moments for all the hydrogen atom orbits when the correct η is applied.

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1. M.L. Coffman, Amer.J.Phys, **33**, p. 820 (1965).

9.7 **Angular Momentum and the Gyromagnetic Ratio**: One further topic that is closely related to the previous developments is that of angular momentum. As discussed in Sections 2.9, 3.6, 5.14 and 7.3 through 7.8, the angular persistence of the frictionless ether produces effects that are similar to the more conventional concept of angular momentum. Since any ether vortex can carry distributed electric energy density or distributed charge distortion or both around with it, some effects of angular momentum and magnetic moment are to be expected. The electron's vortex field is typical of this. In other cases, like the single loop current, the vortex itself has no distributed electric energy or charge, but is inextricably attached to the energy and charge of the particles acting as the source of the field. Other factors also enter into the picture, and all of these conditions must be sorted out.

In Section 3.8 the electron's spin and magnetic moment were derived and related through its *gyromagnetic ratio*,

$$\frac{\mu_s}{\sigma} \cong \frac{e}{m_0 c_0} \quad . \qquad \text{(electron)} \qquad (9.7.1)$$

Since the individual layers in the multi-layer particles have essentially the same structure, they have spin and angular momentum as indicated in Section 5.19. By combining Eqs.(3.6.4), (5.18.6), (5.19.2) and (5.19.4), the gyromagnetic ratio of each layer can be expressed in the form,

$$\frac{\mu_{si}}{\sigma_i} = \frac{q_i}{m_i c_0}\left[\frac{J_i(\infty)\, M_i(\infty)}{L_i(\infty)}\right] \quad . \qquad (9.7.2)$$

Upon checking the values of J_i, M_i and L_i in Appendix I, for a given frequency, the bracketed term is found to be unity (discounting roundoff error) for all possible layers, with the result,

$$\frac{\mu_{si}}{\sigma_i} = \frac{q_i}{m_i c_0} \quad . \qquad \text{(layers)} \qquad (9.7.3)$$

At this point, a reasonable question can be raised about the vortex in the hydrogen ground state. In Section 9.6, it was shown to have a magnetic moment called the Bohr magneton; but *it is not carrying any distributed electric energy*. The only electric energy associated with it is that of the orbiting electron, so what is the relationship between μ_B and p_ψ, the electron's angular momentum? Because of the implications of Eq.(9.4.6), it is natural to seek a relationship between p_ψ and the orbit area. A slight modification of Kepler's second law for elliptic orbits leads to,

$$A = \frac{T}{2m_0} p_\psi \quad , \qquad (9.7.4)$$

where T is the period of the electron in the orbit[1]. From Eq.(9.6.1),

$$iA = \frac{e}{2m_0} p_\psi \quad . \tag{9.7.5}$$

Combining Eqs.(9.4.6) and (9.6.2),

$$\mu = \frac{\eta i}{c_0} A \quad , \tag{9.7.6}$$

so Eqs.(9.7.5) and (9.7.6) then give, for *any* hydrogen orbit,

$$\frac{\mu}{p_\psi} = \eta \frac{e}{2m_0 c_0} \quad . \quad \text{(orbits)} \tag{9.7.7}$$

Thus, for the hydrogen ground state,

$$\frac{\mu}{p_\psi} = \sqrt{2} \frac{e}{2m_0 c_0} \quad . \tag{9.7.8}$$

For the orbit $n = 2$, $n_\psi = 2$,

$$\frac{\mu}{p_\psi} = \sqrt{1.5} \frac{e}{2m_0 c_0} \quad . \tag{9.7.9}$$

As the orbit radius increases, the turning effect becomes smaller, and for very large orbits,

$$\frac{\mu}{p_\psi} \rightarrow \frac{e}{2m_0 c_0} \quad ; \tag{9.7.10}$$

which is the ratio for the single current loop. This can be seen by using the real current in Eq.(9.4.6), since there is no electron turning in the wire loop. The real current can be written,

$$i = \frac{Ne}{T} = \frac{Neu}{2\pi R_0} \quad , \tag{9.7.11}$$

where u is the average electron velocity and N is the total number of electrons circulating in the loop. The resulting magnetic moment is,

$$\mu = \frac{Ne}{2c_0} uR_0 \quad . \tag{9.7.12}$$

The total angular momentum of the N electrons circulating is,

$$p_\psi = Nm_0 uR_0 \quad , \tag{9.7.13}$$

leading to,

$$\frac{\mu}{p_\psi} = \frac{e}{2m_0 c_0} \quad . \quad \text{(single loop)} \tag{9.7.14}$$

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1. F.K. Richtmeyer, Introduction to Modern Physics, p.482 Mc Graw-Hill Book Co. Inc, N.Y (1934).

In the examples discussed so far, many of the basic aspects of magnetostatic fields have been touched upon. *The surprising phenomena associated with them are not in the fields themselves, but in the motion of particles in the fields*, a subject to be investigated in great detail in Chapter 12.

9.8 **Magnetic Energy Storage**: At this point, it is possible to visualize *magnetostatic* energy. It occurs wherever charged particles have been driven in a closed loop. Neglecting, for the moment, the resistance opposing the circulation of charges in a conducting loop, as those charges are accelerated from zero current to some final fixed current, they push the ether so as to produce a circulating ether flow pattern throughout the space surrounding the loop. The long solenoid is typical of the magnetic behavior of closed circuits. The maximum ether flow is at the location of the driving current. Near the center of the coil, the ether flows in a vortex as a rigid body; but farther out the vortex reverses or the flow slips, and at great distances from the coil the flow tapers off to zero. In steady state, the solenoiod current consists of a large number of charged particles moving around the coil; each representing a certain small ether flow, just as it would as a free particle moving at the same velocity; but *the circulating flow is much greater than that involved in the motion of the charges themselves*, and it will continue to flow as long as the charges circulate in the loop. The amount of work done on the charges to accelerate them from zero up to their final velocity is far greater than the kinetic energy they attain (still neglecting resistive losses), and that work is effectively stored in the circulating ether field. If an attempt is made to slow the charges forming the current, the ether pushes on them and is itself slowed, *effectively returning the work or energy stored in the ether circulation pattern*. Unlike the electrostatic energy of Eq.(2.19.1), which is localized, the magnetostatic energy is stored in the motion of the total configuration including the circulating charges and all of the circulating ether out to infinity. Thus, the total energy in the whole can be defined, but no localized energy density is obviously identifiable.

Of course, since a formal relationship between $\mathbf{A}$ and $\overline{\overline{\mathbf{V}}}$ was made in Section 9.2, and since the standard method of calculating the vorticity from the current was invoked in Section 9.3, it is clear that the magnetostatic energy could be visualized as having a distributed density of,

$$\varepsilon_m = 2\frac{\phi_d^2}{c_0^2}\overline{\overline{\mathbf{w}}}^2 \quad , \tag{9.8.1}$$

which implies, in the example of the solenoid of Section 9.3, that only the *inside* circulating ether has magnetic energy stored in it. It is

hard to say, at this point, whether or not this is true; but it is certain that the simple extension of that concept leads to inaccuracies and false results in certain well known cases. Therefore, all that will be assumed here is that magnetostatic energy is stored in a circulating system whenever current in a closed loop produces a field that is described by Eq.(9.3.2). The latter combined with an identity from Table D.6.2 ensures that throughout the field $\nabla\times\nabla\times\overline{\overline{\mathbf{V}}}=0$ or $\nabla\bullet^{s}\Phi=0$; i.e. the curl of the vorticity is zero. In such a system, magnetic energy can be stored and retrieved.

On the other hand, the conventional approach is useful in actual calculations relating to magnetostatic energy. Review again the thinner solenoid of Section 9.3. On the basis of Eq.(9.3.18) the vorticity outside is zero, so in the conventional view, energy is stored inside only. Substituting the velocity found in Section 9.3 into Eq.(9.8.1), the energy density is found to be (everywhere inside) $\varepsilon_m = 9.54\ \text{ergs}/\text{cm}^3$. Therefore, since a 1cm length of the coil has a volume of $0.785\ \text{cm}^3$, the total energy stored in the circulating ether field per cm of coil length is 7.5 ergs/cm. The total kinetic energy of these circulating electrons is 1.576×10^{-9} ergs. Using the old view, that an electron's kinetic energy is its magnetic energy, the magnetic energy of these electrons, found individually, would add up to an amount 4/3 times that value, or 2.101×10^{-9} ergs. In Chapter 3, it was shown that the total energy of a moving electron does not include any magnetic energy. The upshot is that *the energy stored in the circulating magnetic field is* 4.76×10^{9} *times greater than the total kinetic energy of the electrons themselves*.

9.9 **Conventional Magnetic Energy Density**: In defining magnetic energy, *great care must be taken to avoid some of the misconceptions of the conventional theory of magnetic fields*. Conventionally, the "electric" and "magnetic" fields can be written as,

$$\mathbf{E}=-\left(\nabla\bar{\phi}+\frac{1}{c_0^2}\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t}\right)\quad,\quad\mathbf{B}=\frac{1}{c_0}\nabla\times(\overline{\overline{\phi_a\mathbf{V}}})\quad.\qquad(9.9.1)$$

Making the partial time derivative of $\overline{\overline{\phi_a\mathbf{V}}}$ part of the electric field vector introduces the dubious alien idea of "electromotive force", and mixes electric and magnetic phenomena. The conventional definitions of electric and magnetic energy densities as $\mathbf{E}^2/2$ and $\mathbf{B}^2/2$ have led to considerable confusion in describing energy flow in electromagnetic fields (see Chapter 15). The definition of electric energy density given in Eq.(2.19.2) has eliminated that confusion and is Lorentz covariant.

Looked at conventionally, there can be no magnetic effect where $\mathbf{B} = 0$; and wherever $\mathbf{B} \neq 0$, there is supposedly magnetic energy density equal to $\mathbf{B}^2/2$. On this basis, it is not possible to understand the Aharanov-Bohm experiment, and explaining the transformer effect (mutual inductance) or the 4/3 problem requires the introduction of mysterious new concepts such as lines of flux, emf, non-electromagnetic forces inside of particles, etc. The definition of magnetic energy density to be presented here clears up a number of problems. In the general case, *where the ether flow is time variable, it lumps the time variable term with the magnetic field rather than the electric field*. Nature has provided this natural separation of electric and magnetic fields.

9.10 **Magnetic Energy Density**: Unlike electric energy, which is a simple, localized distortion condition at each space point, magnetic energy is only partially localizable and comes in two different forms. To the extent that it can be considered localized, magnetic energy density is given by,

$$\varepsilon_m = \frac{1}{2c_0^2}\left(\left(\nabla \times \overline{\overline{\phi_a}}\mathbf{V}\right)^2 + \frac{1}{c_0^2}\left(\frac{\partial \overline{\overline{\phi_a}}\mathbf{V}}{\partial t}\right)^2\right)_r \quad ; \text{ General} \qquad (9.10.1)$$

where *the subscript r indicates serious restrictions in applying this equation*, which falsely implies that energy is stored in any region where there is vorticity in the flow or the flow is changing with time. For, while the ether has no linear momentum, per se; being a frictionless fluid, it has angular persistence. Once a vortex is formed, it will continue forever unless physically stopped. Therefore, in situations where work is required to generate a vortex and where that work is *recoverable* in stopping the vortex, Eq.(9.10.1) gives the correct energy stored. Conversely, where a vortex exists that required no work to generate it and where no work is recoverable, Eq.(9.10.1) is not applicable., and there is no magnetic *energy* involved.

It is a fact that some ether vortices exist where no interaction can be used to raise, store or retrieve their energies. Particle spin is one example. During formation, *intrinsic* vortices form that remain a part of particles until they are annihilated. In two and three layer particles (bions and trions), each layer can have its own spin vortex and spin orientation. Unlike the vortex surrounded by a closed loop of moving charge, which charge can be used to increase or decrease the enclosed vortex, the spin vortices are permanent unless annihilated by a matching counterspin. Consequently there is no storage of retrievable work, so the spin vortex cannot be said to store energy. On the other hand, there is a mechanism in the ether that causes a spinning, charged particle to align itself in a magnetic field. If that particle is

forced to reorient in the field, a torque is required and energy must be supplied. This energy can be recovered, and the effect is macroscopic, acting on the whole particle and not changing the spin vortex itself.

The criterion for using Eq.(9.10.1) is that the energetic vortex must be in excess of the establishing charges' flow field. The constant velocity positron and electron have internal vortex flow, but do not establish an excess vortex. Thus they have no magnetic energy.

With these cautions about the application of the concept of localized magnetism, Eq.(9.10.1) can be used for all recoverable vortex energy. One important example of this, *not associated with a closed loop of moving charge*, is propagating radiation, which will be discussed in Section 9.12.

The energy density ε_e and the vortex term of ε_m do not describe the condition of energy while it is being converted from one form to another. For example, if a solenoid has no current flow, and a current is built up, electrons carry electric energy distortion from the source into the coil. This drives the ether enclosed in the solenoid into vortex motion; but in the transition from the electrons' ε_e to the ε_m of the coil's vortex, the energy is transiently carried in space in the form of the second RHS term of Eq.(9.10.1). If a second coil is wound around the first, that primary flow acceleration can act on electrons in the secondary and transport energy to them. This is called the *transformer* effect, which also appears in the important case of propagated radiation.

9.11 **Open Questions About Angular Momentum**: In Section 3.8, the electron's spin angular momentum and magnetic moment were derived. In Section 3.10, the constant velocity electron was analyzed; and now, with the definition of the magnetic field presented in Section 9.2, its flow vector can be written, (motion in the x direction),

$$\overline{\overline{\phi_a \mathbf{V}}} = \mathbf{i}\ \gamma u \phi_0 \left(1 - \varepsilon^{-2r_e / r^*}\right) \quad . \tag{9.11.1}$$

The various quantities are the same as defined in Section 3.10. This field has both vorticity and time change; but neither this nor the electron's spin field has retrievable stored energy. In other words, although putting the indicated values into Eq.(9.10.1) results in calculable values for ε_m, the "energy" found is untouchable and does not enter into the energy budgets of observed processes.

In both the atomic orbits and the current loop, the circulating electrons drive an "excess" vortex that has recoverable energy given by the integral of Eq.(9.10.1) throughout the field. So, even though they carry no distributed charge or electric energy density around with them, they do carry the magnetic energy. The question then arises, can ε_m in these cases be used in place of ε_e in Eq.(3.8.11) to find a very small additional angular momentum? Since there is no related

experimental evidence known to the writer, this question remains unanswered.

9.12 **Propagating Transverse Waves**: Just as the structure and motion of layerons involved only electric energy, *propagating radiation involves only magnetic energy*. It is well known that, except in very special circumstances, a system of charges and currents, varying in time and confined to a region of dimensions $d << \lambda$, radiate energy which, at distance $r >> \lambda$, is essentially plane wave. Figure 9.12.1 shows the flow pattern of one sinusoidal component of this transverse radiation. As shown, the wave propagates in the z direction with velocity $\mathbf{u} = c_0\mathbf{n}$, $\mathbf{n}$ being a unit vector. The flow vector $\overline{\overline{\phi_a}}\mathbf{V}$ is constant over any x,y plane, and varies sinusoidally along the axis of propagation. Where the flow is maximum, there is no energy density; but ε_m increases towards the regions of null flow, where the vortex and transformer energy densities are maximum. At each plane along the wave, the energy is half vortex and half flow acceleration. The wave shown is linearly polarized, but it is possible to generate similar waves that corkscrew circularly polarized.

This picture of ether wave propagation differs from the conventional, because the position is taken here that antenna radiation is solely a magnetic phenomenon, requiring only one vector field $\overline{\overline{\phi_a}}\mathbf{V}$ to describe it. *The density* $\overline{\overline{\phi}}$ *is zero*, and the above description says that the *amplitudes* of the vortex and acceleration components of the wave are *equal*, i.e.,

$$\left(\nabla \times (\overline{\overline{\phi_a}}\mathbf{V})\right)_a = \left(\frac{1}{c_0}\frac{\partial \overline{\overline{\phi_a}}\mathbf{V}}{\partial t}\right)_a \quad , \qquad (9.12.1)$$

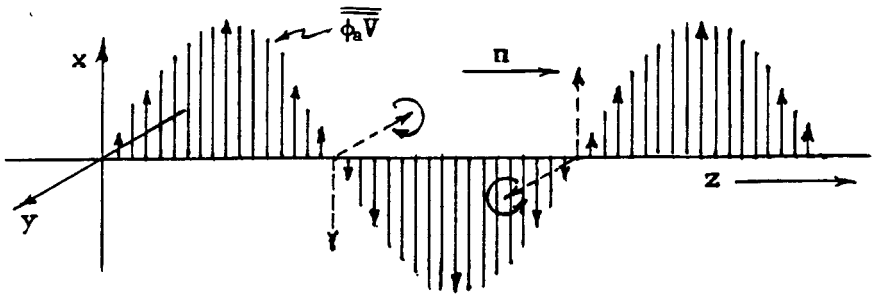

Figure 9.12.1 The flow pattern of plane wave radiation.

so that, from Eq.(9.10.1),

$$\varepsilon_m = \frac{1}{c_0^2}\left(\nabla \times (\overline{\overline{\phi_a \mathbf{V}}})\right)^2 \quad . \tag{9.12.2}$$

The two components are also *perpendicular* to each other and to $\mathbf{n}$, so that,

$$\left(\nabla \times (\overline{\overline{\phi_a \mathbf{V}}})\right) \times \frac{1}{c_0}\frac{\partial \overline{\overline{\phi_a \mathbf{V}}}}{\partial t} = \left(\nabla \times (\overline{\overline{\phi_a \mathbf{V}}})\right)^2 \mathbf{n} \quad . \tag{9.12.3}$$

In Figure 9.12.1 the energy in both components is maximum at the nulls of the $\overline{\overline{\phi_a \mathbf{V}}}$ wave, and zero at the peaks.

Nothing more need be said here about the propagation of radiation. The many fascinating facets of this subject will be set forth in Chapters 10,11,13 and 15.

CHAPTER 10

C-PARTICLES

10.1 **Introduction**: In Chapter 5, the dichotomy dividing fundamental particles into layerons (electric) and c-ons (magnetic) was described, but only the layerons were analyzed there. Now, enough background has been established so that an attempt to describe the c-ons can be made. It will be even less formal than the layeron analysis, but will establish a useful, possible visualization. The evidence comes first from various experiments involving c-ons. The principal impediment to obtaining a rigorous, formal description is the lack of solutions of the ℓ-wave equations. In the future, these will be obtained; but the following is mainly qualitative.

The key to understanding detailed, deterministic c-on structure is the *profound difference between free space antenna radiation and atomic radiation.* Antenna radiation is completely described by Maxwell's macroscopic equations (see Section 11.2), and no case has ever been found where the free space radiation did not *spread out* following a geometrical energy reduction proportional to $1/r^2$. There is no reason to believe that the radiation caused by moving an electron in an antenna is anything more than *the simple wave motion in the ether* described by Maxwell's equations.

Atomically generated photons, on the other hand, travel for untold light years without changing in any way except for a small shift in their wavelength. *This is a profound difference*.

The behavior of antenna radiation is similar to all simple physical wave motions, which exhibit the geometrical spreading. The spreading is essentially independent of any particulate property of the wave medium. On the other hand, *photons behave like particles*. An electron, for example, can travel long distances and still retain its essential properties. Conventionally, photons are treated as point particles, and all transverse wave radiation, including antenna radiation, is assumed to be carried by photons. Here, *this concept is abandoned*. In the present work, for reasons to be discussed in the following, c-on structure applies only to photons and neutrinos; and antenna radiation is seen to be completely free of any photons. It is described in Chapters 11 and 13.

10.2 **C-ons**: Although these ether configurations have wave properties, they are true particles, i.e., stable entities that can maintain their identity only if they move at the velocity c_0 relative to the ether. They come in two basically different varieties, photons and neutrinos; but they are very similar, and both appear to endure forever unless they

interact physically with other particles. For many years very little was known about them, as indicated in Table 10.2.1. Recently there has been speculation that their rest energies and magnetic moments are not zero. However, numerous astounding experiments have only established that these properties are not greater than certain very small maxima.

TABLE 10.2.1

OLD CHARACTERISTICS OF C-ONS

Photon		Neutrino	
Velocity	c_0	Velocity	c_0
Rest Energy	0	Rest Energy	0
Net Charge	0	Net Charge	0
Magnetic Moment	0	Magnetic Moment	0
Spin	1	Spin	$\frac{1}{2}$
Transverse Electro-magnetic Interaction		No Electro-magnetic Interaction	

The first hints as to c-on structure come from the conversion processes that produce them. Table 5.3.1 lists several, such as the μ and τ decays, the $\pi^{\pm}$ decay and the neutron decay, all of which produce neutrinos. Also listed there are the π^0, η and Σ^0 decays which generate photons. Of course, the most common sources of photons are atomic and molecular transitions.

Although photons and neutrinos are, in some respects, almost twins, their differences dictate that the proper study of c-ons begins with a detailed examination of photon generation.

10.3: **Photon Generation**: As far as is known, all photons are generated by *orbiters*. Certainly electrons orbiting in atoms are the primary sources; but the outside orbiter bions are also clear examples. Still questionable are the rare instances where even non-orbiting, concentric bions such as $\pi^{\pm}$ or $K^{\pm}$ have one or two photons as decay products. In these cases the chance of occurrence is less than 10^{-3}, so they probably represent accidental configurations that coincidentally produce an orbiting effect.

In an atom generating a photon, the nucleus absorbs the momentum of the back push as the photon leaves, so a single photon

can be pushed out on an axis perpendicular to the orbit. The choice of which of the two possible directions it takes is made by slight differences in the phase fronts of the ℓ-wave caused by datum fluctuations. In the orbiter bion case, there is no nucleus to absorb the kickback, so two photons are generated, going in opposite directions, again on a line perpendicular to the orbit plane.

In Sections 7.8 and 7.14, several details about photon generation were presented, and they should be reviewed by the reader at this point. It should be emphasized that most of the present discussion will be about the simplest photons produced by *free* atom radiation. Other conditions can result in the production of much more complicated photons; particularly when the radiating atom is part of a more elaborate environment, such as being immersed in a high pressure gas, a solid or a liquid. Though even these complex photons are still roughly similar to the ones to be analyzed, since they result from more violent dumping of energy in a shorter time, they are physically shorter and have larger amplitude waves.

10.4 **The Photon Generator**: To keep the discussion simple, the Hydrogen atom is chosen as the photon generator. Here the task is to visualize *a single, deterministic atom in a field free region*. The basic analysis applies Newton's laws to a "planetary" electron orbiting a proton nucleus. This is not a temporary crutch to be abandoned as the derivation proceeds, but the actual physical mechanism operating. As shown in Chapter 7, certain properties of the *extended* electron control the solution. For example, the difference frequency between the electron's front and back doppler shifted $\ell-$waves establishes the allowed orbits. In Figure 10.4.1, the n shown is the usual

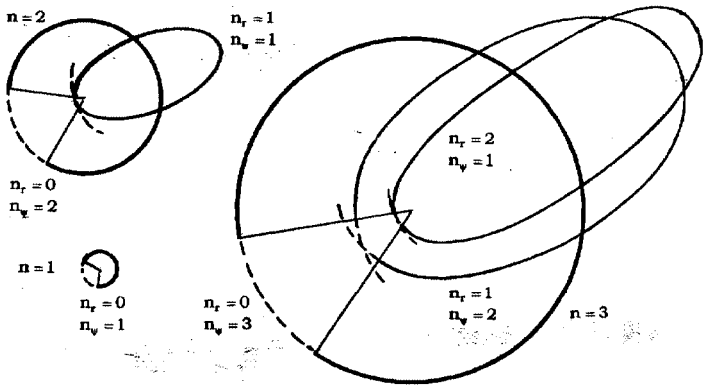

Figure 10.4.1. Orbits

quantum number that fixes orbit size a and energy E,

$$a = \frac{n^2 h^2}{\pi m_0 e^2} \quad (A) \quad , \qquad E = -\frac{e^2}{8\pi a} \quad (B) \qquad (10.4.1)$$

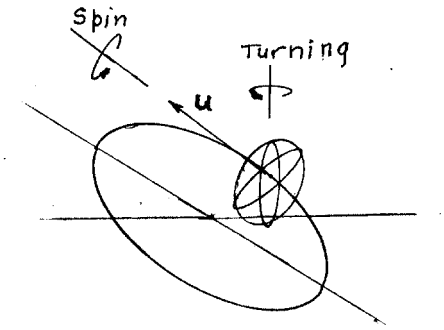

Figure 10.4.2

Orbit shape is set by n_ψ. The electron in orbit is spread out *laterally*, because of its kinetic energy, and takes on an oblate spheroid shape. Also, in the electric field of the proton, it "turns" (in addition to the spin) so its density shape axis is along the orbit. The *spin* aligns itself with the electron *shape* axis. Figure 10.4.2 shows the oblate electron, moving in orbit at velocity **u**, turning and spinning.

Total Vector Angular Momentum

The basic orbital angular momentum is,

$$p_\psi = m_0 a^2 \omega \quad , \qquad (10.4.2)$$

as predicted by Newton's laws. However, because the electron's density/shape "turns", in the plane of the orbit, one turn per electron period, this "turning" angular momentum adds to p_ψ to give a total angular momentum,

$$p = p_\psi + p_t = (1 + K_t)\, p_\psi \quad . \qquad (10.4.3)$$

When atoms are quiescent, the turning factor K_t is related to the orbit shape through,

$$K_t = \frac{1}{n_\psi} \quad ; \qquad (10.4.4)$$

but, during radiation Eq.(10.4.4) is *not applicable*.

Combining Eqs. (10.4.2), (10.4.3) and (10.4.4), the total angular momentum for quiescent atoms, before adding the spin (see Section 7.9), is,

$$p = \sqrt{n_\psi (n_\psi + 1)}\, \frac{h}{2\pi} \quad . \qquad (10.4.5)$$

This has always been a somewhat mysterious form, but is easily explained by the electron's "turning".

Closely following the derivation in Section 7.11, the total vector angular momentum, with the spin added, is,

$$J = \sqrt{n_\psi(n_\psi + 1) + \tfrac{1}{4}}\ \hbar \quad . \qquad (10.4.6)$$

Eqs.(10.4.1) and (10.4.6) give the energy and angular momentum for each orbit. Referring again to Figure 10.4.1, Table 10.4.1 lists some of the values related to those orbits. The radii are given in terms of the *Bohr radius*,

$$a_0 = 5.2918 \times 10^{-9} \quad . \quad \text{cm} \qquad (10.4.7)$$

TABLE 10.4.1

n	n_ψ	a	ω	p_ψ	L = p	J	μ
		cm	s^{-1}	g-cm/s	g-cm/s	g-cm/s	$\frac{\text{ergs}}{\text{hlG}}$
1	1	a_0	2.9233×10^{16}	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
2	1	$4a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	4.2194×10^{15}	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
3	1	$9a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	varies	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
	3	"	1.3260×10^{15}	$2.5981\hbar$	$3.4641\hbar$	$\frac{7}{2}\hbar$	$3\mu_B$

Atomic Magnetic Moment

In Section 9.6, a detailed discussion of the atomic ether vortex caused by an orbiting electron is presented. It is important to the present analysis *because, during photon generation, a portion of the vortex peels off and becomes a critical part of the photon structure*. The atomic vortex is observed as the atom's magnetic moment,

$$\mu = n_\psi \mu_B \quad , \qquad (10.4.8)$$

where μ_B is known as the Bohr magneton,

$$\mu_B = \frac{e}{2m_0 c_0}\hbar = 3.287553 \times 10^{-20} \quad . \quad \text{ergs/hlG} \qquad (10.4.9)$$

Values of μ for the orbits of hydrogen are listed in Table (10.4.1).

10.5 **Transitions and Selection rules**: If an electron transition produces a photon, the photon carries away the energy difference ΔE and the angular momentum difference ΔJ between the two orbits. However, *electron jumps between some orbits are not observed*. A glance at Figure 10.4.1 reveals the basis for what are called *the selection rules*,

$$\Delta n > 0 \quad , \quad \Delta n_{\psi} = \pm 1 \quad . \qquad (10.5.1)$$

The first must be true or ΔE would be zero. The second says that, usually, n_{ψ} changes by just one unit step. *Several physical effects establish this rule*. To better understand *how*, all "possible" transitions between the orbits listed in Table 10.4.1 are shown in Table 10.5.1, along with the important properties carried away if a photon is produced. Only the five jumps designated by the circles in the left-hand column actually produce photons.

TABLE 10.5.1

	Transition	ΔJ	$\Delta\mu$	ΔE
		g-cm/s	$\frac{\text{ergs}}{\text{hlG}}$	ergs
O	$(3,3) \rightarrow (2,2)$	$\hbar$	μ_B	3.0277×10^{-12}
	$(3,3) \rightarrow (2,1)$	$2\hbar$	$2\mu_B$	"
	$(3.3) \rightarrow (1,1)$	$2\hbar$	$2\mu_B$	1.9377×10^{-11}
	$(3,2) \rightarrow (2,2)$	0	0	3.0277×10^{-12}
O	$(3,2) \rightarrow (2,1)$	$\hbar$	μ_B	"
O	$(3,2) \rightarrow (1,1)$	$\hbar$	μ_B	1.9377×10^{-11}
O	$(3,1) \rightarrow (2,2)$	$-\hbar$	$-\mu_B$	3.0277×10^{-12}
	$(3,1) \rightarrow (2,1)$	0	0	"
	$(3,1) \rightarrow (1,1)$	0	0	1.9377×10^{-11}
O	$(2,2) \rightarrow (1,1)$	$\hbar$	μ_B	1.6349×10^{-11}
	$(2,1) \rightarrow (1,1)$	0	0	"

Referring to Table 10.5.1, all of the "allowed" transitions generate photons with $\Delta J = \pm\hbar$. The four that would generate $\Delta J = 0$ cannot produce a photon, because *the peeled off vortex is absolutely essential* to the photon's "particle" structure. The two remaining cases where $\Delta J = 2\hbar$ will be considered, later on, after a discussion of what determines ω_p. Thus, *the only free hydrogen atom transitions possible for the first three energy levels are those with circles* in Table 10.5.1.

The $\pm$ in the selection rule indicates that a photon leaving the atom can have one of two circular polarizations. In the positive case the initial orbit has greater angular momentum than the final orbit; and in the negative case the initial orbit has less.

10.6 **The Radiation Orbit**: When a transition occurs, the electron is suddenly out of lock-step with the initial stable orbit. It spirals inward to the final orbit, releasing energy, but the program it executes is not simple. In the original orbit, the whole field surrounding the electron is locked in a great spiral (see Figures 7.5.1 and 7.5.2). During the transition, this large spiral ether field becomes very irregular and has a strong effect on the electron's emission. Unfortunately, the exact, total field solution is not available. Nevertheless, a reasonably good picture of the radiation orbit that results from this complex interaction has been found.

Fortunately, the orbit Eqs.(10.4.1B), (10.4.2) and (10.4.3) [but not (10.4.1A) or (10.4.4)] are generally applicable, and adequate to describe any path the electron might follow during radiation. As explained in Section 7.14, many orbit cycles are required to generate the photon. *It is assumed that the radiated energy per cycle is small compared to the total photon energy*. The energy radiation rate is proportional to how much of ΔE remains, so that the total energy radiated as a function of time is,

$$E_r = \Delta E\ (1 - \varepsilon^{-t/\tau_p}) \quad , \tag{10.6.1}$$

where $\tau_p = 1.59 \times 10^{23} / \omega_p^2$ and ω_p is the photon angular frequency. It follows that the electron's orbital energy during photon generation is,

$$E = \Delta E\ \varepsilon^{-t/\tau_p} + E_f \quad . \tag{10.6.2}$$

E_f is the energy of the electron in the final orbit. Using Eq.(10.4.1B), the orbit size a, as a function of time, can be found from E.

A typical transition is $(2,2) \rightarrow (1,1)$, and Figure 10.6.1 depicts its energy and orbit size curves. When $t = 5\tau_p$, 99.3% of ΔE has been radiated. Because orbits are energy degenerate, the circular calculations for E and a also apply to the non-circular orbits with the same energy.

Eqs.(10.4.1B) and (10.4.2) come directly from Newton's laws, and specify the *radiation* orbit as a long inward spiral with no surprises. However, certain specific properties of the radiated photon can only be understood by considering the detailed workings of the complicated interaction between the electron shape/motion and the surrounding ether field. Without the complete field solution, this interaction must be pieced together using the following physical observations.

10.7 **Photon Angular Frequency,** ω_p: One of the most interesting photon properties is the very narrow spectral line width, observed in free atom radiation, which indicates that ω_p, *the photon angular frequency, is essentially constant all through the radiation process* (see Section 7.14). *That means the electron's speed,*

$$u_e = \omega_p a \quad , \qquad (10.7.1)$$

is adjusted all along the inward spiral to ensure this. Figure 10.7.1 illustrates the electron orbit velocity curve during the (2,2)---(1,1) transition.

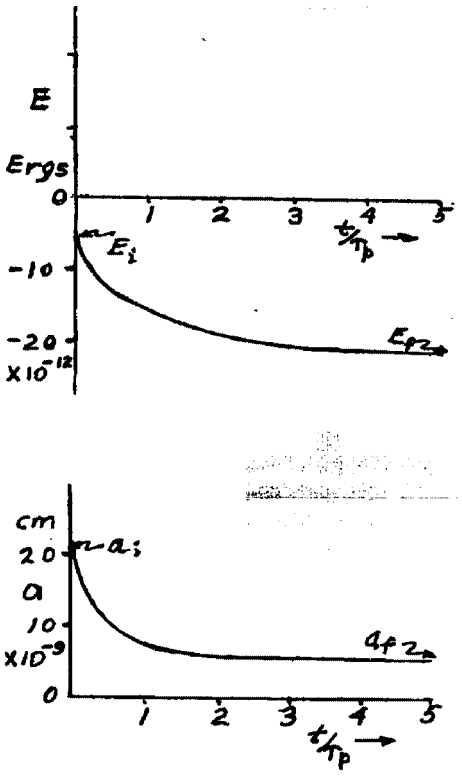

Figure 10.6.1. Transition (2,2)---(1,1) Electron energy and orbit size.

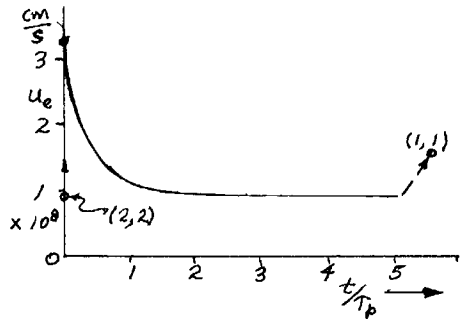

Figure 10.7.1. Transition (2,2)---(1,1) Electron orbit velocity.

The first question to be answered is, what is the value of ω_p? After that, what determines it? Table 7.8.1 shows that the measured photon frequency ω_p is very nearly equal to the average of the initial ω_i and the final ω_f. The *exact* value of ω_p is,

$$\omega_p = \frac{\sqrt{n_f(n_f+1)}\,\omega_f - \sqrt{n_i(n_i+1)}\,\omega_i}{2} \qquad (10.7.2)$$

where ω_f and ω_i refer to the *circular orbits*. Elliptical orbits use the same circular ω_f and ω_i because of the energy degeneracy (see Table 10.5.1).

The answer to the second question is more elusive. Although the reason for this behavior has not yet been established by a rigorous derivation (see Section 13.4), it is likely that, of all the possible unstable orbits between the two stable orbits, the one chosen gives the *maximum rate of radiation*.

In Table 10.5.1, there were two transitions where $\Delta J = 2\hbar$ and $\Delta\mu = 2\mu_B$. For $n > 3$, there are cases where $\Delta J = 3\hbar$ or more. To understand why these transitions do not produce photons, consider Figure 10.7.2. It shows the *stable* orbit frequencies, labeled (3), (2) and (1). The dotted arrows labeled $3 \to 2$, etc. indicate the *radiation* orbit frequencies. They lie roughly half way between the initial and final orbit frequencies, and are given exactly by Eq.(10.7.2). *Normally*

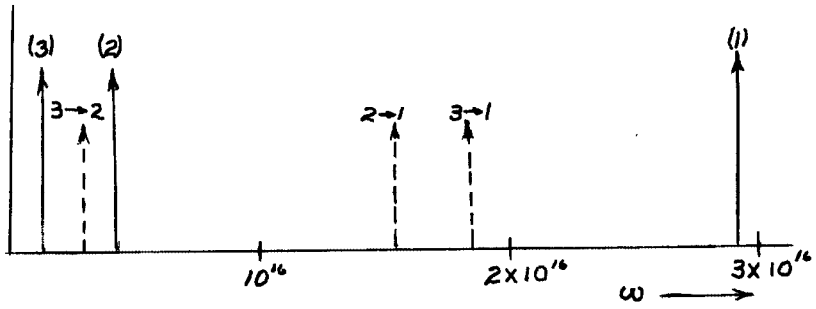

Figure 10.7.2

the photon will be generated at the first radiation frequency reached by the electron as it speeds up, but datum ether fluctuations sometimes cause variations in the orbits that allow bypassing one radiation orbit and going in to another orbit closer to the nucleus. However, in the case $(3,3) \to (2,1)$, the orbits are "shape" *incompatible* (see Figure 10.4.1), requiring the larger adjustment $\Delta J = 2\hbar$. In the transition $(3,3) \to (1,1)$, although the orbits are "shape" *compatible*, the required change $\Delta J = 2\hbar$ is again too difficult to bring about when $(3,3) \to (2,2)$ is encountered first.

The transition $(3,2) \to (2,1)$ is another example of "shape" compatibility. Nevertheless, zero point fluctuations allow some electrons to bypass and go to $(3,2) \to (1,1)$.

10.8 **The Electron/Ether Field Interaction**: By now, the electron's *orbital* motion, during the stable and radiation phases, should be reasonably clear. The analysis it is based upon is, for the most part,

rigorous. At this point, however, a divergence appears between the two phases. In the stable orbit cases, the interaction between the extended electron and the surrounding ether field is represened by the simple "turning" relationship in Eq.(10.4.4),

$$K_t = \frac{p_t}{p_\psi} = \frac{1}{n_\psi} \quad ; \qquad (10.8.1)$$

which expresses the angular momentum contributed by the rotating ether elements relative to the electron's orbital angular momentum. In those cases, K_t has a different *constant* value for each orbit, as specified by n_ψ. On the other hand, *although* $K_t = p_t/p_\psi$ *is still the parameter that describes the interaction during a transition, Eq.(10.4.4) no longer applies, and* K_t *is a widely varying function of time.* At this point it is necessary to derive that function.

Starting with Eqs.(7.3.1) through (7.3.7), it can be shown that, for all orbits, stable and radiating,

$$E = -\tfrac{1}{2}(1 + K_t) m_0 r_{avg}^2 \omega_{avg}^2 \quad , \qquad (10.8.2)$$

where, $r_{avg} = a n_\psi / n$ for *all orbits*. During radiation, $\omega_{avg} = \omega_p$ [taken from Eq.(10.7.2). For stable orbits [quiescent atoms],

$$\omega_{avg} = \sqrt{n(n+1)/\, n_\psi (n_\psi + 1)}\ \omega_{circ} \quad .$$

Combining Eqs.(10.4.1B) and (10.8.2),

$$K_t = -\left(1 + \frac{2E}{m_0 r_{avg}^2 \omega_{avg}^2}\right) \quad . \qquad (10.8.3)$$

Eq.(10.8.3) allows a visualization of the electron/ether-field interaction during photon generation. The fact that, during radiation, $\omega = \omega_p$ and is essentially constant, means that K_t is a function only of E, which is known. Figure 10.8.1 displays the K_t curve for the (2,2)---(1,1) transition, based on the energy curve in Figure 10.6.1.

From Eq.(10.8.3), the sequence of events in Figure 10.8.1 is as follows. At the start, when the electron is in stable orbit (2,2), $K_t = 0.5$, $\omega = 4.22 \times 10^{15}$ and $E = -5.45 \times 10^{-12}$ ergs. Upon breaking orbit-lock, *in a very few cycles*, although E and orbit size, a , remain almost constant, the electron speeds up in orbit to reach $\omega = \omega_p$. This drops K_t, and generally drives it negative, meaning that the electron is turning so fast that the surrounding ether appears to be turning in the opposite direction. Gradually, the circling electron causes the surrounding ether to spin up, just as current in a ring conductor does. After about $5\tau_p$ seconds, 99.3 % of ΔE has been radiated, and the remainder is essentially of the same order of

magnitude as the datum fluctuations. At this time, the electron is orbiting very near to the final orbit size, so lock-in takes place with a rapid increase in ω to ω_f and an equally rapid drop in K_t. Such rapid changes in K_t are possible because the high propagation velocity allows very rapid adjustment of the ether field.

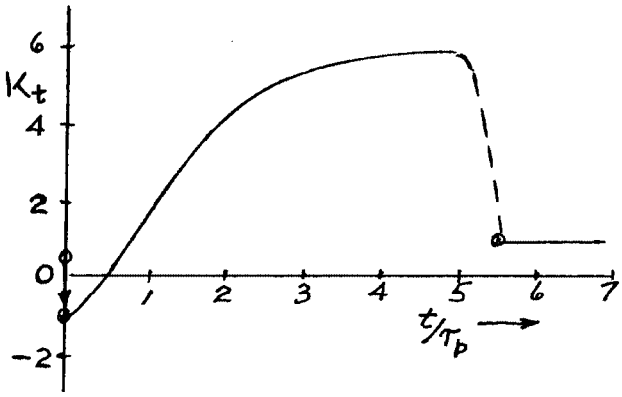

Figure 10.8.1. Transition (2,2)---(1,1)

10.9 **Preliminary Photon Structure**: So far the emphasis has been on the photon generator, i.e. the orbiting electron. Now, the understanding gleaned from a fairly complete extended electron atomic structure theory must be used to assemble an *incomplete* theory of photon structure. Without a full set of non time average $\ell-$wave equations, the picture constructed here will remain incomplete. It involves considerable speculation, but it is important to get as much visualization as possible in the face of the foggy limbo of present day *ensemble* photon theory. Some of the difficult choices remaining will require new and ingenious experiments to settle which way to go. Meanwhile, the following is an attempt (surely with some wrong guesses) to get a useable visualization.

All photons appear to have certain well known characteristics, such as: very small or zero rest energy, zero net charge and zero magnetic moment. They also have spin 1, in agreement with the $1\hbar$ reduction in the radiating atom's total vector angular momentum.

Based on earlier sections, a few other photon characteristics appear well founded. For example, in the (2,2)---(1,1) transition, the photon produced is a long, narrow particle ($L/a > 10^8$) because it takes time to generate it *and it is propagating away from the atomic orbit at velocity* c_0. It has a circularly polarized, energy carrying t-wave; and also, because the radiation process is a disturbance, an energyless, plane $\ell-$wave propagating at velocity c_0.

Of all *atomic photons*, the smallest diameter particle is produced by the (2,2)---(1,1) hydrogen transition. It is roughly 10^{-8}cm across. Its

effective length $L \cong 5c_0\tau_p$ is 99.5 cm, containing 8.2×10^6 cycles of the radiated wave, or 99.3 % of ΔE.

This meager collection of photon attributes is a relatively simple extension of the *conventional* concept of a photon. One further photon property can be invoked to improve that concept considerably.

10.10 **The Photon Vortex**: The proper approach to photon structure is to find the physical mechanism, that antenna radiation does **not** have, by which the photon is held in *particle* form.

The backbone of a photon's particle structure is a long tubular vortex. It is pushed out perpendicularly to the electron's orbit, probably because the orbit radius decreases during radiation. Because of this, the ether vortex has a form close to that of a very long, needle thin solenoid; with ether rotating as a solid body inside, and slipping outside (zero curl field outside). The sharp sheath region takes the shape and size of the "a" radius curve of Figure 10.6.1; *but it is greatly stretched out*, like an unwound spool of thread. Figure 10.10.1 is greatly compressed lengthwise and expanded in diameter.

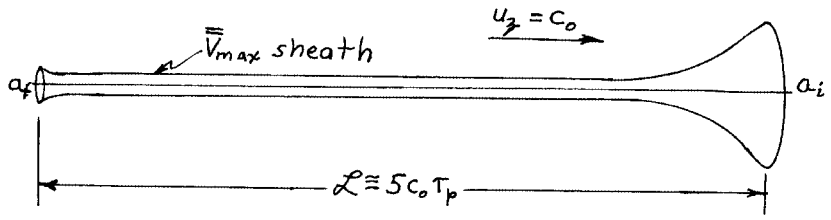

Figure 10.10.1. Photon vortex (compressed in length 10 times and expanded in diameter 10^8 times)

Although originally there is a circulating charge (the orbiting electron *generating* the photon vortex), once the vortex leaves the atom it is a *free* vortex, moving at velocity c_0, *with no attached circulating charge*. This indicates that it is an *energyless magnetic field* (see Sections 9.8ff and 15.6ff). It also explains why a photon has no magnetic moment.

It is well known that vortices can travel considerable distances without significant change. The photon vortex has the advantage that *the ether is a frictionless fluid*, and thus the vortex is essentially indestructible. Only when it finds a compatible particle that unwinds it can it vanish.

Any disturbance in the ether causes the formation of $\ell-$waves (longitudinal waves), so it is assumed that an $\ell-$wave propagates inside the vortex and moves along with it. Photons are also known to exhibit circularly polarized t-wave (transverse wave) characteristics, so there is also a t-wave propagating inside the vortex and moving along with it. Without the vortex, these waves would behave like antenna radiation and would exhibit no particle properties. The remaining task is to describe these waves to the extent present knowledge permits.

10.11 **The Photon ℓ-Wave**: No complete quantitative analysis of the complicated ether flow pattern during the radiation exists. Moreover, no complete quantitative description of the photon is available. However, some progress has been made. Certainly the most significant feature of any photon is its vortex. As described above, the orbiting electron peels off a part of the atom's dipole field as a solenoidal vortex that turns as a rigid body in a small central region but slips outside. It also generates a circularly polarized t-wave, and the disturbance also produces a nearly plane ℓ-wave. Both waves propagate, in the direction of the vortex axis, *inside* the rigid body region. Outside, the slip prevents coherent wave propagation.

At this point, it is necessary to call attention to certain properties of the ℓ-wave equations. For a photon moving in the z direction, assuming a plane ℓ-wave, Eq.(3.10.3) reduces to,

$$\frac{1}{\gamma^2}\left(\frac{\partial^2\bar{\eta}}{\partial z^2}-\frac{1}{\bar{\bar{\eta}}}\left(\frac{\partial\bar{\eta}}{\partial z}\right)^2\right)=\pm\frac{c_0\omega}{\phi_d D}\frac{\partial\overline{\overline{\phi.V.}}}{\partial z} \quad . \qquad (10.11.1)$$

Combining the bridge Eq.(2.14.2) with Eqs.(2.15.1) and (3.10.2), the reduced bridge equation becomes,

$$\overline{\overline{\phi.V.}}=\frac{1}{\gamma^2 b}\frac{\partial\bar{\phi}}{\partial z} \quad . \qquad (10.11.2)$$

Substituting Eq.(10.11.2) into Eq.(10.11.1), the ℓ-wave equation takes the form,

$$\frac{\partial^2\bar{\eta}}{\partial z^2}-\frac{1}{\bar{\bar{\eta}}}\left(\frac{\partial\bar{\eta}}{\partial z}\right)^2=\pm\frac{c_0\omega}{b\phi_d D}\frac{\partial^2\bar{\phi}}{\partial z^2} \quad . \qquad (10.11.3)$$

None of the forms of the ℓ-wave equations that involve time averages allow solving for $\mathbf{V}$. directly, but require trial solutions instead. There could be a more fundamental set of equations not involving time

averages that could be solved for $\mathbf{V}_\bullet$ directly. If that is true, then the time average equations might eliminate terms necessary to correctly describe c-ons. Thus, the visualizations presented here could be wrong, but they probably include most of the major features of the c-ons. For this reason, the simplest possible visualization is sought.

Referring back to Figure 10.10.1, a simple, plane ℓ-wave can be visualized traveling inside the tube with an incremental velocity,

$$\mathbf{V}_\bullet = \mathbf{k}aC \quad , \tag{10.11.4}$$

where a is the amplitude constant, and,

$$C = \cos\omega\left(t - \frac{z}{c_0}\right) \quad . \tag{10.11.5}$$

From the continuity Eq.(2.12.4),

$$\phi_\bullet = \frac{a\phi_d}{c_0}C \quad , \tag{10.11.6}$$

and,

$$\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}} = \frac{a^2\phi_d}{2c_0} = 9.4856\times10^{-37} \quad . \tag{10.11.7}$$

Since $\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}}$ is constant throughout the tube of the vortex,

$$\nabla\bullet\overline{\overline{\phi_\bullet \mathbf{V}_\bullet}} = 0 \quad . \tag{10.11.8}$$

10.12 **The Photon's Bulk Structure**: Because the motion factor γ dropped out of Eq.(10.11.3), the velocity potential can be written as,

$$\overline{\overline{\eta}} = \overline{\overline{\mathbf{V}_\bullet^2}} = \frac{a^2}{2} \quad , \tag{10.12.1}$$

a constant. Eq.(10.11.3) then indicates that the *gradient* of $\overline{\overline{\phi}}$ does not vary throughout the tube. To see how the gradient is related to other parameters, the combination of Eqs.(10.11.7), (2.15.1) and the bridge Eq.(2.14.2) leads to $\mathbf{u} = \mathbf{k}c_0$,

$$\nabla\overline{\overline{\phi}} \cong -\frac{1}{c_0^2}\frac{\partial\overline{\overline{\phi_a \mathbf{V}}}}{\partial t} = -\mathbf{k}\frac{1}{c_0}\frac{\partial\overline{\overline{\phi}}}{\partial t} \quad . \tag{10.12.2}$$

The simplest possible case is one where $\nabla\overline{\overline{\phi}} = 0$ and the flow vector $\overline{\overline{\phi_a \mathbf{V}}}$ and the bulk density $\overline{\overline{\phi}}$ are both constants everywhere in the tube. In that case, both the charge and electric energy densities are zero throughout. Therefore, in this configuration the main body of the photon is just a cylinder of excess (or reduced ?) ether density ϕ_0

spinning on its long axis, containing a longitudinal plane wave and a *circularly polarized* transverse wave moving in the direction of its axis at the velocity c_0.

Any charge, mass or electric energy the bulk carries must be located at the ends of the tube. As it propagates, the ether out in front of it must be compressed until the incremental density reaches

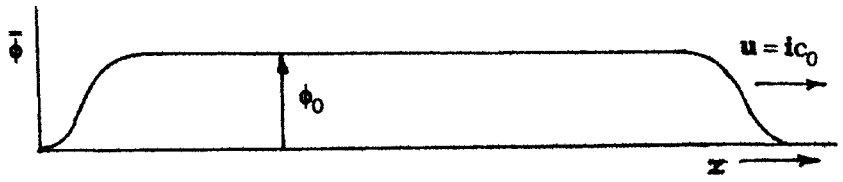

Figure 10.12.1 The neutrino incremental ether density profile.

the ϕ_0 level. At the rear, the density must drop back to the datum. Figure 10.12.1 depicts the overall density profile. No quantitative solution is available at this time, but the general characteristics are represented in Figure 10.12.2. As displayed, there will be equal and opposite layers of charge distortion at each end, so that the net charge is zero.

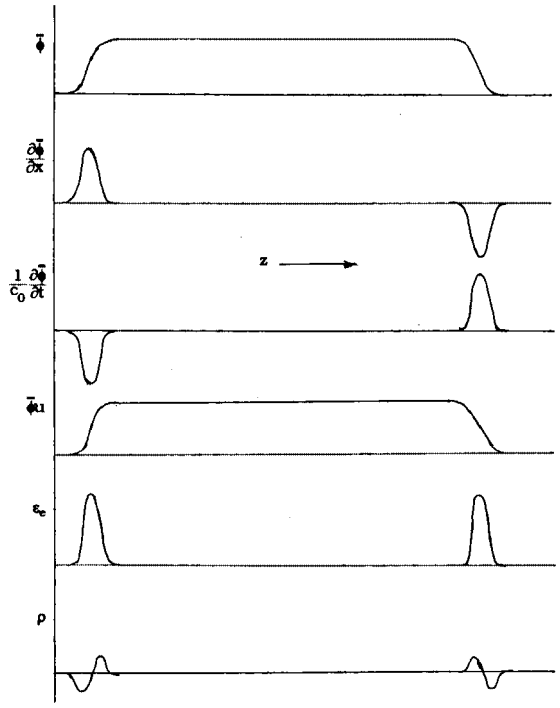

Figure 10.12.2 Photon bulk characteristics.

If, instead of this simplest configuration, there is a constant, non-zero gradient in the tube, then the electric energy being carried away will also be carried in the tube, although no charge density will appear there. Otherwise, the photon will be very much the same as the simple configuration.

10.13 **The Photon t-Wave**: Without a complete set of $\ell-$wave equations, it is difficult to know what is actually happening at the vortex sheath and outside. It appears that the vortex tube acts like a wave guide, but without conducting boundaries or charges. Therefor, *it is assumed that inside the vortex the t-wave has all the properties of an unlimited-space wave*. This allows the t-wave to be described by a circularly polarized, plane wave,

$$\overline{\overline{\phi_a \mathbf{V}}} = \overline{\overline{(\phi_a \mathbf{V})}}_0 D(t)(\mathbf{i}C + \mathbf{j}S) \quad , \tag{10.13.1}$$

where,

$$C = \cos \omega_p \left(t - \frac{z}{c_0} \right) \quad \text{and} \quad S = \sin \omega_p \left(t - \frac{z}{c_0} \right) \quad , \tag{10.13.2}$$

with coordinates as shown in Figure 10.13.1. The $\overline{\overline{\phi_a \mathbf{V}}}$ vector rotates in the direction of α, and the damping factor $D(t)$ is limited by $D(t) \le 1$.

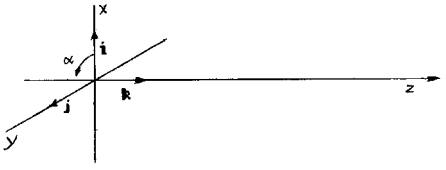

Figure 10.13.1.
Coordinates for the t-wave

It follows that,

$$\frac{\partial \overline{\overline{\phi_a \mathbf{V}}}}{\partial t} = \omega_p \overline{\overline{(\phi_a \mathbf{V})}}_0 D(-\mathbf{i}S + \mathbf{j}C) + \underbrace{\overline{\overline{(\phi_a \mathbf{V})}}_0 \frac{\partial D}{\partial t}(\mathbf{i}C + \mathbf{j}S)}_{\text{small}} \quad , \tag{10.13.3}$$

and

$$\nabla \times \overline{\overline{\phi_a \mathbf{V}}} = \frac{\omega_p}{c_0} \overline{\overline{(\phi_a \mathbf{V})}}_0 D(\mathbf{i}C + \mathbf{j}S) \quad . \tag{10.13.4}$$

Using the modified Poynting theorem of Section 15.8, the t-wave magnetic energy flow vector is,

$$\mathbf{S} = -\frac{1}{c_0^2} \frac{\partial(\overline{\overline{\phi_a \mathbf{V}}})}{\partial t} \times (\nabla \times \overline{\overline{\phi_a \mathbf{V}}}) = \mathbf{k} \frac{\omega_p^2 \overline{\overline{(\phi_a \mathbf{V})}}_0^2}{c_0^3} D^2 \quad , \tag{10.13.5}$$

and the energy density (see Section 9.10) is,

$$\varepsilon_m = \varepsilon_t + \varepsilon_v = \left[\frac{\omega_p^2 \overline{\overline{(\phi_a \mathbf{V})}}_0^2}{2c_0^4} D^2 + \underbrace{\frac{\overline{\overline{(\phi_a \mathbf{V})}}_0^2}{2c_0^4} \left(\frac{\partial D}{\partial t} \right)^2}_{\text{small}} \right] + \frac{\omega_p^2 \overline{\overline{(\phi_a \mathbf{V})}}_0^2}{2c_0^4} D^2 \quad ,$$

or,

$$\varepsilon_m = \frac{\omega_p^2(\overline{\overline{\phi_a \mathbf{V}}})_0^2}{c_0^4} D^2 + \underbrace{\frac{(\overline{\overline{\phi_a \mathbf{V}}})_0^2}{2c_0^4}\left(\frac{\partial D}{\partial t}\right)^2}_{\text{small}} \quad . \qquad (10.13.6)$$

To establish the magnitudes of $(\overline{\overline{\phi_a \mathbf{V}}})_0$, D and $\partial D/\partial t$, it is necessary to go back to Eq.(10.6.1), which gives the known magnetic energy density,

$$\varepsilon_m = \frac{1}{\pi a^2}\frac{dE_r}{dz} = \frac{\Delta E}{\pi c_0 \tau_p}\frac{\varepsilon^{-t/\tau_p}}{a^2} \quad . \qquad (10.13.7)$$

By combining Eqs.(10.4.1B) and (10.6.2) to find a, Eq.(10.13.7) can be written, for transition (2,2)---(1,1),

$$\varepsilon_m = K_0 \varepsilon^{-t/\tau_p}(4/3 - \varepsilon^{-t/\tau_p})^2 \quad , \qquad (10.13.8)$$

where,

$$K_0 = \frac{64\pi(\Delta E)^3}{c_0 \tau_p e^4} = 5.2505 \times 10^3 \quad . \qquad (10.13.9)$$

Eq.(10.13.6) can be transposed and joined with Eq.(10.13.8) as,

$$\left(\frac{dD}{dt}\right)^2 + 2\omega_p^2 D^2 = K_m \varepsilon^{-t/\tau_p}(4/3 - \varepsilon^{-t/\tau_p})^2 \quad , \qquad (10.13.10)$$

with [for (2,2)---(1,1)],

$$K_m = K_0 \frac{2c_0^4}{(\overline{\overline{\phi_a \mathbf{V}}})_0^2} = \frac{8.480 \times 10^{45}}{(\overline{\overline{\phi_a \mathbf{V}}})_0^2} \quad . \qquad (10.13.11)$$

Eq.(10.13.10) can be solved easily for $D(t)$, because $2\omega_p^2 = 4.8019 \times 10^{32}$, so it can be assumed that,

$$\left(\frac{dD}{dt}\right)^2 << 2\omega_p^2 D^2 \quad . \qquad (10.13.12)$$

If Eq.(10.13.12) is true, then,

$$D(t) \cong \sqrt{\frac{K_m}{2\omega_p^2}}\ \varepsilon^{-t/2\tau_p}(4/3 - \varepsilon^{-t/\tau_p}) \quad , \qquad (10.13.13)$$

and,

$$\frac{dD(t)}{dt} \cong \sqrt{\frac{K_m}{2}}\ \frac{1}{\omega_p \tau_p}\varepsilon^{-t/2\tau_p}(\tfrac{3}{2}\varepsilon^{-t/\tau_p} - \tfrac{2}{3}) \quad . \qquad (10.13.14)$$

By substituting Eqs.(10.13.13) and (10.13.14) into Eq.(10.13.12), the *assumption* the latter represents is seen to be correct. As a result, each of the terms labeled "small" in all the preceding equations is *negligible*.

The final step in characterizing the (2,2)---(1,1) photon's t-wave is to establish the magnitude of $(\overline{\overline{\phi_a \mathbf{V}}})_0$. This is done by first arbitrarily choosing the magnitude of the damping factor at $t = 0$ to be $D(0) = 1$. Then, from Eq.(10.13.13), $1 = \sqrt{K_m / 2\omega_p^2}\,/3$; which, when combined with Eq.(10.13.11), determines the amplitude to be $(\overline{\overline{\phi_a \mathbf{V}}})_0 = 1.40 \times 10^6$ hlvolts-cm\s. Thus, the final form of the t-wave vector is,

$$\overline{\overline{\phi_a \mathbf{V}}} = (\overline{\overline{\phi_a \mathbf{V}}})_0 \; D(t) \; (\mathbf{i}C + \mathbf{j}S) \quad , \qquad (10.13.15)$$

where,

$$D(t) = \varepsilon^{-t/2\tau_p}(4 - 3\varepsilon^{-t/\tau_p}) \quad . \qquad (10.13.16)$$

This wave carries all of the energy ΔE taken from the radiating atom.

10.14 **Photon Spin**: The emerging visualization of the photon has three parts, the energyless solenoidal magnetic field that constitutes the vortex, the energyless ℓ – wave that establishes the steady-state bulk ether incremental density distribution and the t-wave that carries the energy ΔE away from the electron in transition.

The final step in describing photon structure is to explain how the angular momentum ΔJ, taken from the electron in transition, is carried along. It would appear that the vortex itself would be the logical carrier, and that will be investigated first. Nevertheless, there are questions that must be answered; and, in some ways, the final determination is still to be made.

Macroscopic angular momentum of a solenoidal vortex is found from (R, α, z coordinates),

$$J = \int_0^a \int_0^{2\pi} \int_0^L \frac{\varepsilon}{c_0^2} u \, R \, dvol \quad , \qquad (10.14.1)$$

where ε is an energy density distribution and u its velocity at each point. Because *the ether has no momentum itself*, great care is needed in applying Eq.(10.14.1) to electric and magnetic fields.

For example, *electron* spin involves an energyless magnetic ether vortex (see Chapter 3, Section 3.8) circulating a known, localized *electric* energy density distribution ε_e at velocity $\overline{\overline{\mathbf{V}}}$, which is substituted into Eq.(3.8.11). In Eq.(10.14.1), u is the velocity of the incremental bulk ether distribution related to the actual ether velocity $\overline{\overline{\mathbf{V}}}$ by,

$$u \cong \frac{\phi_d}{\overline{\overline{\phi}}} \overline{\overline{V}} \quad . \qquad (10.14.2)$$

A long solenoid appears to locally store circulating magnetic energy inside. However, in the magnetic field case, since the energy density is *magnetic* instead of *electric* the *nature of angular momentum is different.* The magnetic field has no angular momentum like the electron's field. It only has the *angular persistence* of a circulating, frictionless fluid (see Section 2.9). For example, the calculation of the long solenoid energy density, ε_m, in Sections 9.3 and 15.11 involves magnetic vortices that are "attached" to circulating charged particles. Angular momentum can only be stored or removed by interaction with the "attached" circulating charged particles, or by injecting other charged particles, into the vortex, which also interact through the original "attached" particles. Nevertheless, J for an energy storing solenoid can be found using ε_m in Eq.(10.14.1).

On the other hand, in the case of an energyless solenoid such as the photon, once it is formed with no "attached" charges, the spin angular momentum cannot be increased or decreased. It can only be transferred in an encounter with another particle. Since it is truly energyless, the spin angular momentum is better written (see Sections 9.3, 9.8 and 15.6) (R, α, z),

$$J = 2\int_0^a \int_0^{2\pi} \int_0^{L} \bar{\bar{w}}^2 \, u \, R \, dvol \quad , \tag{10.14.3}$$

where $\bar{\bar{w}}$ is the ether vorticity.

Referring to Eq.(10.7.1), since the vortex in the photon spins as a rigid body the velocity throughout is,

$$u_\alpha = \omega_p R \quad . \tag{10.14.4}$$

Also, the ether vorticity has the constant value K_s, where, from Eqs.(10.14.2) and (10.14.4),

$$\bar{\bar{w}} = K_s = \frac{\bar{\bar{V}}_\alpha}{R} = \frac{\phi_0}{\phi_d}\omega_p \quad . \tag{10.14.5}$$

Making these substitutions in Eq.(10.14.3) and integrating α and R,

$$J = \pi \frac{\phi_0^2}{\phi_d^2}\omega_p^3 c_0 \tau_p \int_0^{L/c_0\tau_p} a^4 \, d\left(\tfrac{t}{\tau_p}\right) \quad , \tag{10.14.6}$$

where,

$$a = \frac{e^2}{8\pi\Delta E\left(\frac{4}{3} - \varepsilon^{t/\tau_p}\right)} \quad . \tag{10.14.7}$$

Finally,

$$J = \frac{\omega_p^3 c_0 \tau_p e^8}{8^4 \pi^3 \phi_d^2 (\Delta E)^4} \phi_0^2 \int_0^{L/c_0\tau_p} \frac{d\xi}{(\frac{4}{3} - \varepsilon^{-\xi})^4} \quad . \qquad (10.14.8)$$

In the transition (2,2)---(1,1), this becomes,

$$J = 7.1504 \times 10^{-25} \phi_0^2 \int_0^{L/c_0\tau_p} \frac{d\xi}{(\frac{4}{3} - \varepsilon^{-\xi})^4} \quad . \qquad (10.14.9)$$

When the vortex was being generated, lock-in to the (1,1) took place, and the vortex was terminated between $L/c_0\tau_p = 5$ and say 7. At $L/c_0\tau_p = 5$, the integral in Eq.(10.14.9) is 11.9810. At 7, the integral is 12.6194. After that, each integer increase in $L/c_0\tau_p$ adds roughly 0.3164. For the $L/c_0\tau_p = 5$ case,

$$J = 8.5669 \times 10^{-24} \phi_0^2 \quad . \qquad (10.14.10)$$

Without a full set of non time average ℓ – wave equations, ϕ_0 cannot be found directly using this approach. However, if the vortex carries the angular momentum $\hbar$, then $\phi_0 = 1.1095 \times 10^{-2}$ hlvolts. The corresponding ether vorticity $\overline{\overline{w}} = K_s = \phi_0 \omega_p / \phi_d = 1.9138 \times 10^{-7}$ rad/sec. *This completes **one** visualization of the photon.*

10.15 **About Photons**: Einstein was the first to understand that the photon had energy,

$$E_p = \Delta E = h\nu_p = \hbar\omega_p \quad . \qquad (10.15.1)$$

Unlike the neutrinos, which are neutral in almost every respect, the *t-wave in the photon interacts with charged particles, making its detection relatively easy*. This allows photons to be used and observed in many types of processes and experiments. Often these involve lenses, polarizers, analyzers and various retarder plates and beam splitting mirrors. Analyses of beam processing configurations are almost always done on the basis of wave-theory. In most cases the results match the observations; but fundamentally the physics of the beam splitting and polarization changes should be described in terms of the *photon particle* guidance and diversion, which is sadly neglected in modern texts.

Earlier it was said that the photon's t-wave is always circularly polarized. It is probable that *no linearly polarized photon exists in nature*. Certainly the generating atomic electrons and orbiter bion

layers are circling, and it is obvious that the peeling off of energy and spin is compatible with a spiraling circularly polarized wave.[1] Thus, it may be that all linearly polarized beams are composed of paired circularly polarized particles.

There are many unresolved questions about photons, some of which will be discussed in Chapter 11. The visualization presented here does allow some puzzles to be eliminated. As will be seen later on, most or perhaps all of the confusing and paradoxical interpretations of experiments on radiation involving "wave-particle duality" are replaced with simple, intuitive answers.

10.16 **The Neutrino**: To get the proper feeling for the neutrino, it is useful to read, in addition to many sources such as those listed in Section 5.1, Asimov's impressive book, The Neutrino[1]. *In none of these sources, however, is anything available indicating the size or shape of that elusive particle.* The neutrino was first *postulated,* in connection with the conversion of a free neutron into a proton and electron, in order to save the conservation of energy law. However, with hindsight, *its most important function is to allow spin conversion*. This is most easily understood by considering the decay of the μ^- unon, as first described in Table 5.3.1.

The conversion process, in its simplest form, starts with the μ^- at rest, buffeted by the datum fluctuations. After a certain delay time, the μ^- begins to ooze outward from the 2nd layer position; and, at the end of the conversion, an electron with its maximum electric energy at the 1st layer position is formed. Almost all of the rest energy of the μ^- must be carried away as kinetic energy, and the fact that the μ^- vortex velocity distribution is much more compact than the final e^- vortex velocity distribution means that some process of *circulation reconfiguration* must also take place. The method nature uses to do the latter is to simply remove the μ^- spin vortex (ν_μ) from the conversion region, and generate an e^- neutrino/antineutrino pair of spin vortices, one ending up inside the electron and the other ($\bar{\nu}_e$) also leaving the conversion region. All three final particles can carry kinetic energy away with them.

10.17 **The Neutrino Vortex and ℓ-Wave**: The neutrino has almost the same structure as the photon, but it has no transverse wave. It is

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1. I. Asimov, The Neutrino, Avon Books (1966). Wesley Publ. Co., Reading, Mass. (1965).

most often generated by a spinning unon. As described in Section 3.8, the unon spin velocity forms an energyless dipole field that turns as a rigid body in a small central region but slips outside. There is no orbiting electron as in photon generation. During μ^- decay, for example, it is the expansion of the particle to the lower energy electron that causes the spin field to leave the μ^-. Here as in photon generation, the vortex is stretched out, but it is much smaller in cross section ($\leq 0.06\, r_e$) than the photons. The neutrino vortex and its plane ℓ – wave are depicted in Figure 10.17.1.

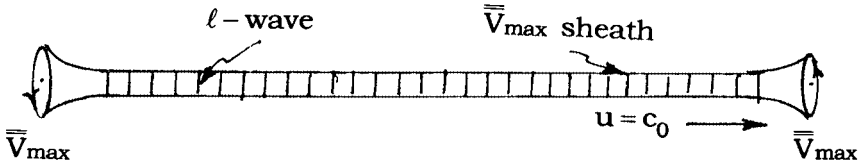

Figure 10.17.1 ℓ-wave inside the propagating ν_μ vortex.

The photon ℓ – wave analysis of Section 10.11 is directly applicable to the neutrino, and the bulk structure discussion and illustrations of Section 10.12 also apply. In light of the discussion in Chapter 9, it does not appear that there is any *recoverable* magnetic energy associated with the neutrino. Since all of its distributed charge is in the end structures, the chance that it has any significant magnetic moment is slight. Nevertheless, it does have the external slip field of $\overline{\overline{\mathbf{V}}}$, so some miniscule magnetic effect might be measurable. Again, this is an open question.

At the present time, there is much speculation that the neutrinos oscillate from one type (ν_e, ν_μ, ν_τ) to another and back. Since nature was forced to remove the ν_μ and replace it with the ν_e in the μ decay, and since it seems to be the rule that one whole spin structure must be replaced with another whole spin structure, it is unlikely that the individual neutrinos can oscillate.

At this point, very little more is known about these untouchable particles; but the visualization presented here does help to understand many of the odd situations that involve neutrinos.

CHAPTER 11

MACROSCOPICS

11.1 **Introduction**: The principal thrust of the first ten chapters was to lay out the properties of the ether and solve the *microscopic* ether equations for the particles and more elaborate structures. From here on the emphasis is on phenomena that are closer to those observed in the every day *macroscopic* world. Commonly this involves certain overt properties of particles or collections of particles, interactions between objects, and interactions between "fields" and objects.

Macroscopic formalisms appeared in two great steps. In the 19th century, Faraday and Maxwell produced Electromagnetic Theory in the deterministic form called Maxwell's equations that describe upwards of 90 percent of all observable physical phenomena. Simultaneously, to handle problems involving large *ensembles* of particles, statistical mechanics was developed (thermodynamics, kinetic energy, etc.) culminating, in the 20th century, in Quantum Physics as expressed by the equations of Schroedinger and Dirac. Unfortunately, *the "field" variables appearing in these statistical quantum mechanics equations did not represent actual physical variables*, so that, in spite of the great power and usefulness of these invaluable formalisms, a gap remained. The ether picture presented here is intended to fill that gap. Here, as in the earlier chapters, very little will be said about the statistical aspects of physics, since the fundamentals are primarily the concern of the deterministic problem.

11.2 **Maxwell's Macroscopic Equations**: Conventionally, Maxwell's equations are written in terms of the **E** and **B** fields of Eqs..(9.9.1), but a more fundamental form is expressed as wave equations for the so called "scalar and vector potentials". The ether equation form is,

$$\nabla^2 \overline{\overline{\phi_a}} - \frac{1}{c_0^2}\frac{\partial^2 \overline{\overline{\phi_a}}}{\partial t^2} = -\rho \quad , \quad \nabla\bullet\, \overline{\overline{\phi_a \mathbf{V}}} + \frac{\partial \overline{\overline{\phi_a}}}{\partial t} = 0 \quad ,$$

$$\nabla^2 \overline{\overline{\phi_a \mathbf{V}}} - \frac{1}{c_0^2}\frac{\partial^2 \overline{\overline{\phi_a \mathbf{V}}}}{\partial t^2} = -\rho\mathbf{u} \quad , \tag{11.2.1}$$

or the incremental alternative,

$$\nabla^2 \bar{\phi} - \frac{1}{c_0^2}\frac{\partial^2 \bar{\phi}}{\partial t^2} = -\rho \quad , \quad \nabla\bullet\, \bar{\phi}\mathbf{u} + \frac{\partial \bar{\phi}}{\partial t} = 0 \quad ,$$

$$\nabla^2 \bar{\phi}\mathbf{u} - \frac{1}{c_0^2}\frac{\partial^2 \bar{\phi}\mathbf{u}}{\partial t^2} = -\rho\mathbf{u} \quad . \tag{11.2.2}$$

Although these equations have a similar appearance to Eqs.(2.20.2) and Eqs.(2.21.2), the profound difference discussed in Chapter 2 must be kept in mind constantly. The *microscopic* equations in Chapter 2 are actually *definitions* of a specific type of distributed distortion (ρ) that results from the presence of a known $\overline{\overline{\phi}}$ distortion distribution. The *macroscopic* Eqs.(11.2.1) and (11.2.2) are just the reverse. They allow finding the $\overline{\overline{\phi}}$ and $\overline{\overline{\phi}}\mathbf{u}$ distributions if ρ and **u** are known everywhere. At the standard level of abstraction, books on E&M generally state explicitly that dimensions less than 10^{-3} cm will be excluded unless special care is taken to account for quantum theory limitations; and the charge density ρ is nothing more than a count of whole charged particles per cubic centimeter times the charge on each particle.

Techniques for solving the macroscopic equations are so common and worked out examples so voluminous that nothing of that sort will be done here except for two simple examples that are useful to illustrate the role of the ℓ-waves.

11.3 **Electrostatics**: Electrostatic fields are the most complex of the basic field types, since they involve lumps and hollows of incremental bulk ether density held in stationary configurations by ℓ-waves. *The solution begins with the assumption that there is a known configuration of whole charged particles from which is to be found the bulk incremental ether density distribution that they produce.* The time average ether flow is zero and the time average fields are not changing, so the conditions of definition of the electrostatic field are:

$$\overline{\overline{\phi}} \neq 0 \quad , \quad \overline{\overline{\phi_a}} = \phi_d + \overline{\overline{\phi}} \quad , \quad \overline{\overline{\phi_a \mathbf{V}}} = 0$$

$$\frac{\partial \overline{\overline{\phi}}}{\partial t} = \frac{\partial \overline{\overline{\phi_a \mathbf{V}}}}{\partial t} = 0$$

All higher time partials of $\overline{\overline{\phi}}$ and $\overline{\overline{\phi_a \mathbf{V}}}$ are zero.

Conditions for Electrostatics (11.3.1)

These conditions reduce Eqs.(11.2.1) and (11.2.2) to,

$$\nabla^2 \overline{\overline{\phi}} = -\rho \quad . \quad \text{Electrostatics} \qquad (11.3.2)$$

All of Maxwell's electrostatics can be obtained by solving for the "potential" or ether density produced by various distributions of charged particles. However, since no stationary configuration of bulk ether can remain unchanged without the support of ℓ-waves,

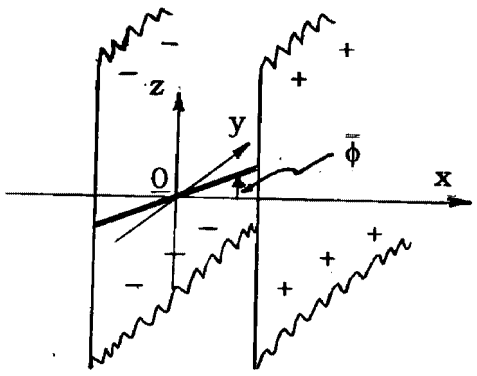

Figure 11.4.1

Incremental ether density between two charge sheets.

the equation is only a partial description of the physical field pattern. Two examples will now be given that will illustrate what is involved.

11.4 **The Parallel Plate Capacitor**: The first example is that of the field between two parallel plane sheets of charge. As seen in Figure 11.4.1, the two sheets are oppositely charged and are of an area great compared with the distance L between them. In the region where edge effects can be neglected, the density field is assumed to be constant in the directions y and z and to vary only with x. In the region between the sheets $\rho = 0$, and double integration of Eq.(11.3.2) yields,

$$\bar{\bar{\phi}} = K_\phi x + K_a \quad . \tag{11.4.1}$$

By setting $\bar{\bar{\phi}} = 0$ at x = 0, K_a is made to be zero. K_ϕ can be found from the voltage applied between the sheets, or from the charge density on the sheets. For example, if the voltage between the sheets is set at Φ hlvolts , then the potential can be written,

$$\bar{\bar{\phi}} = \frac{\Phi}{L} x \quad . \tag{11.4.2}$$

Only to this extent can Maxwell's electrostatics describe the field between the sheets.

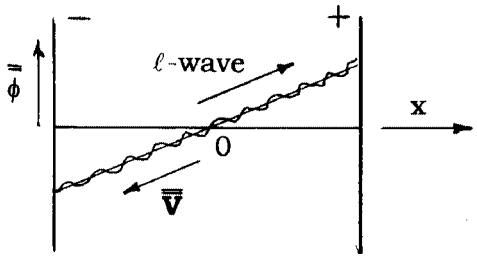

Figure 11.4.2

Ether density sustained by ℓ-wave.

What must be added is the picture of the ℓ-waves. As a trial solution, a plane velocity ℓ-wave is described by,

$$V_{\ell} = V_1 \cos\omega\left(t - \frac{x}{c_0}\right) \quad , \tag{11.4.3}$$

which moves along the x axis as illustrated in Figure 11.4.2. It is a wave of

constant amplitude V_1. The continuity Eq.(2.12.4) gives,

$$\phi_{\cdot} = \frac{\phi_d}{c_0} V_1 \cos \omega \left(t - \frac{x}{c_0} \right) \quad . \tag{11.4.4}$$

Combining Eqs.(11.4.3) and (11.4.4),

$$\overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}} = \frac{\phi_d}{2c_0} V_1^2 \quad , \tag{11.4.5}$$

and,

$$\nabla \bullet \overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}} = 0 \quad . \tag{11.4.6}$$

From Eq.(2.12.3),

$$\bar{\bar{\eta}} = \overline{\overline{\mathbf{V}_{\cdot}^2}} = \frac{V_1^2}{2} \quad , \tag{11.4.7}$$

therefore, since $\bar{\bar{\eta}}$ is constant, the $V_{\cdot}$ of Eq.(11.4.3) is a proper solution of the traveling ℓ-wave Eq.(2.12.1). The bridge equation requires that,

$$K_\phi = \frac{b\phi_d}{2c_0} V_1^2 \quad . \tag{11.4.8}$$

This a good illustration of the condition where there is no net ether flow, i.e. $\overline{\overline{\phi_a \mathbf{V}}} = 0$, but there is a time average velocity $\bar{\bar{\mathbf{V}}}$. Using Eq.(2.8.13),

$$\bar{\bar{\mathbf{V}}} = -\frac{\overline{\overline{\phi_{\cdot} \mathbf{V}_{\cdot}}}}{\phi_d} = -\mathbf{i} \frac{V_1^2}{2c_0} \quad , \tag{11.4.9}$$

which is in a direction opposite to the travel of the original ℓ-wave $V_{\cdot}$.

To get a feeling for some of the magnitudes involved, let the capacitor voltage be $\Phi = 1$ des $= 1$hlvolt $= 1062.74$ volts and the sheet spacing be 1 cm. Then, $K_\phi = 1$ and $V_1 = 6.8 \times 10^{-19}$ cm\sec. The peak of the density wave $\phi_{\cdot}$ is 2.0×10^{-8} des or, in practical units, 21 microvolts.

11.5 **The Spherical Capacitor**: The parallel plane charge sheet case is pathological, in that it cannot exist physically except to some approximation. A real case is that of two concentric spherical surface charge distributions, as shown in Figure 11.5.1. The two spherical shells are oppositely charged. Eq.(11.3.2) is solved first, by double integration, to give the ether density between the spheres as,

$$\bar{\bar{\phi}} = K_\phi \left(\frac{1}{r_0} - \frac{1}{r} \right) \quad . \tag{11.5.1}$$

Here, again, K_ϕ can be found from the charge or voltage. For the case where $L = r_2 - r_1$ and $L << r_0$,

$$\bar{\bar{\phi}} \cong \frac{\Phi}{L}(r - r_0) \quad . \tag{11.5.2}$$

Again, this is the extent to which Maxwell's equations can describe the field.

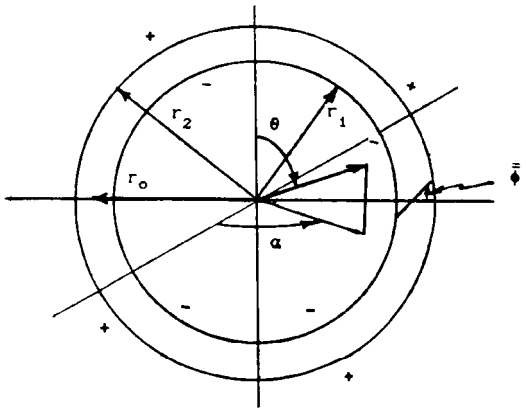

Figure 11.5.1

Two concentric spherical charge sheets.

To obtain the complete solution, the same steps are followed that were used to solve the previous problem. The ℓ-wave moves radially from one sheet to the other as depicted in Figure 11.5.1. It is a wave of decreasing amplitude as r increases (V_1 and K_ϕ are not the same constants found in the previous example). As before, V. is assumed as a trial solution, and takes the form,

$$V_\cdot = \frac{V_1}{r} C \quad , \quad C = \cos\omega\left(t - \frac{r}{c_0}\right) \quad , \tag{11.5.3}$$

which leads to,

$$\phi_\cdot = \frac{\phi_d V_1}{c_0 r}\left(C - \frac{c_0}{\omega r} S\right) \quad . \tag{11.5.4}$$

Then,

$$\overline{\overline{\phi_\cdot V_\cdot}} = \frac{\phi_d}{2c_0 r^2} V_1^2 \quad \text{and} \quad \nabla \bullet \overline{\overline{\phi_\cdot V_\cdot}} = 0 \quad . \tag{11.5.5}$$

The velocity potential is,

$$\bar{\bar{\eta}} = \overline{\overline{V_\cdot^2}} = \frac{V_1^2}{2r^2} \quad . \tag{11.5.6}$$

When $\bar{\bar{\eta}}$ is substituted into the ℓ-wave Eq.(2.12.1), it is found that,

$$\nabla^2 \bar{\bar{\eta}} - \frac{1}{\bar{\bar{\eta}}}\left(\nabla \bar{\bar{\eta}}\right)^2 = -\frac{V_1^2}{r^4} \neq 0 \quad , \tag{11.5.7}$$

so the V. of Eq.(11.5.3) is not an *exact* solution for the ℓ-wave. This

is because for geometries other than parallel plane the bulk deformation is affected by that geometry. To get an exact solution, it would be necessary to consider the geometrical conditions, with the shape factor ψ included. However, for large r_0, the RHS of Eq.(11.5.7) is very small and Eq.(11.5.3) is a good representation of the ℓ-wave field between the shells.

The bridge equation requires that,

$$K_\varphi \cong \frac{b\phi_d}{2c_0} V_1^2 \quad , \tag{11.5.8}$$

or transposed,

$$V_1^2 \cong \frac{2c_0}{b\phi_d} \frac{\Phi}{L} r_0^2 \quad . \tag{11.5.9}$$

11.6 **ℓ-Wave Frequency**: A most important aspect of the visualization of ℓ-waves in these fields is the determination of their frequencies. This question was passed over, in the discussion of the atom in Chapter 7, because it also arises here and can be resolved for both situations together. In the macroscopic electrostatic cases, the neutral atoms that make up the geometry setting conductors are taken apart, usually by removing one electron from each atom and carrying it around to where the negative charge is held. The positively charged atoms remain grouped together in their original places. The bulk $\bar{\bar{\phi}}$ field created between the two oppositely charged regions is held in place by the ℓ-waves leaving the electrons and going over to enter the positive atoms. In many ways this is essentially the same physical arrangement as that of a single hydrogen atom, with one electron separated by some distance from the proton nucleus. The problem of determining the ℓ-wave frequency arises as follows. A free electron has an outgoing ℓ-wave with frequency $\omega_e = 7.7634 \times 10^{20}$ rad/sec, and a free proton has an ingoing ℓ-wave with frequency $\omega_p = 1.4255 \times 10^{24}$ rad/sec. Since they do not match, what happens? Do they both exist in the same region, or does the combination adjust so that only one ℓ-wave goes between them? If the latter is the case, what determines the frequency ω?

This is very similar to the problem encountered in analyzing multi-layer particles in Chapter 5. There it was assumed that the combined layers gave up their individual frequencies and adopted a single $\ell-$wave. Its frequency was obtained from Eq.(5.5.7) which can be

expressed as,

$$\omega = \frac{2\pi}{h} E_0 \quad . \qquad (11.6.1)$$

Eq.(5.5.7) was derived from the basic analysis of the electron which is a solution of the ℓ-wave equation involving the universal constants at the foundation of the ether theory. For this reason, it is not surprising that Eq.(11.6.1) also applied to the layerons, since they too are similar solutions. However, there is no guarantee that exactly the same equation also applies to all other combinations of particles.

At the present time, the paucity of ℓ-wave measuring techniques and experiments leaves no alternative but to assume that Eq.(11.6.1) is a completely general form that applies to the total energy acting in any configuration that is "self contained", where the latter expression is not defined further. Until many basic ℓ-wave experiments to verify this have been devised, it must remain an assumption to be questioned. More will be said about this in connection with the gravitic field of a large mass to be examined in Chapter 14.

11.7 **The Role of Statistics in Macroscopic Physics**: It is not obvious that a firm line can be drawn between *microscopic* and *macroscopic* physics. The general idea underlying the *microscopic* regime is that it deals with the internal structure of the particles, where the charge distortion distribution must be considered in detail. Situations in which charge is in the form of whole charged particles are regarded as *macroscopic*. Atoms, and even nuclei with more than one nucleon are macroscopic systems. *All ensembles of whole particles that do not act together as a single entity are macroscopic.* On the other hand, although the datum is the underpinning of the particles, it is essentially a macroscopic entity.

The basic division is related to whether or not statistical analysis can apply. *Only macroscopic systems can be analyzed statistically*. Microscopic objects are purely deterministic and must be treated as such. Modern physicists believe that quantum statistics are the basic nature of the world. They try to develop statistical theories of particle structure, and even seek the elusive "graviton" in a statistically based gravitic field theory. These attempts are doomed to failure, because particle structure is a microscopic problem, and even the gravitic field of a single large object is the product of one gigantic composite particle. Nevertheless, almost all of statistical quantum mechanics, and a good share of macroscopic deterministic mechanics, is formalized in terms of ensemble statistics, so a few words should be said here in that regard.

There are three types of statistics that are found regularly in physics, Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac.

Conventionally, M-B statistics are considered "classical", whereas B-E and F-D statistics are described as "quantum mechanical", the essential difference being ascribed to the *indistinguishability* of particles in the latter cases. As pointed out by Boyer,[1] indistinguishability of particles is not the point, since to overcome Gibb's paradox in the physics of gasses, the "classical" derivations must include the fact that gas molecules are indistinguishable.[2] It is also generally assumed that the choice of B-E or F-D statistics is determined by the symmetry or anti-symmetry of the wave functions of the ensemble particles. The meaning of this assumption from a deterministic viewpoint requires further probing in the future. Following the suggestion of earlier chapters, the designations "classical" and "quantum mechanical" are abandoned here. *The following rules of thumb are applicable in general.*[3]

In any ensemble where the particles of the ensemble are actually distinguishable, and no restriction is placed on how many particles are in any single-particle state, Maxwell-Boltzmann statistics apply.

In any ensemble where the particles are indistinguishable, and no restriction is placed on how many particles are in any single-particle state, Bose-Einstein statistics apply.

In any ensemble where the particles are indistinguishable, and some further restriction is placed on the particles, a *different* form of statistics applies (i.e. neither M-B nor B-E).

In particular, in any ensemble where the particles are indistinguishable, and only one particle per single-particle state is allowed, Fermi-Dirac statistics apply. This last case occurs wherever the single-solution rule applies to each single-particle state (see Secs. 5.25, 6.6, 7.3 and 7.12), such as electrons in atoms, or nucleon pairs in nuclei.

It was stated above that symmetry or anti-symmetry of the particle's wave function decides whether it is a boson or a fermion. In fact, only in the case of photons has a boson particle type been checked directly; and only in ensembles of electrons, neutrons, protons, and muons, have fermion particle types been checked directly.[4]

Statistics is not divided into such categories as "classical" and "modern". The statistics of ensembles is the straightforward application of very old probability techniques to individual problems, where the basic input conditions are considered properly.

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1. See Section 4,4.
2. F.Reif, Fundamentals of Statistical and Thermal Physics, p. 243, McGraw-Hill Book Co. N.Y. (1995).
3. Op.cit. p 332.
4. D.H.Perkins, Introduction to High Energy Physics, 1st Ed. p 14, Addison-Wesley Publ. Co., Reading, Mass. (1972).

To sumarize: there are three main types of statistics commonly found in particle ensembles, Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac. Conventional presentations imply that something mysteriously related to "quantum mechanics", symmetry/asymmetry and indistinguishability separate "classical" M-B from B-E and F-D. These statistical forms are just three of many possible, based on standard ensemble probability theory, where successively stronger restrictions are placed on the ensemble characteristics.

Before leaving this topic, one well publicized example of the capability of statistics to solve ensemble problems should be mentioned. Notice that the electron's magnetic moment, given by Eq.(3.8.24), is the *intrinsic* or actual flow value for each unon, and is not exactly equal to the measured value listed in Table 5.4.1. The latter is slightly greater *because of the many ways the interaction used in the measurement of* μ_s *can take place*,[1] since the measurement must be made on an ensemble rather than on a single unon. The calculation of this small difference to incredible accuracy is one of the triumphs of quantum electrodynamics,[2] which uses Maxwell's equations combined with Dirac's equation to evaluate ensemble experiments.[3]

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1. R.P.Feynman, Quantum Electrodynamics, p. 130, W.A.Benjamin Inc., N.Y. (1962).
2. ....., QED, Princeton U. Press, Princeton N. J. (1985).
3. R.B.Leighton, Principles of Modern Physics, p. 669 ff, McGraw-Hill Book Co., N.Y. (1939).

CHAPTER 12

MECHANICS

12.1 **Introduction**: Mechanics is customarily defined as the science of the motion of bodies and the causes of change in that motion. In the early days of mechanics, only two influences or forces were known to act upon neutral objects, contact force and gravitation. Now, it is understood that even contact forces are electromagnetic field interactions brought about by close proximity of the constituent charges of the interacting bodies. Although, in the past, the subject of mechanics has had a limited purview, here it will be expanded to include all particle/particle and particle/field *interactions*.

A brief excursion is helpful at this point. It concerns the way in which experiments are performed. First, it will be assumed, unless otherwise explicitly stated, that the ether observer, i.e. one whose inertial system is fixed relative to the ether, is the one who conducts the experiments. The discussion in Section 8.11 gives the reason for this. An experiment can usually be envisioned as performed in a limited region enclosed in an imaginary box. Further, the observer/experimenter has his rods and clocks set up to establish certain starting and end points and times. By having them outside the *region of interaction*, i.e. in the datum ether ϕ_d, he can be sure that, in any practical sense, they will be uninfluenced by the experiment, i.e. by what's inside the box. The moving bodies inside the box are started at known points and at known times. Later they arrive at other known points. Prediction of the arrival times and places based on the initial conditions is the task of mechanics.

The ether observer outside the imaginary box has the same role as any absolute observer, and he examines the complex flows and motions inside the box, including that part of the flow that represents the particles or neutral bodies moving in the fields inside the box. At the same time, the interaction of a particle with the ether flow at each position inside the box is often best established by first knowing what would be seen by a second "ether observer" who, as in Appendix E, uses the moving "fluid" coordinate system. Based on this, the resulting motion is then re-described as seen by the original or absolute ether observer outside the box. As in Appendix E, the transformation from one ether observer $\bar{S}$ to the other absolute ether observer S is written using the ordinary or Galilean transformation.

Neither the Law of Relativity, nor the Lorentz transformation are used in the formulation of mechanics presented here, except in establishing the validity of the *absolute* inertial observer's measurements (see Section 8.11) and using Lorentz covariance to verify the correctness of some derived equations. None of the effects

of variable energy or mass or changing shape, etc., are "relativistic", but merely the result of the basic particle structure and the laws of ether motion as set out in Chapters 2, 3, 5, and 8, and in Appendices A through E.

In the following, the concepts of "force" and "work" are described as fictitious aids to problem solving in situations where the actual physics just involves motion of ether distortion from one configuration to another. Newton's laws are developed, and the Kirkwood and Lorentz forces are derived. Finally particle/particle and particle /field interactions are discussed.

12.2 "**Force" and Work**: Up to this point certain quantities, such as position, velocity, and density, have been defined and observed by an outside, absolute observer as they take on values governed by a set of relatively simple relationships. The entities constructed have been allowed to interact, *but the whole picture is represented by a single continuous solution of the field equations*. Now, in an almost inexcusable way, the picture will be made more subjective. Work somewhat, and force completely, are concepts that arise from the ability of a collection of particles to be aware of its existence and to recognize its own boundaries. When a human "pushes" a stalled car, the contact of the two is conceived by the human to be the means by which a "force" is exerted on the other object; and, if the object moves, "work" is done on it. Yet, there is clearly no difference, other than complexity, between that interaction and the one described in Figure 3.10.3 between two electrons. Force and work are truly superfluous in the electron example, since the solution of the field equations for two dips of ether, kept from filling in by two sets of sustaining waves, is just as proper as for one electron. They will move apart, and their motion is controlled in time by characteristics similar to those of a single particle in motion. Since "work" and "force" are arbitrarily imposed in this case, it is clear that they are arbitrarily imposed in all cases. *There is no real physical motivation for either work or force*.

The fact that it is often easier, in a complex situation, to separate parts of a problem, or to neglect small effects relative to large effects, allows retention of the concepts of force and work in the following. As long as it is clearly kept in mind that their use is an artificial aid in problem solving, no great harm can come to the development of the picture of the world's operation. Nevertheless, every opportunity will be taken to keep the objective picture uppermost. Of primary importance is the realization *that all <u>available</u> equations of motion involving force are essentially approximations*. For example, to correctly derive the force on a single free electron in an electrostatic field requires solving the bulk and ℓ-wave equations, of the electron and the capacitor of Section 11.4 combined, as the electron is

accelerating. To see how difficult this could be, consult the simpler, intractable accelerating *point* charge problem.1 In spite of these difficulties, a good approximation can be found for any case.

12.3 **Newton's Laws**: The picture of an electron's inertia and momentum developed in Sections 3.10, and 3.11 form the basis for Newton's laws. From the ether viewpoint, they are formulated as seen by an absolute ether observer, in a region of constant ether density ϕ_d tracking a body that does not sensibly perturb the ether outside the region occupied by the body. Any body composed of layers (proton, neutron, pion, etc.) or layered particles (atoms, planets, suns, etc.) exhibits inertia and momentum properties similar to the electron.

The first law states:

> Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it.

Each layer or particle that make up the body will continue to move at constant velocity in a straight line unless the boundary conditions are changed by a contact force, e.g. another neutral body approaching close enough to cause the extended particle fields of the two bodies to interact. Thus, Newton's first law, including the special case of a body at rest, is just the extension of the simple ideas of inertia and momentum described in Chapter 3.

The second law states:

> The change of momentum is proportional to the motive force impressed; and is made in the direction of the straight line on which that force is impressed.

Expressed in vector notation, this is equivalent to,

$$\frac{d\mathbf{p}}{dt_.} = \mathbf{F} \quad , \tag{12.3.1}$$

where the momentum is given by,

$$\mathbf{p} = m\mathbf{u} = \frac{E}{c_0^2}\mathbf{u} \quad . \tag{12.3.2}$$

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1. L.Page and N.I.Adams,Jr. Electrodynamics, p. 163, Dover Publications, Inc., New York (1965).

Eq.(12.3.1) is derived, for the electron, in Section 12.5; and extended to other bodies as discussed above. It is the result of applying the relationship between the time rate of change of momentum and the external energy absorbed by each of the body's constituent layers or particles. Questions about the passage of the energy through the body from the point of contact force application, although important and obviously not yet answered, are considered outside the main line of thought here. The "force" in Eq.(12.3.1) is visualized as a contact, through which energy flows from the impressing body structure into the accelerated body, or as a field acting on the body in some way.

The third law states:

> To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

In fact, *this is essentially a statement that there is no such thing as force*, since all "forces" are balanced. The mutual flow pattern of the two objects interacting is all that actually exists. Force is a convenient fiction that allows a mechanics problem to be solved one part at a time.

The application of these three laws to typical mechanics problems is too well known to require any further discussion. The next sections will try to increase the intuitive understanding of these motion processes in terms of the ether.

12.4 **The Kirkwood Force**: To the absolute ether observer, the situations described by Newton's laws are essentially the same as those described conventionally by *inertial system* observers. However, there are other situations of interest where a *neutral* body can be moving through a field of ether velocity that is *neither uniform nor irrotational.* The proper way to deal with this type of problem from the ether viewpoint will now be developed. To get some idea of what is involved here, consider the region of ether flow inside a solenoid (see Section 9.3), as depicted in Figure 12.4.1. A *neutral* body made up of, say, 10^6 atoms would be about 0.1 micron across; and would have negligible effect on the circulating ether inside the solenoid. With no

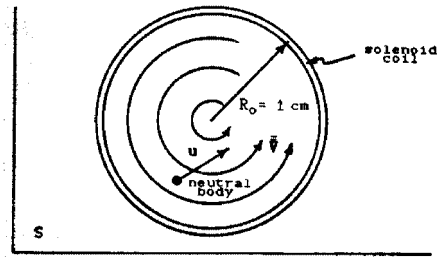


Figure 12.4.1
Neutral body moving inside a solenoid.

external force acting on the body, i.e. no string or miniature "jet engine", it is desired to find the motion of the body, which is initially moving at the velocity $\mathbf{u}$. However, the only knowledge of the body's motion at this stage is that given by Newton's laws for motion in an inertial system. The interior of the solenoid is definitely <u>not</u> an inertial system. Nevertheless, there is a differentially small inertial system $\bar{S}$, fixed to the fluid, i.e. moving with it, with its origin $\bar{0}$ just passing the point where the body is at any given instant. In that differential inertial system, for a differential time, Newton's laws apply (see Appendix E, Figure E.4.1).

With this knowledge, the general motion of the body can be found using Eqs.(E.4.11) and (E.4.17),

$$\frac{d'\bar{\mathbf{u}}}{dt_{\cdot}} = \frac{d'^2\bar{\mathbf{r}}}{dt_{\cdot}} = \frac{d\mathbf{u}}{dt_{\cdot}} - \frac{d\mathbf{V}}{dt} - 2\mathbf{w}\times(\mathbf{u}-\mathbf{V}) \quad , \quad \bar{\mathbf{u}} = \frac{d'\bar{\mathbf{r}}}{dt} = \mathbf{u}-\mathbf{V} \quad . \qquad (12.4.1)$$

Rewriting the acceleration equation as,

$$\frac{d\mathbf{u}}{dt_{\cdot}} = \frac{d\mathbf{V}}{dt} + 2\mathbf{w}\times(\mathbf{u}-\mathbf{V}) + \frac{d'\bar{\mathbf{u}}}{dt_{\cdot}} \quad , \qquad (12.4.2)$$

the acceleration of any moving point as seen by S is composed of the three RHS components; the acceleration of the *origin* of $\bar{S}$, the acceleration resulting from the *rotation* of $\bar{S}$ and the acceleration of the m.p. as seen by $\bar{S}$. *This is a strictly kinematic relationship*. To permit the use of the dynamic law represented by Eq.(12.3.1), Eq.(12.4.2) must be augmented using Eqs.(E.3.18) and (12.4.1), which yield,

$$(\mathbf{u}-\mathbf{V})\frac{dm}{dt_{\cdot}} = \bar{\mathbf{u}}\frac{d'm}{dt_{\cdot}} \qquad \text{where} \qquad m = \gamma m_0 = \frac{m_0}{\sqrt{1-\dfrac{(\mathbf{u}-\mathbf{V})^2}{c_0^2}}} \quad . \qquad (12.4.3)$$

Adding the identity Eq.(12.4.3) to Eq.(12.4.2) multiplied through by m results in,

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = m\frac{d\mathbf{V}}{dt} + 2m\mathbf{w}\times(\mathbf{u}-\mathbf{V}) + \mathbf{V}\frac{dm}{dt_{\cdot}} + \frac{d'(m\bar{\mathbf{u}})}{dt_{\cdot}} \quad . \qquad (12.4.4)$$

which is still a *kinematic* transformation of the quantity $m\bar{\mathbf{u}}$ in $\bar{S}$ into the quantity of the *same kind* $m\mathbf{u}$ in S. To convert to a *dynamical* relationship, Newton's second law in the form of Eq.(12.3.1) is applied to $\bar{S}$ at the origin where the body is just passing through. The result is the general equation for the time rate of change of the body's momentum as seen by S, which has the form,

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = m\frac{d\mathbf{V}}{dt} + 2m\mathbf{w}\times(\mathbf{u}-\mathbf{V}) + \mathbf{V}\frac{dm}{dt_{\cdot}} + \mathbf{F} \quad . \qquad (12.4.5)$$

Here **F** represents any *external* force, such as a string or "jet engine" that acts on the body to change its momentum as seen by $\bar{S}$. Following the steps leading up to Eq.(E.4.16), Eq.(12.4.5) can be shown equivalent to,

$$\mathbf{F} = \frac{d}{dt_{\cdot}}\big(m(\mathbf{u} - \mathbf{V})\big) + m(\nabla\mathbf{V})\bullet(\mathbf{u} - \mathbf{V}) \quad , \qquad (12.4.6)$$

one form of the Kirkwood force equation that appeared in one of the most important papers written in the twentieth century.1 Many of the subsequent derivations to be presented owe a debt to the light shed by that dissertation.

Because of the manner in which they were constructed, both Eqs.(12.4.5) and (12.4.6) apply to fields where $\mathbf{V}_{\cdot} << \overline{\overline{\mathbf{V}}}$; but for cases where $\mathbf{V}_{\cdot} >> \overline{\overline{\mathbf{V}}}$ or where a body has no rest mass, the derivation must be modified slightly. Examples of these variations will appear later on. To enable certain simplifications to be made in subsequent examples, the artificial concept of 'force" will be extended to describe the effect of the ether on the body. In agreement with the Third Law, that force would be,

$$\mathbf{F}_e = -\mathbf{F} = -\frac{d}{dt_{\cdot}}\Big(m(\mathbf{u} - \overline{\overline{\mathbf{V}}}\Big) - m(\nabla\overline{\overline{\mathbf{V}}})\bullet(\mathbf{u} - \overline{\overline{\mathbf{V}}}) \quad , \quad (\mathbf{V}_{\cdot} = 0) \quad , \quad (12.4.7)$$

which says that if external force **F** is applied to a neutral body, the ether opposes that force with a counter force $\mathbf{F}_e$. As will now be seen, that is the nature of centrifugal force, for example. Both Eqs.(12.4.6) and (12.4.7) apply in any case where the zero time average component of the external **V** field is zero.

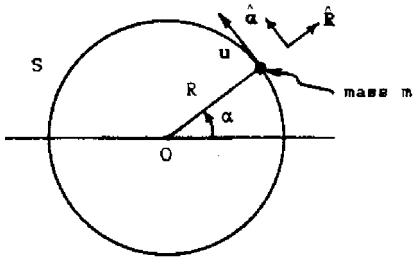


Figure 12.4.2
Circling mass on a string.

Example 1

Ether at rest everywhere. Small mass on a string executing steady circular motion:

This is a common experience, as described in Figure 12.4.2. The ether velocity $\mathbf{V} = 0$ everywhere and the ether density is ϕ_d everywhere. It can be seen from Eq.(12.4.3), that because **V** is zero and,

$$\mathbf{u} = u\hat{\alpha} = w_0 R\hat{\alpha} \quad , \qquad (12.4.8)$$

where w_0 is the constant angular velocity of the body about the point 0, the mass m is constant and equal to,

$$m = m_0 / \sqrt{1 - u^2 / c_0^2} \quad . \qquad (12.4.9)$$

1. R.L.Kirkwood, Phys.Rev., **92,** 1557 (1953).

Since $\nabla\mathbf{V}$ is also zero, Eq.(12.4.7) gives,

$$\mathbf{F}_e = -\,m\frac{d\mathbf{u}}{dt}\quad , \qquad (12.4.10)$$

for the force *exerted by the ether on the body*. Now, using Eq.(12.4.8),

$$\mathbf{F}_e = -\,mw_0R\frac{d\hat{\alpha}}{dt}\quad , \qquad (12.4.11)$$

but,

$$\frac{d\hat{\alpha}}{dt} = -\,w_0\hat{\mathbf{R}}\quad ; \qquad (12.4.12)$$

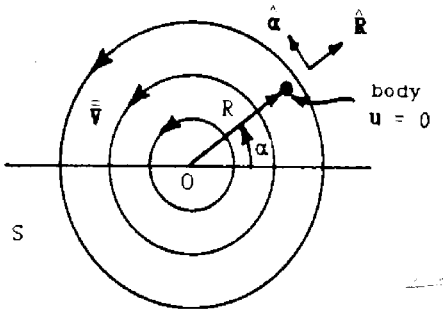


Figure 12.4.3
Body at rest in a vortex flow field.

where w_0 is the angular velocity of the ether. Moreover, Eq.(12.4.7) reduces to,

$$\mathbf{F}_e = +m(\nabla\bar{\bar{\mathbf{V}}})\cdot\bar{\bar{\mathbf{V}}}\quad . \qquad (12.4.15)$$

Because,

$$\frac{\partial\hat{\alpha}}{\partial\alpha} = -\,\hat{\mathbf{R}}\quad , \quad \frac{\partial\bar{\bar{\mathbf{V}}}}{\partial R} = w_0\hat{\alpha}\quad ,$$

so,

$$\mathbf{F}_e = +mw_0^2R\hat{\mathbf{R}}\quad . \qquad (12.4.13)$$

This shows that the ether "pushes" the body in a radial direction with a centrifugal force given by Eq.(12.4.13). To maintain the circular motion of the body, an opposing, external centripetal force must be applied to the body by the string.

Example 2

Ether circulating about 0. Particle and 0 at rest in S:

This is the solenoid flow field of Figure 12.4.1, with the *neutral* body at rest in S as described in Figure 12.4.3. Since neither $\mathbf{V}$ nor $\mathbf{u}$ is changing where the body is located, m is a constant,

$$m = \frac{m_0}{\sqrt{1-\frac{w_0^2R_0^2}{c_0^2}}}\quad , \qquad (12.4.14)$$

then, from Table D.5.1,

$$\nabla\bar{\bar{\mathbf{V}}} = w_0(\hat{\mathbf{R}}\hat{\alpha} - \hat{\alpha}\hat{\mathbf{R}})\quad ; \qquad (12.4.16)$$

so combining Eqs.(12.4.15) and (12.4.16),

$$\mathbf{F}_e = +mw_0^2R\hat{\mathbf{R}}\quad . \qquad (12.4.17)$$

The two situations in Examples 1 and 2 are *identical* as far as the ether force that appears on the moving body. From symmetry, this is not too surprising, but it emphasizes how *care must be applied in thinking of the concept of force caused by ether acceleration*. In case 2, the ether is accelerating radially *inward* at any point with acceleration $\mathbf{a_e} = -\,w_0^2R\hat{\mathbf{R}}$. However, *this is not the only effect that determines the*

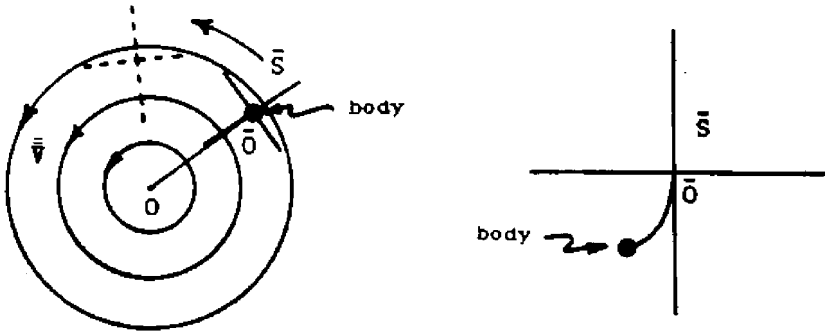
force on the body, even though m is constant. The ether force is established by the acceleration of the ether relative to the body as seen by $\bar{S}$, a frame moving with the ether at the point $\bar{0}$ where the body is instantaneously located. It should be pointed out that Eq.(12.4.7) is a compact form of,

$$\mathbf{F}_e = -\frac{d}{dt_.}\left(m(\mathbf{u}-\bar{\bar{\mathbf{V}}})\right) - m(\mathbf{u}-\bar{\bar{\mathbf{V}}})\cdot\nabla\bar{\bar{\mathbf{V}}} + 2m\mathbf{w}\times(\mathbf{u}-\bar{\bar{\mathbf{V}}}) \ , \ (\mathbf{V}_. = 0) \quad (12.4.18)$$

where the last two terms on the RHS correspond to acceleration components resulting from the motion of the particle to regions of different external field ether velocities. Substituting the values from Example 2 into Eq.(12.4.18),

$$\mathbf{F}_e = -\, mw_0^2 R\hat{\mathbf{R}} + 2mw_0^2 R\hat{\mathbf{R}} \quad . \qquad (12.4.19)$$

The first term on the RHS represents the effect of the $\mathbf{V}$ field's acceleration *inwards* as it rotates, as seen by the general observer S. The second term is an acceleration of the ether past the body, as seen in the frame $\bar{S}$, due to that frame's *rotation* as it moves with the ether. In Figure 12.4.4, the ether frame $\bar{S}$ is shown in two successive positions. First, with the body at $\bar{0}$, and second with $\bar{S}$ further on, so that the body now appears in the third quadrant. Thus, $\bar{S}$ sees the body moving *inward*, towards 0, relative to the ether, or an *outwardly* accelerating ether force, as given by Eqs.(12.4.19) and (12.4.17). Though it is often difficult to get the correct picture from the viewpoint of $\bar{S}$, the careful use of Eqs.(12.4.5) and (12.4.6) will always give the correct result for fields where $\mathbf{V}_.$ is zero, and will help to form the proper visualization.



a. b.

Figure 12.4.4

Motion of the ether relative to the fixed body as seen by $\bar{S}$.

12.5 **The Lorentz Force - Electric**: The Kirkwood force, derived in the preceding section, *applies to all neutral and charged bodies under all conditions*, but it is a very weak force. In the *presence* of an external electromagnetic field, the Kirkwood force still applies to all bodies, charged and uncharged; but bodies with a net electric charge experience an additional "Lorentz" force. This Lorentz force is so much greater than the Kirkwood force that the latter is always neglected when an external electromagnetic field is present and acting on a *charged* body.

The Lorentz force on an electron/positron will be derived now, but a "full field solution" is not available at this time. Some shortcuts are necessary, but the derivation presented here reveals all of the essential fundamentals involved. It is carried out in two parts, one showing the effect of an *external electric field*, and the other the effect of a magnetic field. The electric case will come first.

Take the simple case of a positron momentarily at rest in an electrostatic field between the plates of a parallel plane capacitor (see Section 11.4). Spacing between the plates, presumed here to be in an evacuated tube, might be as little as a few microns; yet, since 98 percent of all the energy in the positron lies within a sphere of radius 200 r_e, or about 10^{-11} cm, the positron occupies a negligible part of the volume between the plates. If a considerable charge is displaced from one plate to another, the positron also has little effect on the bulk ether density gradient between them.

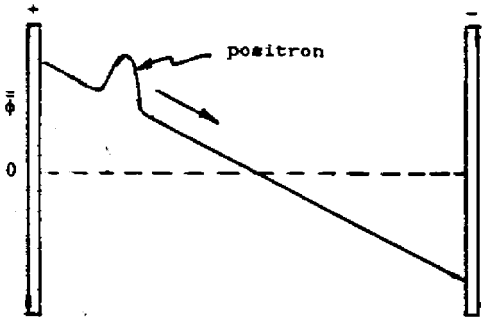


Figure 12.5.1
Positron between charged parallel plates.

Figure 12.5.1 displays an exaggerated picture of the density. Although the sustaining waves are omitted from the figure, they play an important role. First, a large plane sustaining wave moves from the - to the + plate to maintain the constant density gradient. Second, the positron sustaining wave pattern moves from the outer regions towards the positron's center and maintains its identity. The two waves have no effect on each other. It can be shown, using the equations of Section 11.2, that any dense region of ether not supported by ℓ-waves will flow out to the lower density volumes until it equalizes with them. This can be viewed as a kind of diffusion process. Now, the positron's sustaining wave prevents this *within* the positron and the "external" field's sustaining wave prevents the + end of the field from flowing into

the – end, but nothing holds the bump representing the positron from going down hill, i.e. from filling in the lower density region near the – plate. The positron, therefore, *flows* towards that plate, while each of the sustained fields maintains its integrity. However, because the positron can only move by increasing its velocity, *the motion cannot be instantaneous* but is controlled by how fast energy of *interaction* stored at the start can be imparted to the moving positron. Before making this quantitative, the case of an electron will be illuminating.

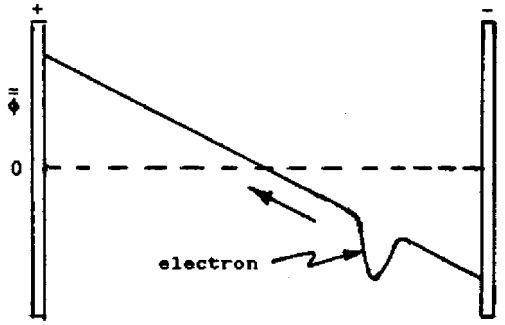


Figure 12.5.2
Electron between charged parallel plates.

Figure 12.5.2 shows the identical capacitor with an electron replacing the positron. Again, the external field's sustaining wave and the electron's sustaining wave (outward in this case) keep those identities intact; but there is nothing to keep the ether just above the lip of the electron from flowing into its volume, to fill it up on the low side. This leaves less ether in the higher region, and in effect the electron has moved closer to the + plate. In both cases, the configurations resulted in ether *flow* not "force". The rate at which this flow and energy exchange takes place will now be found. To keep the signs simple, the positron case will be used in the derivation. The electron case follows directly.

The development is in two parts; first, relating the stored interaction energy to the positron position, and second, showing how fast the positron can take on that energy as it speeds up. The *interaction* energy is approached by a method nicely written up by Armstrong, etc.1 First, the energy of either the external field or the positron taken separately, can be written with the help of Eq.(2.19.1) as,

$$E = \int_{space} \varepsilon \, dvol = \tfrac{1}{2} \int_{space} (\nabla \bar{\bar{\phi}})^2 \, dvol \quad . \qquad (12.5.1)$$

Applying Gauss' theorem (see Table D.6.2) to an identity from the same table, $\nabla\bullet(\psi\nabla\phi) = \nabla\psi\bullet\nabla\phi + \psi\nabla^2\phi$, another form of Green's theorem

1. H.L.Armstrong, Amer.J.Pbys., **23**, 582 (1955). N.S.Japolsky, Phil.Mag., **20**, 417 and 641 (1935), **22**, 537 (1936). B.Podolsky, Phil.Mag., **22**, 998 (1936).

is expressed as,

$$\int_v \nabla\psi\cdot\nabla\phi \; dv = \int_S \psi \mathbf{n}\cdot\nabla\phi \; dS - \int_v \psi\nabla^2\phi \; dv \quad . \qquad (12.5.2)$$

Eq.(12.5.2) can be used to rewrite Eq.(12.5.1) in the form,

$$E = \tfrac{1}{2} \int_{space} (\nabla\bar{\bar{\phi}})^2 \; dvol = -\tfrac{1}{2} \int_{space} \bar{\bar{\phi}}\nabla^2\bar{\bar{\phi}} \; dv + \tfrac{1}{2} \int_{surface} \bar{\bar{\phi}}\mathbf{n}\cdot\nabla\bar{\bar{\phi}} \; dS \quad . \qquad (!2.5.3)$$

Since the integrals are taken over all space, the surface is so far out from the sources of the field that the surface integral is zero. This, coupled with the charge density of Eq.(2.20.1), results in,

$$E = \tfrac{1}{2} \int_{space} \rho\bar{\bar{\phi}} \, dv \quad , \qquad (12.5.4)$$

which is an alternative form of integral that relates the total stored energy to the deformation ρ. When applied to the positron *alone*, substitution of ρ and $\bar{\bar{\phi}}$ from Eqs.(3.7.2) and (3.5.2) results in the positron's rest energy of Eq.(3.6.3). When applied to the parallel plane capacitor *alone*, the charge densities concentrated near the two plates make the major contribution to the energy stored in the parallel plane field.

Similarly, for the positron and external field *superimposed*, by identifying the external and positron densities as $\bar{\bar{\phi}}_1$ and $\bar{\bar{\phi}}_2$ respectively, the field energy is,

$$E = \frac{1}{2} \int_{space} (\nabla\bar{\bar{\phi}}_1 + \nabla\bar{\bar{\phi}}_2)^2 \, dv = \frac{1}{2} \int_{space} (\nabla\bar{\bar{\phi}}_1)^2 \, dv + \frac{1}{2} \int_{space} (\nabla\bar{\bar{\phi}}_2)^2 \, dv + \int_{space} \nabla\bar{\bar{\phi}}_1\cdot\nabla\bar{\bar{\phi}}_2 \, dv \qquad (12.5.5)$$

The first two integrals on the RHS of Eq.(12.5.5) are of passing interest only, since they represent the energies of the two fields when each is *alone*. The third integral clearly represents the added interaction distortion that enters when the two fields are brought together. Here again, Green's theorem can be used to convert the interaction energy into,

$$E_i = \int_{space} \rho_2 \bar{\bar{\phi}}_1 dv \quad . \qquad (12.5.6)$$

In the present case, the positron or *test* charge is assumed to be a very small volume entity, i.e. its main distortion region occupies a negligible volume of the external field. This permits any change in $\bar{\bar{\phi}}_1$ over that region to be neglected, so that,

$$E_i = \bar{\bar{\phi}}_1 \int_{space} \rho_2 dv \quad . \qquad (12.5.7)$$

Dropping the subscripts as unnecessary,

$$E_i = q\bar{\bar{\phi}} \quad , \tag{12.5.8}$$

which states that the presence of a test charge q in an external field of incremental ether density results in a stored energy between them, in excess of their own separate energies, equal to the product in Eq.(12.5.8).

To complete the first half of the present development, it will be assumed that the positron flows along the path of steepest descent, i.e. parallel to the gradient of $\bar{\bar{\phi}}$. The unit vector $\hat{\mathbf{s}}$ is used to denote the direction of motion; and, from Eq.(12.5.8),

$$\hat{\mathbf{s}} \frac{dE_i}{ds} = q\nabla\bar{\bar{\phi}} \quad . \tag{12.5.9}$$

Referring to Figure 12.5.1, Eq.(12.5.9) indicates that the positron moving to the right a distance ds, because $\nabla\bar{\bar{\phi}}$ is negative and q is positive, reduces the stored interaction energy by the amount $q\, d\bar{\bar{\phi}}/ds$. To do this, it must take on the same amount of kinetic energy (see Chapter 15).

The second part of this development determines how *fast* a positron can take on the kinetic energy. This is found from Eqs.(3.10.23) and (3.10.24) by taking the change in the positron's energy as it moves a distance ds,

$$\frac{dE}{ds} = \gamma^3 \frac{u}{c_0^2} E_0 \frac{du}{ds} = \gamma^3 \frac{E_0}{c_0^2} \frac{du}{dt_.} \quad . \tag{12.5.10}$$

This can be put in a more recognizable form by using,

$$\frac{dEu}{dt_.} = E_0 \frac{d\gamma u}{dt_.} = \gamma^3 E_0 \frac{du}{dt_.} \quad ; \tag{12.5.11}$$

which, substituted into Eq.(12.5.10) yields,

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2} u\right) = \frac{dE}{ds} \quad . \tag{12.5.12}$$

Finally, *recognizing that the decrease in stored interaction energy must be equal and opposite to the increase in kinetic energy* (see Chapter 15), Eqs. (12.5.12) and (12.5.9) can be combined so that,

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2} \mathbf{u}\right) = -\, q\nabla\bar{\bar{\phi}} \quad . \tag{12.5.13}$$

Since the quantity in the bracket on the LHS is the positron's momentum, Eq.(12.5.13) declares that, in an external electrostatic field, a small test charge gains momentum at a rate proportional to both the charge and the external field's gradient. Brief speculation will convince the reader that the electron example of Figure 12.5.2 obeys the same relationship. Furthermore, the restriction of a

constant gradient field can be removed without changing the derivation in any way. Thus, Eq.(12.5.13) is quite general.

12.6 **The Lorentz Force - Magnetic**: An important point in the preceding development *is the change in direction of the particle's motion with the sign of the charge*. Referring to Figures 12.5.1 and 12.5.2, the positron moved downhill because it was an excess of ether flowing into a depleted region; whereas, the electron moved uphill because it was a depleted region into which the higher density ether flowed, in effect moving the depleted region to the higher density portion of the external field. *A similar directional effect results from the motion of electrons and positrons in regions where no external ether density gradient exists*. Figure 12.6.1 shows this effect for a very much oversimplified pair. Each particle is represented in the figure as a constant density, spherical distortion, in which the total ether flow due to the particle's motion is given by,

$$\bar{\bar{\phi}}_a \bar{\bar{\mathbf{V}}}_t = \phi_0' \mathbf{u} \quad , \tag{12.6.1}$$

inside the particle and zero outside. It is clear from the figure that, in the electron, the total ether velocity vector $\bar{\bar{\mathbf{V}}}_t$ is moving in a direction opposite to the electron's velocity $\mathbf{u}$. This is the one phenomenon that accounts for the (until now) mysterious motion caused by a magnetostatic field. All other aspects of the physical mechanism are intuitively understandable in terms of simple fluid motion.

The derivation proceeds very much like that for the electrostatic field, since again there is no rigorous solution of the field equations available for the total combination of an "external" magnetic field and a moving positron. A significant difference is the fact that an energy tracking procedure is doomed because, neglecting radiation effects, the magnetic field is known not to change the energy of a charged particle. Therefore, it is necessary to try to understand the fluid motion pattern in more detail and to find some pseudo criterion of a "separate" charged particle that describes what its flow does relative to the total solution.

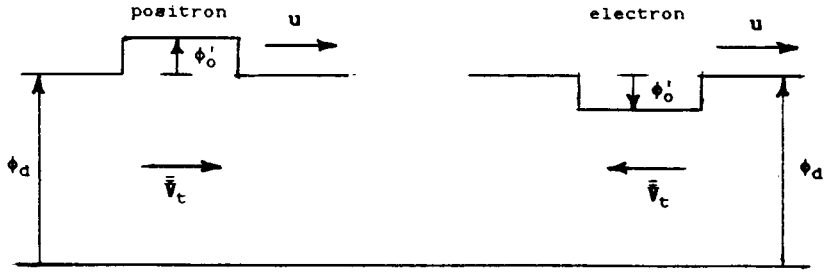


Figure 12.6.1 Flow reversal with charge reversal.

When the real positron and an external magnetostatic field are combined, since the major part of the positron's distortion energy lies inside a sphere about 10^{-11} cm in radius, the external field is essentially unchanged. To help visualize the flow, the inside of a solenoid (see Section 9.3 and figure 12.4.1) will be used as an example. There, the ether velocity $\overline{\overline{\mathbf{V}}}$ is circulating in vortex motion, with the fluid turning as a rigid body. Consider the motion of positrons in a plane perpendicular to the solenoid axis. Figure 12.6.2 depicts the motion of several positrons and one electron. Neglecting radiation effects, the steady state motion of each is observed to be in a circular path of radius proportional to its speed u; with electrons circulating in the same mode as the vorticity $\overline{\overline{\mathbf{w}}}$ of the external field $\overline{\overline{\mathbf{V}}}$ and positrons in the opposite mode. It is this motion that requires explanation.

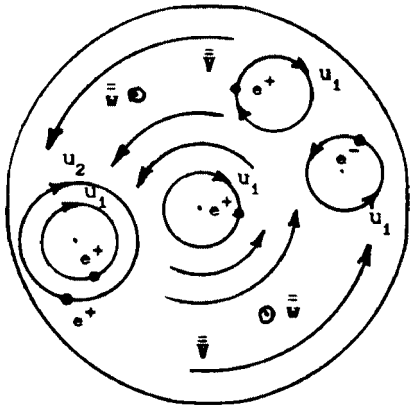


Figure 12.6.2
Charges moving in a magnetic field.

First, it is useful to consider the total flow at the very center of the constant density spherical *model* positron, as expressed by,

$$\bar{\phi}_a \overline{\overline{\mathbf{V}}}_t = \phi_0' \mathbf{u} + \phi_d \overline{\overline{\mathbf{V}}} \quad . \tag{12.6.2}$$

This can be written as,

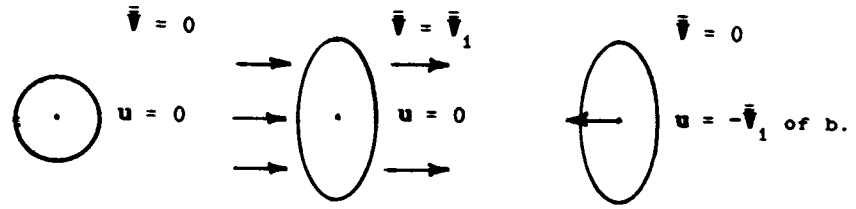
$$\frac{\phi_0'}{\phi_d} \mathbf{u} \cong \overline{\overline{\mathbf{V}}}_t - \overline{\overline{\mathbf{V}}} \quad , \tag{12.6.3}$$

where the neglected quantities are down by 10^{-18}. Taking the *moving point* derivatives, following along with the particle,

$$\frac{\phi_0'}{\phi_d} \frac{d\mathbf{u}}{dt_.} = \frac{d\overline{\overline{\mathbf{V}}}_t}{dt_.} - \frac{d\overline{\overline{\mathbf{V}}}}{dt_.} \quad . \tag{12.6.4}$$

The external field $\overline{\overline{\mathbf{V}}}$ is known, so it is now only necessary to specify the time rate of change of the total ether flow inside the particle to obtain the magnetostatic force.

Now, consider the positron as an extended entity. A better understanding of its present action can be obtained by seeing the relative control of the various velocity or flow components on its shape. From Chapter 3 it is known that the equi-density contours of an e/p at rest in ether that is at rest are spherical, and it was shown



Positron and ether at rest
a.

Positron at rest ether moves right
b.

Positron moves left ether at rest
c.

Figure 12.6.3 Positron shape versus velocity.

that the shape changed to an oblate spheroid (expanding laterally) when the particle moved at velocity $\mathbf{u}$. As illustrated in Figure 12.6.3(c), if the positron moves to the left at the velocity $\bar{\bar{\mathbf{V}}}_1$ in ether that is at rest, the shape is exactly the same as for the case in (b) where the positron is at rest and the ether moves to the right at velocity $\bar{\bar{\mathbf{V}}}_1$. Thus, a positron at rest just inside the solenoid of Section 9.3, is distorted a negligible amount, inasmuch as the ether velocity there was only 5.9123×10^{-10} cm/sec. In all magnetostatic fields commonly dealt with, the ether velocity of the external field is of roughly that same order of magnitude, and a particle at rest will have essentially a spherical shape. On the other hand, the velocities of moving electron/positrons in typical experiments, are anywhere from 1 cm/sec to just under the speed of light, and *so a significant shape change is controlled and varied by the particle velocity* $\mathbf{u}$. Figure 12.6.4 sums up the idea that the shape of a moving particle is essentially independent of the external field velocity. A more formal way to say this is to express the velocity of the particle in terms of

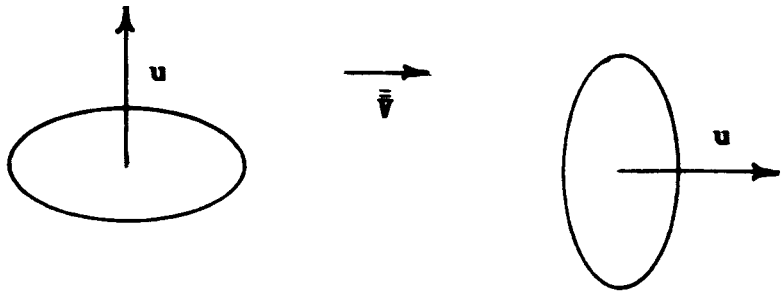


Figure 12.6.4 Particle shape is independent of the external $\bar{\bar{\mathbf{V}}}$ field.

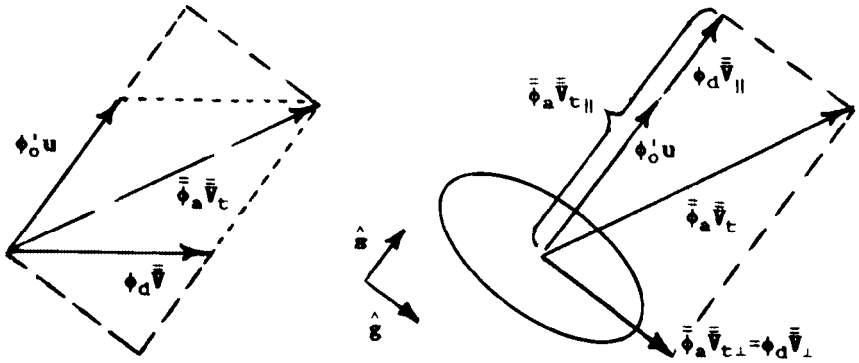
ether coordinates, as seen by $\bar{S}$ (see Sections E.3 and E.4),

$$\bar{\mathbf{u}} = \mathbf{u} - \bar{\bar{\mathbf{V}}} \quad , \tag{12.6.5}$$

and then realize that $\bar{\bar{\mathbf{V}}}$ is negligibly small compared to $\mathbf{u}$, so that,

$$\bar{\mathbf{u}} \cong \mathbf{u} \quad , \tag{12.6.6}$$

to an extremely good approximation. With this understanding, it is now possible to go back to Eq. (12.6.2) and sketch the flow vectors in a positron moving through a magnetostatic field (in a plane perpendicular to the axis as in Figure 12.6.2. Figure 12.6.5(a) represents the basic flow at the center. Some care must be observed



a. Basic flow

b. Parallel and perpendicular components

Figure 12.6.5 Flow at the positron center.

in interpreting this diagram and the accompanying Eqs.(12.6.2) and (12.6.3). Inasmuch as the external ether field is known *everywhere* inside the solenoid, for example, space derivatives of $\bar{\bar{\mathbf{V}}}$ can be taken at every point in the field. Conversely, in these equations, $\mathbf{u}$ and $\mathbf{V}_t$ are only specified at the *center* of the positron so that only moving point derivatives are allowed. Eq.(12.6.3), for example, cannot be operated on by ∇ and related space derivative functions. Figure 12.6.5(b) shows the convenient component breakdown, where the direction of $\mathbf{u}$, i.e. $\hat{\mathbf{s}}$, and of those quantities perpendicular to $\mathbf{u}$, i.e. parallel to $\hat{\mathbf{g}}$, are indicated. Note that the axis of the electron is specified and controlled by $\mathbf{u}$.

The derivation now proceeds by returning to Eq.(12.6.4), and first considering the component form of the LHS,

$$\frac{d\mathbf{u}}{dt_.} = u \frac{d\hat{\mathbf{s}}}{dt_.} + \frac{du}{dt_.} \hat{\mathbf{s}} \quad . \tag{12.6.7}$$

Earlier, in Section 3.11, it was indicated that unless an external distortion is applied to a positron, its own shape and internal distortion cannot change. Consequently, its *speed* cannot change, although in more complicated situations, its *velocity* can vary by having the direction of motion turn. In the present example, the correct, and rigorous solution of the total field, although unknown, hangs over the derivation and must be compatible. Therefore, the fact that a steady state solution is required where no external distortion is added, forces the conclusion that the positron, which is the only distortion in the picture, cannot change shape or speed, so that,

$$\frac{du}{dt_{\cdot}} = 0 \quad , \quad \frac{d\mathbf{u}}{dt_{\cdot}} = u\,\frac{d\hat{\mathbf{s}}}{dt_{\cdot}} \quad . \tag{12.6.8}$$

An immediate consequence is that the positron's path is a *circle*, and this can be made quantitative by finding $d\overline{\overline{\mathbf{V}}}_t / dt_{\cdot}$ to substitute in Eq.(12.6.4). Because $\frac{d\mathbf{u}}{dt_{\cdot}}$ is perpendicular to $\mathbf{u}$, the LHS of Eq.(12.6.4) can be eliminated by taking the dot product of each term of the equation by $\mathbf{u}$, with the result,

$$\mathbf{u} \cdot \frac{d\overline{\overline{\mathbf{V}}}_t}{dt_{\cdot}} = \mathbf{u} \cdot \frac{d\overline{\overline{\mathbf{V}}}}{dt_{\cdot}} \quad . \tag{12.6.9}$$

Resulting restrictions placed on $d\overline{\overline{\mathbf{V}}}_t / dt_{\cdot}$ can be found by replacing $d\overline{\overline{\mathbf{V}}} / dt_{\cdot}$ by its equivalent (see Eq.E.4.4),

$$\frac{d\overline{\overline{\mathbf{V}}}}{dt_{\cdot}} = \frac{\partial \overline{\overline{\mathbf{V}}}}{\partial t} + \mathbf{u} \cdot \nabla \overline{\overline{\mathbf{V}}} \quad . \tag{12.6.10}$$

Keeping in mind that $\partial \overline{\overline{\mathbf{V}}} / \partial t = 0$ in this magnetostatic field, and separating $\nabla \overline{\overline{\mathbf{V}}}$ into its symmetric (deformational) and anti-symmetric (rotational) components (see Eq.E.6.10),

$$\frac{d\overline{\overline{\mathbf{V}}}}{dt_{\cdot}} = \mathbf{u} \cdot \overline{\overline{\Phi}} + \mathbf{u} \cdot \overline{\overline{\Omega}} \quad ; \tag{12.6.11}$$

so Eq.(12.6.9) becomes,

$$\mathbf{u} \cdot \frac{d\overline{\overline{\mathbf{V}}}_t}{dt_{\cdot}} = \mathbf{u} \cdot \overline{\overline{\Phi}} \cdot \mathbf{u} + \mathbf{u} \cdot \overline{\overline{\Omega}} \cdot \mathbf{u} \quad . \tag{12.6.12}$$

This automatically reduces to (see Table C.5.1),

$$\mathbf{u} \cdot \frac{d\overline{\overline{\mathbf{V}}}_t}{dt_{\cdot}} = \mathbf{u} \cdot \overline{\overline{\Phi}} \cdot \mathbf{u} \quad . \tag{12.6.13}$$

Although $\overline{\overline{\Phi}}$ is zero in the inside solenoidal field, it is not zero either outside or in the field between parallel plane current sheets, etc., so it

will be carried along in this derivation. Solving Eq.(12.6.13),

$$\frac{d\overline{\overline{\mathbf{V}}}_t}{dt_.} = \mathbf{u}\cdot\overline{\overline{\Phi}} + \mathbf{G} \quad , \qquad (12.6.14)$$

where $\mathbf{G}$ is an arbitrary vector of the type $\mathbf{u}\cdot\mathbf{G} = 0$. Note the similarity of the form of Eq.(12.6.14) to that of (12.6.11). At this point it might be tempting to guess that they were identical, but it is easy to see from Eq.(12.6.4) that this is not correct. On the other hand, since $\overline{\overline{\Phi}}$ is also its own conjugate, another equally reasonable guess might be that $\mathbf{G}$ is the "conjugate" of $\mathbf{u}\cdot\overline{\overline{\Omega}}$. That this is actually the case will be seen shortly, but it will be deduced in a somewhat more rational manner.

The quantity $\mathbf{u}\cdot\overline{\overline{\Omega}}$ in Eq.(12.6.11) represents a component of the ether acceleration caused by the rotation of the ether at the moving point. Clearly, the quantity $\mathbf{G}$ is a component of $d\overline{\overline{\mathbf{V}}}_t / dt_.$ resulting from the particle flow attempt to either match or counter the "external" rotational flow or otherwise be compatible with the rotational flow of the total field equation solution. To see how this match might be described, take the simplest possible case where ether rotation is involved, namely, a positron at rest *on the axis* of a solenoid. An observer looking at the external field and the positron together would see, if the positron did not rotate, a flow pattern inside the positron that was circulating; so that rather than the solution for a positron at rest, there would be a spiraling in of the sustaining waves. On the other hand, if the positron's incremental density were circulating counter to the external field at every point, the effective rotational component of ether flow inside would be zero, and the already accepted solution for the positron at rest would apply. Notice that the rotation rate of the positron would be equal and opposite to the vorticity $\overline{\overline{\mathbf{w}}}$ of the "external" field. It will be assumed that this reverse turning will occur not only at the center of the solenoid, but anywhere that there is a vorticity or rotational component of ether flow. In particular, if the positron has a velocity $\mathbf{u}$ other than zero, its motional axis will turn at the rate $-\mathbf{u}\cdot\overline{\overline{\Omega}}$, opposite to the mode of the vorticity. Clearly, an electron would rotate in the same direction as the ether rotation.

Going back to Eq.(12.6.14),with $\mathbf{G} = -\mathbf{u}\cdot\overline{\overline{\Omega}}$ (see Eq.D.4.12),

$$\frac{d\overline{\overline{\mathbf{V}}}_t}{dt_.} = \mathbf{u}\cdot\overline{\overline{\Phi}} - \mathbf{u}\cdot\overline{\overline{\Omega}} = \mathbf{u}\cdot(\overline{\overline{\mathbf{V}}}\nabla) \quad . \qquad (12.6.15)$$

Substituting this into Eq.(12.6.4),

$$\frac{\phi_0'}{\phi_d}\frac{d\mathbf{u}}{dt_.} = -\mathbf{u}\bullet(\nabla\overline{\overline{\mathbf{V}}} - \overline{\overline{\mathbf{V}}}\nabla) \quad ; \qquad (12.6.16)$$

or, with the help of Eqs.(C.5.4) and (C.5.13), a more recognizable form,

$$\frac{\phi_0'}{\phi_d}\frac{d\mathbf{u}}{dt_.} = -\mathbf{u}\bullet 2\overline{\overline{\Omega}} = \mathbf{u}\times(\nabla\times\overline{\overline{\mathbf{V}}}) \quad . \qquad (12.6.17)$$

As derived, this equation applies only to a particle moving in the plane of the vorticity; but since a velocity component perpendicular to that plane does not change (no reason to), the motion in a general magnetostatic field is given by Eq.(12.6.17). To bring that equation into the form of Eq.(12.5.13), some of the relationships from Chapter 3 can be used to show that the energy of a constant speed positron is the constant $E = \gamma\phi_0 e/4$, where ϕ_0 is the center point incremental ether density of the positron in motion. Combining this with Eq.(12.6.17),

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2}\mathbf{u}\right) = \left(\gamma\frac{\phi_0}{4\phi_0'}\right)\frac{e}{c_0^2}\mathbf{u}\times\nabla\times(\phi_d\overline{\overline{\mathbf{V}}}) \quad . \qquad (12.6.18)$$

Now, ϕ_0 represents the ether density at the real positron's center; whereas ϕ_0' represents an effective value for the spherical model of Figure 12.6.1. The spurious RHS quantity in brackets results from the use of the simple model. From experience its value is essentially unity.

It is possible to derive the Lorentz force using the Lorentz transformation equations given in Section 8.13. When this is done, neglecting radiation, the magnetic term found is,

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2}\mathbf{u}\right) = \frac{q}{c_0^2}\mathbf{u}\times\nabla\times(\overline{\overline{\phi_a\mathbf{V}}}) \quad , \qquad (12.6.19)$$

so Eq.(12.6.19) is the finaI form of this component of the magnetic force in the general case.

It is well known that from Eq.(12.6.19) it can be shown that a charged particle injected at low velocity $\mathbf{u}$ into a constant vorticity field will execute a helical motion of radius,

$$R = \frac{mu_\perp}{2q\overline{\overline{w}}} \quad , \qquad (12.6.20)$$

where $u_\perp$ is the component of the velocity perpendicular to $\overline{\overline{\mathbf{w}}}$.

12.7 **The Lorentz Force - Transformer Effect**: One more component of the Lorentz force equation remains to be found. It is the basis for understanding the "Transformer Effect". In the simplest case,

visualize a region where the ether is at rest in the field. Now, externally, cause the ether at the point where the positron rests to speed up. The description of this external field at the location of the positron is given by $\partial\overline{\overline{\mathbf{V}}}/\partial t \neq 0$. This term was dropped out of the magnetostatic force derivation, and there, the particle moves to maintain a steady state field solution. Here, a *transient* effect controls, but the motion is still given by Eq.(12.6.4); except *that the positron moves to maintain its original flow vector* as expressed by,

$$\frac{d\overline{\overline{\mathbf{V}}}_t}{dt_.} = 0 \quad . \tag{12.7.1}$$

Therefore, Eq.(12.6.4) reduces to,

$$\frac{\phi_0'}{\phi_d}\frac{d\mathbf{u}}{dt_.} = -\frac{\partial\overline{\overline{\mathbf{V}}}}{\partial t} \quad . \tag{12.7.2}$$

Proceeding in exactly the same manner that led to Eq.(12.6.19),

$$\frac{E}{c_0^2}\frac{d\mathbf{u}}{dt_.} = -\left(\gamma\frac{\phi_0}{4\phi_0'}\right)\frac{e}{c_0^2}\frac{\partial(\phi_d\overline{\overline{\mathbf{V}}})}{\partial t} \quad . \tag{12.7.3}$$

One major difference is that, because the particle *accelerates in line with its motion*, the energy E *varies*; and therefore the LHS of Eq.(12.7.3) is not quite the same as that of Eq. (12.6.18). However, combining Eqs.(3.6.3), (3.7.3) and (12.5.11), Eq.(12.7.3) becomes,

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2}\mathbf{u}\right) = -\left(\gamma^3\frac{\phi_0}{4\phi_0'}\right)\frac{e}{c_0^2}\frac{\partial(\phi_d\overline{\overline{\mathbf{V}}})}{\partial t} \quad . \tag{12.7.4}$$

Here, again, the spurious factor is the result of using the simple model, and the most general case is written,

$$\frac{d}{dt_.}\left(\frac{E}{c_0^2}\mathbf{u}\right) = -\frac{q}{c_0^2}\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t} \quad . \tag{12.7.5}$$

This derivation is not restricted to the simple case above but applies to any moving charge in a field specified by $\partial\overline{\overline{\phi_a\mathbf{V}}}/\partial t \neq 0$.

12.8 **The Lorentz Force - Total**: The form of the Lorentz force equation commonly used in present day texts is the combination of the Eqs.(12.5.13), (12.6.19), and (12.7.5), where the "transformer" component is grouped with the electrostatic force even though, from the present viewpoint, it is clearly a "magnetic" effect. Here, the Lorentz force on a *charged* particle is expressed in the form,

$$\mathbf{F}_L = q\left(-\nabla\bar{\phi} + \frac{1}{c_0^2}\mathbf{u}\times\nabla\times(\overline{\overline{\phi_a\mathbf{V}}}) - \frac{1}{c_0^2}\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t}\right) \quad . \tag{12.8.1}$$

The preceding derivations of "forces" on charged particles were not intended to be rigorous in any way, since it has been maintained all along that only the total field flow solutions will be rigorous. The attempt to make the results come out in the form of the Lorentz equation was based on the hope that the field approach could be broken down into a particle/force model in the same fortuitous way that the mass of a large body produces an effect that can be considered generated by a point mass. At any rate, the derivations do serve to clarify what physical mechanisms are involved. Although the electric and transformer force derivations are fairly intuitive, the magnetic force component is more involved. A simple example can make the magnetic derivation more understandable.1

Example

If the current in the first solenoid of Section 9.3 is raised until the field inside is B = 0.02 (Tesla) = 200 (Gauss)find the radius of the path of a 1Kev positron circling inside, with path centered on the solenoid axis.

The conversion factors relating magnetic field to angular rotation rate of the ether are:

$$\overline{\overline{w}} = \frac{1}{2c_0\sqrt{4\pi}} B = 4.7048\times10^{-12} B \text{ (Gauss) rad/sec}$$

$$\overline{\overline{w}} = \frac{10^4}{2c_0\sqrt{4\pi}} B = 4.7048\times10^{-8} B \text{ (Teslas) rad/sec}$$

Therefore, $\overline{\overline{w}} = 4.7048\times10^{-12}\times200 = 9.4096\times10^{-10}$ rad/sec. Also, for a 1Kev positron, $\gamma = 1+(E_k/E_0) = 1+(10^{-3}/0.511) = 1.0020$, and $u = c_0\sqrt{1-\gamma^{-2}} = 0.0632c_0$. Using Eq.(12.6.20), R = 0.54 cm.

The external ether velocity at the orbit is, $\overline{\overline{V}} = \overline{\overline{w}}R = 5.08\times10^{-10}$ cm/sec, and the ether velocity *due to the positron alone* ($\phi_0'/\phi_d = \gamma m_0/e$ in Eq. 12.6.3) is, $\overline{\overline{V}}_p = (\gamma m_0/e)\,u = 1.016\times10^{-9}$ cm/sec. The total ether velocity inside the positron is, $\overline{\overline{\mathbf{V}}}_t = \overline{\overline{\mathbf{V}}}_p + \overline{\overline{\mathbf{V}}}$; but, from Figure 12.8.1, the two components point in opposite directions, so that, $\overline{\overline{V}}_t = \overline{\overline{V}}_p - \overline{\overline{V}} = 5.08\times10^{-10}$ cm/sec.

1. R.H.Dishington, Physics, p 258 ff, Beak Publications, Pacific Palisades, CA. (1989).

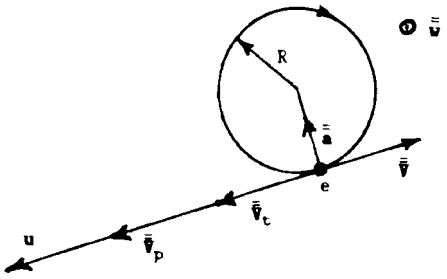


Figure 12.8.1
Positron circling inside a solenoid.

The external field ether acceleration at the positron orbit is inward (centripetal), $\bar{\bar{a}} = \bar{\bar{w}}^2 R = 4.78 \times 10^{-19}$, which is *insignificant* relative to the ether accelerations seen moving with the positron. These are given by a slightly modified Eq.(12.6.4),

$$\frac{m}{e}\frac{d\mathbf{u}}{dt_\bullet} = \frac{d\bar{\bar{\mathbf{V}}}_t}{dt_\bullet} - \frac{d\bar{\bar{\mathbf{V}}}}{dt_\bullet} \quad , \qquad (12.8.2)$$

or,

$$\frac{d\bar{\bar{\mathbf{V}}}_p}{dt_\bullet} = -\mathbf{u}\bullet\bar{\bar{\Omega}} - \mathbf{u}\bullet\bar{\bar{\Omega}} \quad .$$

The LHS term is the centripetal acceleration of ether, due just to the positron, seen at the m.p. The first RHS term is the centripetal acceleration of the total ether velocity due to the positron's *turning*, as seen at the m.p., $\bar{\bar{a}}_t = u\bar{\bar{w}} = 1.78 \text{ cm/sec}^2$. The second RHS term is the centripetal ether acceleration due to the external field, as seen at the m.p., $\bar{\bar{a}}_{ex} = u\bar{\bar{w}} = 1.78 \text{ cm/sec}^2$. Thus, combining the RHS terms, the centripetal ether acceleration due to the positron alone, as seen at the m.p., is, $\bar{\bar{a}}_p = 2u\bar{\bar{w}} = 3.65 \text{ cm/sec}^2$.

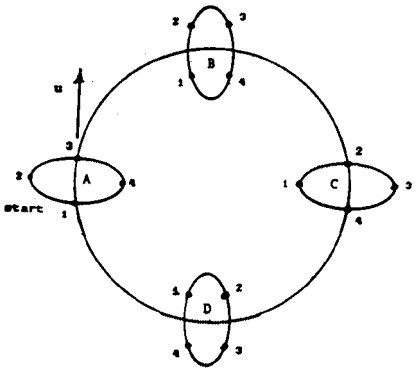


Figure 12.8.2
Partial ether turning inside a solenoid.

It was stated earlier that the lateral extension of the particle's shape is always perpendicular to its velocity u. Thus, as seen in Figure 12.8.2, the shape turns at the angular rate, $w_s = u/R = 3.51 \times 10^9 \text{ rad/sec}$. The equivalent *effective* ether rate is, $(m/e)\, w_s = 1.88 \times 10^{-9} \text{ rad/sec}$, or $(m/e)\, w_s = 2\bar{\bar{w}}$. This indicates that the fluid elements undergo distortion as the positron circles, because *the shape turns twice as fast* as $\bar{\bar{\mathbf{V}}}_t$, the total flow in the particle. It is easier to see graphically if, instead, the turning of $\bar{\bar{\mathbf{V}}}_t$ is converted to the equivalent incremental flow,

$w_{p_{eff}} = (e/m)\,\overline{\overline{w}} = 1.76 \times 10^9\,\text{rad/sec}$, or $w_{p_{eff}} = w_s/2$. This is used to plot the motion of the numbered fluid elements in Figure 12.8.2. Their relative displacement is obvious. Section 12.9 will examine the significance of this particular turning condition in detail.

Outside the solenoid, $\overline{\overline{\mathbf{w}}} = 0$, so a positron will move in a straight line at constant velocity $\mathbf{u}$. However, whereas in the previous case the ether rate of strain tensor $\overline{\overline{\Phi}}$ was zero, in this case it is *not*. Consider the situation exhibited in Figure 12.8.3, where a positron located at $R = 5$ cm travels towards the center of the field at a velocity $u = 0.1c_0$. The applicable form of Eq.(12.8.2) can be written,

$$\frac{d\overline{\mathbf{V}}_p}{dt} = \mathbf{u} \bullet \overline{\overline{\Phi}} - \mathbf{u} \bullet \overline{\overline{\Phi}} = 0 \quad . \tag{12.8.3}$$

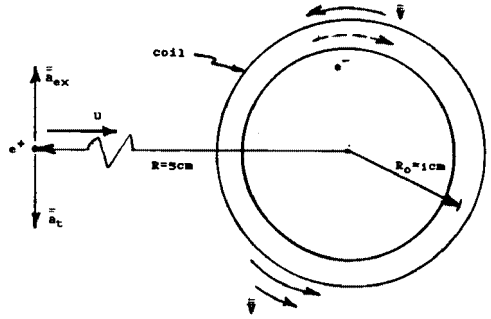


Figure 12.8.3
Positron approaching a solenoid.

Referring back to Section 9.3 and table D.5.2, the rate of strain dyadic outside the solenoid is, $\overline{\overline{\Phi}} = -R_0\,\overline{\overline{V}}_0/R^2\ (\hat{\mathbf{R}}\hat{\alpha} + \hat{\alpha}\hat{\mathbf{R}})$. At the point shown in Figure 12.8.3, the positron velocity is, $\mathbf{u} = -\hat{\mathbf{R}}\,u$. Thus, $\mathbf{u} \bullet \overline{\overline{\Phi}} = \hat{\alpha}\,(R_0\,\overline{\overline{V}}_0/R^2)\,u$, or, $\mathbf{u} \bullet \overline{\overline{\Phi}} = \hat{\alpha}\ 0.113\,\text{cm/sec}^2$. In Eq.(12.8.3), the first RHS term is the acceleration, $\mathbf{a}_t = \hat{\alpha}\ 0.113\ \text{cm/sec}^2$, of total ether velocity due to shear at the m.p. The second RHS term is the acceleration of external field ether velocity seen at the m.p. As indicated in Figure 12.8.3, *these two shear accelerations cancel.* The positron always moves so that the two rotations add and the two shears cancel. In the case of the parallel current sheets described in Section 9.3, neither $\overline{\overline{\Omega}}$ nor $\overline{\overline{\Phi}}$ is zero, so the accelerations acting would be the combination of Figures 12.8.1 and 12.8.3.

12.9 **Particle Turning**: This example has combined the customary method of using the Lorentz force equation to describe the motion of a charged particle in external fields with certain added calculations to *illustrate internal physical phenomena not normally known*. Next, one or two loose ends relating to force will be considered. First, in regard to the Lorentz force equation, the magnetic component was derived in such a way that the action of the external field on both a charge at

rest or moving was included. This was sloughed over in the derivation of the electrostatic gradient component, inasmuch as the effect of the electron's possible initial motion was not explicitly included. Going back over the steps leading up to Eq.(12.5.13), and using the motional configurations of Section 3.10, it is possible to show that even for an electron with an initial velocity, Eq.(12.5.13) still holds. Based on this knowledge, an important characteristic of the *electrostatic*-e/p interaction can be found. Rewriting Eq.(12.5.13) as,

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = -q\,\nabla\bar{\phi} \quad ; \tag{12.9.1}$$

and recalling that the internal ether flow in the positron in the absence of an external $\bar{\bar{\mathbf{V}}}$ field is,

$$\bar{\bar{\mathbf{V}}}_p = \bar{\bar{\mathbf{V}}}_t = (m/q)\,\mathbf{u} \quad , \tag{12.9.2}$$

(see the Example above), the rate of change of $\bar{\bar{\mathbf{V}}}_t$ is,

$$\frac{d\bar{\bar{\mathbf{V}}}_t}{dt_{\cdot}} = -\nabla\bar{\phi} \quad . \tag{12.9.3}$$

Here, use has been made of the fact that the charge q of a moving particle is constant.

Eqs.(12.9.1) and (12.9.3) together contain a very important fact about the turning of an electron/positron in an *electrostatic* field. To see the full import, the components of Eq.(12.9.1) are examined first. Letting, $\mathbf{u} = u\,\hat{\mathbf{s}}$, the change in momentum is,

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = mu\frac{d\hat{\mathbf{s}}}{dt_{\cdot}} + \frac{d(mu)}{dt_{\cdot}}\hat{\mathbf{s}} = -q\,\nabla\bar{\phi} \quad . \tag{12.9.4}$$

The two components represent the changes in direction and magnitude. In the same sense as used in Figure 12.6.5, the gradient of Eq.(12.9.4) can be put in the component form,

$$\nabla\bar{\phi} = \hat{\mathbf{g}}(\hat{\mathbf{g}}\bullet\nabla\bar{\phi}) + \hat{\mathbf{s}}(\hat{\mathbf{s}}\bullet\nabla\bar{\phi}) \quad ; \tag{12.9.5}$$

which combines with Eq.(12.9.4) to the end that,

$$\frac{d(mu)}{dt_{\cdot}} = -q\,(\hat{\mathbf{s}}\bullet\nabla\bar{\phi}) \quad , \quad \frac{d\hat{\mathbf{s}}}{dt_{\cdot}} = -\frac{q}{mu}(\hat{\mathbf{g}}\bullet\nabla\bar{\phi})\hat{\mathbf{g}} \quad . \tag{12.9.6}$$

The turning rate of the positron's shape, given by Eq.(12.9.6) is the focus of the present development. Going back to Eq. (12.9.3), the same procedure will be used to find the turning rate of the total ether flow inside the positron. Letting, $\bar{\bar{\mathbf{V}}}_p = \bar{\bar{V}}_p\,\hat{\mathbf{s}} = (m/q)u\,\hat{\mathbf{s}}$, the change in ether velocity is,

$$\frac{d\bar{\bar{\mathbf{V}}}_p}{dt_{\cdot}} = \bar{\bar{V}}_p\frac{d\hat{\mathbf{s}}}{dt_{\cdot}} + \frac{d\bar{\bar{V}}_p}{dr}\hat{\mathbf{s}} = -\nabla\bar{\phi} \quad .$$

Now, in conjunction with Eq.(12.9.5),

$$\frac{d\overline{\overline{V}}_p}{dt_\bullet} = -(\hat{\mathbf{s}}\bullet\nabla\bar{\phi}) \quad , \text{and} \quad \frac{d\hat{\mathbf{s}}}{dt_\bullet} = -\frac{1}{\overline{\overline{V}}_p}(\hat{\mathbf{g}}\bullet\nabla\bar{\phi})\hat{\mathbf{g}} \quad . \tag{12.9.7}$$

From Eqs.(12.9.6), (12.9.7) and (12.9.1), it follows that,

$$\left.\frac{d\hat{\mathbf{s}}}{dt_\bullet}\right|_{\text{shape}} = \left.\frac{d\hat{\mathbf{s}}}{dt_\bullet}\right|_{\substack{\text{total}\\ \text{ether}\\ \text{flow}}} \quad ; \tag{12.9.8}$$

in other words, *in an electrostatic field a moving charge flows with <u>full turning</u>*. This has profound implications in radiation theory.

12.10 **Newton's Second Law - Neglecting Radiation**: The equation of motion for any small body, charged or neutral, moving in a general ether flow field and *neglecting radiation* from the body, is written as,

$$\frac{d(m\mathbf{u})}{dt_\bullet} = \mathbf{F}_K + \mathbf{F}_L + \mathbf{F}_{ext} \quad , \tag{12.10.1}$$

where $m = \gamma m_0$, $\mathbf{F}_L$ is the Lorentz force given by Eq.(12.8.1), $\mathbf{F}_{ext}$ is any external force (e.g. a string or jet engine), and the Kirkwood force, from Eq.(12.4.5), is,

$$\mathbf{F}_K = m\left(\bar{\mathbf{a}} + 2\overline{\overline{\mathbf{w}}}\times(\mathbf{u} - \overline{\overline{\mathbf{V}}}_e)\right) + \overline{\overline{\mathbf{V}}}_e\frac{dm}{dt_\bullet} \quad . \tag{12.10.2}$$

For a charged particle in an electromagnetic field, the Lorentz force far exceeds the Kirkwood force and $\mathbf{F}_K$ is negligible. If no electromagnetic field is present, the motion of any charged or neutral particle is determined by the Kirkwood force, $\mathbf{F}_K$. The quantity $\overline{\overline{\mathbf{V}}}_e$, in Eq.(12.10.2) is *the effective velocity of the primary inertial system at each point in the field*, so that the motion factor becomes,

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{(\mathbf{u} - \overline{\overline{\mathbf{V}}}_e)^2}{c_0^2}}} \quad . \tag{12.10.3}$$

In most situations, the primary inertial system velocity is $\overline{\overline{\mathbf{V}}}$; but, in the gravitic field, $\overline{\overline{\mathbf{V}}} = 0$ and the field manifests itself through $\mathbf{V}_\bullet$ (see Chapter 14).

12.11 **Newton's Second Law - Including Radiation**: The exact way to apply Newton's law, in circumstances where *radiation* from an accelerated body *is not negligible*, is unknown at this time. In Chapter 13, the details of the radiation process are investigated, and the present status of the theory is presented. Here, only a brief sketch of the results will be stated.

Radiation occurs when charged particles undergo certain specific types of acceleration. Their fields distort in such a way that the deformation exceeds the "bound" electric energy distortion that defines the basic particle. The excess goes off into space. In neutral particles, the positively and negatively charged regions manage to interchange distortion so that this does not take place. Therefore, only charged bodies are considered here.

One feasible form of Newton's second law can be written,

$$\frac{d\mathbf{P}}{dt_{\bullet}} = \mathbf{F}_L + \mathbf{F}_R + \mathbf{F}_{ext} \quad , \qquad (12.11.1)$$

where, $\mathbf{F}_L$ is the Lorentz force of Eq.(12.8.1), $\mathbf{F}_{ext}$ is any other external force (contact, etc.) applied, and $\mathbf{F}_R$ is the radiation reaction force that the outgoing, free distortion exerts back on the charged body. As derived in Chapter 13, a reasonable form for the radiation reaction of an e/p takes the form,

$$\mathbf{F}_R = -\frac{e^2}{6\pi c_0^5}\gamma^4\left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\bullet\dot{\mathbf{u}})^2\right)\mathbf{u} \quad , \qquad (12.11.2)$$

where $\mathbf{u}$ is the velocity of the e/p and $\dot{\mathbf{u}}$ its acceleration. The "total" momentum $\mathbf{P}$ is defined as,

$$\mathbf{P} = \mathbf{p} + \mathbf{p}_a \quad , \qquad (12.11.3)$$

where $\mathbf{p} = \gamma m_0 \mathbf{u}$, the "bound", conventional momentum associated with a constant velocity body. The unknown lies in $\mathbf{p}_a$, the momentum associated with the accelerating field. More will be said about this in Chapter 13.

12.12 **Particle-Particle Interactions**: Using the energy, momentum, and force equations derived for unons in *fields*, and extrapolated to all layerons in *fields*, the exchanges of energy and momentum in elastic collisions between layerons are considered to be well understood. The discovery of the Compton effect in the collision between a photon and an electron, i.e. the particle like behavior of the photon, has led to the conventional point of view about wave-particle duality that light is "neither" particle nor wave.[1] From the ether point of view, it is "both", with no confusion. Formal description of the Compton effect is so well covered in the available literature that it need not be reproduced here.[2] Simply, it is the collision of two different particle types, a unon and a photon. The unon's ℓ-waves and bulk structure interact with

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1. R.P.Feynman, R.B.Leighton, M.Sands, Lectures on Physics, **3**, p 1-1, Addison-Wesley Publ. Co., Reading, MA (1965).
2. R.L.Armstrong & J.D.King, The Electromagnetic Interaction, p 3O2 ff, Prentice-Hall Inc. N.J. (1973). K.H.Spring, Photons and Electrons, Ch-3, Methuen & Co. Ltd, London (1950).

the photon's ℓ-waves and bulk structure to deflect both particles; but energy, momentum, and spin, are conserved, while both particles change direction and make adjustments in their wave structures. The best possible deterministic description of any particle is a "wave-particle duality".

As long as particle-particle collisions involve only a relatively weak field interaction, the exchange of energy and momentum is *elastic* and can be dealt with in the customary fashion. However, if enough energy is involved, the collision may be *inelastic*; i.e. the two particles may merge and splatter. This is a more difficult problem, formally, because the exact structure of most of the particles is not yet available. This is one of the gaps yet to be filled.

12.13 **Particle-Field Interactions**: The most common of these, i.e. layerons in electrostatic and magnetostatic fields, have been discussed in some detail in earlier sections. Some of the concepts were broadened to include neutral bodies. However, the important interaction between particles and the gravitic field has been avoided so far. Mainly, the reason for this is that it is a less familiar discipline and involves a considerably more sophisticated approach. This omission will be corrected further on, where a complete chapter (14) is devoted to the gravitic field.

12.14 **Work**: So far, this chapter has concentrated on force and interactions, and nothing has been said about work. Once the idea of force is allowed, the interaction of two particles, the "worker" and "worked on", or of a particle in a field, where the latter "works" on the former, follows naturally. From the correct and rigorous point of view, there is a transfer of interaction energy into the "worked on" particle, and work is defined as that transferred energy. In earlier sections, the transfer process was described as a single solution of the field equations; but inasmuch as a "worked on" particle can afterwards be separated from the "worker" field and it retains the transfer energy as part of its own separate moving field solution, it is possible to divorce work and force. Saying that a field works on a charge just means that energy is being transferred. Only when it is desired to calculate the energy transfer in terms of the "force" is the conventional definition of work necessary.

Formally, work is defined as follows. Starting with Eq.(12.3.1), the component forms are written,

$$\frac{dp}{dt_{\cdot}}\hat{\mathbf{s}} + p\frac{d\hat{\mathbf{s}}}{dt_{\cdot}} = (\hat{\mathbf{s}}\bullet\mathbf{F})\hat{\mathbf{s}} + (\hat{\mathbf{g}}\bullet\mathbf{F})\hat{\mathbf{g}} \quad . \qquad (12.14.1)$$

Since the component related only to changing *direction* does not change the particle shape (or energy or momentum), the energy

change must be found from,

$$\frac{dp}{dt_{\bullet}} = \hat{\mathbf{s}} \bullet \mathbf{F} = \frac{dE}{ds} \quad , \tag{12.14.2}$$

(see Eq.12.5.12). From this, the transferred energy is,

$$E = \int \mathbf{F} \bullet d\mathbf{s} = W \quad , \tag{12.14.3}$$

which is the definition of work. In conclusion, the concept of work, while it can be separated from force, is not needed in the overall structure of physics. It appears only as a convenience in thinking in the conventional way. Force has no meaning whatsoever, but is again a convenient approximation in simplifying problems. One final point of importance is that in this and all of the preceding sections of this chapter, no "so called" relativistic assumption has been made. Nevertheless, many of the expressions derived are loaded with the "so called" relativistic factors and forms, all simply the result of the solutions of the overall field equations, as seen by an absolute observer.

CHAPTER 13

WAVES

13.1 **Introduction**: Ether waves come in two varieties, *longitudinal* and *transverse*. Both types have been discussed in some detail in earlier chapters, yet there are several aspects of each that require further description. That is the goal of this chapter.

Although ℓ-waves carry no energy, they cause deflection and interference effects, are a possible component of datum fluctuations (see Chapter 4) that trigger various actions and determine radiation line widths (see Sections 7.13, 7.14), and they hold the bulk shapes of layeron particles together (see Chapters 3 and 5). Their frequencies can be determined (see Sections 5.5 and 11.6). Beyond these few properties, not much more is known about them. Nevertheless, some apparently *strange* phenomena can be explained very nicely by what *is* known about them.

A great deal of knowledge about *t* -waves has been accumulated over the last 150 years. Their principal function is to carry *energy* from one region to another. They do this in two basically different forms, *photon* and *antenna* radiation. Generally, the word "radiation" does not apply to ℓ-waves, because it implies energy transport. There are two different aspects to radiation, its *generation* (see Sections 3.12, 3.13, 7.8, 9.6, 12.8, 12.9 and 12.11) and the *form* it takes in traveling away from the source (see Section 9.12 and Chapter 10).

In the following, the generation of *t* -waves is investigated in more detail; and several conventionally puzzling phenomena are explained on the basis of ℓ-waves.

13.2 **Types of Radiation**: The two types of radiation, *photon* and *antenna*, differ both in the way they are generated and the form they take. Photons, which are true particles, are probably all generated by *orbiting* processes. Their particle nature is dependent upon a *magnetic vortex core*, and they are long, needle like objects (length/diameter > 10^3) that travel at speeds $c_0 \pm \Delta c$. Inside the vortex core, both an ℓ-wave and a *t* -wave propagate at the speed of the core. The *t* -wave carries the energy and is circularly polarized, because it turns with the vortex around the propagation axis.

Antenna radiation is quite different in form. *It is not particulate, i.e. it has no photons in it. It has no vortex core. It is a pure wave phenomenon*, a solution of Maxwell's macroscopic equations, as presented in Section 11.2. Running through all of the earlier sections listed above, is the concept that, although Maxwell's macroscopic equations can properly describe the actual propagating t-waves, those equations cannot *alone* describe the microscopic interface where the

actual radiation takes place. They are necessary but not sufficient. The propagating t-wave solutions are well known and understood. It is the microscopic generating process that needs elaboration.

13.3 **Some Microscopic Radiation Generators**: Radiation is generated by accelerating charged particles. Once this process is understood for the electron/positron, it is easily extended to other charged objects. In Sections 3.12 and 3.13 a simple rule of thumb about when and where e/p radiation will be generated was stated. The following will illustrate its application in certain specific cases.

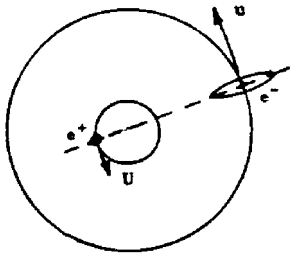

Figure 13.3.1
Hydrogen atom ground state.

Atomic Ground State Orbits: The simplest *non-radiating* accelerating electron example is that of the ground state atomic orbit. Figure 13.3.1 portrays the hydrogen atom in its ground state condition. The proton (e^+) and the electron (e^-) are circling about their common center of energy as a rigid body. The total field equation solution is this steady state, rigid body type rotation. In Section 12.9, Eq.(12.9.8), it was shown that an electron moving in an *electrostatic* field exhibits full turning (see also Section 3.13); which is to say that its bound density does not distort as it changes its direction of motion. Since the electron's speed is not changing, there is no distortion change whatsoever; and the same applies to the proton. Therefore, such a system cannot radiate, regardless of what Maxwell's equations of the far field say. The non-radiating atom, with its extended particles, is part of the boundary condition for that solution that is not included in the *conventional* solution of Maxwell's equations for accelerating point charges.

Electron Circling in a Magnetic Field: Several types of particle accelerators have as their basic element a space constant magnetic field that holds a particle's motion on a circular path while it is accelerated by other means. In a cyclotron, for example, protons are accelerated twice in each circuit by passing briefly through high electrostatic gradient regions. In a betatron, the magnetic field itself is increased, allowing the transformer effect to accelerate the charge. If, at any instant, the *driving* field in either of those machines were to be turned off, the circling particles, particularly at high velocities, would continue to circle in the steady magnetic field. When driven, the particles accelerate in-line with their motion as well as centripetally. The in-line acceleration produces some radiation, but by far the greater amount of radiation is produced in the simple

circling motion. Even with the in-line driving fields turned off, this radiation continues until the particles are damped down to a low velocity. The cause of the circling or *synchrotron* radiation is the only-partial-turning of the particle's ether elements as its shape rotates. As discussed in Section 3.13 (see Figure 3.13.1), partial turning results in the motion of bound ether elements relative to one another; and, especially for high speeds, inevitably leads to lost or freed distortion. In the example in Section 12.8, in association with Figure 12.8.2, it was shown that, in an e/p circling in a constant magnetic field, *the ether elements are rotating only half as fast as the shape turns*, so that there is an oscillating change in distortion of the bound field, the source of the radiation.

Linear Accelerators: Uniform acceleration of an electron in a straight line has significance in two senses. One, because a basic class of particle accelerating machines uses that format; and the other because the investigators into the conventional approach to radiation reaction have used that example as a work horse. The analysis in Section 12.5 can be used to help visualize the conditions (see Figure 12.5.1). In that case, radiation reaction was ignored. Here, it will be emphasized. This can be done by looking more closely at the shape of the positron's field during the acceleration. Assuming that the positron appears in the external electrostatic field with no velocity and spherical in shape (there are questions here, but they will not affect the following), it starts to flow into the lower density regions. It does not do this as a rigid body, but begins to deform from the outside first, as delineated in Figure 13.3.2. As the particle accelerates, it not only takes on the interaction distortion by extending laterally, but its equi-density contours also bunch *asymmetrically* as a result of the accelerating flow. Thus, there are *two* bound components of the momentum, a *velocity* related and an *acceleration* related part. If, as shown in the figure, the positron passes out of the external field and is no longer accelerated; just as it leaves, a brief readjustment of the shape back to the symmetrical constant velocity form of Section 3.10 and Eq.(3.10.16) takes place, and the acceleration part of the bound field is radiated away. The total radiation during the whole process consists of the continuous free radiation during the acceleration process plus the final disposal of the bound asymmetrical acceleration momentum.

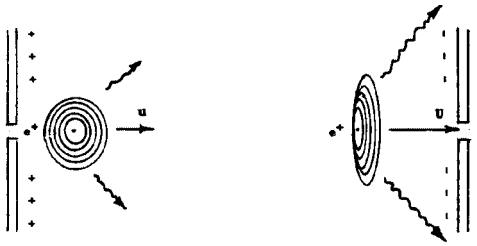

Figure 13.3.2
The positron under linear acceleration.

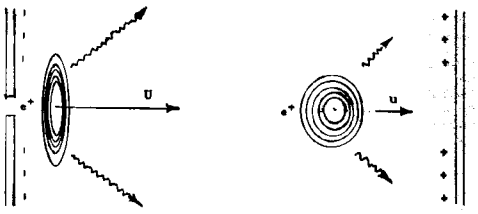

Figure 13.3.3
Positron under linear deceleration.

Braking Radiation: Whether a speeding positron is slowed by passing into a field, such as that portrayed in Figure 13.3.3, or by passing the surface atoms of a lattice structure as it enters the lattice, for example, the deceleration it experiences is caused by its slowing in some form of electrostatic gradient. Here again there are two forms of the bound deformation, the laterally expanded symmetrical *velocity* component and the *asymmetrical acceleration* component. As the equidensity $\bar{\bar{\phi}}$ contours bunch up, the kinetic energy of the braking positron is converted partly into the interaction energy between the positron and the external field and partly into free radiation. The description of collision energy transfer and radiation has been nicely summarized by Jackson and by Panofsky and Phillips, for example.[1]

Free Electron Moving in a Conductor: In a conductor free-standing in space, to which no charge or potential has been applied, the conduction electrons are moving along varied and twisted orbits involving many nuclei, being permanently attached to none. Neglecting effects of impurities and lattice defects, the resistance to electron motion comes in the form of collisions of the electron with thermally agitated atoms. Thus each conduction electron snakes its way between the nuclei, making hyperbolic shifts in direction and alternately (on the average) adding to or subtracting from the energy of interaction with the lattice, while at the same time, executing full turning. The net effect being that no free radiation leaves the conductor except that establishing its thermal equilibrium with its surroundings.

Driven Electrons Moving in a Wire: The application of an alternating potential difference between the ends of a piece of wire causes the electrons, in addition to their random snaking motions, to drift in unison, to and fro, along the wire, alternately accelerating and decelerating. Figure 13.3.4 shows only the driven component of the motion and its effect on a single electron. At T_1, its velocity is a maximum with no acceleration. Subsequently, it decelerates and radiates until at T_2 its "center" is at rest and it is radiating at

1. J.D.Jackson, Classical Electrodynamics, J.Wiley & Sons, N.Y. (1962).
W.K.H.Panofsky & M.Phillips, Classical Electricity and Magnetism, Addison-Wesley Publ.Co., Cambridge, MA (1955).

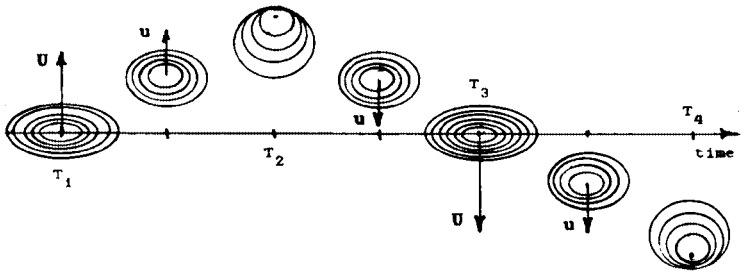

Figure 13.3.4 A single oscillating electron.

maximum rate, since outer regions of its field are unable to come back or reverse direction. It continues to accelerate, although less and less, till at T_3 it again has maximum velocity and no acceleration. Beyond this, the same sequence is repeated in the opposite direction, until one cycle is completed. This is a case where Eqs.(13.4.1) and (13.4.2) hold, because ether elements are clearly moving relative to one another; and the velocity distortion at T_1, for example, is converted into acceleration distortion at T_2 some of which cannot be retrieved.

Pair Production and Annihilation: One of the most remarkable experiments supporting the existence of the ether is the complete conversion of an energetic photon into a positron-electron pair. If the photon energy exceeds 3.2747×10^{-6} ergs (2.044 Mev), i.e. if it has enough dynamic distortion, then upon coming into the immediate vicinity of a large particle, the photon can be collapsed into a chaotic distortion that resolves itself into two distinct entities. One is a region from which ether has been removed and the other is a region, close by, where the removed ether has been deposited. During the fluctuating formation period, longitudinal waves are set up, *ingoing* in the excess ether region, and *outgoing* in the depleted region, which allow these deformations to remain stable and independent to the extent that any excess energy (deformation) in the process appears as kinetic energy of motion separating the two. These are the electron and positron, and since they are charged, they attract. Under some circumstances it is possible that they fall back together, and the excess ether just fills the depleted region, the ingoing and outgoing waves cancel and the dynamic distortion energy generates two new particles (both photons) going away from the collision point in opposite directions with equal energies. This is called the "annihilation" of the pair. The usual visualization of this process involves the formation of a primitive structure called positronium, consisting of the two particles circling around their common center of

energy. Much like a hydrogen atom, they have certain pseudo-stable orbits from which they more or less quickly fall to the next inner state, radiating as they do. Although the hydrogen atom has a ground state in which it is in equilibrium with the datum fluctuations, positronium does not; so the pair is totally destroyed and radiation is produced.

Atomic Radiation

Here again, even the non-circular orbits have full turning, but the electron's shape changes with its orbital velocity, and thus it radiates and decays. A certain set of these (certain ellipses) have low distortion and low self interference, and these are pseudo-stable, having long lifetimes and small variations in orbit energy (narrow line widths). Any circular orbit also has full turning, which would allow stability except for those self interfering ℓ-wave orbits that radiate. The non-self interfering circular orbits are almost stable, being perturbed only by the small zero-point fluctuations. Thus, these are very long lifetime orbits and have sharp linewidths (very small variations in orbit energy). Only one orbit is actually stable. All the others radiate photons.

13.4 **Electron Radiation and Reaction Force**: The standard approach to this problem is to first solve Maxwell's equations for the radiated fields from charges moving in explicit ways. Using this to define the net radiated energy leaving a charge, that energy is associated with the *acceleration* of the charge. Moreover, it is assumed that the departing energy exerts a back "force" or reaction "force" on the charge. Analyses of this "force" have been made assuming a *point charge* as the radiator. Although some truly brilliant work has been done recently[1] this whole approach is basically as unsound as the force and work concepts of the preceding chapter; because Maxwell's macroscopic equations represent only the weak field, so that even if they are adequate to handle the *free* radiation, they cannot be used for the interface between the electron and the radiation. Furthermore, using a *point charge* defeats this approach from the start. The strong field equations must be solved to see whether the total field has a free or radiated part. Nevertheless, it is possible to form the same type of crude separatist picture for this process as was done for the Lorentz "force".

Before developing the ether description of electron radiation, the conventional approach will be inspected more closely. The whole conventional idea of radiation from an accelerating electron started with the derivation of the radiation from a dipole of electric charge

1. C.Teitelboim, Phys.Rev. D**1**, 1572 (1970); D**2**, 1763 (1970). F.Rohrlich, Classical Charged Particles, Addison-Wesley, Reading, MA (1965).

(one fixed, one in rectilinear motion) as found by Hertz in 1888. The lobed radiation pattern was integrated over a sphere of large radius surrounding the dipole by Larmor (1900) to give a total rate of energy radiation of,

$$\frac{dE_R}{dt} = \frac{e^2 \dot{\mathbf{u}}^2}{6\pi c_0^3} \quad . \qquad (u \ll c_0) \qquad (13.4.1)$$

From that day to this, the *erroneous* statement is always made that Eq.(13.4.1) is true in general because the field quantities it is based upon are independent of the kind of motion specified. Quite the contrary is true, and Eq.(13.4.1) can be said to be *correct only for acceleration in the direction of the electron's velocity*, unless other special circumstances exist. Nevertheless, to continue the conventional approach, following Jackson,[1] a Lorentz 4-vector generalization of Eq.(13.4.1) leads to,[2]

$$\frac{dE_R}{dt} = \frac{e^2}{6\pi c_0^3}\gamma^4 \left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2 \right) \quad ; \qquad (13.4.2)$$

where again the claim is made that the result is valid for arbitrary acceleration of the electron, and where again the claim is *false*. Up to this point, the approach has been proper except for the careless statements about the radiation equations being generally applicable. At this juncture, the conventional picture defines a radiation reaction "force" doing "work" on the electron equal to the negative of the energy released by radiation,[3]

$$\int_{t_1}^{t_2} \mathbf{F}_R \cdot d\mathbf{s} = -\int_{t_1}^{t_2} \frac{dE_R}{dt}\, dt \quad . \qquad (13.4.3)$$

When Eq.(13.4.1) is substituted into Eq.(13.4.3), and the RHS is integrated by parts, if $\mathbf{u}\cdot\dot{\mathbf{u}} = 0$ at t_1 and t_2 or if the motion is periodic, then,

$$\mathbf{F}_R = \frac{e^2}{6\pi c_0^3}\ddot{\mathbf{u}} \quad . \qquad (u \ll c_0) \qquad (13.4.4)$$

The resulting Abraham-Lorentz equation of motion,

$$\frac{d(m_0\mathbf{u})}{dt_.} = \mathbf{F} + \mathbf{F}_R \quad \text{or} \quad \frac{d(m_0\mathbf{u})}{dt_.} - \frac{e^2}{6\pi c_0^3}\ddot{\mathbf{u}} = \mathbf{F} \quad , \qquad (13.4.5)$$

being second order, leads immediately to runaway solutions. Again,

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1. J.D.Jackson, Classical Electrodynamics, p 469, J.Wiley&Sons, N.Y. (1962).
2. A pre-relativistic derivation was given by Lie'nard (1898).
3. A.Sommerfeld, Electrodynamics, p 293, Academic Press, N.Y. (1952).
   J.D.Jackson, loc.cit. p 582.

contrary to the usual claims of generality, it is valid only for rectilinear motion except in very special circumstances. Note also, that some confusion exists in the literature[1, 2] as to which side of the equation the new term belongs on.

The preceding process leading from Eq.(13.4.1) to Eq.(13.4.4) is bypassed in finding the radiation reaction force corresponding to Eq.(13.4.2). Instead, following Abraham (v.Laue and Pauli), as described in Sommerfeld,[3] the Lorentz 4-vector derivation is extended; and it results in the following Lorentz-Dirac equations of motion,

$$\frac{d\mathbf{p}}{dt_{\cdot}} = \mathbf{F} + \frac{e^2}{6\pi c_0^3}\gamma^2\left[\ddot{\mathbf{u}} + 3\frac{\gamma^2}{c_0^2}\dot{\mathbf{u}}(\mathbf{u}\cdot\dot{\mathbf{u}}) + \frac{\gamma^2}{c_0^2}\mathbf{u}\left(\mathbf{u}\cdot\ddot{\mathbf{u}} + 3\frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2 + \dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2\right)\right] - \frac{e^2}{6\pi c_0^5}\gamma^4\left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2\right)\mathbf{u}$$

$$\frac{dE}{dt_{\cdot}} = \mathbf{F}\cdot\mathbf{u} + \frac{e^2}{6\pi c_0^3}\gamma^4\left[\mathbf{u}\cdot\left(\ddot{\mathbf{u}} + 3\frac{\gamma^2}{c_0^2}\dot{\mathbf{u}}(\mathbf{u}\cdot\dot{\mathbf{u}})\right) + \dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2\right] - \frac{e^2}{6\pi c_0^3}\gamma^4\left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2\right) \quad . \quad (13.4.6)$$

The second line of the momentum equation cancels, as does that of the energy equation; but because of later developments, these equations have been written in this unaccustomed way. Conventionally, the non-relativistic approximations to Eqs.(13.4.6) are found by setting $\gamma = 1$, and neglecting all but the $\ddot{\mathbf{u}}$ terms, reducing them to Eq.(13.4.5) and a corresponding energy equation for the particle,

$$\frac{dE}{dt_{\cdot}} = \mathbf{F}\cdot\mathbf{u} + \mathbf{F}_R\cdot\mathbf{u} \quad . \quad (13.4.7)$$

Here, $\mathbf{F}_R$ reduces to that in Eq. (13.4.4). The energy radiated from the particle (last term) should be the negative of the energy radiation found in Eq.(13.4.1), but this can only be shown working backwards through Eqs.(13.4.4) to (13.4.3) and imposing the special conditions mentioned there. *The full Lorentz-Dirac* Eqs.(13.4.6) *have the same second order runaway solution problem that the reduced equations*

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1. Loc.cit.
2. F.Rohrlich, Classical Charged Particles, p 12 Addison-Wesley, Reading, MA (1965).
3. A.Sommerfeld, loc-cit. p 297 ff.

have. Recently, great progress in countering that aspect of the difficulties has been made, as will be discussed presently. First, however, an overwhelming difficulty of the *conventional* approach will be examined.

Unsatisfied with certain aspects of the above line of thought, later investigators, following Abraham and Lorentz in their attempt to make the analysis more fundamental and rigorous, constructed an elaborate theory which could not succeed. It included the self fields of the electron as well as the external fields; and assuming Maxwell's macroscopic equations applied in the interior of an extended electron, with its distributed charge, integrated those self field forces over the electron interior. The resulting series of terms included the coulomb forces *internal* to the electron, the Larmor radiation, and many higher order terms that involved the radius of the charge. Only by letting that radius go to zero could the higher order terms be eliminated exactly, but this resulted in an infinite "electromagnetic" mass, etc. Beset with innumerable difficulties of this nature, the whole conventional process related to this last "rigorous" refinement is worthless and misleading. For more basic reasons not known to or understood by past and present investigators, that aspect of the conventional approach must be abandoned. The inapplicability of Maxwell's equations in the strong charge regions, the non-existence of "force" precluding integration of it over a volume, the absence of "forces" between elements of a distributed charge (i.e., the basic nature of charge itself), etc., all require its abandonment.

Before presenting the ether picture of radiation reaction, one further brilliant development in the approximate conventional theory should be discussed. Teitelboim[1] reworked the point-charge theory to show rigorously that a natural split in the terms of the equations that come from the Lorentz 4-vector approach settles the problems about which terms are forces, or radiation reactions, and which terms are changes in momentum and energy. He showed that some of the terms represent physical entities *bound* to the point charge and that others are not bound and represent energy and momentum *free* to leave the charged particle. Specifically, the square bracketed terms on the RHS of Eqs. (13.4.6) are the *bound* terms, whereas the second bracketed terms are the free terms; and those equations are better written,

$$\frac{d\mathbf{P}}{dt_{\cdot}} = \mathbf{F} - \frac{e^2}{6\pi c_0^5}\gamma^4\left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\cdot\dot{\mathbf{u}})^2\right)\mathbf{u} \quad , \qquad (13.4.8)$$

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1. C.Teitelboim, Phys.Rev. D**1**, 1572 (1970); D**2**, 1763 (1970). Ch.G.vanWeert, Phys. Rev. D**9**, 339 (1974). P.Rowe, Phys.Rev. D**12**, 1576 (1975).

and,

$$\frac{dE_t}{dt_.} = \mathbf{F}\bullet\mathbf{u} - \frac{e^2}{6\pi c_0^3}\gamma^4\left(\dot{\mathbf{u}}^2 + \frac{\gamma^2}{c_0^2}(\mathbf{u}\bullet\dot{\mathbf{u}})^2\right) \quad . \qquad (13.4.9)$$

The $\mathbf{P}$ and E_t in these equations are defined as,

$$\mathbf{P} = \mathbf{p} + \mathbf{p}_a = \gamma m_0\mathbf{u} - \frac{e^2}{6\pi c_0^3}\gamma^2\left(\dot{\mathbf{u}} + \frac{\gamma^2}{c_0^2}(\mathbf{u}\bullet\dot{\mathbf{u}})\mathbf{u}\right) \quad , \qquad (13.4.10)$$

and,

$$E_t = E + E_a = \gamma m_0 c_0^2 - \frac{e^2}{6\pi c_0^3}\gamma^4\mathbf{u}\bullet\dot{\mathbf{u}} \quad . \qquad (13.4.11)$$

This grouping of terms suggested by Teitelboim has a number of advantages. First, the radiation reaction terms on the RHS of Eqs.(13.4.8) and (13.4.9) *are exactly the same in form as the original radiation rate from* Eq.(13.4.2). Since Maxwell's equations are adequate to find the far field radiation, when it occurs, *it is probable that those terms are final and not to be improved upon*. Second, since Maxwell's equations are not adequate to specify the change in momentum internal to or bound in the electron, the subtractive terms on the RHS of Eqs.(13.4.10) and (13.4.11) *are probably incorrect*. But since even the present *conventional* approach tries to modify those terms by an iterative technique,[1] where the first iteration is considered by some to be the complete and rigorous form, those terms pinpoint the area where improvements must be made; and the advantage comes from the fact that, using the first iteration, runaway solutions are eliminated.[2] Third, the first iteration also eliminates the pre-acceleration problem.[2]

At present, various "interpretations" of these equations are being put forward, and because they are made from the conventional point of view, problems discussed above appear to anyone taking the ether viewpoint. Several of the workers in this field have suggested that $\mathbf{P}$ in Eq.(13.4.10) is the *measured* momentum in experiments rather than $\gamma m_0\mathbf{u}$. These, and other questions will be discussed now from the ether standpoint.

It is possible to state with some confidence that the momentum and energy radiation specified in Eqs.(13.4.8) and (13.4.9) are probably as good as is obtainable short of full field solutions. It is

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1. L.Landau,E.Lifshitz, The Classical Theory of Fields, p 222, Addison- Wesley Press, Cambridge,MA (195l). C.Teitelboim, D.Villarroel, and Ch.G.vanWeert, Riv.Nuovo Cimento, **3**, p 1 (1980). E.N.Glass, J.Huschilt, and G.Szamosi, Am.J.Phys. **52**, p 445 (1984). R.J.Cook, Am.J.Phys. **52**, p 894 (1984).
2. W.E.Baylis and J.Huschilt, Pbys.Rev. D**13**, 3237 and 3262 (1976).

only when Eqs.(13.4.10) and (13.4.11) are reconsidered *that the proper replacements are not obvious*.

Correct use of Eqs.(13.4.8) and (13.4.9) involves *first ascertaining*, by comparing with the foregoing cases of radiation generation or by considering the basic rule of radiation generation by distortion of ether elements, *whether or not an accelerating charge can actually produce radiation*. If it can, Eqs.(13.4.8) and (13.4.9) are used very much as in the texts referred to earlier. The *true momentum* of the accelerating particle is increasing by the difference between the externally applied force and the radiation reaction force, and the actual energy change is the difference between the rate of the external field's work and the reaction force loss. *If the accelerating charge cannot produce radiation, these equations are just not used*.

From the ether point of view, the true momentum and energy $\mathbf{P}$ and E_t are not to be found from Eqs.(13.4.10) and (13.4.11) but are given by,

$$\mathbf{P} = \mathbf{p} + \mathbf{p}_a = \gamma m_0 \mathbf{u} + \mathbf{p}_a \quad \text{and} \quad E_t = E + E_a = \gamma m_0 c_0^2 + E_a \ , \qquad (13.4.12)$$

where each has a velocity and an acceleration component, but the exact form of the acceleration part is not known. The velocity form is just that part of the electron distortion that remains after acceleration ceases, no matter when that occurs.

13.5 **Angular Momentum in Radiation**: It is well known that all forms of radiation have *linear* momentum in the direction of propagation. It is also well known that photons have angular momentum (vortex spin 1).[1] However, there is some confusion about the nature of antenna radiation. The conventional approach is to assume that all radiation is composed of photons, and therefore has angular momentum. That this is not true can be demonstrated by considering an ordinary, circularly polarized electromagnetic wave, a solution of Maxwell's macroscopic equations. The straightforward application of Poynting's Theorem (see Chapter 15) shows that the only momentum in the wave is along the propagation axis. As stated earlier, contemplation of the generation mechanisms for photons and antenna radiation appears to support the difference in the waves. In the last 60 years or so, more than a few journal articles have attempted to explain angular momentum carried by antenna radiation. A surprising number of these papers have appeared in the recent past.

The discussion began with the recognition that, according to the Poynting theorem, a circularly polarized plane wave, of *infinite* extent, carries no angular momentum. In 1943, Humblet[2] pointed out that

1. G.I.Taylor, Proc.Camb.Phil.Soc. **15**, 14 (1909).
2. Humblet, J., Physica, **10**, 585-603 (1943).

there is no such wave. All real waves come in bounded packets. One faction in the discussion takes this as the starting point for the solution.

Others point out that, far from an antenna, a circularly polarized wave becomes extremely close to being a plane wave, and several have used the example that an electron in that plane wave will take up a circular or helical path. Feynman[1] and Yurchenko[2] have discussed this, and Yurchenko suggested that the electron's circular motion is initiated at the start up of the wave.

A third group have attempted to show the presence of angular momentum in the wave by measuring the torque on the antenna involved. Proponents of this approach are Carrara[3] and also Chute[4] .

Many other papers have appeared involving light beams and electron spin. They are not included here because there is no argument about electron spin or photons carrying angular momentum[5]. Only antenna radiation presents a problem.

From the view point of the present writer, *the question as to whether antenna radiation carries angular momentum is still open.* In the first place, experiments on *transmitting* antenna torque do not help, because a transmitting antenna is in the "near" field. There, energy and momentum are passed back and forth and exchanged with the surroundings to complicate the problem. The often used argument that an induction motor transfers angular momentum is spurious because of the "near" field problem. To get useful experimental data on antenna torque, a *receiving* antenna in the "far" field must be used, and this is very difficult to do with any confidence.

Accepting the fact that a free electron, initially at rest and then acted upon by a circularly polarized plane wave packet, circles a fixed point: if exponential start up and shut down periods are included in Yurchenko's derivation of that circular path, for start up and shut down times that include at least 20 cycles, the coefficients in the equation of circular motion are the same, at all times, as for his calculation without start up and shut down. It can then be seen that, although the electron is circling between start up and shut down, after the wave packet passes, the electron is again at rest. No net angular momentum is transferred to the electron. In this case, while the electron is circling, it may not have true angular momentum, but just *persistence* in a lossless system. If, while it is circling, the electron is engaged in some way to remove angular momentum, the latter may be created with a counter angular momentum that leaves

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1. Feynman, R. P., The Feynman Lectures, **3**, Ch.17, pg 10 (1965).
2. Yurchenko, V. B., Am. J. Phys., **10**, No.6, June (2002).
3. Carrara, N., Nature, **164**, pg 882 (1949).
4. Chute, F. S., IEEE Trans. on Ant. and Prop., pg 585, July (1967).
5. Beth, R. A., Phys. Rev., **50**, pg 115-25 (1936).

in very much the same way two opposite electron neutrinos are created during decay of the μ meson.

In spite of all those difficulties, if it is still assumed that a circularly polarized plane wave packet carries angular momentum, then the conventional consensus must adopt a difficult approach. Many writers[1] have presented more or less complete descriptions of the accepted form of the conventional theory, which begins by stating that, since a propagating wave packet is purely magnetic, i.e. $\bar{\phi} = 0$ and $\nabla \cdot \mathbf{A} = 0$, the $\mathbf{A}$ field must return upon itself. It is assumed that this occurs out at the "edges" of the packet, and the Pointing theorem makes possible an angular momentum circulating around the wave packet far out. Since there is no vortex in the conventional picture, this angular momentum must be attributed to the t-wave. However, since little is known about the wave packet structure at the "edges" and farther out, a rather involved process is used to estimate the angular momentum.

The most detailed description of this process is found in Rohrlich's book, but many are not familiar with the notation. The same can be said for Heitler, but Jackson and also Becker and Sauter are very brief. In the end, a fair picture of the approach can be pieced together from the papers of Chute, Ohanian, Gough, Yurchenko and Gasponer. The following is a short form of Rohrlich's work and the others'.

The coordinates are those of Figure 10.13.1. Although the form of the wave packet can be quite general, for simplicity a long cylindrical shape will be considered here. In fact, the t-wave of Section 10.13 will be used, but the packet radius will be taken as R_0 all along.

According to Eq.(10.13.5), the only energy and momentum flow inside the packet is in the direction of the packet radiation. At this point in the derivation it is assumed that the unknown region outside the packet has a component of energy and momentum flow circulating in the direction of α shown in Figure 10.13.1. The total angular momentum contributed by this flow is,

$$\mathbf{J} = \frac{1}{c_0^2} \int_{\text{space}} \mathbf{R} \times \mathbf{S} \; \text{dvol} \quad , \qquad (13.5.1)$$

where $\mathbf{S}$ is taken from Eq.(10.13.5). By manipulating $\mathbf{S}$, and then the integral, an alternative integral is found that allows finding $\mathbf{J}$.

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1. F.J. Belinfante, Phisics, **6**, pg 887 (1939).
W. Heitler, The Quantum Theory of Radiation, pg 401 (1954).
J.D. Jackson, Classical Electrodynamics, problem 6.9 (1962).
R. Becker, F. Sauter, Electromagnetic Fields and Interactions, Vol II, pg320 (1964).
F. Rohrlich, Classical Charged Particles, pg.94 ff, Addison-Wesley Publ. Co. (19650
H.C, Ohanian, Am. J. Phys., **54**, pg 500, June (1986).
W. Gough, Eur. J. Phys., **7**, pg 81-87 (1986).
A. Gesponer, arXiv; physics/0308027 v.3 10 Sept. 2003.

The method is similar to one used in electrostatics to find the interaction energy between an electron and an external electric field it rests in. It is described in Section 12.5. By the use of an equivalent integral that does not directly involve the correct interaction energy density, if the integration is carried out over all space, the correct total interaction energy is obtained. The same approach is used here.

First, employing a well known identity, $\mathbf{S}$ can be written,

$$\mathbf{S} = \frac{\partial \mathbf{A}}{\partial t} \bullet (\nabla \mathbf{A}) - (\nabla \mathbf{A}) \bullet \frac{\partial \mathbf{A}}{\partial t} \quad . \tag{13.5.2}$$

This leads to,

$$\mathbf{J} = \frac{1}{c_0^2} \int_{space} \mathbf{R} \times \frac{\partial \mathbf{A}}{\partial t} \bullet (\nabla \mathbf{A}) \, dvol + \frac{1}{c_0^2} \int_{space} (\nabla \mathbf{A}) \bullet \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{R} \, dvol \quad . \tag{13.5.3}$$

The first integral is then integrated by parts, with $\nabla \bullet (\partial \mathbf{A}/\partial t) = 0$, and the total angular momentum becomes,

$$\mathbf{J} = \overset{spin}{\mathbf{J}_\sigma} + \overset{surface}{\mathbf{J}_s} + \overset{offset}{\mathbf{J}_o} \quad , \tag{13.5.4}$$

where,

$$\mathbf{J}_\sigma = \frac{1}{c_0^2} \int_{space} \mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t} \, dvol \quad , \quad \mathbf{J}_s = \frac{1}{c_0^2} \int_{surface} \mathbf{n} \bullet \left[\frac{\partial \mathbf{A}}{\partial t} \bullet (\mathbf{R} \times \mathbf{A}) \right] dS$$

$$\mathbf{J}_o = \frac{1}{c_0^2} \int_{space} (\nabla \mathbf{A}) \bullet \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{R} \, dvol$$

The offset integral represents an angular momentum due to choosing the z axis other than the axis of symmetry of the field. In the present example, the z axis is centered on the cylinder, and $\mathbf{J}_o = 0$. By taking the surface integral very far out where the field is weak ($\mathbf{R} \to \infty$), $\mathbf{J}_s = 0$. Thus the total angular momentum of the wave packet is in the intrinsic angular momentum called "spin",

$$\mathbf{J} = \mathbf{J}_\sigma = \frac{1}{c_0^2} \int_{space} \mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t} \, dvol \quad . \tag{13.5.5}$$

Just as in the electrostatic example mentioned before, the integrand in Eq.(13.5.5) is not the local angular momentum density, so this integration must be taken over all space to get the correct total angular momentum. Actually, the greatest contribution seems to be made in the interior region, but there are still some questions.

The conventional approach generally winds up relating $\mathbf{J}_\sigma$ and packet energy. According to Eqs.(10.13.1), (10.13.3) and (10.13.12),

$$\mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t} = \mathbf{k}\omega_p A_0^2 D^2 \quad . \tag{13.5.6}$$

Here, D is a simple exponential damping. Substituting this in Eq.(13.5.5) gives,

$$\mathbf{J} = \mathbf{J}_\sigma = \frac{\omega_p A_0^2 D^2}{c_0^2} \int_{space} dvol \quad . \tag{13.5.7}$$

Fom Eqs.(10.13.6) and (10.13.12), the total energy in the packet is,

$$\mathcal{E} = \frac{\omega_p^2 A_0^2 D^2}{c_0^2} \int_{space} dvol \quad . \tag{13.5.8}$$

The conventional approach avoids the integration by combining Eqs.(13.5.7) and (13.5.8), with the result,

$$\mathbf{J}_\sigma = \frac{\mathcal{E}}{\omega_p} \quad . \tag{13.5.9}$$

The preceding considerations convince the author that the problems of photon structure and antenna radiation are far from being solved satisfactorily. They require more experimental work and more analysis. These problems are of sufficient fundamental importance to warrant the effort.

13.6 **Low Level Radiation Interference**: In 1909, Taylor[1] performed a most important experiment. Using an incoherent light source falling on a slit from which radiation passed through two narrow parallel slits, he observed the interference pattern on a screen. The set up is shown in Figure 13.6.1. It was essentially Young's experiment with the intensity of the source reduced to the lowest possible levels. With long exposure time, identical interference patterns appeared on the film for all intensities. This was no problem as long as radiation was considered to be waves; but starting with J.J. Thomson[2] in 1903, and

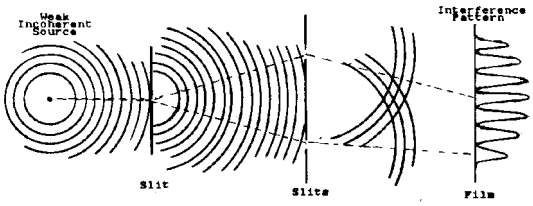

Figure 13.6.1
Taylor's weak light interference experiment

1. G.I.Taylor, Proc.Camb.Phil.Soc. **15**, 14 (1909).
1. J.J.Thomson, Electricity and Matter, p 63 ff, ... (1904);
_____, Conduction of Electricity Through Gasses, p 258, ... (1903).

augmented by Einstein[1] plus many others, the particle like properties of radiation could not be denied. So, how could they be reconciled with Taylor's results; particularly when the intensity level was reduced to the point where only one photon was present in the apparatus at any given time? In just a few years, many explanations were offered and proven objectionable.

Ever since, a steady interest has been maintained in this and related questions[2]. The slits were replaced by a grating[3] in 1927, and the source by a coherent laser beam forty years later[4]. Further refinements followed[5]. This experiment was not the only contributor to the wave-particle dilemma[6] and all of them together gave resounding support to both aspects.

One way to explain the interference experiments with low Intensity levels was suggested by Einstein[7] in one form and De Broglie[8] in another. Both used *waves that carry no energy* to guide photon particles that do. From the ether point of view, this is the correct approach even if the details of their analyses must be modified. Basically, the idea is that the weak incoherent source emits individual photons in all directions, but only rarely does one pass through the first slit. The ratio of photons emitted into the whole sphere about the source to the few that pass through the slit is enormous. At the time that each photon is generated, a separate spherical longitudinal wave is generated by the disturbance; and all of these ℓ-waves, produced by *the total number of atoms emitting photons in all directions*, travel outward and reach the slit continuously. Thus, in spite of the geometrical attenuation, a fairly sizable wave impinges on the first slit. From there on, a normal, *steady-state* interference pattern is formed after the ℓ – waves pass through the pair of slits. This ℓ-wave interference pattern carries no energy, but *guides* any photon that

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1. A.Einstein, Ann.d.Phys. **17**, 132 (1905).
2. R.Gans & A.P.Mignez, Ann.Physik, **52**, 291 (1917); P.Zeeman, Physica, p 329, Nov. (1925).
3. A.J.Dempster, H.F.Batho, Phys.Rev. **30**, 644 (1927).
4. R.L.Pfleegor & L.Mandel, Phys.Rev. **159**, 1084 (1967); J.Opt.Soc.Amer. **58**, 946 (1968).
5. G.T.Reynolds et al, Nuovo Cimento, **61**, 355 (1969); D.N.Pinder & R.N. Gould, Nature, **221**, 460, 1 Feb. (1969).
6. A.E.Ruark & H.C.Urey, Atoms, Molecules and Quanta, 2nd Rev.Ed. p 82 ff, McGraw-Hill Book.Co. N.Y. (1930), also see Dover Publ.Inc. N.Y. (1964); E.Whittaker, A History of the Theories of Aether and Blectricity, Vol.2, p 93 ff, Thomas Nelson & Sons,Ltd. London (1953).
7. A.Einstein, ...?.....; H.A.Lorentz, Problems of Modern Physics, p 156 ff, Ginn and Co. MA (1927), also see Dover Publ. Co. (1967).
8. L.De Broglie, Phys.Rev. **172**, 1284 (1968), also see, Ondes Electromagnetiques et Photons, Gauthier-Villars, Paris (1968), for De Broglie'a earlier publications.

goes through the first slit. The spread due to the finite source size plus the variation in path due to the datum fluctuations in the ether causes the photons that enter the first slit to be diffracted with about equal numbers going through each of the pair of slits. From there, each photon is guided towards the screen, and deflected so that all of the photons are deposited properly to form the observed fringe pattern. It is a totally deterministic process with the datum fluctuations having a small effect. Each photon goes through only one of the paired slits. Of course, all of the conventional discussions of perturbing the photons if they are "observed" in transit are correct, but without such interference the process is clearly understandable. While the mechanism just described is understandable, it is clear that the actual "guiding" process has not yet been worked out.

13.7 **Beam Splitting and the Aspect Experiment**: Interferometer beam splitting experiments have become so popular in the last few years that the literature has burgeoned. Loose language about individual photons splitting and going through both branches of the system is used to create a paradox or mystery. From the ether point of view, the ℓ-waves from the source do pass through both branches, and set up a standing wave pattern just as they did in the double slit experiment described in Section 13.6. *The individual photons go through one branch or the other.* Almost all variations on this theme have essentially the same explanation.

There is one possible exception, i.e. experiments of the type conducted by Aspect, et. al.[1] Without knowing the exact positions of all the optical elements in the setup, it is difficult to say whether or not the observations can be explained on the basis of ℓ-waves; but most experiments have more elements than appear in the conventionalized diagrams usually presented in the literature. Until further experimental work is done on ℓ-waves, there does not appear to be any reason to jump to supernatural causes in a few experiments still unexplained.

13.8 **Electron Interference Effects**: The presence of the electron's ℓ-waves results in a number of other interference effects that are presently ascribed to the "wave-like" properties of the electron (the mathematical wave). It is better to know that an "actual wave" property of the electron causes the phenomena. Early evidence for the wave effects was provided by Davisson and Germer[2] in 1927, and

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1. A.Aspect, P.Grangier &G. Roger, Phys.Rev.Lett. **47**, 460 (1981), **49**, 91 (1982). A.Aspect, J.Dalibard & G.Roger, Phys.Rev.Lett. **49**, 1804 (1982).
2. C.J.Davisson & L.H.Germer, Nature, **119**, 558 (1927); Phys.Rev. **30**, 705 (1927); Proc.N.A.S. **14**, 317 & 619 (1926). C.J.Davisson, J.Franklin Inst. **205**, 597 (1928); B.S.T.J. **11**, 546 (1932).

shortly thereafter by Thomson and Reid[1] and also by Kikuchi[2]. These experiments involved impinging an electron beam on targets with single crystal layers, thin metal films and thin mica films respectively; but all involved passage of the electrons through the target with the associated complexities. The first "clean" observation of electron diffraction was made in 1940 by Boersch,[3] who measured the diffraction pattern of a beam of electrons passing an opaque edge (see Figure 13.8.1). Because the electrons that pass behind the edge enter it and are quickly absorbed, the pattern of diffracted electrons appearing on the plane of observation can be regarded as being strictly the result of the beam electrons themselves. According to Boersch, the maxima are found to lie a distance from the edge position given by,

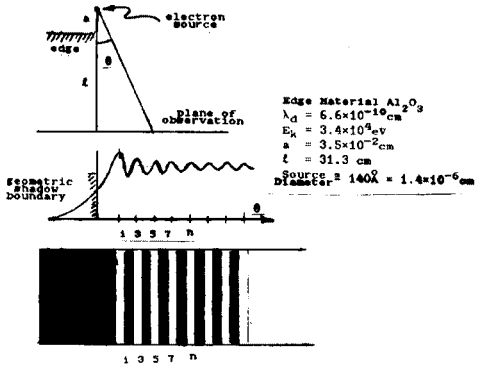


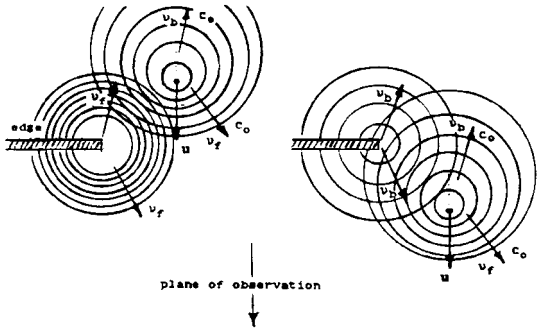
Figure 13.8.1
Opaque edge electron diffraction.

$$d_m = (a+1)\sqrt{\frac{c_0}{a\nu_d}(n-\tfrac{1}{4})}$$

where n = 1, 3, 5, ... ; and this is used as support for the conventional relationship of Eq.(3.14.9). Here, again, it is the *difference frequency* that appears to govern the process. Since, from the conventional point of view, that is the only relevant frequency associated with the electron, the data appear to support the concept. From the ether viewpoint it is not as clear. For example, taking an oversimplified look at the process of an electron passing the edge, Figure 13.8.2 diagrams the combined sustaining ℓ-waves and diffracted ℓ-waves as an electron approaches and leaves the edge. *Before the electron arrives*, only the diffracted *front* wave interacting with the wave gives the difference frequency, whereas the pattern in $\phi_\bullet$ at the plane of observation is strictly the front wave interfering with itself. Of course, *after the electron passes the edge*, the pattern at the plane of observation is composed of the diffracted back wave and the front wave, so there is a difference

1. G.P.Thomson & A.Reid, Nature, **119**, 890 (1927). G.P.Tbomson, Proc.Roy. Soc. A, **117**, 600 (1928). A.Reid, Proc.Roy.Soc. A, **119**, 663 (1928). R.Ironside, Proc.Roy. Soc.A, **119**, 668 (1928). G.P.Thomson, Proc.Roy.Soc A, **128**, 641 (1930).
2. S.Kikuchi, Japan Jour.Phys. **5,** 83 (1928); Proc.Tokyo Ac. **4**, 271, 354, 471 (1928).
3. H.Boersch, Die Naturwiss. **28**, 709, 711 (1940). W.Glaser, Grundlagen der Electronenoptic, p 498, 542 ff, Springer-Verlag, Wien (1952). G.Heber & G.Weber, Fundamentals of Modern Quantum Physics, vol 1, p 44-46, Asia Publishing House, Bombay (1959).

frequency. Also, the edge is really not an edge but one or another of the bound atomic electrons with which the ℓ-waves of electrons in the beam interact. Moreover, little is known about how the bulk $\bar{\bar{\phi}}$ of the electron responds in the ϕ. standing wave diffraction pattern.



a. Before b. After

Figure 13.8.2
Electron edge interference.

13.9 **The Aharanov-Bohm Experiment**: At first glance, the phenomena exhibited by this experiment might be thought to be just the result of a more complex form of the edge effect observed by Boersch; and, in the preliminary stage of the process, that is true. However, the final phenomenon is probably one of the most direct verifications of the ether's presence available, but it could not be included until after the discussion of the interference effects.

The first stage of the setup involves two minute slits and a source of electrons similar to that used in Boersch's arrangement. Figure 13.9.1 gives an idea of the components needed, although it fails to communicate the fantastically difficult minuteness of the various parts. For example, the aluminized quartz fiber used by Chambers[1] had a diameter of about 1.5 microns, with commensurate spaces out to the edges. In some respects, this arrangement is the two slit equivalent of Boersch's edge test, but it involves an electrostatic bi-prism to split and refocus the beam. Even when the electron rate is low enough so that only one electron at a time is in the space between the source and screen, the individual electrons produce the interference field with their remote ℓ-waves that ensures the fringe pattern observed on the screen. Here again, the *difference frequency* prevails.

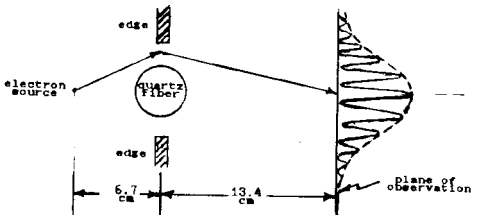


Figure 13.9.1
Chambers' setup before magnet insertion.

1. R.G.Chambers, P{hys.Rev.Lett **5**, 3 (1960).

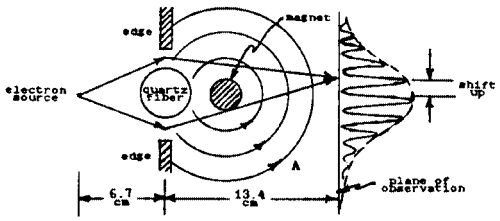


Figure 13.9.2
Effect of magnet insertion on fringe position.

The main point of the experiment suggested by Aharanov and Bohm was not to examine the diffraction pattern per se, nor its mechanism of generation; but, to show the effect, on the beams, of the region where $\mathbf{B} = 0$ outside a magnet placed in the shadow of the quartz fiber, as sketched in Figure 13.9.2. Chambers used a magnetized iron "whisker", about 1μ in diameter. The effect was observed; i.e. *although the envelope of the distribution remained the same, the fringes shifted*. Nevertheless, irregularities in the magnetic "whisker", although small, combined with the dimensions chosen for the layout, were enough to throw doubt on the result. Later, however, the experiment was performed elegantly by Moellenstedt and Bayh[1] using a miniscule 4.7μ diameter solenoid coil for the magnet; and their many results have been accepted as convincing evidence for the effect.[2]

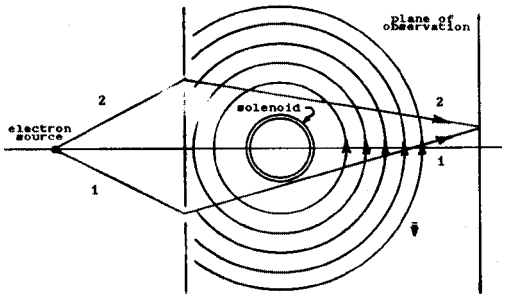


Figure 13.9.3
The Aharanov-Bohm ether phase shift.

To understand the physics, reference must be made to the ether velocity field outside a long solenoid as described in Section 9.3, Eq.(9.3.17),

$$\overline{\overline{\mathbf{V}}} = \hat{\boldsymbol{\alpha}} \frac{R_0 \overline{\overline{V}}_0}{R} \quad . \qquad \text{(outside)}$$

Figure 13.9.3 illustrates the essential elements of the geometry. Here, as in the Boersch case, the difference frequency establishes the spacings of the overall diffraction pattern (with no current in the solenoid). When the current in the coil is raised from zero, it is not difficult to see why a change in the phase shifts will produce the shift

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1. G.Moellenstedt and W.Bayh, Naturwiss. **49**, 81 (1962).
2. G.Moellenstedt and H.Lichte, in Neutron Interferometry, Ed. U.Bonse and H.Rauch, p 363-388, 1.Electron Interferometry, Oxford Science Publications; (Proceedings of an International Workshop, 5-7 June 1978, Institut Max von Laue-Paul Langevin, Grenoble). Despite the generous help of Prof. Dr. G. Moellenstedt, the present author's inexperience in this area prevents him from doing justice here to the inspiring work done by several investigators in this field. Prof. Moellenatedt's papers should be consulted for many detailed references.

difficult to see why a change in the phase shifts will produce the shift of the pattern of fringes that results. Since the ℓ-wave propagation velocity along path 1 is $c_0 +$ and that along path 2 is $c_0 -$, the phases along the two paths in the presence of the $\overline{\overline{\mathbf{V}}}$ field can only be made to match (produce the maximum bright fringe) if path 1 is lengthened and path 2 shortened. This produces the fringe shift observed.

Because of the lack of detailed knowledge regarding the guiding process of the interfering ℓ-waves, it is difficult to speculate about the result of a similar future experiment using a positron. The true picture must await the working out of the ℓ-wave guiding principles. Nevertheless, it is clear that this experiment is one of the most basic and important in verifying the ether's existence, since it could possibly offer the understanding needed to decide the polarity of ϕ and $\mathbf{V}$ in an absolute sense.

13.10 **More About t-Waves**: The visualization of neutrino generation is roughly understandable. The picture of photon generation in atoms is very clear. The distinction between antenna radiation, which lacks a vortex core, and c-on generation, which needs the vortex core to prevent geometric expansion, is also reasonable. However, there are several processes that generate radiation where it is not obvious which classification that radiation should fall into. Examples are braking, synchrotron, Cherenkov radiation, etc. In these cases, it is not clear how a spin core can be generated in the process if photons are the result. Not much light can be shown on this question at this time.

13.11 **Controlled Synchrotron Radiation**: Earlier it was shown that whether or not an accelerating electron radiates depends on the behavior of the incremental ether elements in the moving particle. If they turn along with the *shape* of the electron, their own shapes do not change and no distortion (energy) can be radiated. If they do not turn with the electron's shape, they distort; and the more violently they do the more distortion escapes. Since, in a fixed magnetic field the ether elements of a circling electron turn at only half the rate of the electron's shape (see the example in Section 12.8) synchrotron radiation is produced. However, in a circling electron guided solely by an *electrostatic* field, as in the hydrogen atom ground state, its ether elements have *full turning* and no distortion is radiated. Clearly, *some mix of magnetic and electric fields will eliminate the radiation in a storage ring*; and other mixes should provide various levels of radiation controlled smoothly by the electrostatic and magnetostatic field strengths.

Figure 13.11.1 shows the physical layout of the electron's orbit in a ring with combined fields, *but depicts only a small segment of the positive and negative electrodes* that extend all around the ring, supplying the electrostatic field. Combining Eqs.(12.8.1) and (E.6.11), the equation of motion of the electron moving at velocity $\mathbf{u}$ is given by,

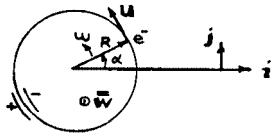

Figure 13.11.1
Circling electron.

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = +\, e\nabla\bar{\bar{\phi}} - 2e\mathbf{u}\times\bar{\bar{\mathbf{w}}} \quad , \qquad (13.11.1)$$

where, $m = \gamma m_0$, $\mathbf{u} = u\left(\mathbf{i}(-\sin\omega t) + \mathbf{j}\cos\omega t\right)$, and both m and u are constant in time. Also, $\bar{\bar{\phi}} = (\text{Volts}\times 9.4097\times 10^{-4})$ des, $\bar{\bar{\mathbf{w}}}$ is the ether vorticity of the steady magnetic field and $\bar{\bar{\mathbf{w}}} = (\text{Teslas}\times 4.7048\times 10^{-8})$ rad/sec. Furthermore, $\bar{\bar{\mathbf{w}}} = \mathbf{k}\bar{\bar{w}}$, $\nabla\bar{\bar{\phi}} = \hat{\mathbf{R}}\dfrac{\partial\bar{\bar{\phi}}}{\partial R}$, $\hat{\mathbf{R}} = \mathbf{i}\cos\omega t + \mathbf{j}\sin\omega t$ and $\mathbf{u}\times\bar{\bar{\mathbf{w}}} = u\bar{\bar{w}}\,\hat{\mathbf{R}}$. From Eq.(13.11.1),

$$\frac{d\mathbf{u}}{dt_{\cdot}} = \left(\frac{e}{m}\frac{\partial\bar{\bar{\phi}}}{\partial R} - 2\frac{e}{m}u\bar{\bar{w}}\right)\hat{\mathbf{R}} \quad . \qquad (13.11.2)$$

Differentiating the expression for $\mathbf{u}$ given after Eq.(13.11.1) above,

$$\frac{d\mathbf{u}}{dt_{\cdot}} = -u\omega\,\hat{\mathbf{R}} \quad . \qquad (13.11.3)$$

Therefore, combining Eqs.(13.11.2) and (13.11.3),

$$\frac{m}{e}\omega = 2\bar{\bar{w}} - \frac{1}{u}\frac{d\bar{\bar{\phi}}}{dR} \quad . \qquad (13.11.4)$$

Since the electron *shape* rotates with ω, $w_s = \omega$. The effective *ether* equivalent of the shape rotation is (see Section 12.8),

$$w_{eff} = \frac{m}{e}w_s = 2\bar{\bar{w}} - \frac{1}{u}\frac{d\bar{\bar{\phi}}}{dR} \quad . \qquad (13.11.5)$$

In a purely magnetic ring ($d\bar{\bar{\phi}}/dR = 0$),

$$w_{eff} = \frac{m}{e}w_s = 2\bar{\bar{w}} \quad , \qquad (13.11.6)$$

which says that the *shape* turns twice as fast as the incremental ether elements, so normal synchrotron radiation occurs.

The idea now is to change the magnetic field B and the gradient $d\bar{\bar{\phi}}/dR$ together in such a way that u, m and R remain constant, but the rate at which the ether elements rotate differs from the factor of 2

in Eq.(13.11.6). To find the correct values for B and $d\bar{\bar{\phi}}/dR$ let,

$$w_{eff} = \frac{m}{e} w_s = N\bar{\bar{w}} \quad . \qquad (13.11.7)$$

Then, from Eq.(13.11.5),

$$\frac{d\bar{\bar{\phi}}}{dR} = (2 - N)\, u\bar{\bar{w}} \quad \text{and} \quad R = \frac{mu}{Ne\bar{\bar{w}}} \quad . \qquad (13.11.8)$$

Using the conversion factors given earlier, the required controlling gradient is,

$$\frac{dV}{dR} = 1.499 \times 10^6 (2 - N)\, \beta B \quad \text{volts/cm} \quad , \qquad (13.11.9)$$

where, $\beta = u/c_0$ and B is in Teslas. Substituting the proper units, Eq.(13.11.8) can be written (for electrons),

$$R = 0.3409\ \frac{\gamma\beta}{NB} = 0.3409 \frac{\gamma\beta}{B_1} \quad , \qquad (13.11.10)$$

where B_1 is the field corresponding to N=1, or twice the value of B when the electric field is zero. Figure 13.11.2 gives plots of the normalized gradient,

$$\frac{\dfrac{dV}{dR}}{1.499 \times 10^6 \beta B_1} = \left(\frac{2}{N} - 1\right)$$

and the normalized B field,

$$\frac{B}{B_1} = \frac{1}{N}$$

required as a function of N. Thus, a single "knob" could control the two fields and the amount of radiation would be tunable from zero to a value somewhat higher than that produced in a magnetic field alone.

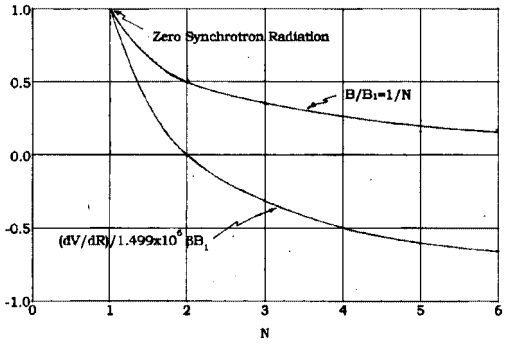

Figure 13.11.2
Normalized plots of the gradient and B field.

As an example that helps to understand the various magnitudes, assume a source of 20 Mev electrons in a ring with maximum field $B_1 = 0.01$ Tesla. Then, $\gamma = 40.14$, $\beta = 0.9997$, $u \cong c_0$ and $R = 1.37 \times 10^3$ cm. When the settings are N=1 and $B = B_1$, then, $dV/dR = 1.499 \times 10^6 \beta B_1 \cong 15{,}000$ volts/cm across the beam, and *there is no synchrotron radiation*. For N=2, $B = B_1/2 = 0.005$ Tesla, $dV/dR = 0$, the radiation is normal for a simple magnetic field. For

N=5, $B = 2 \times 10^{-3}$ Tesla, $dV/dR = -9000$ volts/cm and the radiation is 2.5 times the N=2 case.

It might be thought that, for very energetic electrons and large magnetic fields, the Eq.(13.11.1) should be augmented by the radiation reaction term of Eq.(13.4.8), but a sampling of calculations shows that over a wide range the term is negligibly small.

CHAPTER 14

THE GRAVITIC FIELD

14.1 **Introduction**: Around 1590, Galilei measured the acceleration of falling bodies. In 1687, after thinking about the problem on and off for about twenty years, Newton published his Principia, which expounded the Law of Universal Gravitation (attraction between large neutral bodies). To explain *why* neutral bodies attracted each other, Einstein "geometrized" space-time in his 1916 General Theory of Relativity. From that time to this, investigators have been trying to combine that space-time geometrization with a theory of electricity and magnetism; but this has failed. In the 1950's Kirkwood[1] adopted the ether as the gravitic medium, and succeeded in developing a gravitic field theory that takes the same form as Maxwell's equations.

The present chapter is essentially an elaboration of Kirkwood's gravitic field theory with minor modifications. The full importance of his theory is not recognized at present[2], but its influence is strong in every chapter of this work.

In the following, first the *sources* of gravitic fields are discussed, and then various interactions between particles and the fields are described. Finally, the concept of gravitic energy is considered.

14.2 **The Gravitic Field**: Layerons have gravitic fields, c-ons do not. In Section 3.9, the gravitic field of the most elementary layeron, the electron, was shown to be a *standing* ℓ-wave with density $\phi_{\bullet s}$ and velocity $V_{\bullet s}$ (coexistent with the *traveling* ℓ-wave) that just quivers in and out at frequency ω_e. All the other layerons also have similar standing wave fields varying at their own characteristic frequencies. The travelling ℓ-wave fields of *charged* layerons produce bulk distortions that interact so strongly with other charged layerons that the gravitic forces are negligible. However, in neutral layerons, neutral atoms and larger neutral composites, the *traveling* ℓ-waves go directly from the negative to the positive layers, and so *only a short distance away from the neutral combination, the standing gravitic wave is dominant.*

Conventionally, gravitation is approached as a "force" between neutral bodies. The ether view is that, since there is no "force", the interaction is just a condition of acceleration of the primary inertial

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1. Kirkwood, R.L., PhD Thesis, *Stanford U. Physics Dept.*, (1950).
   ....., Phys. Rev., **92**, 1557 (1953). Phys. Rev., **95**, 1051 (1954).
2. Kirkwood, R.L., *Project RAND*, D-7210, (1960). *The RAND Corp.*, RM-3146-RC, (1962). J. Math. Phys., **11**, 2983 (1970). Int. J. Theor. Phys., **6**, 133 (1972). Loc.cit. **7**, 391 (1973).

system at each point in the field. Establishing the motion of the primary inertial system about a neutral body is the key to the problem.

14.3 **Gravitostatics**: Here the emphasis will be on the field around a "large", spherical, neutral (uncharged) body at rest relative to the absolute observer. The conditions for a gravitostatic field are:

$$\overline{\overline{\mathbf{V}}} = \overline{\phi} = 0 \quad , \quad \overline{\phi}_a = \phi_d \quad , \quad \frac{\partial \overline{\overline{\mathbf{V}}}}{\partial t} = \frac{\partial \overline{\phi}}{\partial t} = 0$$

All higher time partials of $\overline{\overline{\mathbf{V}}}$ and $\overline{\phi}$ are zero.

Conditions for Gravitostatics (14.3.1)

When these conditions are substituted into the standing ℓ-wave Eq.(2.12.5), that equation reduces to,

$$\nabla^2 \overline{\overline{\mathbf{V}_\bullet^2}} = 0 \quad . \tag{14.3.2}$$

It was shown, in Section 3.9, that Eq.(14.3.2) has a solution,

$$\mathbf{V}_\bullet = \hat{\mathbf{r}} \sqrt{\frac{K_g}{r}} \cos \omega t \quad , \tag{14.3.3}$$

where K_g and ω are, as yet, *unspecified* and r is greater than the source mass radius. Again, from Section 3.9,

$$\phi_\bullet = \frac{3\phi_d}{2\omega} \frac{\sqrt{K_g}}{r^{3/2}} \sin \omega t \quad , \tag{14.3.4}$$

and as discussed there, this standing ℓ-wave has a time average acceleration,

$$\overline{\overline{\mathbf{a}}} = \overline{\overline{\mathbf{V}_\bullet \cdot \nabla \mathbf{V}_\bullet}} = - \hat{\mathbf{r}} \frac{K_g}{4r^2} \quad , \tag{14.3.5}$$

where K_g is not the same as the electron's. Here again, it is the time average acceleration of the neutral body standing ℓ-wave that produces the gravitic effect on other bodies.

14.4 **Sources of the Gravitic Field**: A hydrogen atom is composed of a proton and an electron. To act together as a single object they must adjust their individual frequencies ω_p and ω_e to a single composite ℓ-wave frequency that is slightly greater than ω_p

($E_{p0} = 1.50328 \times 10^{-3}$ ergs) because of the added electron rest energy ($E_{e0} = 0.00082 \times 10^{-3}$ ergs) (see Eq.11.6.1). The negative interaction energy ($E_{ep} = -4.3574 \times 10^{-11}$ ergs) is so small that the reduction from $\omega_h = 1.42626 \times 10^{24}$ rad/sec is not measurable. If another hydrogen atom is added to the first, the two often combine to form a molecule, and *all the particles must adjust to a new ℓ-wave frequency*, roughly twice the original ω_h, because of the increased energy. However, when many atoms are joined in a large, neutral body they don't appear to go to higher frequencies, but simply combine their fields. The exact cutoff is not yet understood.

Nevertheless, it is well known that the time average acceleration field of a composite body is,

$$\overset{=}{\mathbf{a}} = -\hat{\mathbf{r}}\ \frac{GM}{4\pi r^2} \quad , \tag{14.4.1}$$

where M is just the sum of the rest masses of all the components. So, working back to Eq.(14.3 3), the ℓ-wave velocity field is then,

$$\mathbf{V}_\bullet = \hat{\mathbf{r}}\sqrt{\frac{GM}{\pi r}}\cos\omega t \quad . \tag{14.4.2}$$

If the body consists of N hydrogen atoms, then,

$$\mathbf{V}_\bullet = \hat{\mathbf{r}}\sqrt{\frac{NK_g}{r}}\cos\omega t \quad , \tag{14.4.3}$$

where K_g is the constant for a single atom.

14.5 **The Gravitic Primary Inertial Systems**: Earlier chapters have provided characteristics of particles, e.g., rest energy, frequency, charge, etc., as seen in inertial systems or by an absolute observer. In Chapter 12, the technique for dealing with bodies moving in non-inertial regions, e.g. inside the vortex of a solenoid, was examined. It used the fact that, at any point in the flow, laws of physics held in the differential inertial system that translated and rotated with the fluid at the point. Then, the motion in that differential system was transformed to the absolute system by a Galilean transformation.

In considering particle/field interactions, particularly neutral particle/field interactions, one controlling influence is the particle's motion factor (or distortion factor), discussed in Sections 3.10, 8.3, 12.4 and 12.10. Its rigorous definition is given in Eq.(12.10.3) as,

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{(\mathbf{u} - \overset{=}{\mathbf{V}}_e)^2}{c_0^2}}} \quad , \tag{14.5.1}$$

where $\overline{\overline{\mathbf{V}}}_e$ is the *effective* velocity of the differential primary inertial system at the particle's location and $\mathbf{u}$ is the particle's velocity, both as seen by the absolute observer conducting the measurements. From another viewpoint, the absolute observer can find the primary inertial system at any point in the field by finding a test particle's velocity that results in a γ of unity (only rest energy in the particle). In Section 3.10, $\overline{\overline{\mathbf{V}}}_e = \overline{\overline{\mathbf{V}}} = 0$ because the datum ether is at rest relative to the absolute observer. In Section 8.3, the same conditions applied. However, in Section 12.4, $\overline{\overline{\mathbf{V}}}_e = \overline{\overline{\mathbf{V}}} \neq 0$. *In most cases, the time average ether velocity* $\overline{\overline{\mathbf{V}}}$ *establishes the primary inertial system velocity*.

One outstanding exception to this is the gravitic field of a large neutral body, because there $\overline{\overline{\mathbf{V}}} = 0$ and the test particle distortion that establishes $\overline{\overline{\mathbf{V}}}_e$ is caused by *opposing* the acceleration $\overline{\mathbf{a}}$. In Section 13.3, several figures show various accelerated particles and the asymmetrical bunching distortion in their fields. The gravitic acceleration produces just such distorted particles if their motion in step with $\overline{\mathbf{a}}$ is *impeded* or *augmented*. To determine the primary inertial system in a spherical gravitic field, allow a free-space test particle at very large distance from the source body to free-fall toward it. As it falls, its velocity $\mathbf{u} = \overline{\overline{\mathbf{V}}}_e$ increases. At each point in the field, the primary inertial system *inward* velocity can be found by integrating $\overline{\mathbf{a}}$ from $r = \infty$ to r, with the result,

$$\overline{\overline{\mathbf{V}}}_e = -\hat{\mathbf{r}}\sqrt{\frac{GM}{2\pi r}} \quad . \tag{14.5.2}$$

Any test body that moves at that inward velocity is in free-fall *and is at rest in the differential primary inertial system*. It feels no acceleration and has only its rest distortion ($\mathbf{u} = \overline{\overline{\mathbf{V}}}_e$, γ=1).

One of the strangest facts about the gravitic field is that *at each point there appear to be* <u>two</u> *primary inertial systems*. To see this, start a test body at the source surface with the *outward* <u>escape</u> velocity given by,

$$\overline{\overline{\mathbf{V}}}_e = +\hat{\mathbf{r}}\sqrt{\frac{GM}{2\pi r}} \quad , \tag{14.5.3}$$

and let it free-fall to infinity where its velocity will be zero. All during that *outward* free-fall, *the test body is at rest in the differential primary inertial system*. It feels no acceleration and has only its rest distortion ($\mathbf{u} = \overline{\overline{\mathbf{V}}}_e$, γ=1).

In practical problem solving, the velocity $\mathbf{u}$ is not always along a radius, but varies both in magnitude and direction as the body moves through the $\bar{\mathbf{a}}$ field. Since $\bar{\bar{\mathbf{V}}}_e$ appears in the Kirkwood Eq.(12.10.2) that applies to the test body's motion, at any given point the proper sign of $\bar{\bar{\mathbf{V}}}_e$ must be selected. It is determined by the sign of the r component of $\mathbf{u}$, i.e. by $u_r = dr/dt$. *The sign of* $\bar{\bar{\mathbf{V}}}_e$ *must always be chosen to be the same as the sign of* $u_r = dr/dt$. For example, in elliptic orbit problems, for the half of the orbit where r is increasing, $\bar{\bar{\mathbf{V}}}_e$ is positive; but, for the other half, r is decreasing and $\bar{\bar{\mathbf{V}}}_e$ is negative.

14.6 **Test Body Mass in a Gravitic Field**: Once the primary inertial system motion field is established, Eq.(14.5.1) leads directly to the energy and mass relationships,

$$E = \gamma E_0 \quad , \quad m = \gamma m_0 \quad , \qquad (14.6.1)$$

where E_0 and m_0 are the tests body's *rest* energy and mass. The concept of a body's *rest* energy, in all preceding chapters, was fixed by its energy distortion content when it was at *rest* in the datum ether. That definition still applies in a gravitic field. However, a semantic problem arises when a test body is at "rest" ($\mathbf{u} = 0$) in a gravitic field, since Eqs.(14.5.1) and (14.6.1) indicate that its *rest* mass is augmented by its motion relative to the primary inertial system in the field. Here, to avoid confusion, *a test body with zero velocity, as seen by the absolute observer, will be described as* "fixed" *in the gravitic field*.

Notice that, when a body is "fixed" in the field ($\mathbf{u} = 0$), Eq.(14.5.1) gives the same value of γ for both the inward and outward inertial system values of $\bar{\bar{\mathbf{V}}}_e$,

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{GM}{2\pi c_0^2\, r}}} \quad . \qquad (\mathbf{u} = 0) \qquad (14.6.2)$$

Eq.(14.6.1) then indicates that a test body held fixed at smaller distances from the source has greater mass. It is smallest at $r \to \infty$, $m = m_0$; and increases, when it is fixed at the surface, to,

$$m = m_0 \left(1 - \frac{GM}{2\pi c_0^2\, r_s}\right)^{-\frac{1}{2}} \quad , \qquad (14.6.3)$$

where r_s is the source body radius.

14.7 **Clock Rate in a Gravitic Field**: Now it is possible to understand the observed slowing of clocks in a gravitic field. Just as in Section 8.4, only one simple circling mass-on-a-string clock will be analyzed. In the gravitic case, for simplicity, the circling is in a plane perpendicular to the $\overline{\overline{\mathbf{a}}}$ field, so that the only effect of the field on the mass particle is the change in energy and momentum. Figure 14.7.1 shows the physical layout. At $r \to \infty$, the circling mass has energy $\gamma_1 E_0$ and momentum $u_1\gamma_1 E_0 / c_0^2$. If the clock is moved toward the source body, and held fixed at the distance r_2, the energy of the circling mass changes to $E_2 = \gamma_2 E_0$, but its momentum $u_1\gamma_1 E_0 / c_0^2$ remains the same, since no force was applied in the plane of the orbit. Therefore, $u_2 E_2 = u_2\gamma_2 E_0 = u_1\gamma_1 E_0$ and, because $\mathbf{u}_i$ is perpendicular to $\overline{\overline{\mathbf{V}}}_e$,

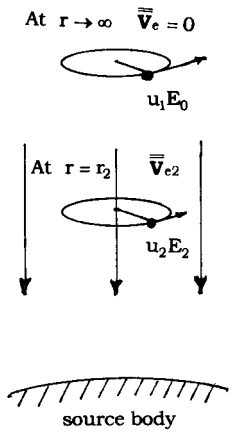

Figure 14.7.1
Circling mass clock.

$$\frac{u_1}{\sqrt{1 - \frac{u_1^2}{c_0^2}}} = \frac{u_2}{\sqrt{1 - \frac{u_2^2}{c_0^2} - \frac{\overline{\overline{\mathbf{V}}}_e^2}{c_0^2}}} \quad .$$

This can be solved to show that,

$$u_2 = \frac{u_1}{\gamma} \quad , \quad T_2 = \gamma T_1 \quad , \tag{14.7.1}$$

where,

$$\gamma = \frac{1}{\sqrt{1 - \frac{\overline{\overline{\mathbf{V}}}_e^2}{c_0^2}}} = \frac{1}{\sqrt{1 - \frac{GM}{2\pi c_0^2\, r}}} \quad , \tag{14.7.2}$$

so the clock runs slower, closer to the source body. This is true of all types of clocks.

If this process is reversed, that is, start the clock at the surface and then move it far from the source ($r \to \infty$), the clock runs *faster* at greater distances.

The lowered clock rate at the surface of a star, for example, explains what is known as the "gravitational red shift". A photon emitted at the star surface has a lower frequency than the value measured on the less massive earth (red shifted), and that frequency remains constant as the photon travels outward from the source body because $\overline{\overline{\phi}}_a = \phi_d$ and $c = c_0$.

14.8 **A Test Body Energy Equation**: Before applying Kirkwood's force equation to test body motion in a gravitic field, a related energy equation will be obtained. The derivation begins by removing m_0 from Eq.(12.4.6) , and recalling Eq.(12.4.3),

$$\frac{d}{dt_{\cdot}}\left(\gamma(\mathbf{u}-\mathbf{V})\right) = -\gamma(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V}) + \frac{\mathbf{F}}{m_0} \quad . \qquad (14.8.1)$$

Now, dotting both sides by $\gamma(\mathbf{u}-\mathbf{V})$, and manipulating the derivative,

$$\frac{d}{dt_{\cdot}}\left(\gamma^2(\mathbf{u}-\mathbf{V})^2\right) = -2\gamma^2(\mathbf{u}-\mathbf{V})\bullet(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V}) + \frac{2\gamma}{m_0}(\mathbf{u}-\mathbf{V})\bullet\mathbf{F} \quad , \quad (14.8.2)$$

a form related to the energy equation. Squaring the γ factor of Eq.(12.4.3) and taking the derivative,

$$\frac{d\gamma}{dt_{\cdot}} = \frac{1}{2\gamma c_0^2}\frac{d}{dt_{\cdot}}\left(\gamma^2(\mathbf{u}-\mathbf{V})^2\right) \quad . \qquad (14.8.3)$$

Combining Eqs.(14.8.2) and (14.8.3),

$$\frac{d\gamma}{dt_{\cdot}} = -\frac{\gamma}{c_0^2}(\mathbf{u}-\mathbf{V})\bullet(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V}) + \frac{1}{m_0 c_0^2}(\mathbf{u}-\mathbf{V})\bullet\mathbf{F} \quad , \quad (14.8.4)$$

which is a form of the energy equation, as can be seen by multiplying through by $m_0 c_0^2$.

14.9 **Test Body Motion in a Gravitic Field**: The application of Kirkwood's force equation to the motion of a test body in a gravitic field will be carried out for those cases where *the vorticity has no periodic component* ($\mathbf{w}_{\cdot} = 0$) so that $\mathbf{w} = \overline{\overline{\mathbf{w}}}$. This guarantees that in following the ether the differential system $\bar{S}$ is not rotating with a rapid oscillation, but rather with a smoothly turning motion or not at all.

The form of the force equation to be used is given in Eqs.(12.10.2) and (12.10.3),

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = m\bar{\mathbf{a}} + 2m\overline{\overline{\mathbf{w}}}\times(\mathbf{u}-\overline{\overline{\mathbf{V}}}_e) + \overline{\overline{\mathbf{V}}}_e\frac{dm}{dt_{\cdot}} + \mathbf{F} \quad , \qquad (14.9.1)$$

where, with the help of Eq.(14.8.4), the third RHS term can be written,

$$\overline{\overline{\mathbf{V}}}_e\frac{dm}{dt_{\cdot}} = -\frac{m}{c_0^2}\overline{\overline{\mathbf{V}}}_e(\mathbf{u}-\overline{\overline{\mathbf{V}}}_e)\bullet(\nabla\overline{\overline{\mathbf{V}}}_e)\bullet(\mathbf{u}-\overline{\overline{\mathbf{V}}}_e) + \frac{\overline{\overline{\mathbf{V}}}_e}{c_0^2}(\mathbf{u}-\overline{\overline{\mathbf{V}}}_e)\bullet\mathbf{F} \quad . \quad (14.9.2)$$

Eqs.(14.9.1) and (14.9.2) give a complete description of test body motion in a gravitic field. Using them, several examples will be discussed that bring out the various ideosyncracies of gravitation, some that have stirred feelings of mystery for almost a century.

14.10 **A Test Body Held Fixed in a Gravitic Field**: An external force is required to hold a test body fixed ($\mathbf{u} = 0$) against the acceleration of a source body gravitic field. The field is assumed to be irrotational ($\overline{\overline{\mathbf{w}}} = 0$), and because $\mathbf{u} = 0$ and r is fixed, Eq.(14.6..2) indicates that γ is constant and it follows that $dm/dt_{\cdot} = 0$. Thus, Eq.(14.9.1) reduces to, $\mathbf{F} = -m\overline{\mathbf{a}}$. Taking the value of $\overline{\mathbf{a}}$ from Eq.(14.4.2),

$$\mathbf{F} = \hat{\mathbf{r}}\frac{GMm}{4\pi r^2} = \hat{\mathbf{r}}\gamma\frac{GMm_0}{4\pi r^2} \quad . \tag{14.10.1}$$

This is the external force that must be exerted on any body of mass γm_0 held fixed in the gravitational field of a source body. More fundamental is the acceleration of the object when that support is removed.

14.11 **A Satellite Orbiting a Massive Source Body**: One of the phenomena that were prominent in gravitation discussions in the last century was the advance of the orbit of Mercury. Einstein's geometrized space-time gave the correct value for it. The ether theory also accounts for it, as Kirkwood demonstrated.

Assume that $\overline{\overline{\mathbf{w}}} = 0$, $\mathbf{F} = 0$ and $\mathbf{u} \neq 0$. To apply Eqs.(14.9.1) and (14.9.2) to the *up* side of the orbit ($\dot{r} > 0$),

$$\mathbf{u} = \hat{\mathbf{r}}\,\dot{r} + \hat{\boldsymbol{\theta}}\,r\dot{\theta} \quad \text{and} \quad \overline{\overline{\mathbf{V}}}_e = \hat{\mathbf{r}}\sqrt{\frac{GM}{2\pi r}} \quad . \tag{14.11.1}$$

From these conditions,

$$(\mathbf{u} - \overline{\overline{\mathbf{V}}}_e) = \hat{\mathbf{r}}\left(\dot{r} - \sqrt{\frac{GM}{2\pi r}}\right) + \hat{\boldsymbol{\theta}}\,r\dot{\theta} \quad , \tag{14.11.2}$$

and following Eq.(D.5.4) and Table D.5.1,

$$\nabla\overline{\overline{\mathbf{V}}}_e = -\tfrac{1}{2}\left(I - 3(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}})\right)\sqrt{\frac{GM}{2\pi r^3}} \quad . \tag{14.11.3}$$

Then, combining Eqs.(14.11.2) and (14.11.3),

$$(\mathbf{u} - \overline{\overline{\mathbf{V}}}_e)\cdot\nabla\overline{\overline{\mathbf{V}}}_e\cdot(\mathbf{u} - \overline{\overline{\mathbf{V}}}_e) = -\tfrac{1}{2}\left((\mathbf{u} - \overline{\overline{\mathbf{V}}}_e)^2 - 3(r\dot{\theta})^2\right)\sqrt{\frac{GM}{2\pi r^3}} \quad , \tag{14.11.4}$$

so that Eq.(14.9.2) reduces to,

$$\overline{\overline{\mathbf{V}}}_e\frac{dm}{dt_{\cdot}} = \hat{\mathbf{r}}\frac{GMm}{4\pi c_0^2\, r^2}\left((\mathbf{u} - \overline{\overline{\mathbf{V}}}_e)^2 - 3(r\dot{\theta})^2\right) \quad . \tag{14.11.5}$$

Finally, substituting Eqs.(14.4.2) and (14.11.5) into Eq.(14.9.1), the equation of motion becomes,

$$\frac{d(m\mathbf{u})}{dt_{\cdot}} = -\hat{\mathbf{r}}\frac{GMm}{4\pi r^2}\left(\frac{1}{\gamma^2} + \frac{3}{c_0^2}(r\dot{\theta})^2\right) \quad , \tag{14.11.6}$$

where γ is represented by Eq.(14.5.1).

Since it is the *shape* of the orbit and its orientation that is of primary interest, it is customary at this point to simplify the equation by changing the time variable to the so called "proper" time, $d\tau_{\bullet} = dt_{\bullet} / \gamma$. Recalling that, $\mathbf{u} = d\mathbf{r} / dt_{\bullet}$ and factoring out m_0, Eq.(14.11.6) becomes,

$$\frac{d^2\mathbf{r}}{d\tau_{\bullet}^{\,2}} = -\hat{\mathbf{r}}\frac{GM}{4\pi r^2}\left(1 + \frac{3}{c_0^2}r^2\left(\frac{d\theta}{d\tau_{\bullet}}\right)^2\right) \quad . \qquad (14.11.7)$$

The first bracketed term gives the basic orbit of Kepler and Newton. The second term leads directly to a precession of the orbit that figured as one of the three famous tests of Einstein's General Relativity theory. No curved space, relativity, or other exotic concept was used to find this result. The preceding derivation of Eq.(14.11.7) is essentially Kirkwood's ether gravitation theory derivation with only minor modifications.

14.12 **The C-on Energy Equation**: Because c-ons have no *rest* mass and are quite different from layerons, the usual motion factor effects do not apply to them directly. Nevertheless, c-ons do carry energy and interact with the ether around them in the gravitic field. Since the exact solution for a c-on's structure and behavior in moving through the ether is not available, its equation of motion must be obtained by roughly paralleling the derivation of the test body motion equation using an unspecified factor ξ that plays a role similar to layeron mass or energy.

Because c-ons are long, needle like objects, certain approximations are required. For example, the source body is taken to be very large relative to the c-on length, so that the c-on is treated as a point object in the field. The kinematic acceleration of a moving point, as seen by the general observer S, in terms of its acceleration as seen by the fluid observer $\bar{S}$, is found from Eq.(12.4.2) to be,

$$\frac{d\mathbf{u}}{dt_{\bullet}} = \frac{d\mathbf{V}}{dt} + 2\mathbf{w}\times(\mathbf{u} - \mathbf{V}) + \frac{d'\bar{\mathbf{u}}}{dt_{\bullet}} \quad , \qquad (14.12.1)$$

where all quantities and conditions are the same as in Section 14.8. Here a scalar function $\xi(x, y, z, t)$ will be introduced, that will not be further identified at this point. It will, according to Eq.(E.3.18), behave as any scalar, so,

$$\frac{d\xi}{dt_{\bullet}} = \frac{d'\xi}{dt_{\bullet}} \quad . \qquad (14.12.2)$$

This can be used with Eq.(12 .4.1) to give,

$$(\mathbf{u}-\mathbf{V})\frac{d\xi}{dt_{\cdot}}=\bar{\mathbf{u}}\,\frac{d'\xi}{dt_{\cdot}} \quad . \tag{14.12.3}$$

Now, multiplying Eq.(14.12.1) by ξ and combining with Eq.(14.12.3),

$$\frac{d(\xi\mathbf{u})}{dt_{\cdot}}=\xi\frac{d\mathbf{V}}{dt}+2\xi\mathbf{w}\times(\mathbf{u}-\mathbf{V})+\mathbf{V}\frac{d\xi}{dt_{\cdot}}+\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad , \tag{14.12.4}$$

which resembles Eq.(12.4.4). Again, with the aid of Eqs.(E.4.12) and (E.4.15),

$$\frac{d}{dt_{\cdot}}\big(\xi(\mathbf{u}-\mathbf{V})\big)=-\xi(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V})+\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad , \tag{14.12.5}$$

which is the c-on counterpart of Eq.(12.4.6).

The c-on velocity is designated by Eq.(12.4.1), $\bar{\mathbf{u}}=\mathbf{u}-\mathbf{V}$, where no restriction is placed on $\bar{\mathbf{u}}$ as yet; since, under some circumstances, the c-on velocity can vary both in magnitude and direction. Therefore,

$$(\mathbf{u}-\mathbf{V})^2=\bar{u}^2 \quad , \tag{14.12.6}$$

where $\bar{u}$ is the *speed* of the c-on (sans direction). Now, carrying out the differentiation in Eq.(14.12.5) and transposing,

$$\frac{d\xi}{dt_{\cdot}}(\mathbf{u}-\mathbf{V})=-\xi(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V})-\xi\frac{d}{dt_{\cdot}}(\mathbf{u}-\mathbf{V})+\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad . \tag{14.12.7}$$

Dotting both sides by $(\mathbf{u}-\mathbf{V})$, and using Eq.(14.12.6),

$$\frac{d\xi}{dt_{\cdot}}=-\frac{\xi}{\bar{u}^2}(\mathbf{u}-\mathbf{V})\bullet(\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V})-\frac{\xi}{\bar{u}}\frac{d\bar{u}}{dt_{\cdot}}+\frac{(\mathbf{u}-\mathbf{V})}{\bar{u}^2}\bullet\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad , \tag{14.12.8}$$

which is the desired counterpart of Eq.(14.8.4). Thus, ξ acts as a kind of motion factor that relates c-on energy to the effect motion through the ether produces on it.

14.13 **C-on Motion in a Gravitic Field**: Following along the same steps used in Section 14.9, the equation of motion to be used in the gravitic field is, from Eq.(14.12.4),

$$\frac{d(\xi\mathbf{u})}{dt_{\cdot}}=\xi\bar{\bar{\mathbf{a}}}+2\xi\bar{\bar{\mathbf{w}}}\times(\mathbf{u}-\bar{\bar{\mathbf{V}}}_e)+\bar{\bar{\mathbf{V}}}_e\frac{d\xi}{dt_{\cdot}}+\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad , \tag{14.13.1}$$

which parallels Eq.(14.9.1). The complete equation results if Eq.(14.12.8) is substituted in the third RHS term of Eq.(14.13.1), so,

$$\begin{aligned}\frac{d(\xi\mathbf{u})}{dt_{\cdot}}=\xi\bar{\bar{\mathbf{a}}}&-\frac{\xi}{\bar{u}^2}\bar{\bar{\mathbf{V}}}_e(\mathbf{u}-\bar{\bar{\mathbf{V}}}_e)\bullet(\nabla\bar{\bar{\mathbf{V}}}_e)\bullet(\mathbf{u}-\bar{\bar{\mathbf{V}}}_e)+2\xi\bar{\bar{\mathbf{w}}}\times(\mathbf{u}-\bar{\bar{\mathbf{V}}}_e)\\&-\frac{\xi}{\bar{u}}\bar{\bar{\mathbf{V}}}_e\frac{d\bar{u}}{dt_{\cdot}}+\bar{\bar{\mathbf{V}}}_e\frac{(\mathbf{u}-\bar{\bar{\mathbf{V}}}_e)}{\bar{u}^2}\bullet\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}}+\frac{d'(\xi\bar{\mathbf{u}})}{dt_{\cdot}} \quad .\end{aligned} \tag{14.13.2}$$

To derive this equation it was necessary to allow the c-on speed $\bar{u}$ to vary, even though in regions where $\bar{\bar{\phi}}_a = \phi_d$, it does not; because, until all differentiations were completed, making the assumption that $\bar{u}$ is constant would cause some of the ether effects to be lost from the equations. However, in the gravitic field specified by Eq.(14.11.1), $\bar{\bar{\phi}}_a = \phi_d$, so the *speed* of propagation in $\bar{S}$ is $\bar{u} = c_0$. Also, $\bar{\bar{\bar{\mathbf{w}}}} = 0$, so all quantities in Eq.(14.13.2) are known except for the value of ξ. There is no further information available to determine ξ, so it must be chosen to make Eq.(14.13.2) match observed phenomena.

If ξ is chosen to be $\xi = 1/c_0$, and $\mathbf{u} = c_0\hat{\mathbf{s}}$, Eq.(14.13.2) reduces to,

$$\frac{d\mathbf{u}}{dt_\cdot} = \bar{\bar{\mathbf{a}}} - \frac{\bar{\bar{\bar{\mathbf{V}}}}_e}{c_0^2}(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)\bullet(\nabla\bar{\bar{\bar{\mathbf{V}}}}_e)\bullet(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e) - \bar{\bar{\bar{\mathbf{V}}}}_e\frac{1}{c_0}\frac{dc_0}{dt_\cdot} + \bar{\bar{\bar{\mathbf{V}}}}_e\frac{(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)}{c_0}\bullet\frac{d'\hat{\mathbf{s}}}{dt_\cdot} + c_0\frac{d'\hat{\mathbf{s}}}{dt_\cdot} \quad . \qquad (14.13.3)$$

Since c_0 is a constant and in $\bar{S}$, a primary inertial system, c-ons do not change direction, the last two terms are zero, and the final c-on orbit (*up* side) can be found from,

$$\frac{d\mathbf{u}}{dt_\cdot} = \bar{\bar{\mathbf{a}}} - \frac{\bar{\bar{\bar{\mathbf{V}}}}_e}{c_0^2}(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)\bullet(\nabla\bar{\bar{\bar{\mathbf{V}}}}_e)\bullet(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e) \quad . \qquad (14.13.4)$$

Since Eq.(14.13..4) does describe c-on motion in a gravitic field, the choice of $\xi = 1/c_0$ was correct.

14.14 **C-on Path Bending in a Source Body Field**: In addition to the "red shift" (Section 14.7) and the perihelion advance (Section 14.11), a third "test" that Einstein's geometrized space theory met was a bending of a light ray as it passed the limb of a star. Kirkwood's gravitic field theory also explains the effect successfully.

Using values from Eqs.(14.4.2) and (14.11.1),

$$\frac{d^2\mathbf{r}}{dt_\cdot^2} = -\frac{GM}{4\pi r^2}\hat{\mathbf{r}} + \frac{GM}{4\pi r^2 c_0^2}\left((\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)^2 - 3(r\dot{\theta})^2\right)\hat{\mathbf{r}} \quad , \qquad (14.14.1)$$

which parallels Eq.(14.11.7). But whereas in that earlier case $(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)^2$ was a very small number, because of the relatively slow satellite motion, from Eq.(14.12.6) the high speed c-on has $(\mathbf{u} - \bar{\bar{\bar{\mathbf{V}}}}_e)^2 = c_0^2$, thus cancelling out the ordinary gravitational acceleration acting on the c-on. The remaining terms give,

$$\frac{d^2\mathbf{r}}{dt_\cdot^2} = -\frac{3GM}{4\pi c_0^2}\dot{\theta}^2\,\hat{\mathbf{r}} \quad , \qquad (14.14.2)$$

which differs from Eq.(14.11.7). From Eq.(14.14.2), Einstein's formula for the orbit of the photon follows in a straightforward way. Again, this result is essentially Kirkwood's ether theory derivation with minor modifications, and again no use of relativity was needed to obtain it.

14.15 **Gravitic Energy**: In educational institutions the gravitic field is presented in two completely different ways. Undergraduates are given a simple picture of "force" and "work" that leads directly to the concept of "potential energy" in the *field*. The farther apart two neutral objects are, the greater the potential energy stored between them. On the other hand, graduate students are introduced to the General Theory of Relativity, and bombarded with such a horde of math symbolisms that the fact that the experimental evidence precludes gravitic energy stored in the field is lost. The ether theory allows this to be demonstrated quite simply.

Since the gravitic field is just an acceleration in a standing ℓ-wave field, and since ℓ-waves carry no energy, it appears, right from the start, unlikely that there can be energy stored in the field. In the following it will be shown that all energy observed in gravitic field experiments appears to be electric or possibly magnetic.

14.16 **Force and Work in a Gravitic Field**:[1] The nature of gravitic energy can be pursued by studying a small, neutral test body moving *radially* in the field of a large mass M, where $\overline{\overline{\mathbf{w}}} = 0$. The motion is described by the reduced equation,

$$\frac{d(\gamma m_0 \mathbf{u})}{dt} = -\hat{\mathbf{r}}\left(\frac{GMm_0}{4\pi r^2}\left(\frac{1}{\gamma}\right) - \left(\frac{1}{\gamma^2} + \frac{1}{c_0^2}\sqrt{\frac{GM}{2\pi r}}\;|\,\dot{r}\,|\right)F_{ext}\right) \quad . \quad (14.16.1)$$

where $\mathbf{F}_{ext}$ is radially outward. Only three specific cases are needed to describe the gravitic energy problem.

The first is a test body fixed in the Earth's field. Eq.(14.10.1) indicates that the external upward force required to hold the body fixed is,

$$\mathbf{F}_{ext} = \hat{\mathbf{r}}\,\gamma\frac{GMm_0}{4\pi r^2} \quad . \quad (14.16.2)$$

Present day interpretations of energy in relativity are a strange mix of Newtonian ideas and "relativistic" motion factors. Conventionally, the work (energy) required to *slowly* raise such a test body from the

1. R.H.Dishington, Apeiron, **5**, 1 (1998). Presented at Symposium *The Present Status of the Quantum theory of Light*, York University, Toronto, Canada, August (1995).

Earth's surface to infinity is defined as,

$$W = \int_{r_{ea}}^{\infty} F_{ext}\ dr = m_0 c_0^2 \left(\frac{\gamma_s - 1}{\gamma_s} \right) \quad , \qquad (14.16.3)$$

where γ_s, is the value at the Earth's surface.

In gravitic energy situations, γ_s is usually so close to unity that it is useful to introduce the increment $\delta = \gamma - 1$ instead. To get some idea of the amounts of the energies involved, Eq.(14.16.3) becomes $W \cong \delta_s m_0 c_0^2 = \delta_s E_0$, where $\delta_s \cong 7 \times 10^{-10}$. Thus, energies involved in test body motion are smaller than the body's rest energy by a factor of about 10^{-9}, and essentially a negligible fraction of the source body energy. With this in mind, if a test body is slowly *lifted* from the Earth's surface to outer space, using an hypothetical elevator attached to Earth, work or energy $\delta_s E_0$ (Newtonian) or slightly *less* than $\delta_s E_0$ (relativistic, Eq.14.16.3) is required conventionally. Actually the test body energy is *lowered* from $\gamma_s E_0$ to E_0, and the elevator gives $\delta_s E_0$ back to the source (Earth).

The second case is that of a test body free-falling in the field from $r \to \infty$ to the Earth's surface, neglecting air friction. Since $F_{ext} = 0$, if the initial velocity is $\mathbf{u} = 0$, *then the test body remains at rest in the primary inertial system all the way down*, so that $\mathbf{u} = \overline{\overline{\mathbf{V}}}_e$ and, from Eq.(14.5.1), $\gamma = 1$. This reduces Eq. (14.16.1) to,

$$\frac{d\mathbf{u}}{dt_.} = -\hat{\mathbf{r}} \frac{GM}{4\pi r^2} \quad . \qquad (14.16.4)$$

Just before making contact with the Earth's surface, the test body velocity is,

$$\mathbf{u}_s = -\hat{\mathbf{r}} \sqrt{\frac{GM_{ea}}{2\pi r_{ea}}} \quad . \qquad (14.16.5)$$

In test body cases, the slick Newtonian approximation obscures the true problem. It describes a free-falling mass as converting "potential" to "kinetic" energy and carrying the latter to the source body, which ultimately absorbs the "kinetic" energy as heat. However, in connection with Eqs.(14.16.4) and (14.16.5), the free-falling body (E_0 at ∞) is at *rest* in an inertial system, its $\gamma = 1$, *undergoing no physical change all the way to the ground*. After the inelastic collision with the Earth, the test body's energy is $\gamma_s E_0$ (See Section 14.6), so it has *gained* energy $\delta_s E_0$. This is borne out by the observed change-of-clock-rate and red-shift in a gravitic field. In addition, there is an *equal* amount of heat generated, so a total of $2\delta_s E_0$ *suddenly* appears

in the collision. Clearly *it comes from the source, not the test body*, meaning that all of these energies are electric, localized in the bodies and conserved. Since the "binding" energy is just that lost to heat, it also is localized and electric in nature. So, the "kinetic" and "potential" energies of Newtonian theory are just artificial bookkeeping tricks to allow easy calculation of the heat energy generated, ignoring the energy increase of the body after it is stopped.

The third case is that of a test body *shot* vertically from the earth's surface with a velocity the negative of that given in Eq.(14.16.5), again neglecting air friction. With $F_{ext} = 0$, it follows that,

$$\mathbf{u} = \overline{\overline{\mathbf{V}}}_e = \hat{\mathbf{r}}\sqrt{\frac{GM}{2\pi r}} \quad , \tag{14.16.6}$$

$\gamma = 1$, and the body decelerates to 0 as $r \to \infty$. From the instant it is free, its energy is E_0 *with all other energy adjusted out through the driving mechanism*. Being at rest in the primary inertial system, it rises with no energy change, neither "kinetic" nor "potential", and escapes with energy E_0.

At present, most of what conventionally appear to be gravitic energy phenomena actually are localized electric energy exchanges. Without a few new solutions to certain presently intractable accelerating charge problems, the final word on localized, stored gravitic energy cannot be said.

14.17 **Transmission of Electric Energy Between Gravitic Fields**: There is an example of interaction between two large neutral bodies that, as yet, has no simple explanation. Figure 14.17.1 illustrates the experiment. The two bodies are accelerating towards each other. A jet engine, located to push one of the bodies away from the other, is turned on. The bodies continue to attract one another. *The center of mass of the two now accelerates in the direction of the jet's push.* If the acceleration caused by the jet is greater than the rate at which the two bodies were accelerating together, the jet propelled body will stop and turn around and the second body will chase after it. Clearly, the second body is receiving electric energy just as the propelled body is. There is probably a way to understand this in terms of $\partial\overline{\overline{\mathbf{V}}}_e / \partial t$ which would be a kind of magnetic effect; but obviously more work remains to be done.

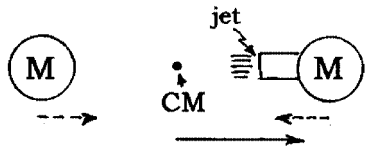

Figure 14.17.1
Neutral body energy transfer.

14.18 **Artificial Laboratory Gravitic Fields**: In a speculative vein, there is a possibility that gravitic fields can be created in the laboratory. What is required is a configuration of conductors that have oscillating currents which create an oscillating **A** field with certain specific characteristics. If a *standing* $\mathbf{V}_{\bullet}$ ℓ-wave is set up, where $\overline{\mathbf{a}} = \overline{\overline{\mathbf{V}_{\bullet} \bullet \nabla \mathbf{V}_{\bullet}}} \neq 0$, then tiny neutral test particles should be deflected in their motion by that $\overline{\mathbf{a}}$ gravitic field. One simple configuration would be a block of metal, into which a cone shaped cavity has been cut, with electric current running up and down the sides of the cone. Another could have an alternating electric field between two electrodes, one very large (perhaps an enclosing sphere) and the other point like to greatly concentrate the field. Although the numbers are discouragingly small, some such configuration might be devised to accelerate neutrons, for example.

14.19 **Mach's Principle**: The contents of this chapter serve to show that several widely held views about gravitics are not only intuitively unsatisfactory, but simply wrong. *Space* is not, in itself, curved; and gravitics is not geometry. The ether is "curved"; that is, it is variable from place to place in euclidean space. Neutral bodies move as their constituent particles move, and these have what appear to be extended particle characteristics such as energy and momentum. The change in momentum a neutral body undergoes depends only on its local motion with respect to the ether where it is presently located. If all matter were eliminated from the universe, a single electron, or any other particle, would still exhibit its energy and momentum characteristics. Mach's principle is a non-intuitive fallacy, and should be rejected from physics. Geometrized space has no place in physics either; so let the confusion end, by eliminating it from the picture.

CHAPTER 15

THE CONSERVATION LAW

15.1 **Introduction**: For more than 150 years, starting with mechanical systems, the fact that certain quantities such as energy, momentum, etc. are constant in physical processes has led to an increasing number of conservation laws. With the advent of quantum physics, new conserved quantities, such as baryon and lepton numbers, have been found. In these new cases, the question of just what is being conserved arises. Moreover, it is clear that the same lack of understanding applies to the "classical" laws, since no conventional theory explains just what "energy" or "momentum" really are, for example.

Recently, much emphasis has been placed on the related transformation symmetry properties, and the realization that gauge transformation symmetries are the source of certain quantum conservation laws. However, in spite of the insight this approach has provided, in no case has true understanding of "what it is" that is conserved been forthcoming.

The following account suggests that, rather than the multiplicity of conservation laws now in use, a single conservation law produces all of the effects now ascribed to the many; and further, the one quantity that is being conserved is shown to be the ether.

15.2 **The Conservation Law**:[1] In Section 2.5 the *conservation of ether* was described by the *kinematic* relationship known as the continuity equation,

$$\nabla\bullet\left(\phi_a \mathbf{V}\right) = -\frac{\partial \phi_a}{\partial t} \quad ; \tag{15.2.1}$$

and in Section 2.16 its separated forms were given as,

$$\nabla\bullet\left(\overline{\overline{\phi_a}}\,\mathbf{V}\right) + \frac{\partial \overline{\overline{\phi}}_a}{\partial t} = 0 \quad , \quad \nabla\bullet\left\{\phi_a \mathbf{V}\right\}_{\bullet} + \frac{\partial \phi_{\bullet}}{\partial t} = 0 \quad . \tag{15.2.2}$$

Each of these equations is a derived conservation law that holds because of the basic ether conservation law of Eq.(15.2.1). The second equation indicates that ether is conserved during the passage of any ℓ-wave. The first equation ensures that ether is conserved in bulk motion. It is often written as,

$$\nabla\bullet(\overline{\overline{\phi}}\mathbf{u}) + \frac{\partial \overline{\overline{\phi}}}{\partial t} = 0 \quad . \tag{15.2.3}$$

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1. R.H.Dishington, Apeiron, **5**, p.1, Jan-Apr (1998)

With Eqs.(2.15.1) and (9.2.2), the continuity equation becomes the Lorentz "gauge" condition. Thus, rather than an arbitrary choice for convenience, the Lorentz gauge is the only one that has physical *significance.* The bulk conservation equation it represents is the basis for all of the other conservation laws now known. Several important examples of this will now be presented.

15.3 **Charge Conservation**: The conservation of charged *particles* is well known and easily derived using Maxwell's macroscopic equations. It has an exact parallel in the microscopic case. To show that the distributed charge distortion in the ether is conserved, add the divergence of Eq.(2.21.2) to the partial time derivative of Eq.(2.20.2) and transpose the signs, with the result,

$$\nabla\bullet\left(\rho\mathbf{u}\right)+\frac{\partial\rho}{\partial t}=-\nabla^2 C+\frac{1}{c_0^2}\frac{\partial^2 C}{\partial t^2} \quad , \tag{15.3.1}$$

where,

$$C=\nabla\bullet(\overline{\overline{\phi_a}}\,\mathbf{V})+\frac{\partial\overline{\overline{\phi}}_a}{\partial t}=0 \quad . \tag{15.3.2}$$

So because, and only because, ether is conserved according to Eq.(15.2.2), distributed charge distortion is conserved. It is easy to show that this means that moving along with the incremental density, the ratio $\rho/\overline{\overline{\phi}}$ is constant, so $\mathbf{u}$ is the velocity of ρ as well as $\overline{\overline{\phi}}$.

15.4 **Conventional E&M Energy Conservation**: It is instructive to review the current status of Electrodynamics. Conventionally the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are defined in terms of forces on whole charged particles. In terms of the ether, they are defined in Eqs.(9.9.1). Maxwell's "force field" equations, i.e. written in terms of $\mathbf{E}$ and $\mathbf{B}$, can be derived, for the case of free charges in space (absence of matter), from the bulk Eqs.(11.2.1), with the result,

$$\nabla\bullet\mathbf{E}=\rho-\frac{1}{c_0^2}\frac{\partial C}{\partial t} \quad , \quad \nabla\times\mathbf{B}=\frac{1}{c_0}\left(\rho\mathbf{u}+\frac{\partial\mathbf{E}}{\partial t}\right)+\frac{1}{c_0}\nabla C \quad , \tag{15.4.1}$$

and the identities,

$$\nabla\bullet\mathbf{B}=0 \quad , \quad \nabla\times\mathbf{E}=-\frac{1}{c_0}\frac{\partial\mathbf{B}}{\partial t} \quad . \tag{15.4.2}$$

Eqs.(15.4.1) reduce to the usual forms only because ether is conserved ($C=0$). Both Eqs.(15.4.1) and (15.4.2) are valid macroscopically and microscopically. Looked at conventionally and macroscopically, energy flow is handled by the use of Poynting's

theorem, which can be derived in the usual way from Eqs.(15.4.1) and (15.4.2), leading to,

$$\nabla\bullet\mathbf{S} + \frac{\partial\varepsilon}{\partial t} + \rho\mathbf{u}\bullet\mathbf{E} = -\nabla\mathcal{C}\bullet\mathbf{E} = 0 \quad , \tag{15.4.3}$$

where,

$$\mathbf{S} = c_0\mathbf{E}\times\mathbf{B} \quad , \quad \varepsilon = \tfrac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) \quad . \tag{15.4.4}$$

Here again, *because ether is conserved*, Eq.(15.4.3) reduces to the usual form. There is no question about the validity of Eqs.(15.4.3) and (15.4.4), since they constitute an identity derived directly from the field equations. The problem with Poynting's theorem is that, contrary to common practice, **S** and ε *do not represent energy flow and density except in very restricted cases*. The strange descriptions of energy flow cited in the literature result from use of the theorem as an energy conservation law when it *cannot* be.

Butler points out (see Section 3.10) that, although the field and force equations of electrodynamics are *covariant* (see Sections B.8 and D.7) under Lorentz transformations, as all valid physical laws must be, Poynting's theorem fails this test in many cases. Generally, energy and momentum form a covariant 4-vector; but, except in very special circumstances the quantities ε, and **S** in Eq. (15.4.4) do not. So even though the relationship in Eq.(15.4.3) holds, ε and **S** cannot be identified as energy density and flow unless they form a covariant 4-vector. Butler has shown that *the condition for this is the total absence of free charge and current sources* ($\rho = 0$) in Eqs.(15.4.1) and (15.4.3). This means that *only in the case of radiation* can ε and **S** represent energy density and flow. There are a few spurious non-radiation cases, where free charge and currents are present but at rest, that allow ε and **S** to transform properly, but appear to predict strange energy flow patterns. However, the correct flow is not given by ε and **S** in these cases, but it can be found by carefully considering the physical condition of the energy as it flows from the sources. So only in the case of radiation will Eqs.(15.4.3) and (15.4.4) represent conservation of energy, and the implications of this from the ether viewpoint will be discussed next.

15.4 **Energy Conservation**: In contrast to the simplicity of charge conservation, energy conservation is complicated. First, energy comes in so many forms. Second, no *conventional* visualization of the internal mechanism is available in many situations. Finally, the whole conventional structure of equivalent energies in the different forms is based on "forces". As discussed in Chapter 12, in the ether, there are no forces. Particles flow. Here the concept of work (force) will be used as a convenience, whereas the different forms of energy have been given a visualizable mechanism (see Eqs.2.19.2 and 9.10.1). Still,

there is, as yet, no overall energy conservation equation, and each of these different forms must be dealt with individually, as is the custom. There are still certain aspects of energy that are not understood, even in the context of the unified field theory; but the following will help to correct some of the present misunderstanding. Only two types of energy, electric and magnetic require discussion here. Most other forms of energy can be understood in terms of those.

15.5 **Electric Energy Conservation**: The most common type of energy is electric. Even the nuclear interaction or so called "strong force" has the same kind of interaction energy mechanism (see Section 6.3). In Section 2.19, the electric energy density was defined as,

$$\varepsilon_e = \frac{1}{2}\left(\left(\nabla\bar{\bar{\phi}}_a\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial\bar{\bar{\phi}}_a}{\partial t}\right)^2\right) \quad , \quad \varepsilon_e = \frac{1}{2}\left(\left(\nabla\bar{\bar{\phi}}\right)^2 - \frac{1}{c_0^2}\left(\frac{\partial\bar{\bar{\phi}}}{\partial t}\right)^2\right) \qquad (15.5.1)$$

It is of interest to examine under what conditions the electric energy distortion is conserved. The usual continuity relationship can be expressed as,

$$\nabla\bullet(\varepsilon_e\mathbf{u}) + \frac{\partial\varepsilon_e}{\partial t} = \varepsilon_e\nabla\bullet\mathbf{u} + \frac{d\varepsilon_e}{dt_\bullet} \quad , \qquad (15.5.2)$$

where $d/dt_\bullet$ is the m.p. derivative (sometimes called convective or material, see Section E.4),

$$\frac{d\varepsilon_e}{dt_\bullet} = \frac{\partial\varepsilon_e}{\partial t} + \mathbf{u}\bullet\nabla\varepsilon_e \quad . \qquad (15.5.3)$$

Using Eq.(15.5.1),

$$\nabla\bullet(\varepsilon_e\mathbf{u}) + \frac{\partial\varepsilon_e}{\partial t} = \varepsilon_e\nabla\bullet\mathbf{u} + \nabla\bar{\bar{\phi}}\bullet\frac{d(\nabla\bar{\bar{\phi}})}{dt_\bullet} - \frac{1}{c_0^2}\frac{\partial\bar{\bar{\phi}}}{\partial t}\frac{d}{dt_\bullet}\left(\frac{\partial\bar{\bar{\phi}}}{\partial t}\right) \quad . \qquad (15.5.4)$$

Expanding the last two terms of Eq.(15.5.4), with some manipulation,

$$\begin{aligned}\nabla\bullet(\varepsilon_e\mathbf{u}) + \frac{\partial\varepsilon_e}{\partial t} = {} & \varepsilon_e\nabla\bullet\mathbf{u} + \nabla\bar{\bar{\phi}}\bullet\left(\nabla(\bar{\bar{\phi}}\nabla\bullet\mathbf{u}) + (\nabla\mathbf{u})\bullet\nabla\bar{\bar{\phi}}\right) \\ & + \frac{1}{c_0^2}\frac{\partial\bar{\bar{\phi}}}{\partial t}\left(\frac{\partial}{\partial t}(\bar{\bar{\phi}}\nabla\bullet\mathbf{u}) + \frac{\partial\mathbf{u}}{\partial t}\bullet\nabla\bar{\bar{\phi}}\right) - \nabla\bar{\bar{\phi}}\bullet\nabla C - \frac{1}{c_0^2}\frac{\partial\bar{\bar{\phi}}}{\partial t}\frac{\partial C}{\partial t}\end{aligned} \qquad (15.5.5)$$

Eq.(15.5.5) shows that even in the few special cases where the first three RHS terms go to zero, *electric energy is conserved only because conservation of ether makes the last two RHS terms zero.* Several special cases will now be considered to illustrate certain important ideas.

The first is that of a particle moving at constant velocity. Even a neutral particle, like a neutron, has an internal ρ distribution, including both positive and negative distortion, which integrates to zero over all space. Although this ρ distribution is uneven in concentration, if the particle moves at constant velocity, the $\mathbf{u}$ vector field throughout the particle is constant in both space and time. Thus, all RHS terms of Eq.(15.5.5) are zero, and electric energy is conserved. When electric energy is conserved, the ratio $\varepsilon_e / \overline{\overline{\phi}}$ is constant, and $\mathbf{u}$ is also the velocity of ε_e.

A more interesting case is that of an electron in a hydrogen atom. The electron and proton orbit about their center of energy as a rigid body, always presenting the same faces to each other. If they are in circular orbits, their angular velocity is constant, say $\mathbf{w}$. At each point the velocity field is $\mathbf{u} = \mathbf{w} \times \mathbf{r}$, which is *constant* in time. The result is that $\nabla \bullet \mathbf{u} = 0$, $\partial \mathbf{u} / \partial t = 0$ and $\nabla \overline{\overline{\phi}} \bullet \nabla \mathbf{u} \bullet \nabla \overline{\overline{\phi}} = 0$, so Eq.(15.5.5) indicates that ε_e is conserved. It follows that all circular orbits will be stable and non-radiating if no other ether condition disturbs the flow.

A final case is that of a positron being accelerated in a straight line by a second higher energy positron approaching from the rear (see Section 3.10). As the accelerating particle's velocity increases, that particle expands laterally by taking on more of the distortion energy ε_e from the driving particle. Since the $\mathbf{u}$ field is changing with time and has divergence, by Eq.(15.5.5) energy ε_e, is *not* conserved. Some of the distortion converts into another form called magnetic energy and is lost by both particles as radiation.

In the previous case of the hydrogen atom, the datum fluctuations (zero-point fluctuations) buffeting the orbiting electron are large enough, in all but the ground state, to force radiation to occur. In other situations, similar failures of conservation of electric energy can result when conversion of ε_e to magnetic energy occurs.

15.6 **Magnetic Energy Conservation**: Unlike electric energy, which is a simple, localized distortion condition at each space point, magnetic energy is only partially localizable and comes in two different forms. In Section 9.10 magnetic energy density is defined as,

$$\varepsilon_m = \frac{1}{2c_0^2}\left(\left(\nabla \times \overline{\overline{\phi_a}} \mathbf{V} \right)^2 + \frac{1}{c_0^2}\left(\frac{\partial \overline{\overline{\phi_a}} \mathbf{V}}{\partial t} \right)^2 \right)_r \quad , \qquad (15.6.1)$$

where *the subscript r indicates serious restrictions in applying this equation*, which falsely *implies* that energy is stored in any region where there is vorticity in the flow or the flow is changing with time.

In situations where work is required to generate a vortex and where that work is *recoverable* in stopping the vortex, the first term in Eq.(15.6.1) gives the correct energy stored. Conversely, where a vortex exists that required no work to generate it and where no work is *recoverable*, the first term in Eq.(15.6.1) is not applicable (see Sections 9.8---9.10). Assuming, for the moment, that the correct magnetic energy density distribution is known, it might be supposed that a form similar to Eq.(15.5.5) should follow.

According to Eq.(15.5.5), electric energy is conserved only when all RHS terms are zero, otherwise some of the energy is being converted into another form. Similarly, if a given magnetic configuration conserves energy, the conservation criterion would be expressed as,

$$\nabla\bullet\left(\varepsilon_m \mathbf{u}\right)+\frac{\partial\varepsilon_m}{\partial t}=0 \quad ; \qquad (15.6.2)$$

and the problem of writing a conservation equation with localized magnetic energy appears at once. In looking at electric energy conservation, its *bulk* nature allows the observer to "see" where the energy is and where it is going; and the meaning and measure of its velocity **u** is obvious. Not always so in the magnetic case. In magnetic problems, visualizing a *velocity* of energy flow is a very abstract process. Simple illustrations will make this clear.

15.7 **Energy in a Long, Straight Current Conductor**: A simple example discussed in the literature is that of a long, straight conducting wire through which current is steadily driven by a battery. If the leads connecting the wire to the voltage source have negligible resistance, then the potential, neglecting end effects, is uniform across the wire's interior and the electric field inside and just outside the wire is uniform, with the equipotentials perpendicular to the length of the wire. As long as the current is steady state, the energies stored in the magnetic field around the wire and the electric field are constant, so no energy from the battery goes into the vortex or the electric field after they are established. The energy that goes into heat (collisions of conduction electrons with atoms) is carried by the electrons as their individual electric kinetic energies. It is stored in a sphere about each electron that is less than 10^{-11}cm in radius and none of it gets near the region outside the wire. Clearly, in this case, the Poynting theorem makes no sense; since it indicates that the steady state energy from the battery doesn't go directly into the wire with the electrons, but leaves the source and travels through the space around the wire, entering it radially through its long cylindrical surface.

If the battery is shorted, so that the current decays, the vortex is slowed and the excess vortex energy contributes to the heating of the wire. In both this situation and during the vortex startup, it is

natural to imagine the vortex energy moving radially inward or outward; but in the steady state, Poynting's theorem fails to correctly describe energy flow. Even in the transient periods, it requires modification.

To understand the problem here, consider the *steady state*, where conventionally the energy flow of Eq.(15.4.4) would be written,

$$\mathbf{S} = -c_0 \nabla \bar{\bar{\phi}} \times \mathbf{B} \neq 0 \quad . \tag{15.7.1}$$

In the *transient* case, the conventional flow takes the form,

$$\mathbf{S} = -c_0 \left(\nabla \bar{\bar{\phi}} + \frac{1}{c_0} \frac{\partial \mathbf{A}}{\partial t} \right) \times \mathbf{B} \neq 0 \quad . \tag{15.7.2}$$

Eq.(15.7.2) *correctly* indicates there is radial energy flow into and out of the wire in the transient periods, but Eq.(15.7.1) *falsely* indicates that energy flows radially into the wire during steady state. The latter could be remedied if it is concluded that *the steady gradient and vortex have no direct influence on each other.* In fact, even in the transient case they don't. Thus, a possible correct form for the magnetic energy flow vector would be,

$$\mathbf{S} = -\frac{\partial \mathbf{A}}{\partial t} \times \mathbf{B} \neq 0 \quad . \tag{15.7.3}$$

This not only corrects the long straight wire problem, but clears up a number of other questions related to magnetic energy conservation.

15.8 **A Modified Poynting Relationship for the Magnetic Field**: It is important to understand that the Poynting theorem has nothing to do with electric energy density. It describes a magnetic process. Neglecting, for the moment, any *restrictions* on magnetic energy, from Eq.(15.6.1) the magnetic energy density can be characterized by,

$$\varepsilon_m = \varepsilon_v + \varepsilon_t \quad , \tag{15.8.1}$$

where the vortex and transformer energy density components are represented by,

$$\varepsilon_v = \frac{1}{2c_0^2} (\nabla \times \bar{\bar{\phi}}_a \mathbf{V})^2 \quad \text{and} \quad \varepsilon_t = \frac{1}{2c_0^4} \left(\frac{\partial \bar{\bar{\phi}}_a \mathbf{V}}{\partial t} \right)^2 \quad . \tag{15.8.2}$$

Now, following Eq.(15.7.3), let,

$$\mathbf{S} = -\frac{1}{c_0^2} \frac{\partial \bar{\bar{\phi}}_a \mathbf{V}}{\partial t} \times (\nabla \times \bar{\bar{\phi}}_a \mathbf{V}) \quad , \tag{15.8.3}$$

and its divergence,

$$\nabla\bullet\mathbf{S} = -\frac{1}{c_0^2}\nabla\bullet\left(\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t}\times(\nabla\times\overline{\overline{\phi_a\mathbf{V}}})\right) \quad . \qquad (15.8.4)$$

Aided by an identity from Table D.6.2, the divergence can be expanded to yield,

$$\nabla\bullet\mathbf{S} = -\frac{1}{c_0^2}\left((\nabla\times\overline{\overline{\phi_a\mathbf{V}}})\bullet\nabla\times\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t} - \frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t}\bullet(\nabla\times\nabla\times\overline{\overline{\phi_a\mathbf{V}}})\right) \quad . \qquad (15.8.5)$$

After some manipulation, assuming $\overline{\overline{\phi}} = 0$,

$$\nabla\bullet\mathbf{S} = -\frac{\partial}{\partial t}\left(\frac{(\nabla\times\overline{\overline{\phi_a\mathbf{V}}})^2}{2c_0^2}\right) - \frac{1}{c_0^2}\frac{\partial\overline{\overline{\phi_a\mathbf{V}}}}{\partial t}\bullet\nabla^2\overline{\overline{\phi_a\mathbf{V}}} \quad . \qquad (15.8.6)$$

Combining Eqs.(11.2.1), (15.8.1) and (15.8.6), assuming no free charge present ($\rho = 0$),

$$\frac{\partial}{\partial t}(\varepsilon_v + \varepsilon_t) = -\nabla\bullet\mathbf{S} \quad . \qquad (15.8.7)$$

Except for the absence of charge here, Eq.(15.8.7) has the same *form* as Eq.(15.4.3), but the definitions of **S** and ε are different. *Both equations represent rigorously correct identities derived directly from Maxwell's equations.* Both sometimes represent conservation of energy. Eq.(15.8.7) produces fewer spurious descriptions of energy flow, but even it is not necessarily the basic *magnetic* energy conservation law. If it is assumed that Eqs.(15.8.1) and (15.8.2) correctly represent the magnetic energy density, then, in cases where Eq.(15.8.7) fails as a conservation law, **S** cannot simply represent the magnetic energy *flow*. What then is the meaning of the two identities in Eqs.(15.4.3) and (15.8.7)? In each, $-\nabla\bullet\mathbf{S}$ *gives the rate at which the* <u>*designated*</u> *"density" is changing at a given point in the field*. In Eq.(15.4.3), the quantity $(\mathbf{E}^2 + \mathbf{B}^2)/2$ is <u>*not*</u> the magnetic energy density, but still is a measurable quantity, and $-\nabla\bullet\mathbf{S}$ gives its time rate of change. In Eq.(15.8.7) the quantity $\varepsilon_m = \varepsilon_v + \varepsilon_t$ <u>*is*</u> the magnetic energy density. However, since ε_m can change by both a flow, toward or away from the point, and also by a conversion, from or to another form of energy, $-\nabla\bullet\mathbf{S}$ represents the time rate of change of ε_m due to both the flow and the conversion. On the other hand, the magnetic energy conservation Eq.(15.6.2) represents only the flow.

15.9 **The Magnetic Energy Conservation Relationship**: By following along the same path taken in Section 15.5, the correct magnetic energy conservation relationship should be derivable. Replacing ε_e in Eqs.(15.5.2) and (15.5.3) with ε_m, and using Eqs.(15.8.1) and (15.8.2), the magnetic equivalent of Eq.(15.5.4) becomes,

$$\nabla\bullet(\varepsilon_m \mathbf{u}) + \frac{\partial \varepsilon_m}{\partial t} = \varepsilon_m \nabla\bullet \mathbf{u} + \frac{1}{c_0^2}(\nabla \times \overline{\overline{\phi_a}}\mathbf{V})\bullet\frac{d}{dt_\bullet}(\nabla \times \overline{\overline{\phi_a}}\mathbf{V}) + \frac{1}{c_0^4}\frac{\partial \overline{\overline{\phi_a}}\mathbf{V}}{\partial t}\bullet\frac{d}{dt_\bullet}\left(\frac{\partial \overline{\overline{\phi_a}}\mathbf{V}}{\partial t}\right) \quad . \qquad (15.9.1)$$

Unfortunately, no useful expansion of Eq.(15.9.1) to the magnetic equivalent of Eq.(15.5.5) has yet been found; but even if it had been, no practical method for determining $\mathbf{u}$ is available. It may be that the resolution of this problem must await a derivation that involves the simultaneous conservation of at least electric and magnetic energy, including radiation.

In spite of the apparent difficulties in visualizing magnetic energy processes, there is still much that can be understood. This is brought out by considering several special cases.

15.10 **Energy Conservation in Radiation**: Because of the sketchy knowledge of the internal structures of the c-ons, no energy conservation formalism is available for them. However, *antenna* radiation fits well into the magnetic energy conservation picture. Considering the discussion in Section 9.12, and combining Eqs.(15.8.1), (15.8.3), (9.12.2) and (9.12.3), the antenna radiation conservation equation can be written ($\rho = 0$),

$$\nabla\bullet(\varepsilon_m \mathbf{u}) + \frac{\partial \varepsilon_m}{\partial t} = \nabla\bullet \mathbf{S} + \frac{\partial \varepsilon_m}{\partial t} = 0 \quad . \qquad (15.10.1)$$

In this case, *the modified Poynting theorem is a true energy conservation equation, and the energy and momentum it gives are the correct values according to the ether theory*.

In Eq.(15.10.1) the velocity is $\mathbf{u} = u\mathbf{n}$, where $\mathbf{n}$ is a unit vector in the direction of propagation, and ε_m has *equal* contributions from both terms of Eq.(15.8.1) as described in Section 9.12. To find $\mathbf{u}$, set,

$$\mathbf{u} = \frac{\mathbf{S}}{\varepsilon_m} \quad . \qquad (15.10.2)$$

Combining Eqs.(9.12.3) and (15.8.3),

$$\mathbf{S} = \frac{1}{c_0}(\nabla \times \overline{\overline{\phi_a}}\mathbf{V})^2 \mathbf{n} \quad . \qquad (15.10.3)$$

Now, substituting Eqs.(9.12.2) and (15.10.3) into Eq.(15.10.2),

$$\mathbf{u} = c_0\mathbf{n} \quad . \tag{15.10.4}$$

In this case it is well known that the energy travels with velocity c_0.

15.11 **Solenoid With Decaying Current**: If one of the long solenoids of Section 9.3, wound with high resistance wire, has a D.C. voltage applied to the ends of the coil with very low resistance connecting leads, the steady state physics is very similar to that of the long straight wire of Section 15.7, except that the vortex energy is stored inside the coil. A sudden short circuiting of the coil produces a result similar to the straight wire case. The potential gradient between the ends of the coil is reduced essentially to zero and remains that way. In a somewhat oversimplified visualization, *the only physical actions on the electrons now are their stopping upon colliding with atoms, and the decreasing* $\overline{\overline{\phi_a \mathbf{V}}}$ *in the vortex field*. When an electron passes its kinetic energy to an atom, and stops, it finds the primary inertial system accelerating in the direction of the average electron motion, so the electron accelerates along with it and gains kinetic energy again. The process repeats for all conduction electrons until the vortex is gone. If the vortex energy is to be considered actually localized, then there will be a flow of magnetic energy *outward* to the electrons in the coil, but *the velocity* $\mathbf{u}$ *is not determined easily*.

Using the geometry of Section 9.3, the transient decay solution of Eqs.(11.2.1) for the flow field $\overline{\overline{\phi_a \mathbf{V}}}$ is found in terms of Bessel functions to be,

$$\overline{\overline{\boldsymbol{\phi}_a \mathbf{V}}} = \hat{\boldsymbol{\alpha}}(\overline{\overline{\boldsymbol{\phi}_a \mathbf{V}}})_0 \frac{I_1(\mathcal{Z})}{I_1(\mathcal{Z}_0)} \boldsymbol{\varepsilon}^{-t/\boldsymbol{\tau}}\, \varepsilon^{-t/\tau} \quad , \tag{15.11.1}$$

where $\mathcal{Z} = R/c_0\tau$, $\mathcal{Z}_0 = R_0/c_0\tau$, R_0 is the coil radius and τ is the time constant of the decay ($\tau = \mathbb{L}/\mathbb{R}$, where $\mathbb{L}$ is the coil inductance and $\mathbb{R}$ the coil resistance, both per unit coil length). The flow is zero at the coil axis and $(\overline{\overline{\phi_a \mathbf{V}}})_0$ at R_0.

From Eq.(15.11.1),

$$\varepsilon_v = \frac{(\overline{\overline{\phi_a \mathbf{V}}})_0^2}{2c_0^4\tau^2} \frac{\left[I_0(\mathcal{Z})\right]^2}{\left[I_1(\mathcal{Z}_0)\right]^2} \varepsilon^{-2t/\tau}, \text{ and } \varepsilon_t = \frac{(\overline{\overline{\phi_a \mathbf{V}}})_0^2}{2c_0^4\tau^2} \frac{\left[I_1(\mathcal{Z})\right]^2}{\left[I_1(\mathcal{Z}_0)\right]^2} \varepsilon^{-2t/\tau} \quad , \tag{15.11.2}$$

leading to the magnetic energy density,

$$\varepsilon_m = \frac{(\overline{\overline{\phi_a \mathbf{V}}})_0^2}{2c_0^4\tau^2} \frac{\left[I_0(\mathcal{Z})\right]^2 + \left[I_1(\mathcal{Z})\right]^2}{\left[I_1(\mathcal{Z}_0)\right]^2} \varepsilon^{-2t/\tau} \quad . \tag{15.11.3}$$

The ratio of transformer energy density to vortex energy density is,

$$\frac{\varepsilon_t}{\varepsilon_v} = \frac{\left[I_1(\not{Z})\right]^2}{\left[I_0(\not{Z})\right]^2} \quad . \tag{15.11.4}$$

For wide range of coil sizes, $\not{Z} = R / c_0\tau$ is the order of 10^{-4} or smaller, so the ratio in Eq.(15.11.4) is less than 10^{-7}.

The picture that unfolds is this. The steady state solution has no transformer energy, only vortex; but when the coil is shorted and the electric field collapses, a small residual *transformer* energy is produced as the decay starts. According to Eq.(15.11.4), ε_t remains extremely small relative to ε_v, which is quite different from the radiation case of Section 15.10, where ε_t and ε_v were *equal*. This is a good indication that the modified Poynting theorem is extremely close to being a magnetic energy conservation law in this case; because, if any radiation takes place during the decay, the vortex part of the radiation will be of the same order as ε_t, which makes the radiation essentially negligible.

It is possible to say, in this case, that $\mathbf{S} = \varepsilon_m \mathbf{u}$, where ε_m includes both the coil energy and the radiation energy. In that case, the determination of $\mathbf{u}$ is not obvious, because the radiation component velocity is c_0, but the coil vortex component would have a lower speed.

Where the radiation is negligible, $\mathbf{u}$ can be calculated using Eq.(15.10.2), with the result,

$$\mathbf{u} \cong \hat{\mathbf{R}} \frac{R}{\tau} \quad . \tag{15.11.5}$$

In some ways this is a satisfying result, but in others it is unconvincing. The velocity is zero at $R = 0$, and increases linearly out to R_0, which is not surprising, but it is not a function of time, which seems odd. Clearly, much more work on magnetic energy conservation is needed.

15.12 **Other Conservation Laws**: It is clear from the earlier discussion that the conventional description of the quantities being conserved required revisions. The other conservation laws can be understood by considering those same revisions. For example, as described in Section 3.11, momentum and inertia derive from the fact that it takes *time* for electric energy distortion in a particle to physically redistribute itself into another particle and be separated from the first. Ether conservation is again central to the process. Even in cases involving presently unexplained phenomena such as

baryon and lepton number conservation, it is simple to show ether conservation to be the basis.

In particle experiments, conversion occurs when a single pseudo-stable particle redistributes to a less distorted configuration, or splatter produces a number of by-products during a cataclysmic collision in an interaction between particles (see Sections 5.27 and 12.12). In analyzing which interactions are possible and which are not, it has been found that certain numbers assigned to particles are always conserved, leading to baryon and lepton number conservation. What is conserved in these interactions is ether.

A simple example of this is one used by Feynman.[1] Proton-proton bombardment is used to produce anti-protons by the reaction,

$$P + P \rightarrow P + P + P + \bar{P} \quad ; \qquad (15.12.1)$$

but not by,

$$P + P \not\rightarrow P + P + \bar{P} \quad \text{or} \quad P + P \not\rightarrow P + \bar{P} \quad . \qquad (15.12.2)$$

Violation of baryon number conservation is the conventional explanation. However, from Chapter 5, P and $\bar{P}$ have opposite charge distortions and the $\bar{\bar{\phi}}$ ether density patterns are also opposites. Thus, $P + \bar{P}$ represents zero net ether increment, whereas $P + P$ represents a *large* ether increment. To conserve ether, there must be the original $P + P$ increment and no more. $P + \bar{P}$ adds no more. Both of the interactions of Eq.(15.12.2) violate ether conservation. In fact, if vortex conservation is included, all of the cases of baryon and lepton conservation, as conventionally described, are seen to be cases of ether conservation.

All conservation laws can be traced back to the single conservation of ether law. In the future, new conservation laws can be found by examining phenomena in the light of the ether physics involved. Still to come will be a *total* energy conservation law, probably to be derived using a four dimensional formalism along the lines investigated by Kirkwood.

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1. R.P.Feynman, R.B.Leighton, and M.Sands, The Feynman Lectures on Physics, **3**, p 25-4, Addison-Wesley Publ. Co., Reading, Mass. (1965).

APPENDIX A

REPRESENATION OF FIELDS

A.1. **Introduction**: In the next few appendices some tools for working with fields are presented. Although such things as scalar, vector, and dyadic fields will be discussed in some detail later on, it is assumed that the reader already has a good grounding in vector analysis and dyadics, preferably from Weatherburn's two little books, "Elementary Vector Analysis" and "Advanced Vector Analysis.[1] In this appendix, the main *physical* characteristics of fields are defined in terms of scalars, vectors and dyadics. Such fundamental concepts as divergence and circulation are discussed, and the Laplacian is singled out as the most important field property.

A.2 **Physical Scalars, Vectors, and Tensors**: Many space distributed physical entities are representable by scalars, vectors, or tensors (particularly second order tensors). The simplest of these are the *scalars*, which are used to describe such fields as the temperature at each point in a room or the density of a gas at each point in some volume, i.e., fields completely characterized by a single magnitude at each point in space and nothing more.

The velocity flow pattern on the surface of a stream of water is familiar to almost everyone, and it is easy to visualize its extension to the space distributed flow pattern beneath the surface. At each point in the space, the fluid has a specific speed in some direction. The combination of a magnitude and a direction at each point in space typifies a *vector* field. In this case it is the velocity vector.

Next in this hierarchy are the *dyadics* (second order tensors). There are few good examples, common to lay experience, to illustrate what a dyadic symbolizes. Going back to the scalar that represents the density ϕ of a gas, *if a particular point in space is designated*, the scalar field function ϕ specifies the density at that point. In the vector case representing the velocity $\mathbf{V}$, *if a particular point in space is selected*, the vector field function $\mathbf{V}$ specifies the magnitude and direction of the velocity at that point. In the case of a dyadic, *if a particular point in space is selected and an "input" or cause vector* $\mathbf{a}_1$ *at that point is specified (both magnitude and direction)* the diadic Ψ specifies a resultant or effect vector $\mathbf{b}_1$ at that point which has a magnitude and direction that can be completely different from those of the initial vector given. Furthermore, *if at that same point* a different initial vector $\mathbf{a}_2$ is considered, the "output" vector $\mathbf{b}_2$ given by the

1. C.E.Weatherburn, Elementary Vector Analysis, (1948); Advanced Vector Analysis, (1947), G.Bell and Sons, London.

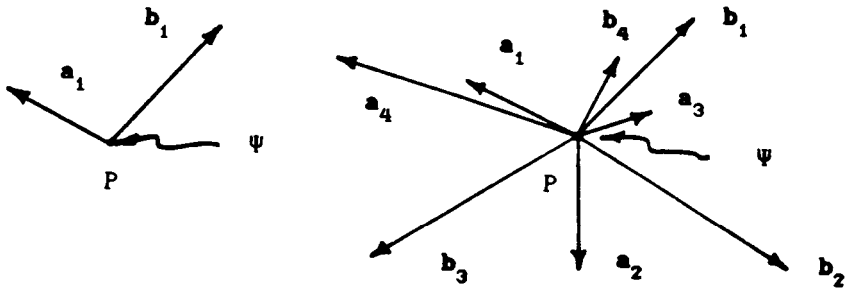

Figure A.2.1 Dyadic input-output vector family.

field dyadic Ψ can also be different from the previous "output" vector. This is illustrated in Figure A.2.1. In fact, a dyadic is a field function such that, for every cause vector $\mathbf{a}_i$, *specified at a particular point in space*, a corresponding effect vector $\mathbf{b}_i$ at that point is given by the dyadic. At a different point in space Ψ is different, and the cause-effect relationship, although still relating the total cause family to the total effect family, can be completely different from that at any other space point.

The merry-go-round is a familiar example of a physical situation describable by a dyadic. If a person on the turning platform walks radially outward at constant speed, since he passes to a region of the platform that moves more rapidly, he must accelerate in a direction perpendicular to his own instantaneous radial velocity. He can "feel" the opposition to this acceleration. In this case, his radial velocity is the cause vector and the effect vector is the acceleration he experiences. If, on the other hand, he walks perpendicularly to the radius in the direction of the platform motion, he will be accelerated inwardly along the radius. In fact, the effect vector acceleration $\mathbf{a}$ at a point is always at right angles to the cause velocity $\mathbf{u}$ of his motion relative to the platform, regardless of which direction the cause velocity vector takes. A planar dyadic Ω can be used to relate $\mathbf{u}$ to $\mathbf{a}$ for all points on the platform and for any $\mathbf{u}$.

A.3 **The Perfect Continuum, Ether**: The symbols to be presented in the following describe *physical* processes that can be intuitively visualized in terms of the ether which is the only perfect continuum. It is this picture that transfers over to the particulate fields mentioned before and the field mathematics applies to the latter only to the extent that for some range of parameters those fields (gases, liquids, solids, etc.) appear to fit the continuum model. Only in the case of the ether will it be assumed that the reality and symbols match identically.

The most important of these properties of fields are the "gradient", the "divergence", the "curl" and the "laplacian". Each one represents a simple, visualizable physical description of some condition in the field. Each one can also be calculated using a mathematical equation that differs from the others. To truly understand fields, it is essential that the *physical* visualizations be comprehended unencumbered by the equations. Around the end of the 19th century, science writers actually used two ways to designate each of these properties. For example the "gradient of ϕ" was called "gradϕ", to indicate the physical visualization, and $\nabla\phi$ (pronounced del phi) to mathematically represent the gradient in equations and calculations. This practice has been abandoned, for simplicity; but it places a burden on the practitioner to keep in mind the *physical* visualisation of $\nabla\phi$ when generally describing the field, reserving the mathematical concept for calculation.

A given field is described by an equation that tells its value at each space point as a function of time, but nothing more. Several important properties of a field are uncovered by a detailed examination of the conditions in a differential volume around any chosen space point at any instant of time. This is done using a differential vector of length ds that points in the direction of an arbitrary unit vector $\hat{\mathbf{s}}$, so that,

$$d\mathbf{s} = \hat{\mathbf{s}}ds \quad . \tag{A.3.1}$$

The region that can be examined around each space point is a small sphere of radius ds. This is the starting point for all of the physical functions to be described in the following sections.

A.4 **The Directional Derivative and Gradient of a Scalar Field**: A valuable property of any smoothly varying scalar field is that its *space* derivatives describe another *vector* field, call it $\mathbf{G}$, that permits easy calculation of the *directional derivative* of ϕ,

$$\frac{d\phi}{ds} = \hat{\mathbf{s}}\cdot\mathbf{G} \quad , \tag{A.4.1}$$

where the ordinary dot product is implied. It is easy to show that the magnitude of $\mathbf{G}$ represents the maximum *space* rate of change of ϕ at the point, defined as,[1]

$$\frac{d\phi}{dn} = \left(\frac{d\phi}{ds}\right)_{max} \quad , \tag{A.4.2}$$

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1. See the neat derivation given in Vector Analysis, J.W.Gibbs, E.B.Wilson, p. 139, Dover Publications, Inc, N.Y.; also C.E.Weatherburn, Advanced Vector Analysis, G.Bell and Sons, London (1947).

where the direction of the maximum is set by the unit vector $\mathbf{n}$, and,

$$\mathbf{G} = \mathbf{n}\frac{d\phi}{dn} \quad . \tag{A.4.3}$$

Combining Eqs.(A.4.1) and (A.4.3),

$$\frac{d\phi}{ds} = \hat{\mathbf{s}}\bullet\mathbf{n}\frac{d\phi}{dn} = \frac{d\phi}{dn}\cos\theta_{sn} \quad , \tag{A.4.4}$$

so that, if $\hat{\mathbf{s}}$ is in the direction of $\mathbf{n}$, then Eq.(A.4.2) is satisfied. If $\theta = 90°$, $d\phi/ds = 0$, which shows that $\mathbf{n}$ is *normal* to the surfaces of constant ϕ in the field. Because Eq.(A.4.3) has a similarity to the idea of the slope of a hillside, $\mathbf{G}$ *is called the gradient of* ϕ.

There is a mathematical function $\nabla\phi$ (called del phi) that involves space derivatives of ϕ. It permits calculation of the gradient in Eq.(A.4.3) because, as discussed in Appendix C, in any coordinate system,

$$\mathbf{G} = \nabla\phi \quad . \tag{A.4.5}$$

Modern usage, where the emphasis is on computation rather than visualization, omits $\mathbf{G}$, and calls $\nabla\phi$ the gradient; but it is important to remember that Eq.(A.4.3) is what the gradient of a scalar field truly represents and the mathematical equation for $\nabla\phi$ is just a calculation aid. From here on the symbol $\nabla\phi$ will have two meanings, that of both $\mathbf{G}$ and the mathematical form $\nabla\phi$.

A.5 **The Line Integral of a Vector, and the Scalar Potential**: If a curved path cuts through a region in which some vector field $\mathbf{V}$ is present, the projection of $\mathbf{V}$ onto any differential segment of the path $d\mathbf{s}$ is $\mathbf{V}\bullet d\mathbf{s}$. The integral of the projection along the path between any two points on the path is called the line integral of $\mathbf{V}$, and the result is generally a function of the points and the path. It can be shown that *in the special case where the vector is the gradient of a scalar field* ϕ,

$$\int_1^2 \nabla\phi\bullet d\mathbf{s} = \phi_2 - \phi_1 \quad . \tag{A.5.1}$$

The importance of this relationship lies in the fact that first, since the value of the integral depends only on the values of ϕ at the two end points, it is independent of the path along which $\nabla\phi\bullet d\mathbf{s}$ is integrated, which is the basis for defining the concept of the "potential"; and

second, if the path is closed, i.e., the integration is around a loop that starts and ends at the same point, then $\phi_2 = \phi_1$ and,

$$\oint \nabla\phi \bullet d\mathbf{s} = 0 \quad . \tag{A.5.2}$$

Eqs. (A.5.1) and (A.5.2) express two remarkable properties of the gradient of a scalar field.

The converse of Eq.(A.5.2) is also of fundamental importance. If the line integral of a vector field $\mathbf{V}$ is zero *around every closed loop*, i.e., if,

$$\oint \mathbf{V} \bullet d\mathbf{s} = 0 \quad , \tag{A.5.3}$$

then there is a scalar field ϕ of which $\mathbf{V}$ is the gradient; i.e. $\mathbf{V} = \nabla\phi$.

If the condition Eq.(A.5.3) is satisfied by a vector $\mathbf{V}$, it is equally well satisfied by the vector $\mathbf{E} = -\mathbf{V}$, and since the vector actions caused by fields are often "downhill", i.e. opposite in direction to the gradient, the relationship just following Eq.(A.5.3) is just as often written $\mathbf{E} = -\nabla\phi$.

A.6 **The Directional Derivative of a Vector Field**: It is just as easy to visualize a directional derivative in a *vector* field $\mathbf{V}$, since it is clear that a small displacement results in a new vector $\mathbf{V}$, and the change in $\mathbf{V}$ is the difference. Even the *expression* for the directional derivative,

$$\frac{d\mathbf{V}}{ds} = \hat{\mathbf{s}} \bullet \Psi \quad , \tag{A.6.1}$$

is similar to Eq.(A.4.1), except that the "change " vector $\mathbf{G}$ must be replaced with a "change" *dyadic* so that the dot product will be a *vector*. Not too surprising is the fact that the "change" dyadic Ψ can again be calculated using the same kind of mathematical operation, as expressed by,

$$\Psi = \nabla\mathbf{V} \quad . \tag{A.6.2}$$

Here, however, the correspondence ends. There is no simple physical analogy to the *gradient* of a scalar.

Since $\mathbf{V}$ is a vector field, it has not only a magnitude at each point, but a direction, and when the change in $\mathbf{V}$ is determined from some displacement, it must reflect both the change in magnitude and the change in direction. For this reason, only in special cases can a gradient of a vector field have the same intuitive meaning that a scalar field gradient has.

The simple laminar flow field inside a straight cylindrical tube conducting fluid illustrates this. The flow can be described best in cylindrical coordinates, (R, α, z), and is three dimensional with

rotational symmetry about the flow axis z. The velocity vector $\mathbf{V}$ is smaller as the radial distance increases. The direction $\hat{\mathbf{R}}$ is always radially outward from the flow axis, $\mathbf{k}$ is parallel to the axis, and $\hat{\boldsymbol{\alpha}}$ is tangent to circles about the axis that are centered on that axis. If a differential displacement ds is taken in the direction of $\hat{\mathbf{R}}$, $\mathbf{V}$ decreases in magnitude and $d\mathbf{V}$ is a negative vector, i.e. pointing in a direction opposite to $\mathbf{k}$. Because the velocity is assumed not to vary in a direction parallel to the axis, if ds is taken in the direction of $\mathbf{k}$, $d\mathbf{V}$ is zero. Similarly, by the rotational symmetry, ds taken in the direction of $\hat{\alpha}$ gives no change in $\mathbf{V}$, so $d\mathbf{V} = 0$. In this special case, then, the velocity vector $\mathbf{V}$ is constant on cylindrical surfaces, and a displacement in position can, in some directions, produce a change in the magnitude of $\mathbf{V}$, but never results in a change in the direction of $\mathbf{V}$. The latter is obvious, since the velocity field originally defined has only one direction. In a field as simple as this, where $\mathbf{V} = \mathbf{k}V$, the *scalar* V has a gradient ∇V, and the concept can easily be extended to $\mathbf{V}$. However, in general fields, more often than not, any change in $d\hat{\mathbf{s}}$ produces a change in direction of a component of flow as well as a change in magnitude. Thus, *there is no simple dyadic analogy to the "gradient" of a scalar field*.

A.7 **The Line Integral of a Dyadic, and the Vector Potential**: A parallel can be drawn between the line integral of a field dyadic and that discussed in Section A.5 for a vector. If a curved path cuts through a region in which some dyadic field Ψ is present, the projection of Ψ onto any differential segment of the path $d\mathbf{s}$ is $\Psi \cdot d\mathbf{s}$. The integral of the projection along the path between any two points on the path is the line integral of Ψ, and the result is again a function of the points and the path. It can be shown that *in the special case where the dyadic is the gradient of a vector field* $\mathbf{V}$,

$$\int_1^2 d\mathbf{s} \cdot \nabla \mathbf{V} = \mathbf{V}_2 - \mathbf{V}_1 \quad . \tag{A.7.1}$$

Again, the integral is independent of the <u>path</u> along which the integral is taken, which is the basis for the concept of the "vector potential". Also, if the path is <u>closed</u>, i.e., the integration is around a loop that starts and ends at the same point, then $\mathbf{V}_2 = \mathbf{V}_1$ and,

$$\oint d\mathbf{s} \cdot \nabla \mathbf{V} = 0 \quad . \tag{A.7.2}$$

Eqs. (A.7.1) and (A.7.2) express two remarkable properties of $\nabla \mathbf{V}$. The converse of Eq.(A.7.2) is also of fundamental importance. If the

line integral of a dyadic field Ψ is zero *around every closed loop*, i.e., if,

$$\oint \Psi \cdot d\mathbf{s} = 0 \quad , \tag{A.7.3}$$

there is a vector field $\mathbf{V}$ of which $\Psi = \nabla\mathbf{V}$.

A.8 **The Divergence of a Vector Field**: Another simple but very important property of a vector field is its divergence. Symbolically, the divergence of $\mathbf{F}$ is written $\nabla\cdot\mathbf{F}$ (pronounced del dot $\mathbf{F}$ or divergence of $\mathbf{F}$ interchangeably). Here again, $\nabla\cdot$ is not to be considered separately from $\mathbf{F}$. The total symbol $\nabla\cdot\mathbf{F}$ represents the physical idea "divergence of $\mathbf{F}$". The divergence of a vector is a scalar.

Physically, the divergence of $\mathbf{F}$ is defined as,

$$\nabla\cdot\mathbf{F} = \lim_{S\to 0} \frac{1}{V} \int \mathbf{F}\cdot\mathbf{n}\, dS \quad , \tag{A.8.1}$$

where the symbols are identified in Figure A.8.1. Eq.(A.8.1) states that the divergence of a vector field $\mathbf{F}$ at any point is the sum, over a *closed surface* surrounding the point, of the component of $\mathbf{F}$ normal outward to the surface at each point, averaged over the volume enclosed by the surface as that volume approaches the limit zero.

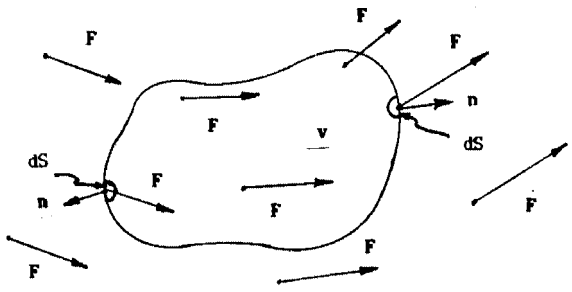

Figure A.8.1
The integration volume in the flow field $\mathbf{F}$.

To get some feeling for the utility of the divergence, consider a region in which the ether field velocity $\mathbf{V}$ is known. At any point in this region, the ether flow density is given by $\mathbf{F} = \phi_a\mathbf{V}$ the product of the density and the velocity. Since the flow density is defined as the quantity of ether flowing past a unit area, perpendicular to the flow in unit time, if $\mathbf{S}$ is an area through which flow is passing, $(\phi_a\mathbf{V})\cdot\mathbf{S}$ represents the total quantity of ether passing through $\mathbf{S}$ in unit time. With this in mind, visualize a small volume of arbitrary shape through which the ether flows (see Figure A.8.1). At some points on the surface of this volume, the flow is outward; at some points it is inward. If $(\phi_a\mathbf{V})\cdot\mathbf{n}ds$ is summed over the total surface, something very useful can be found out about what is going on inside. If, for example, there is a net positive outflow, then one of two things must be happening inside, either the ether density is decreasing with time

or in some unexplained fashion, ether is being "created". If the divergence is negative, i.e., the outflow is negative or net flow is inward, then either the density inside is increasing with time or, by some magic, ether is being "annihilated" within. Notice that n is defined as the *outward* normal.

Although this example of ether flow gives an intuitive picture of the idea of divergence in one particular case, Eq.(A.8.1) is quite general and applies to vector fields even when such a simple interpretation is not possible. Most writers emphasize the inflow-outflow viewpoint of divergence because, in fields where sources and sinks exist, the divergence is helpful in indicating their presence and density. However, many important fields have divergence where no sources exist. An example of this is the class of geometrical fields given by $\mathbf{F} = \mathbf{r} / r^n$, where n is some integer.

A.9 **The curl of a Vector field**: In fields where there is circulation or vortex motion, the concept of the curl is basic. Physically, the curl of $\mathbf{F}$ is related to the line integral of $\mathbf{F}$ around a small closed loop in the field. To find the curl of $\mathbf{F}$ at a point, the first step is to define a very small, plane contour about the point. The orientation of the plane is arbitrary, and the shape of the contour is arbitrary. Now, if the line integral of $\mathbf{F} \cdot d\mathbf{s}$ is taken around the contour, a certain magnitude results. This magnitude is divided by the area enclosed by the small contour on the plane, and the surface area is then taken to the limit zero. Mathematically, the resulting quantity is expressed as,

$$\lim_{S \to 0} \frac{1}{S} \int \mathbf{F} \cdot d\mathbf{s} \quad .$$

Clearly, except in very special fields, as the plane is oriented in all possible ways about the point, the quantity will differ with different orientation, and *one orientation will exist which makes the integral a maximum at that point.* The opposite orientation will give the same value as the maximum but with a negative sign, since it is equivalent to running the line integral backwards around the contour.

All of the important elements of the curl are now apparent, the maximum magnitude of the differential circulation, the orientation of the plane contour that gives the maximum, and the direction of integration around the contour. Surprisingly, no one has yet thought of a good vector method to represent the orientation of a plane and the direction of a contour on that plane that is both simple and yet does not introduce some extraneous property that has no correspondence to anything in the field.

The simplest representation of the curl uses a vector. Symbolically, the vector curl of $\mathbf{F}$ is written $\nabla \times \mathbf{F}$ (pronounced del cross $\mathbf{F}$ or curl of $\mathbf{F}$ interchangeably). The symbol $\nabla \times \mathbf{F}$ represents the physical idea "curl of $\mathbf{F}$". By arbitrarily assigning a unit vector $\mathbf{n}$

normal to the plane contour and adopting the convention that the integration is taken around the contour in a direction such that a right handed screw turned that way would advance in the direction of $\mathbf{n}$, the vector curl can be defined as the vector having the direction of $\mathbf{n}$ for which the line integral is maximum and having the magnitude of the maximum,

$$\nabla \times \mathbf{F} = \left[\lim_{S \to 0} \frac{1}{S} \int \mathbf{F} \cdot d\mathbf{s} \right]_{max} \mathbf{n} \quad . \tag{A.9.1}$$

Since this process can be carried out at each point in the field, and a magnitude and direction can be obtained for each point, $\nabla \times \mathbf{F}$ is a field vector.

One important property of the curl is that, at any point in the field, the line integral of $\mathbf{F}$ around any differentially small loop oriented arbitrarily is,

$$\int \mathbf{F} \cdot d\mathbf{s} = (\nabla \times \mathbf{F}) \cdot d\mathbf{A} \quad , \tag{A.9.2}$$

where $d\mathbf{A} = \mathbf{n} dS$, i.e. the line integral is zero if $d\mathbf{A}$ is perpendicular to $\nabla \times \mathbf{F}$ and otherwise it varies as the cosine of the angle that $\mathbf{n}$ makes with the curl vector.

There is an alternative integral form for the vector curl that is often more convenient to use, although less intuitively understandable, that is equivalent to Eq.(A.9.1). It can be written,

$$\nabla \times \mathbf{F} = \lim_{V \to 0} \frac{1}{V} \int \mathbf{n} \times \mathbf{F} \, dS \quad , \tag{A.9.3}$$

where the integration is carried out over a *closed* surface.

The vector representation of the curl suffers one very serious drawback. It implies that something physical is happening along the direction $\mathbf{n}$, when in reality all the direction does is establish an axis and indicate *by convention* which way the field is physically "circulating" about that axis. This is such a serious misrepresentation that it would be better not to use the vector form. The only alternative is to use a dyadic characterization which, although it is rigorously correct, and doesn't give a false representation, is not nearly as intuitively clear (this might not be true if the dyadic approach were taught from the beginning and no reference were ever made to the vector form, but there is no assurance of this).

A.10 **The Laplacian**: By far the most important quantity in field theory is the Laplacian. In scalar fields, it is defined as the divergence of a particular vector field, namely the gradient of the scalar, and is designated $\nabla^2 \phi$ (pronounced Laplacian of ϕ or del squared ϕ). The Laplacian of a scalar field is itself a scalar field obtained by first

forming the gradient $\nabla\phi$ according to Eq.(A.4.3), and then taking the divergence of the vector field $\nabla\phi$ according to Eq.(A.8.1) to give,

$$\nabla^2\phi = \nabla\cdot\nabla\phi = \lim_{v\to 0}\frac{1}{v}\int \mathbf{n}\cdot\nabla\phi \, dS \quad . \tag{A.10.1}$$

By analogy, it is possible to obtain the Laplacian of a *vector* field also, and it has paramount importance in vector field theory. Paralleling Eq.(A.8.1), the divergence of the dyadic Ψ is,

$$\nabla\cdot\Psi = \lim_{v\to 0}\frac{1}{v}\int \mathbf{n}\cdot\Psi \, dS \quad . \tag{A.10.2}$$

As in Section A.4, from the definition of a dyadic given in Section A.2, having specified a point and $\mathbf{n}$ at that point, $\mathbf{n}\cdot\Psi$ gives another vector at that point. Thus, $\nabla\cdot\Psi$ is a vector found from the integral of the vector $\mathbf{n}\cdot\Psi$ over a differentially small volume v.

The *vector* Laplacian is now obtained by essentially the same steps as Eq.(A.10.1),

$$\nabla^2\mathbf{F} = \nabla\cdot\nabla\mathbf{F} \quad , \tag{A.10.3}$$

i.e. the Laplacian of $\mathbf{F}$ is the divergence of $\nabla\mathbf{F}$, where the latter is a dyadic (see Eq. A.6.1). Using $\nabla\mathbf{F} = \Psi$, and Eq.(A.10.2), the definition of the vector Laplacian is,

$$\nabla^2\mathbf{F} = \nabla\cdot\nabla\mathbf{F} = \lim_{v\to 0}\frac{1}{v}\int \mathbf{n}\cdot\nabla\mathbf{F} \, dS \quad . \tag{A.10.4}$$

The formal definitions of the scalar and vector Laplacians do not yield a very clear intuitive picture of their physical meanings; but *a simple picture is available*. This can be seen by considering the field in the neighborhood surrounding a specified point. In the scalar case, let $\dddot{\phi}_v$ represent the volume average of ϕ within a very small sphere of radius R drawn about the point at which ϕ and $\nabla^2\phi$ are observed. The average is given by,

$$\dddot{\phi}_v = \frac{1}{\frac{4}{3}\pi R^3}\int \phi \, dv \quad . \tag{A.10.5}$$

It can be shown that,

$$\nabla^2\phi = \lim_{R\to 0}\frac{10}{R^2}(\dddot{\phi}_v - \phi) \quad . \tag{A.10.6}$$

This leads to naming the Laplacian the <u>surrounding function</u>, because it gives the state of the average field surrounding a point relative to the field at that point. For example, if ϕ is a density and the scalar $\nabla^2\phi$ is positive at a point, it means that the density, on the average, surrounding that point is greater than the density ϕ at that point. If

$\nabla^2\phi$ is negative, it means that the average density surrounding the point is less then ϕ at the point.

In exactly the same way, if the volume average $\ddot{\ddot{\mathbf{F}}}$ of a vector field $\mathbf{F}$ *surrounding* a point is taken, then the vector $\nabla^2\mathbf{F}$ at that point is a measure of the difference between the average $\ddot{\ddot{\mathbf{F}}}$ and $\mathbf{F}$ at that point. $\ddot{\ddot{\mathbf{F}}}$ and $\mathbf{F}$ can be different in both *magnitude* and *direction*. Eqs.(A.10.5) and (A.10.6) apply in the *vector* case by replacing ϕ with $\mathbf{F}$ throughout.

One useful fact can be gained from Eq.(A.10.6), and that is that if the surrounding function is zero everywhere in some region, no maximum or minimum value of the field variable can exist in the region. The converse is not true, since even where the surrounding function is not zero, more often that not there are no maxima or minima.

The importance of the surrounding function in field theory is that it designates the presence or absence of *equilibrium* in the field, and in addition is a measure of the extent of the departure from equilibrium. For example, if $\nabla^2(\ ) = 0$, at every point the value of $(\ )$ is equal to the average of $(\ )$ surrounding that point. This does not mean that $(\ )$ is constant everywhere, but that it has a space distribution that is in equilibrium everywhere. If $\nabla^2(\ ) = \text{fcn}(x, y, z, t)$ then the magnitude and direction of fcn(x,y,z,t) measures the deviation from equilibrium. It generally determines the rate or acceleration of the return to equilibrium.

APPENDIX B

COORDINATE SYSTEMS

B.1 **Introduction**: All of the relationships given in Appendix A are independent of any coordinate system. The entities defined are visualized in terms of magnitudes and/or directions at points in space. These entities can be manipulated to show many important ideas and "rules" about field representation, and some of these field relationships will be given further on. However, any description of a field in space will be little more than qualitative unless a coordinate system is adopted. This allows magnitudes and directions at each point to be specified quantitatively. The next few sections will describe certain basic properties of coordinate systems.

B.2 **Coordinate Systems:** A coordinate system is an arrangement that relates each point in space to a unique set of three numbers. This is done by establishing three separate families of surfaces that fill all space. Figure B.2.1 illustrates the three families (x,y,z) in the most elementary coordinate system, which is called "Cartesian" or "rectangular". The x family is a set of parallel planes that extend infinitely. Each plane is identified with one of the real numbers, and the reference plane is marked zero. For any given point in space, only one value of x can be specified, corresponding to the x plane passing through that point. The addition of a second family of parallel planes, y, perpendicular to the x planes, allows specifying one value of y for each space point. By adding a third set of z planes, each of which is parallel to all others in the z set and perpendicular to both the x and y

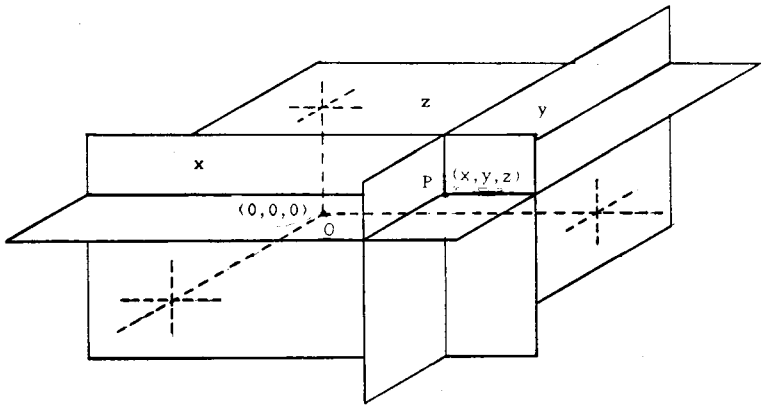

Figure B.2.1 The general point (x,y,z) in Cartesian coordinates.

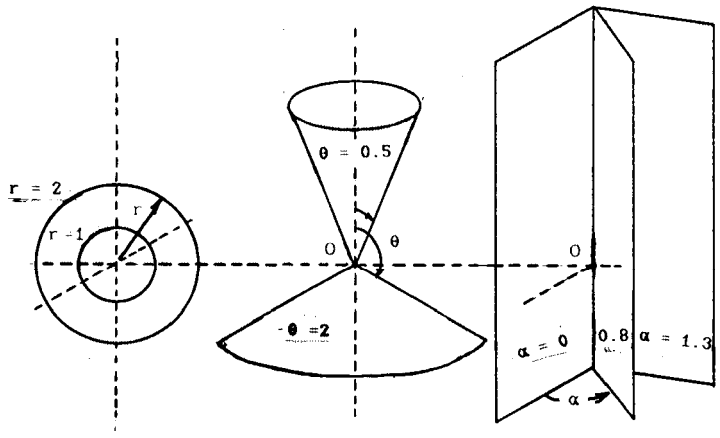

Figure B.2.2 Three families used in spherical coordinates.

sets, every point is space is uniquely defined by the intersection of three planes specified by the three numbers x, y, and z. A general, arbitrary point (x,y,z) is then represented relative to the reference point (0,0,0) by three intersecting planes as shown in Figure B.2.1.

Another set of three families that compose what are called "spherical" coordinates is given in Figure B.2.2. The first is the family of concentric spheres of radius r for all values of r between 0 and infinity. The second is the family of cones with vertices at the reference point 0, called the origin, and extending infinitely. Each cone is represented by a value of the coordinate θ, the angle between the surface and the cone axis. θ varies between zero and π. The third is a family of half infinite planes originating along the cone axis and fanning out at different angles α measured from a reference plane as shown in Figure B.2.2. Each plane is identified uniquely by the coordinate value α, which varies from zero to 2π. A general, arbitrary point in this system is located by the intersection of the three surfaces (r, θ, α), as drawn in Figure B.2.3.

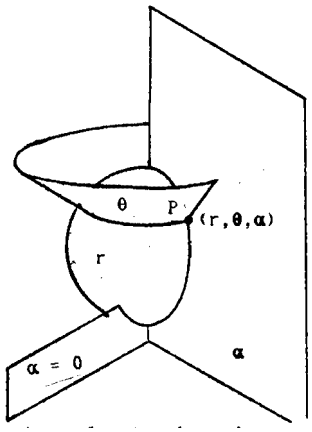

Figure B.2.3
The general point (r, θ, α) in spherical coordinates.

A third set of coordinates that points up certain special characteristics that systems can have is a composite of one family of parallel planes (designated ζ),

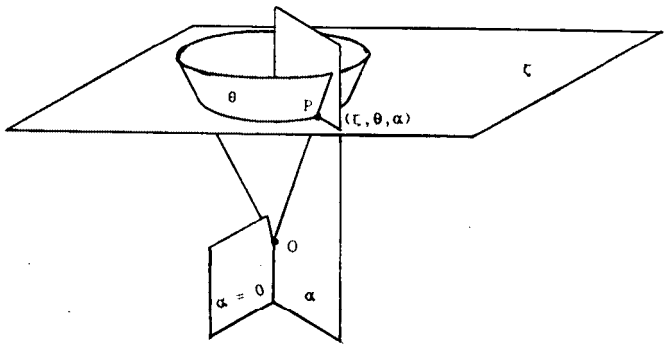

Figure B.2.4 Cone coordinates

and the cones and half planes used in Figure B.2.2 for spherical coordinates. The system, illustrated in Figure B.2.4, is called the "cone" coordinate system. Looking at the three systems discussed so far, i.e., Cartesian, spherical, and cone, two required properties of the families of surfaces are brought out. First, the members of any one family do not cross members of the same family except at special points or curves. Second, no member of one family is tangent to any member of either of the other two families except at special surfaces. The latter condition implies that although two surfaces of different families generally cannot be tangent, they can be nearly tangent. Thus, while it is clear from Figures B.2.1 and B.2.3 that the surfaces x,y,z or r, θ, α are all perpendicular or *orthogonal*, Figure B.2.4 makes it equally clear that, although the surfaces ζ and α are orthogonal as are θ and α, ζ and θ are orthogonal only where $\theta = 0$ and π. Systems in which all three coordinate surfaces are mutually orthogonal are called "orthogonal" systems. All others are called "non-orthogonal". Cartesian coordinates form a very special case in which all of the surfaces are planes. Coordinate systems involving curved surfaces are called "curvilinear".

B.3 **The Position and Differential Length Vectors**: The intuitively understood ability of an individual to act as a reference point and specify all other positions in space by actually pointing in their respective directions and stating how far away they are corresponds to the idea of representing points in space by the reference vector $\mathbf{r}$. The separation of $\mathbf{r}$ into direction and magnitude is expressed by writing $\mathbf{r} = \hat{\mathbf{r}}r$, where $\hat{\mathbf{r}}$ is the unit vector giving the *direction* to a point P from 0, the origin, and r is the magnitude of the distance from 0 to P. Since much of the physics of fields is based on effects resulting from small changes in position, it is convenient to obtain an

expression for the vector d**r** that represents a differential change in position from P as portrayed in Figure B.3.1. Differentiating the expression for **r**,

$$d\mathbf{r} = \hat{\mathbf{r}}\, dr + r\, d\hat{\mathbf{r}} \quad , \tag{B.3.1}$$

so the differential vector d**r** can be expressed in terms of two components, one due to the change in the magnitude r and one due to the change in direction of the unit vector $\hat{\mathbf{r}}$. Since a unit vector always remains unit in length, it can only change in direction, and the change is always perpendicular to the original unit vector. The ideas represented by Eq.(B.3.1) are completely independent of any choice of coordinate system. However, to go much farther in understanding how **r** and d**r** are used in actual problems, it is necessary to use a coordinate system. For purposes of illustration, the cone system of Figure B.2.4 will be used, but the general conclusions derived will apply to any curvilinear coordinate system.

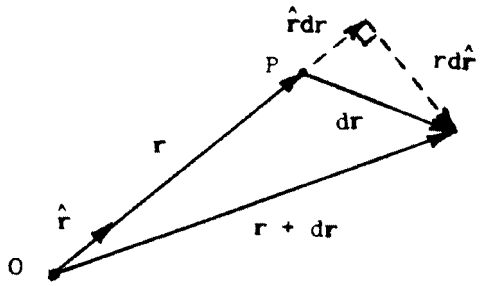

Figure B.3.1
The components of d**r**

The technique for bringing together the vector position concept and the coordinates is to notice that the position vector $\hat{\mathbf{r}}r$ can be used to identify any point in space which can also be identified by the three numbers of the coordinate surfaces that intersect there. Thus, **r** can be written as a function of the generalized coordinates $(\overset{1}{\xi}, \overset{2}{\xi}, \overset{3}{\xi})$ giving,

$$\mathbf{r} = \mathbf{r}(\overset{1}{\xi}, \overset{2}{\xi}, \overset{3}{\xi}) \quad . \tag{B.3.2}$$

Comparing Eq.(B.3.2) and $\hat{\mathbf{r}}r$, both $\hat{\mathbf{r}}$ and r are generally functions of $(\overset{1}{\xi}, \overset{2}{\xi}, \overset{3}{\xi})$ but this is not always true. In the special case of cone coordinates, for example, Eq.(B.3.2) becomes $\mathbf{r} = \mathbf{r}(\zeta, \theta, \alpha)$, where,

$$r = r(\zeta, \theta) \qquad \text{and} \qquad \hat{\mathbf{r}} = \hat{\mathbf{r}}(\theta, \alpha) \quad . \tag{B.3.3}$$

This can be seen best by observing Figure B.2.4. If θ and α are held constant and ζ increases by $d\zeta$, r increases whereas $\hat{\mathbf{r}}$ is

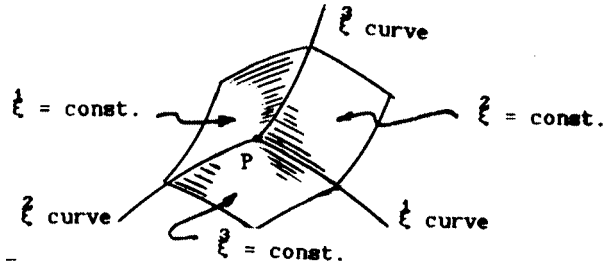

Figure B.3.2 Three coordinate numbers match coordinate surfaces at P.

unchanged. If ζ and α are constant and θ increases by $d\theta$, r increases and $\hat{\mathbf{r}}$ swings outward to point along the new surface $\theta + d\theta$. Finally, if ζ and θ are constant and α increases by $d\alpha$, although $\hat{\mathbf{r}}$ swings along the conical surface θ, r is unchanged.

The changes in the components of $d\mathbf{r}$ just described help in understanding a very basic property of coordinates. At each space point P, identified by three coordinate numbers, three surfaces intersect. Each coordinate number identifies one of the surfaces (see Figure B.3.2). The intersection defines three curves that pass through the point P, and each curve represents a path along which one of the coordinate variables changes while the other two are held constant. In Figure B.3.2, for example, ξ^1 varies along the curve ξ^1; but ξ^2 and ξ^3 do not, since the ξ^1 curve stays on the same ξ^2 and ξ^3 surfaces. Figure B.3.3 shows the variable curves for a particular point P in terms of cone coordinates.

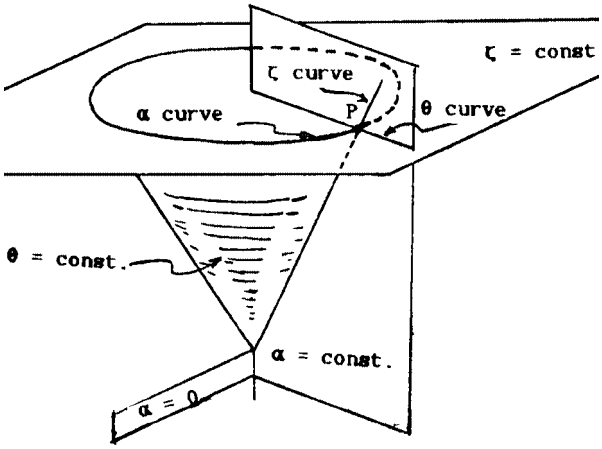

Figure B.3.3
Variable curves for the point P.

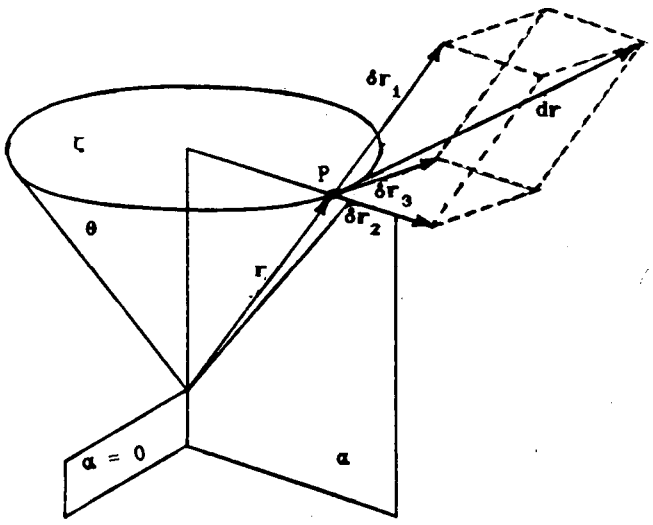

Figure B.4.1 Parallelogram components of d**r**

B.4 **Unit Vectors, Unitary Vectors and Scale Factors**: In the previous Section B.3, when discussing changes in **r** in Figure B.2.4, the differential changes d**r** were taken purposely along only one of the coordinate curves at P at a time. A more general case is pictured in Figure B.4.1, where the displacement d**r** is arbitrary. By definition, the components of a vector are any vectors whose sum equals the vector. In the present case, d**r** has been expressed in term of three components, each taken along one of the variable curves ζ, θ and α (see Figure B.3.3). These vector components add according to the standard vector parallelogram rule, which is expressed mathematically in the form,

$$d\mathbf{r} = \delta\mathbf{r}_1 + \delta\mathbf{r}_2 + \delta\mathbf{r}_3 \quad . \qquad (B.4.1)$$

Equation (B.4.1) and the parallelogram rule apply in the most general case where the variables are $\overset{1}{\xi}$, $\overset{2}{\xi}$ and $\overset{3}{\xi}$, and since each of the component displacement vectors is made up of a magnitude ds_i and a *unit* vector $\mathbf{k}_i$, Eq.(B.4.1) becomes,

$$d\mathbf{r} = \mathbf{k}_1 ds_1 + \mathbf{k}_2 ds_2 + \mathbf{k}_3 ds_3 \quad , \qquad (B.4.2)$$

where the ds_i factors are the actual distances along the variable curves, and the unit vectors are often not orthogonal.

To express $d\mathbf{r}$ in terms of the coordinates, the function $\mathbf{r}(\xi^1, \xi^2, \xi^3)$, first introduced in Eq.(B.3.2), is used to find the total derivative,

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial \xi^1} d\xi^1 + \frac{\partial \mathbf{r}}{\partial \xi^2} d\xi^2 + \frac{\partial \mathbf{r}}{\partial \xi^3} d\xi^3 \quad . \tag{B.4.3}$$

Each term of Eq.(B.4.3) has a vector $\partial \mathbf{r} / \partial \xi^i$ tangent to the variable curve at P and a scalar factor $d\xi^i$, and the "apparent" similarity to Eq.(B.4.2) makes it tempting to equate the vector and scalar factors. That this cannot be done is easily seen by writing Eq.(B.4.3) in cone coordinates,

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial \zeta} d\zeta + \frac{\partial \mathbf{r}}{\partial \theta} d\theta + \frac{\partial \mathbf{r}}{\partial \alpha} d\alpha \quad . \tag{B.4.4}$$

In Eq.(B.4.2), the ds_i were defined as the actual <u>distances</u> along the variable curves, whereas the $d\xi^i$, as exemplified by $d\zeta$, $d\theta$, or $d\alpha$ are basically <u>not</u> distances but changes in coordinate variable numbers. The relationship between distances ds_i and the $d\xi^i$ factors is brought out in Figure B.4.2. For example, $d\zeta$ appears as a distance but not equal to distance ds_1 since $ds_1 = \sec\theta \, d\zeta$.

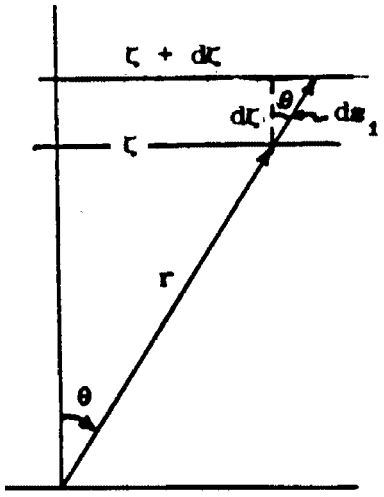

Figure B.4.2
A differential displacement component.

The method by which components of Eqs.(B.4.3) and (B.4.4) are made compatible with those of Eq.(B.4.2) is to use <u>scale factors</u>. The basic Eq.(B.4.3) becomes,

$$d\mathbf{r} = \left(\frac{1}{h_1} \frac{\partial \mathbf{r}}{\partial \xi^1} \right) h_1 d\xi^1 + \left(\frac{1}{h_2} \frac{\partial \mathbf{r}}{\partial \xi^2} \right) h_2 d\xi^2 + \left(\frac{1}{h_3} \frac{\partial \mathbf{r}}{\partial \xi^3} \right) h_3 d\xi^3 \quad , \tag{B.4.5}$$

which allows the unit vectors $\mathbf{k}_i$ to he identified as,

$$\mathbf{k}_1 = \frac{1}{h_1}\frac{\partial \mathbf{r}}{\partial \overset{1}{\xi}} \quad , \quad \mathbf{k}_2 = \frac{1}{h_2}\frac{\partial \mathbf{r}}{\partial \overset{2}{\xi}} \quad , \quad \mathbf{k}_3 = \frac{1}{h_3}\frac{\partial \mathbf{r}}{\partial \overset{3}{\xi}} \quad . \qquad \text{(B.4.6)}$$

and the differential lengths to be,

$$ds_1 = h_1 d\overset{1}{\xi} \quad , \quad ds_2 = h_2 d\overset{2}{\xi} \quad , \quad ds_3 = h_3 d\overset{3}{\xi} \quad . \qquad \text{(B.4.7)}$$

The h_i are chosen to be the *scale factors* that *convert differential changes in the coordinate variables into differential lengths*. The cone coordinate *scale factors* are,

$$h_1 = \sec\theta \quad , \quad h_2 = \zeta \sec^2\theta \quad , \quad h_3 = \zeta \tan\theta \quad , \qquad \text{(B.4.8)}$$

found as, for example, h_1 was found from Figure B.4.2. Unit vectors $\mathbf{k}_i$ are now tangent to the variable curves, unit in length, and not necessarily orthogonal, as shown for cone coordinates in Figure B.4.3, where $\mathbf{k}_1 = \hat{\mathbf{r}}$, $\mathbf{k}_2 = \hat{\boldsymbol{\theta}}_c$, $\mathbf{k}_3 = \hat{\boldsymbol{\alpha}}$.

Sometimes, for convenience in certain mathematical procedures, the distance and unit vector approach is not used, and the forms like Eq.(B.4.3) are used instead. For this reason, the variables $\overset{i}{\xi}$ have been given a special name, in this case <u>contravariant</u>, and the vectors $\partial \mathbf{r} / \partial \overset{i}{\xi}$ are called <u>unitary</u> to indicate that although they are parallel to the unit vectors they are not unit in length but involve the scale factors. The super-superscript numbers indicate that *contravariant* components are being used. However, here a warning about notation should be made. In most writing of this type, the <u>unitary</u> vectors are written with subscripts like those used in the preceding equations for unit vectors. In this work, the normal subscripts are reserved for actual length or distance type quantities, and the conventional subscript and superscript used by others for the unitary vectors and contravariant

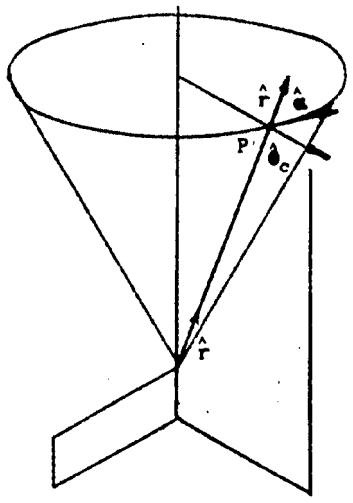

Figure B.4.3
Non-orthogonal unit vectors.

components are replaced by the super-super and sub-sub notation,

$$\underset{1}{\mathbf{b}} = \frac{\partial \mathbf{r}}{\partial \overset{1}{\xi}} \quad , \quad \underset{2}{\mathbf{b}} = \frac{\partial \mathbf{r}}{\partial \overset{2}{\xi}} \quad , \quad \underset{3}{\mathbf{b}} = \frac{\partial \mathbf{r}}{\partial \overset{3}{\xi}} . \qquad \text{(B.4.9)}$$

Although these expressions look simpler than those for the unit vectors in Eq.(B.4.6), since the unit vectors have a *known* magnitude (unity), they can often be written down intuitively, if the directions of the coordinate variable curves can be seen from the geometry, as was just done for cone coordinates. If this is the case, and if the scale factors can be seen from the geometry also, as is often true, then the unitary vectors, if and when needed, can be found from,

$$\underset{1}{\mathbf{b}} = \mathbf{k}_1 h_1 \quad , \quad \underset{2}{\mathbf{b}} = \mathbf{k}_2 h_2 \quad , \quad \underset{3}{\mathbf{b}} = \mathbf{k}_3 h_3 \quad . \qquad \text{(B.4.10)}$$

For cone coordinates the unitary vectors are,

$$\underset{1}{\mathbf{b}} = \hat{\mathbf{r}} \ \sec\theta \ , \ \underset{2}{\mathbf{b}} = \hat{\boldsymbol{\theta}}_c \ \zeta \sec\theta \ , \ \underset{3}{\mathbf{b}} = \hat{\boldsymbol{\alpha}} \ \zeta \tan\theta \quad . \qquad \text{(B.4.11)}$$

SUMMATION CONVENTION

In most work of this type, the appearance of a repeated sub or super-script is used to indicate a sum over several variables. This *summation convention* will <u>not</u> be used here unless specifically noted.

B.5 **Angle Variables and the Line Element**: According to Eq.(A.3.2) for the dot product of two vectors, if a vector is dotted into itself, the result is the square of its magnitude. Applying this to the differential $d\mathbf{r}$ in Eq.(B.3.2),

$$\begin{aligned} ds^2 = d\mathbf{r}\bullet d\mathbf{r} = \ & \mathbf{k}_1\bullet\mathbf{k}_1 ds_1^2 + \mathbf{k}_1\bullet\mathbf{k}_2 ds_1 ds_2 + \mathbf{k}_1\bullet\mathbf{k}_3 ds_1 ds_3 \\ & + \mathbf{k}_2\bullet\mathbf{k}_1 ds_2 ds_1 + \mathbf{k}_2\bullet\mathbf{k}_2 ds_2^2 + \mathbf{k}_2\bullet\mathbf{k}_3 ds_2 ds_3 \\ & + \mathbf{k}_3\bullet\mathbf{k}_1 ds_3 ds_1 + \mathbf{k}_3\bullet\mathbf{k}_2 ds_3 ds_2 + \mathbf{k}_3\bullet\mathbf{k}_3 ds_3^2 \end{aligned} \quad , \qquad \text{(B.5.1)}$$

Since the $\mathbf{k}_i$ are unit vectors, their dot products represent the cosines of the angles between them. These "angle" variables are designated by,

$$q_{ij} = \mathbf{k}_i\bullet\mathbf{k}_j = \cos\theta_{ij} \quad . \qquad \text{(B.5.2)}$$

Recalling that the angle between identical vectors is zero ($\cos\theta = 1$),

the nine q_{ij} in Eq.(B.5.1) can be written,

$$q_{ij} = \begin{bmatrix} 1 & q_{12} & q_{13} \\ q_{12} & 1 & q_{23} \\ q_{13} & q_{23} & 1 \end{bmatrix} \quad , \tag{B.5.3}$$

where the fact that $q_{ij} = q_{ji}$ has been used. When known as functions throughout space, the scale factors h_i together with the angle variables q_{ij} represent much of the important information about a coordinate system.

In terms of cone coordinates, the angle variables are,

$$q_{ij} = \begin{bmatrix} 1 & \sin\theta & 0 \\ \sin\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \tag{B.5.4}$$

as found by observation from Figures B.4.2 and B.4.3, using,

$$q_{12} = \hat{\mathbf{r}}\cdot\hat{\boldsymbol{\theta}}_c = \sin\theta \quad , \quad q_{23} = \hat{\boldsymbol{\theta}}_c\cdot\hat{\boldsymbol{\alpha}} = 0 \quad , \quad q_{13} = \hat{\boldsymbol{\alpha}}\cdot\hat{\mathbf{r}} = 0 \quad . \tag{B.5.5}$$

Two short ways of expressing the "line element" of Eq.(B.5.1) are,

$$ds^2 = \sum_{\substack{i=3 \\ j=3}} q_{ij} ds_i ds_j = q_{ij} ds_i ds_j \quad , \quad (*i, j) \tag{B.5.6}$$

where the very abbreviated *second* form is written using Einstein's summation convention, and any repeated index is to be summed over the range of variables. Conventionally, this summing rule is used without further indication. In the present work, no summation will be carried out on a repeated index unless followed by an asterisk and the variables to be summed, i.e. (*i, j) as in Eq.(B.5.6).

The line element is sometimes written in terms of *contravariant* components,

$$ds^2 = g_{ij}\, d\xi^i d\xi^j \quad , \quad (*i, j) \tag{B.5.7}$$

where the g_{ij} are called *coefficients of the metric tensor.* The g_{ij} are related to the *unitary* vectors by,

$$g_{ij} = \mathbf{b}_i\cdot\mathbf{b}_j \quad , \tag{B.5.8}$$

but are *mixtures* of angle variables and scale factors according to,

$$g_{ij} = h_i h_j q_{ij} \quad , \tag{B.5.9}$$

as can be seen by comparing Eqs.(B.5.1) and (B.5.7) using Eq.(B.4.10). Although the *contravariant* forms are very popular, and

the g_{ij} lead to more symmetrical mathematical expressions, they are somewhat obscure and lack the intuitive value of the scale factors and angle variables.

B.6 **Vector Components**: Just as the components of d**r** can be taken parallel to the coordinate curves, as in Eq.(B.4.1) and Figure B.4.1, any arbitrary vector **V** can be composed of three components (added according to the parallelogram rule) taken in the directions of the coordinate curves. In fact, each coordinate system specifies three separate fields, each field consisting of all the unit vectors corresponding to one variable. It is these unit vectors, three at each point, that give the directions in which the components of a general vector are specified. A vector is fully described then, by,

$$\mathbf{V} = \mathbf{k}_1 V_1 + \mathbf{k}_2 V_2 + \mathbf{k}_3 V_3 \quad , \tag{B.6.1}$$

where the magnitudes V_i correspond to "real length" components and the $\mathbf{k}_i$ are unit vectors.

Some confusion arises when a writer first places too much emphasis on rectangular or Cartesian coordinates because they are the most intuitively natural reference frame. In that system, the components of a vector **V** at any point are parallel to the components of the position vector **r** designating that point. This is <u>not</u> true in curvilinear systems. The unit vectors at the <u>point</u> determine the component directions.

Another blind spot that can develop because of too much exposure to Cartesian coordinates is the concept of the perpendicular projections of a vector onto the coordinate axes as the components of the vector. Although a vector can be resolved into these "components", they often have bizarre properties unless orthogonal coordinates are used, in which case the perpendicular projection components are identical to the parallelogram components that are fundamental in the general case.

The expression for a vector in terms of *unitary* vectors and *contravariant* components is,

$$\mathbf{V} = \mathbf{b}_1 \overset{1}{V} + \mathbf{b}_2 \overset{2}{V} + \mathbf{b}_3 \overset{3}{V} \quad . \tag{B.6.2}$$

These components are often useful for mathematical manipulation, but are not too meaningful intuitively, since they involve both the real length components and the scale factors according to,

$$\overset{1}{V} = \frac{V_1}{h_1} \quad , \quad \overset{2}{V} = \frac{V_2}{h_2} \quad , \quad \overset{3}{V} = \frac{V_3}{h_3} \quad . \tag{B.6.3}$$

In more mathematically oriented discussions of this subject, vectors are often represented solely in terms of their components. For example, V_i or $\overset{i}{V}$ can be used to indicate a vector, where it is understood that i takes on three values, one for each of the three components. This approach is used here on only rare occasions, because it places emphasis on the *components*, which are fundamentally unnecessary, and takes it away from the vector as an entity.

B.7 **Vector Products**: Certain handy vector product rules will now be given, almost without discussion.

The dot product: $$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta \qquad \text{(B.7.1)}$$

gives the perpendicular projection of either vector on the other times that other as a scalar quantity.

The cross product: $$\mathbf{A}\times\mathbf{B} = \mathbf{n}\,AB\sin\theta \qquad \text{(B.7.2)}$$

gives a vector normal to the plane of the two vectors and of magnitude equal to the parallelogram area specified by the two vectors. The unit vector $\mathbf{n}$ points in the direction of motion of a right handed screw rotating from $\mathbf{A}$ to $\mathbf{B}$. If the order of the vectors is reversed, the direction of $\mathbf{n}$ is reversed, i.e. $\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$.

The box product[1] $$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = v \qquad \text{(B.7.3)}$$

(often called the scalar triple product) gives the volume of the parallelepiped specified by the three vectors. The volume is independent of the position of the dot and cross. However, if the cyclic order of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ is changed, the sign is reversed.

The vector triple product $$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \qquad \text{(B.7.4)}$$

is a vector that lies in the plane of $\mathbf{B}$ and $\mathbf{C}$ and has two components, one parallel to $\mathbf{B}$ and one to $\mathbf{C}$. Notice that,

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{A}(\mathbf{B}\cdot\mathbf{C}) \qquad \text{(B.7.5)}$$

The dot product and the vector triple product make it possible to resolve any vector $\mathbf{B}$ into two components, one parallel to and one

1. The name proposed by J.H.Taylor, Vector Analysis, New York (1939).
See L.Brand, Vector and Tensor Analysis, **43**, John Wylie & Sons, N.Y. (1947).

perpendicular to another vector $\mathbf{A}$. The components are given by,[1]

$$\mathbf{B}_{\|} = \frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\,\mathbf{A} \quad , \quad \mathbf{B}_{\perp} = \frac{\mathbf{A}\times(\mathbf{B}\times\mathbf{A})}{\mathbf{A}\cdot\mathbf{A}} \tag{B.7.6}$$

Operations involving cross products are analogous to the vector curl, in that $\mathbf{A}$ and $\mathbf{B}$ define a plane and the cross product specifies an area in the plane. The unit vector $\mathbf{n}$ is superfluous to the physics, as can be seen using a dyadic representation. However, in setting up coordinate systems, the built in convention of the sign related to $\mathbf{n}$ is useful in preserving the "right handedness" of the coordinates, also adopted by convention.

B.8 **Reciprocal Base Systems**: In orthogonal systems, any component of a given vector is obtainable by dotting the corresponding *unit* vector into the given vector, i.e.,

$$V_i = \mathbf{k}_i\cdot\mathbf{V} \qquad \text{(orthogonal coordinates)} \tag{B.8.1}$$

The reason this subtle operation works for orthogonal systems is that all three unit vectors are perpendicular to each other and the dot product of any one with the others is zero, so that for example,

$$\mathbf{k}_1\cdot\mathbf{V} = \mathbf{k}_1\cdot\mathbf{k}_1 V_1 + \mathbf{k}_1\cdot\mathbf{k}_2 V_2 + \mathbf{k}_1\cdot\mathbf{k}_3 V_3 = V_1 \tag{B.8.2}$$

If the same operation is tried in a non-orthogonal system, the $\mathbf{k}_i\cdot\mathbf{k}_j$ can be other than zero, and the resultant is, for example,

$$\mathbf{k}_1\cdot\mathbf{V} = V_1 + q_{12}V_2 + q_{13}V_3 \quad . \tag{B.8.3}$$

This problem can be overcome by defining a so-called "reciprocal" set of base vectors $\mathbf{k}^i$ related to the original base vectors $\mathbf{k}_i$ so that,

$$V_1 = \mathbf{k}^1\cdot\mathbf{V} \qquad \text{(any coordinates)} \tag{B.8.4}$$

Figure B.8.1 shows an arbitrary set of unit vectors $\mathbf{k}_i$ at a given point. It is desired to find another vector $\mathbf{k}_1$, for example, that will obey the following relationships,

$$\mathbf{k}^1\cdot\mathbf{k}_1 = 1 \quad , \quad \mathbf{k}^1\cdot\mathbf{k}_2 = 0 \quad , \quad \mathbf{k}^1\cdot\mathbf{k}_3 = 0 \quad . \tag{B.8.5}$$

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1 J.W.Gibbs-E.B.Wilson, Vector Analysis, **72**, Dover Publications (1960).

The second and third products in Eq.(B.8.5) can be zero only if $\mathbf{k}^1$ is normal to the plane defined by $\mathbf{k}_2$ and $\mathbf{k}_3$, so it must lie along the vertical dotted line. If its length is now chosen so that its perpendicular projection onto $\mathbf{k}_1$ is unit in length, then the first product condition in Eq.(B.8.5) is also satisfied. The direction of $\mathbf{k}^1$ is chosen to give an acute angle between $\mathbf{k}^1$ and $\mathbf{k}_1$. By repeating this procedure for $\mathbf{k}^2$ and $\mathbf{k}^3$, a complete set of reciprocal base vectors is obtained. Clearly, the $\mathbf{k}^i$ are not necessarily unit vectors.

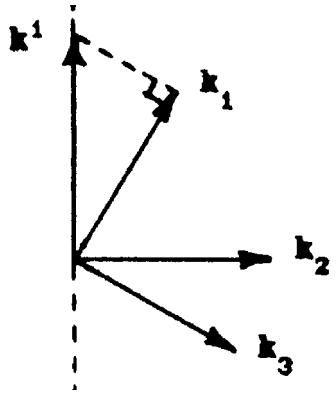

Figure B.8.1
Reciprocal base vector geometry.

Mathematically,

$$\mathbf{k}^i = \frac{\mathbf{k}_j \times \mathbf{k}_k}{\mathbf{k}_i \bullet \mathbf{k}_j \times \mathbf{k}_k} \quad , \qquad \text{(B.8.6)}$$

where i,j and k are given the values 1,2 and 3 in rotation without changing their order. This can be checked by dotting the expressions with $\mathbf{k}_i$ directly,

$$\mathbf{k}^i \bullet \mathbf{k}_j = \delta^i_j \quad , \qquad \text{(B.8.7)}$$

where the Kronecker delta is defined as,

$$\delta^i_j = \begin{cases} 1 \text{ when } j = i \\ 0 \text{ when } j \neq i \end{cases} \quad . \qquad \text{(B.8.8)}$$

From inspection of Eqs.(B.4.2) and (B.4.3), in terms of the unit vectors of cone coordinates, using Eq.(B.7.2),

$$\begin{aligned} \hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}}_c &= \hat{\boldsymbol{\alpha}} \cos\theta \\ \hat{\boldsymbol{\theta}}_c \times \hat{\boldsymbol{\alpha}} &= \mathbf{k} = \hat{\mathbf{r}} \sec\theta - \hat{\boldsymbol{\theta}}_c \tan\theta \\ \hat{\boldsymbol{\alpha}} \times \hat{\mathbf{r}} &= \hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \tan\theta + \hat{\boldsymbol{\theta}}_c \sec\theta \end{aligned} \quad , \qquad \text{(B.8.9)}$$

where $\mathbf{k}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors used in the orthogonal spherical coordinate system. Again, using the box product of Eq.(B.7.3),

$$\mathbf{k}_1 \bullet \mathbf{k}_2 \times \mathbf{k}_3 = \hat{\mathbf{r}} \bullet \hat{\boldsymbol{\theta}}_c \times \hat{\boldsymbol{\alpha}} = \cos\theta \quad . \qquad \text{(B.8.10)}$$

Now, the base system reciprocal to $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}_c$, $\hat{\boldsymbol{\alpha}}$ of cone coordinates is defined by Eqs.(B.8.6), (B.8.9) and (B.8.10) to be,

$$\mathbf{k}^1 = \mathbf{k} \sec\theta \quad , \quad \mathbf{k}^2 = \hat{\boldsymbol{\theta}} \sec\theta \quad , \quad \mathbf{k}^3 = \hat{\boldsymbol{\alpha}} \quad . \qquad \text{(B.8.11)}$$

Since $\mathbf{k}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\alpha}}$ are unit vectors, $\mathbf{k}^1$ and $\mathbf{k}^2$ are *not* and $\mathbf{k}^3$ *is* unit. In this particular case, $\mathbf{k}^1$, $\mathbf{k}^2$ and $\mathbf{k}^3$ are orthogonal.

It is a fundamental property of a base system and its reciprocal that they are mutually reciprocal. It is possible to find components of

any vector such that,

$$\mathbf{V} = \mathbf{k}^1 V^1 + \mathbf{k}^2 V^2 + \mathbf{k}^3 V^3 \quad , \qquad \text{(B.8.12)}$$

where,

$$V^i = \mathbf{k}_i \bullet \mathbf{V} \quad . \qquad \text{(B.8.13)}$$

Now, looking at Eq. (B.8.13), it is clear that, whereas the V_i are the parallelogram projections of $\mathbf{V}$ on $\mathbf{k}_i$, the V^i have the magnitudes of perpendicular projections of $\mathbf{V}$ on $\mathbf{k}_i$. Although these latter have little fundamental significance, they play an almost equal role in many mathematical, manipulatory studies, where they are usually presented as covariant components along with reciprocal *unitary* vectors,

$$\mathbf{V} = \overset{1}{\mathbf{b}} \underset{1}{V} + \overset{2}{\mathbf{b}} \underset{2}{V} + \overset{3}{\mathbf{b}} \underset{3}{V} \quad . \qquad \text{(B.8.14)}$$

Again, caution should be observed not to confuse the reversal of sub and superscript notation with that of other writers.

To round out the picture of the base system reciprocal to the unit vectors, a reciprocal form of Eq.(B.4.2) would be,

$$d\mathbf{r} = \mathbf{k}^1 ds^1 + \mathbf{k}^2 ds^2 + \mathbf{k}^3 ds^3 \quad , \qquad \text{(B.8.15)}$$

where the $\mathbf{k}^i$ are defined by Eq.(B.8.6), and the ds^i are obtained from Eq.(B.8.13), i.e. by dotting the $\mathbf{k}_i$ into Eq.(B.4.2) to give,

$$\begin{aligned} ds^1 &= ds_1 + q_{12} ds_2 + q_{13} ds_3 \\ ds^2 &= q_{12} ds_1 + ds_2 + q_{23} ds_3 \\ ds^3 &= q_{13} ds_1 + q_{23} ds_2 + ds_3 \end{aligned} \quad . \qquad \text{(B.8.16)}$$

Dotting Eq.(B.8.15) into itself gives the line element in terms of reciprocal components,

$$ds^2 = q^{ij} ds^1 ds^2 \quad , \qquad (*i, j) \qquad \text{(B.8.17)}$$

where the reciprocal angle variables are,

$$q^{ij} = \mathbf{k}^i \bullet \mathbf{k}^j \quad . \qquad \text{(B.8.18)}$$

The latter can be found in terms of the q_{ij} with the aid of the defining Eqs.(B.8.6) and (B.7.3),

$$q^{ij} = \frac{q_{ik} q_{jk} - q_{ij}}{q} \quad , \quad q^{ii} = \frac{1 - q_{jk}^{\,2}}{q} \quad , \quad \text{(i, j, k cyclic)} \qquad \text{(B.8.19)}$$

where $q = (\mathbf{k}_i \bullet \mathbf{k}_j \times \mathbf{k}_k)^2$, the square of the volume of the box. It can be shown using only trigonometry, that,

$$q = 1 + 2q_{12}q_{13}q_{23} - (q_{12}{}^2 + q_{13}{}^2 + q_{23}{}^2) \quad . \qquad \text{(B.8.20)}$$

To illustrate some of these ideas, in cone coordinates the square of the parallepiped volume is,

$$q = 1 - \sin^2\theta = \cos^2\theta = \frac{1}{\sec^2\theta} \quad , \qquad \text{(B.8.21)}$$

and the reciprocal angle variables are,

$$q^{ij} = \begin{bmatrix} \sec^2\theta & -\tan\theta\sec\theta & 0 \\ -\tan\theta\sec\theta & \sec^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad . \qquad \text{(B.8.22)}$$

Just as the $\mathbf{k}_i$ and the $\mathbf{k}^i$ obey a reciprocal relationship, Eq.(B.8.7),

$$q_{ik}q^{jk} = \delta_i^j \quad . \qquad \text{(B.8.23)}$$

Finally, the relationships in Eqs.(B.8.4) and (B.8.13) can be used to verify that,

$$\mathbf{k}^i = \mathbf{k}_j q^{ij} \quad , \quad \mathbf{k}_i = \mathbf{k}^j q_{ij} \quad . \quad (*j) \qquad \text{(B.8.24)}$$

In *orthogonal* coordinates, $\mathbf{k}^i = \mathbf{k}_i$.

One further set of relationships that is sometimes useful can be found from Eqs.(B.6.1), (B.8.4), (B.8.12) and(B.8.13). It gives the components of a vector in terms of its reciprocal components,

$$V_i = q^{ij}V^j \quad , \quad V^i = q_{ij}V_j \quad , \quad \underset{i}{V} = g_{ij}\overset{j}{V} \quad , \quad \overset{i}{V} = g^{ij}\underset{j}{V} \quad . \quad (*j) \qquad \text{(B.8.25)}$$

From these and Eq.(B.6.3),

$$\overset{i}{V} = \frac{V_i}{h_i} \quad , \quad \underset{i}{V} = h_i V^i \quad . \qquad \text{(B.8.26)}$$

APPENDIX C

DYADICS

C.1 **Introduction**:[1] Because dyadics are so essential in the description of fields, a fairly complete summary of the mathematics used to represent them is developed here. An effort is made to present this material in a way that increases the *visualization* and intuitive feeling about the field when dyadics are used. In Appendix A dyadics (second order tensors) were described as functions that specify an "output" or effect vector at any point where an "input" or cause vector is given. Actually, there are many kinds of vector functions that meet this requirement, whereas *dyadics represent a very restricted class of functions*. The need for dyadics arises naturally in field physics in those cases where the cause-effect relationship is linear. To understand what this means, consider a cause vector,

$$\mathbf{V} = \mathbf{k}_1 V_1 + \mathbf{k}_2 V_2 + \mathbf{k}_3 V_3 \quad , \tag{C.1.1}$$

and an effect vector,

$$\mathbf{U} = \mathbf{k}_1 U_1 + \mathbf{k}_2 U_2 + \mathbf{k}_3 U_3 \quad . \tag{C.1.2}$$

A dyadic is a function that converts $\mathbf{V}$ into $\mathbf{U}$ in such a way that the contribution of each component of $\mathbf{V}$ to each component of $\mathbf{U}$ is independent of the others, as expressed by the linear relationship,

$$\begin{aligned} \mathbf{U} = \quad & \mathbf{k}_1(\psi_1^1 V_1 + \psi_1^2 V_2 + \psi_1^3 V_3) \\ & + \mathbf{k}_2(\psi_2^1 V_1 + \psi_2^2 V_2 + \psi_2^3 V_3) \\ & + \mathbf{k}_3(\psi_3^1 V_1 + \psi_3^2 V_2 + \psi_3^3 V_3) \end{aligned} \quad . \tag{C.1.3}$$

The nine ψ_i^j constitute the components of the dyadic. To write Eq.(C.1.3) in a form which clearly separates the dyadic Ψ from the vector $\mathbf{V}$, substituting for the V_i from Eq.(B.8.4) and factoring $\mathbf{V}$, Eq.(C.1..3) becomes,

$$\mathbf{U} = \Psi \bullet \mathbf{V} \quad , \tag{C.1.4}$$

where,

$$\begin{aligned} \Psi = \quad & \mathbf{k}_1\mathbf{k}^1\psi_1^1 + \mathbf{k}_1\mathbf{k}^2\psi_1^2 + \mathbf{k}_1\mathbf{k}^3\psi_1^3 \\ & + \mathbf{k}_2\mathbf{k}^1\psi_2^1 + \mathbf{k}_2\mathbf{k}^2\psi_2^2 + \mathbf{k}_2\mathbf{k}^3\psi_2^3 \\ & + \mathbf{k}_3\mathbf{k}^1\psi_3^1 + \mathbf{k}_3\mathbf{k}^2\psi_3^2 + \mathbf{k}_3\mathbf{k}^3\psi_3^3 \end{aligned} \quad . \tag{C.1.5}$$

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1. This Appendix follows closely: Weatherburn, Brand, Gibbs-Wilson, loc.cit.; and A.P.Wills, Vector Analysis With an Introduction to Tensor Analysis, Prentice Hall, New York (1938).

Strong emphasis has been placed on the independence of *vectors* from coordinates or components they may be expressed in terms of. Particularly, the physical entity $\mathbf{V}$ is equally well represented by,

$$\mathbf{V} = \mathbf{k}_1 V_1 + \mathbf{k}_2 V_2 + \mathbf{k}_3 V_3 = \mathbf{k}^1 V^1 + \mathbf{k}^2 V^2 + \mathbf{k}^3 V^3 \quad . \qquad \text{(C.1.6)}$$

The same is true of *dyadics*, which represent physical processes independent of the choice of the coordinates or components used to express them. For example, if Eq.(C.1.3) had been written,

$$\begin{aligned} \mathbf{U} = \quad & \mathbf{k}_1(\psi_{11} V^1 + \psi_{12} V^2 + \psi_{13} V^3) \\ + \ & \mathbf{k}_2(\psi_{21} V^1 + \psi_{22} V^2 + \psi_{23} V^3) \quad , \\ + \ & \mathbf{k}_3(\psi_{31} V^1 + \psi_{32} V^2 + \psi_{33} V^3) \end{aligned} \qquad \text{(C.1.7)}$$

then by substituting for the V^i from Eq.(B.8.13),

$$\begin{aligned} \Psi = \quad & \mathbf{k}_1\mathbf{k}_1\psi_{11} + \mathbf{k}_1\mathbf{k}_2\psi_{12} + \mathbf{k}_1\mathbf{k}_3\psi_{13} \\ + \ & \mathbf{k}_2\mathbf{k}_1\psi_{21} + \mathbf{k}_2\mathbf{k}_2\psi_{22} + \mathbf{k}_2\mathbf{k}_3\psi_{23} \quad . \\ + \ & \mathbf{k}_3\mathbf{k}_1\psi_{31} + \mathbf{k}_3\mathbf{k}_2\psi_{32} + \mathbf{k}_3\mathbf{k}_3\psi_{33} \end{aligned} \qquad \text{(C.1.8)}$$

Now, although the nine ψ_{ij} are different from the ψ_i^j of Eq.(C.1.5), Ψ is the same in both cases, and it converts $\mathbf{V}$ into the same $\mathbf{U}$ according to Eq.(C.1.4), no matter which set of components is used to write it out. In fact, Ψ can be expressed in terms of co and contravariant components also, so that,

$$\begin{aligned} \Psi &= \mathbf{k}_i\mathbf{k}_j\psi_{ij} = \mathbf{k}_i\mathbf{k}^j\psi_i^j = \mathbf{k}^i\mathbf{k}^j\psi^{ij} = \mathbf{k}^i\mathbf{k}_j\psi^i_j \\ &= \mathbf{b}_i\mathbf{b}_j\psi^{ij} = \mathbf{b}_i\mathbf{b}_j\psi^{j\,i} = \mathbf{b}^i\mathbf{b}^j\psi_{ij} = \mathbf{b}^i\mathbf{b}^j\psi_{j\,i} \quad , \quad (*i,j) \end{aligned} \qquad \text{(C.1.9)}$$

are all equivalent representations. The fourth set of mixed components of each set in Eq.(C.1.9) will not be carried through in the following. Some care is required when using components of mixed indices.

Just as the reciprocal and co and contravariant components of vectors are related as in Eqs.(B.8.25) and (B.8.26), a similar set of

relationships holds for dyadics,

$$\begin{aligned}
&\psi_{ij} = q^{ik} q^{jm} \psi^{km} \quad , \quad \psi^{ij} = q_{ik} q_{jm} \psi_{km} \qquad (*k,m) \\
&\psi_{ij} = q^{jk} \psi_i^k \quad , \quad \psi^{ij} = q_{ik} \psi_k^j \qquad (*k) \\
&\psi_i^j = q^{ki} \psi^{jk} \quad , \quad \psi_i^j = q_{jk} \psi_{ik} \qquad (*k)
\end{aligned} \tag{C.1.10}$$

and,

$$\begin{aligned}
&\overset{ij}{\psi} = \overset{ik}{g}\, \overset{jm}{g}\, \underset{km}{\psi} \quad , \quad \underset{ij}{\psi} = g\, g\, \overset{km}{\psi} \qquad (*k,m) \\
&\overset{i}{\psi} = \overset{ik}{g}\, \underset{k}{\psi} \quad , \quad \underset{ij}{\psi} = g\, \overset{k}{\psi} \qquad (*k) \\
&\overset{i}{\psi}_j = \overset{ik}{g}\, \psi_{jk} \quad , \quad \underset{j}{\psi} = g\, \overset{i}{\psi}_k \qquad (*k)
\end{aligned} \tag{C.1.11}$$

Also,

$$\overset{ij}{\psi} = \frac{\psi_{ij}}{h_i h_j} \quad , \quad \underset{ij}{\psi} = h_i h_j \psi^{ij} \quad . \tag{C.1.12}$$

These component equations are complicated. Fortunately, the most important use of dyadics, as well as of vectors, is in visualizing field relationships. In that role, they are seldom resolved into components; but, if they are, tensors in a specific case often reduce to a considerably smaller set of components than in the general case.

From Eqs.(C.1.5), (C.1.8) and (C.1.9) it is clear that dyadics are expressible as one or more dyads ($\mathbf{ij}$, $\mathbf{ab}, \mathbf{PQ}$), each with a coefficient. In addition to this "component" form any dyadic can be written in the "reduced" form,

$$\Psi = \mathbf{aA} + \mathbf{bB} + \mathbf{cC} \quad . \tag{C.1.13}$$

Here, the <u>antecedents</u> $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and the <u>consequents</u> $\mathbf{A}, \mathbf{B}, \mathbf{C}$, none of which is necessarily a unit vector, can be derived from the component form of Ψ. If, for example, in Eq.(C.1.8), the consequents $\mathbf{k}_i$ and components ψ_{ij} are written as components of three vectors,

$$\begin{aligned}
\mathbf{A} &= \mathbf{k}_1 \psi_{11} + \mathbf{k}_2 \psi_{12} + \mathbf{k}_3 \psi_{13} \\
\mathbf{B} &= \mathbf{k}_1 \psi_{21} + \mathbf{k}_2 \psi_{22} + \mathbf{k}_3 \psi_{23} \quad , \\
\mathbf{C} &= \mathbf{k}_1 \psi_{31} + \mathbf{k}_2 \psi_{32} + \mathbf{k}_3 \psi_{33}
\end{aligned} \tag{C.1.14}$$

then,

$$\Psi = \mathbf{k}_1 \mathbf{A} + \mathbf{k}_2 \mathbf{B} + \mathbf{k}_3 \mathbf{C} \quad . \tag{C.1.15}$$

Instead, taking the antecedents $\mathbf{k}_i$ in Eq.(C.1.9) with the components ψ_{ij}, to form $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$,

$$\Psi = \mathbf{a}\mathbf{k}_1 + \mathbf{b}\mathbf{k}_2 + \mathbf{c}\mathbf{k}_3 \quad , \tag{C.1.16}$$

where,

$$\begin{aligned} \mathbf{a} &= \mathbf{k}_1\psi_{11} + \mathbf{k}_2\psi_{21} + \mathbf{k}_3\psi_{31} \\ \mathbf{b} &= \mathbf{k}_1\psi_{12} + \mathbf{k}_2\psi_{22} + \mathbf{k}_3\psi_{32} \\ \mathbf{c} &= \mathbf{k}_1\psi_{13} + \mathbf{k}_2\psi_{23} + \mathbf{k}_3\psi_{33} \end{aligned} \quad . \qquad \text{(C.1.17)}$$

Notice that **A**, **B** and **C** are composed of the *rows* of Ψ, whereas **a**, **b** and **c** come from the *columns*. Breaking each ψ_{ij} into two factors, sets of **A**, **B**, **C** and **a**, **b**, **c** can be formed having none of the $\mathbf{k}_i$ appearing explicitly. The reduced form allows properties of dyadics to be deduced or explained in a simple fashion. In fact, often a single dyad **AB** is sufficient for some demonstrations. Except in special cases,

$$\mathbf{V}\cdot\Psi \neq \Psi\cdot\mathbf{V} \quad , \qquad \text{(C.1.18)}$$

which easily can be seen using a single dyad $\Psi = \mathbf{AB}$, since the vectors resulting are, $\mathbf{V}\cdot\Psi = (\mathbf{V}\cdot\mathbf{A})\,\mathbf{B}$ and $\Psi\cdot\mathbf{V} = \mathbf{A}\,(\mathbf{B}\cdot\mathbf{V})$. Note that the bracketed quantities are scalars. Actually, all of the ordinary operations of addition, subtraction, multiplication, etc. can be performed using scalars, vectors, and dyadics *except those operations that require an interchange of the order of operation*. The difference between the two vectors resulting above when **V** is used first as a <u>prefactor</u> and then as a <u>postfactor</u> in the dot product is an example of the non-commutativity referred to. In all operations using scalars or vectors with dyadics the scalar or vector is applied to each dyad in the tensor. To illustrate, combining Eqs.(C.1.8) and (C.1.6),

$$\begin{aligned} \Psi\cdot\mathbf{V} = \quad & \mathbf{k}_1(\psi_{11}V_1 + \psi_{12}V_2 + \psi_{13}V_3) \\ & + \mathbf{k}_2(\psi_{21}V_1 + \psi_{22}V_2 + \psi_{23}V_3) \\ & + \mathbf{k}_3(\psi_{31}V_1 + \psi_{32}V_2 + \psi_{33}V_3) \end{aligned} \quad . \qquad \text{(C.1.19)}$$

The cross product of **V** as a prefactor of the reduced form in Eq.(C.1.13) gives another <u>dyadic</u>,

$$\mathbf{V}\times\Psi = (\mathbf{V}\times\mathbf{a})\mathbf{A} + (\mathbf{V}\times\mathbf{b})\mathbf{B} + (\mathbf{V}\times\mathbf{c})\mathbf{C} \quad , \qquad \text{(C.1.20)}$$

the bracketed quantities being vectors.

Reversing each dyad in a tensor gives what is called the <u>conjugate</u> dyadic. The conjugates of the reduced form of Eq.(C.1.13) and the component form of Eq.(C.1.8) are, respectively, $\Psi_c = \mathbf{Aa} + \mathbf{Bb} + \mathbf{Cc}$ and

$$\begin{aligned} \Psi_c = \quad & \mathbf{k}_1\mathbf{k}_1\psi_{11} + \mathbf{k}_1\mathbf{k}_2\psi_{21} + \mathbf{k}_1\mathbf{k}_3\psi_{31} \\ + \quad & \mathbf{k}_2\mathbf{k}_1\psi_{12} + \mathbf{k}_2\mathbf{k}_2\psi_{22} + \mathbf{k}_2\mathbf{k}_3\psi_{32} \\ + \quad & \mathbf{k}_3\mathbf{k}_1\psi_{13} + \mathbf{k}_3\mathbf{k}_2\psi_{23} + \mathbf{k}_3\mathbf{k}_3\psi_{33} \end{aligned} \quad . \qquad \text{(C.1.21)}$$

The effect of the latter form is to interchange the components of the rows and columns of Ψ. Using Eqs.(C.1.6) and (C.1.21) it is easy to show that,

$$\mathbf{V}\bullet\Psi = \Psi_c\bullet\mathbf{V} \quad , \text{ and } \quad \Psi\bullet\mathbf{V} = \mathbf{V}\bullet\Psi_c \quad . \tag{C.1.22}$$

Clearly,

$$\Psi = (\Psi_c)_c \quad , \quad (\Psi+\Phi)_c = \Psi_c + \Phi_c \quad , \quad (\Psi\bullet\Phi)_c = \Psi_c\bullet\Phi_c \quad . \tag{C.1.23}$$

If, $\psi_{ij} = \psi_{ji}$, the dyadic is <u>self conjugate</u> or <u>symmetric</u>, in which case,

$$\Psi\bullet\mathbf{V} = \Psi_c\bullet\mathbf{V} \quad . \qquad (\Psi \text{ symmetric}) \tag{C.1.24}$$

A most important self conjugate tensor is the *idemfactor* I, which acts in a manner similar to unity in algebra; so that, when dotted into a vector $\mathbf{V}$, it gives that vector. For example,

$$\mathrm{I}\bullet\mathbf{V} = \mathbf{V} \quad , \quad \mathbf{V}\bullet\mathrm{I} = \mathbf{V} \quad . \tag{C.1.25}$$

The idemfactor takes several forms, some of which are,

$$\begin{aligned} \mathrm{I} &= \mathbf{k}_i\mathbf{k}_j(q^{ij}) = \mathbf{k}_i\mathbf{k}^j\delta^j_i = \mathbf{k}^i\mathbf{k}^j(q_{ij}) \\ &= \mathbf{b}_i\mathbf{b}_j g^{ij} = \mathbf{b}_i\mathbf{b}^j\delta^i_j = \mathbf{b}^i\mathbf{b}^j g_{ij} \quad . \quad (*i,\ j) \end{aligned} \tag{C.1.26}$$

This is a convenient place to point out a basic difference between the co-contra/unitary system of $\mathbf{b}_i$ and Ψ^{ij} and the distance/unit vector system of $\mathbf{k}_i$ and ψ_{ij}. Eq.(C.1.9) shows that the $\mathbf{b}_i\mathbf{b}_j\psi^{ij}$ and $\mathbf{b}^i\mathbf{b}^j\psi_{ij}$ are always matched sub to super and super to sub. In the distance/unit vector system, in order to preserve the sub notation for both *magnitudes* and *unit* vectors in the greatest number of equations using those quantities, the $\mathbf{k}_i\mathbf{k}_j\psi_{ij}$ and $\mathbf{k}^i\mathbf{k}^j\psi^{ij}$ are always matched sub to sub and super to super. These conventions carry throughout both systems except for the two cases in Eq.(C.1.26) involving the (q_{ij}) and (q^{ij}) which are shown in brackets to emphasize that in this dyadic I, $\psi_{ij} = (q^{ij})$ and $\psi^{ij} = (q_{ij})$. Actually, confusion can arise only when the *angle variables* are written in index notation. When actual variables are used the problem does not appear. For example, when written out in cone coordinates, from Eq.(B.8.22),

$$\mathrm{I} = \mathbf{k}_1\mathbf{k}_1\sec^2\theta - (\mathbf{k}_1\mathbf{k}_2 + \mathbf{k}_2\mathbf{k}_1)\tan\theta\sec\theta + \mathbf{k}_2\mathbf{k}_2\sec^2\theta + \mathbf{k}_3\mathbf{k}_3 \quad ; \tag{C.1.27}$$

from Eq.(B.8.8),

$$\mathrm{I} = \mathbf{k}_1\mathbf{k}^1 + \mathbf{k}_2\mathbf{k}^2 + \mathbf{k}_3\mathbf{k}^3 \quad ; \qquad \text{(C.1.28)}$$

and from Eq.(B.5.4),

$$\mathrm{I} = \mathbf{k}^1\mathbf{k}^1 + (\mathbf{k}^1\mathbf{k}^2 + \mathbf{k}^2\mathbf{k}^1)\sin\theta + \mathbf{k}^2\mathbf{k}^2 + \mathbf{k}^3\mathbf{k}^3 \quad , \qquad \text{(C.1.29)}$$

and the confusion of indices has disappeared. In any case, *the form of* Eq.(C.1.28) *is usually the simplest to use in demonstrations.*

The idemfactor also acts as unity in the dot product with a tensor, so that,

$$\mathrm{I}\cdot\Psi = \Psi \quad , \quad \Psi\cdot\mathrm{I} = \Psi \quad . \qquad \text{(C.1.30)}$$

It also helps define the reciprocal of a tensor Ψ^{-1} through,

$$\Psi\cdot\Psi^{-1} = \mathrm{I} \quad . \qquad \text{(C.1.31)}$$

C.2 **Products of Vectors and Diadics**:[1] Here, a number of product rules will be given. Going from the dot product,

$$\mathbf{V}\cdot\Psi = \mathbf{V}\cdot(\mathbf{AB}) = (\mathbf{V}\cdot\mathbf{A})\mathbf{B} = \text{ a vector} \quad , \qquad \text{(C.2.1)}$$

which was used to define dyadics, it is straightforward to show that,

$$\Phi\cdot\Psi = (\mathbf{ab})\cdot(\mathbf{CD}) = \mathbf{aD}(\mathbf{b}\cdot\mathbf{C}) = \text{a dyadic} \quad . \qquad \text{(C.2.2)}$$

For two dyadics in component form, each dyad of Φ is dotted into each dyad of Ψ. In Eq.(C.1.20) it was observed that,

$$\mathbf{V}\times\Psi = \mathbf{V}\times(\mathbf{ab}) = (\mathbf{V}\times\mathbf{a})\mathbf{b} = \text{a dyadic} \quad ; \qquad \text{(C.2.3)}$$

but, when two dyadics are used,

$$\Phi\times\Psi = (\mathbf{ab})\times(\mathbf{CD}) = \mathbf{a}(\mathbf{b}\times\mathbf{C})\mathbf{D} = \text{a triadic} \quad , \qquad \text{(C.2.4)}$$

since $\mathbf{b}\times\mathbf{C}$ is a vector. A triadic is three vectors standing together, a tetradic would be four, etc. These higher order forms are quite legitimate mathematically, although it is not always simple to get a physical interpretation. Keeping this in mind, a general product is,

$$\mathbf{V} * \Psi * \Phi * \ldots\ldots * \Omega * \mathbf{A} \quad , \qquad \text{(C.2.5)}$$

where either or both vectors can be left off, and any asterisk represents either a dot or a cross product. As long as no dyad or vector changes its position in the chain, any arbitrary grouping of factors is permissible. An example is,

$$\left[\mathbf{V}\cdot(\Psi\cdot\Phi)\right]\cdot\Omega = \left[(\mathbf{V}\cdot\Psi)\cdot\Phi\right]\cdot\Omega = (\mathbf{V}\cdot\Psi)\cdot(\Phi\cdot\Omega) = \mathbf{V}\cdot(\Psi\cdot\Phi\cdot\Omega) \quad . \qquad \text{(C.2.6)}$$

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1. See Weatherburn, Brand, Wills, Gibbs-Wilson, etc., loc.cit.

Examples of mixed cross and dot products are,

$$(\mathbf{V}\times\Phi)\cdot\Psi = \mathbf{V}\times(\Phi\cdot\Psi) \quad \text{and} \quad \Phi\cdot(\Psi\times\mathbf{V}) = (\Phi\cdot\Psi)\times\mathbf{V} \quad . \qquad \text{(C.2.7)}$$

Although the order of the vectors and dyads must be preserved in product operations, there are some cases in which a dot and cross are interchangeable, just as in the box product of Eq.(B.7.3). Therefore,

$$\Phi\cdot(\mathbf{V}\times\mathbf{U}) = (\Phi\times\mathbf{V})\cdot\mathbf{U} \quad \text{and} \quad (\mathbf{V}\times\mathbf{U})\cdot\Phi = \mathbf{V}\cdot(\mathbf{U}\times\Phi) \quad . \qquad \text{(C.2.8)}$$

Another similar relationship is,

$$\mathbf{V}\times(\mathbf{U}\cdot\Phi) = -(\mathbf{U}\cdot\Phi)\times\mathbf{V} = -\mathbf{U}\cdot(\Phi\times\mathbf{V}) \quad . \qquad \text{(C.2.9)}$$

Most of these can be proven using the box product and a single dyad.

Since dyads are composed of two vectors, the products possible between two dyads also include certain double products for which a notation has been developed.[1] The simplest, most useful of these is,

$$(\mathbf{ab}):(\mathbf{cd}) = \mathbf{a}\cdot\mathbf{c}\ \mathbf{b}\cdot\mathbf{d} = \mathbf{c}\cdot(\mathbf{ab})\cdot\mathbf{d} = \mathbf{a}\cdot(\mathbf{cd})\cdot\mathbf{b} \quad \text{scalar} \quad . \qquad \text{(C.2.10)}$$

Products involving the idemfactor I are found helpful, the simplest being,

$$\mathrm{I}\cdot\mathrm{I} = \mathrm{I} \quad \text{and} \quad \mathrm{I}:\mathrm{I} = 3 \quad . \qquad \text{(C.2.11)}$$

Another important dyadic is,

$$\mathbf{a}\times\mathrm{I} = \mathrm{I}\times\mathbf{a} = (\mathrm{I}\times\mathbf{a})_c = -\mathrm{I}\times\mathbf{a} \quad . \qquad \text{(C.2.12)}$$

Now, using Eqs.(C.2.8) and (C.2.12),

$$(\mathrm{I}\times\mathbf{a})\cdot\mathbf{r} = \mathrm{I}\cdot(\mathbf{a}\times\mathbf{r}) = \mathbf{a}\times\mathbf{r} \quad \text{and} \quad \mathbf{r}\cdot(\mathbf{a}\times\mathrm{I}) = \mathbf{r}\cdot(\mathrm{I}\times\mathbf{a}) = \mathbf{r}\times\mathbf{a} \quad . \qquad \text{(C.2.13)}$$

These equations show that the dot product of a vector $\mathbf{r}$ with $\mathrm{I}\times\mathbf{a}$ or $\mathbf{a}\times\mathrm{I}$ can be replaced by the cross product with $\mathbf{a}$. This can be extended to the dot product of a dyadic Ψ and a vector $\mathbf{a}$,

$$(\mathbf{a}\times\mathrm{I})\cdot\Psi = (\mathrm{I}\times\mathbf{a})\cdot\Psi = \mathbf{a}\times\Psi \quad \text{and} \quad \Psi\cdot(\mathbf{a}\times\mathrm{I}) = \Psi\cdot(\mathrm{I}\times\mathbf{a}) = \Psi\times\mathbf{a} \quad . \qquad \text{(C.2.14)}$$

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1. See Weatherburn, loc. cit.

Finally, the dyadic form of the vector triple product of Eq.(B.7.4) is,

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\bullet\mathbf{C}) - \mathbf{C}(\mathbf{A}\bullet\mathbf{B}) = \mathbf{A}\bullet(\mathbf{CB} - \mathbf{BC}) \quad , \qquad \text{(C.2.15)}$$

which can be used with Eq.(C.2.13) to give the very important relationship,

$$\mathrm{I}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{CB} - \mathbf{BC} \quad . \qquad \text{(C.2.16)}$$

C.3 **Invariants of a Dyadic**: The idea has been stressed, throughout the earlier discussion on coordinates, that scalars, vectors, and dyadics are physical entities independent of coordinates in terms of which they might be expressed. Implicit is the idea that the field in question is stationary (at least momentarily frozen while being examined), and quantities with this independence property are called <u>invariants</u>, meaning that they are invariant to <u>space coordinate transformations</u>. Each dyadic $\Psi = \mathbf{aA} + \mathbf{bB} + \mathbf{cC}$ has several associated quantities that exhibit this same space coordinate invariance, and five of these are important enough to have been given special names. Three are scalars, generally labeled Ψ_1, Ψ_2 and Ψ_3, one is a vector Ψ_v and one is a dyadic Ψ_a called the "adjoint" of Ψ.

The "scalar" of Ψ is:

$$\Psi_s = \Psi_1 = \mathbf{a}\bullet\mathbf{A} + \mathbf{b}\bullet\mathbf{B} + \mathbf{c}\bullet\mathbf{C} \quad ,$$

or, (C.3.1)

$$\Psi_s = \Psi_1 = q_{ij}\psi_{ij} = \delta^j_i\psi^j_i = q^{ij}\psi^{ij} \quad . \quad (*i,\ j)$$

In terms of double product notation,

$$\Psi_s = \Psi_1 = \Psi : \mathrm{I} = \mathrm{I} : \Psi \quad . \qquad \text{(C.3.2)}$$

The "vector" of Ψ is:

$$\Psi_v = \mathbf{a}\times\mathbf{A} + \mathbf{b}\times\mathbf{B} + \mathbf{c}\times\mathbf{C} \quad ,$$

or, (C.3.3)

$$\psi_v = \sqrt{q}\left[\mathbf{k}^1(\psi_{23} - \psi_{32}) + \mathbf{k}^2(\psi_{31} - \psi_{13}) + \mathbf{k}^3(\psi_{12} - \psi_{21})\right]$$

$$= \frac{1}{\sqrt{q}}\left[\mathbf{k}_1(\psi^{23} - \psi^{32}) + \mathbf{k}_2(\psi^{31} - \psi^{13}) + \mathbf{k}_3(\psi^{12} - \psi^{21})\right] \quad .$$

Clearly, from Eq.(C.3.3), any self conjugate tensor has a vector equal to zero. Conversely, any dyadic with a zero vector is symmetric.

The "second" of Ψ is (see Eq.C.1.5):

$$\Psi_2 = \psi^1_1\psi^2_2 + \psi^2_2\psi^3_3 + \psi^3_3\psi^1_1 - \psi^2_1\psi^1_2 - \psi^3_1\psi^1_3 - \psi^3_2\psi^2_3 \quad , \qquad \text{(C.3.4)}$$

and the "third" of Ψ is:

$$\Psi_3 = \psi^1_1\psi^2_2\psi^3_3 + \psi^2_1\psi^3_2\psi^1_3 + \psi^1_2\psi^2_3\psi^3_1 - \psi^1_1\psi^3_2\psi^2_3 - \psi^2_2\psi^1_3\psi^3_1 - \psi^3_3\psi^2_1\psi^1_2 \quad . \qquad \text{(C.3.5)}$$

Mixed index components were used above because they yield the simplest form.

The "adjoint" of Ψ is:

$$\Psi_a = \mathbf{k}_i \mathbf{k}_j \frac{A_{ij}}{\sqrt{q}} \quad , \qquad (*i,\ j) \qquad (C.3.6)$$

where the A_{ij} are defined as the cofactors or signed minors of Ψ_c. A useful relationship between the third scalar invariant of a dyadic and its adjoint is,

$$\Psi \bullet \Psi_a = \Psi_a \bullet \Psi = \Psi_3 I \quad , \qquad (C.3.7)$$

so, if the reciprocal Ψ^{-1} exists,

$$\Psi^{-1} = \Psi_a / \Psi_3 \quad . \qquad (C.3.8)$$

These five invariants have numerous uses related to classifying and characterizing dyadics, as will be brought out later on. Their values for the idemfactor are,

$$\begin{aligned} &I_1 = I_s = 3 \quad , \quad I_2 = 3 \quad , \quad I_3 = 1 \quad , \\ &I_v = 0 \qquad , \quad I_a = I = I_c = I^{-1} \quad . \end{aligned} \qquad (C.3.9)$$

Other invariants of a dyadic Ψ can be found, although they are often more complicated and not particularly useful. However, there is another useful invariant of dyadics called the "scalar magnitude". It will be described in the next section on dyadic classification.

C.4 **Classification of Dyadics**: Physically, dyadics describe at each point the properties of a field that relate an input or *cause* vector to an output or *effect* vector. *If the family of input vectors includes all magnitudes and directions, then one class of dyadics produces families of output vectors that also include all magnitudes and directions*. Dyadics of this class are called "complete". All others are called "incomplete". Among the incomplete dyadics are those that, regardless of the input vector, produce output vectors restricted to a plane or a line or those of zero length. *Only complete dyadics have "reciprocals"*.

The classification to which a dyadic Ψ belongs can be determined readily using two of the invariants defined in the preceding section:

$$\begin{aligned} &\Psi_3 \neq 0 \quad , && \text{Complete} \\ &\Psi_3 = 0 \quad \& \quad \Psi_a \neq 0 \quad , && \text{Planar} \\ &\Psi_a = 0 \quad \& \quad \Psi \neq 0 \quad . && \text{Linear} \end{aligned} \qquad (C.4.1)$$

These conditions are both necessary and sufficient. A <u>complete</u> dyadic cannot be reduced to less than three dyads,

$$\Psi = \mathbf{aA} + \mathbf{bB} + \mathbf{cC} \quad , \qquad (C.4.2)$$

and **a**, **b**, **c** and **A**, **B**, **C** form two sets of non-coplanar vectors. A planar dyadic can be reduced to the sum of two dyads (not less),

$$\Psi = \mathbf{uU} + \mathbf{vV} \quad , \tag{C.4.3}$$

where **u** and **v**, and **U** and **V** form two sets of non-parallel vectors. Finally, a linear dyadic is reducible to a single dyad,

$$\Psi = \mathbf{gh} \quad . \tag{C.4.4}$$

The lowest class is $\Psi = 0$, the null dyadic.

Clearly from Eq.(C.4.3) the planar dyadic dotted into a vector as a *post-factor* produces another vector in the plane defined by **u** and **v**. The dot product with a vector *pre-factor* produces a vector in the **U** / **V** plane. The linear dyadic of Eq.(C.4.4) dotted by a vector gives an output vector parallel to **g** or **h** depending upon whether the input vector is a post or pre-factor respectively. Table C.4.1 lists dot product relationships between the classes.

TABLE C.4.1

DYADIC CLASS PRODUCTS

$$\begin{aligned}
&C\bullet C \rightarrow C && P\bullet L \rightarrow \begin{cases} L \\ 0 \quad (\text{only when } \perp) \end{cases} \\
&C\bullet P \rightarrow P && \\
&C\bullet L \rightarrow L && P\bullet 0 \rightarrow 0 \\
&C\bullet 0 \rightarrow 0 && \\
&P\bullet P \rightarrow \begin{cases} P \\ L \quad (\text{only when } \perp) \end{cases} && L\bullet L \rightarrow \begin{cases} L \\ 0 \quad (\text{only when } \perp) \end{cases} \\
& && L\bullet 0 \rightarrow 0
\end{aligned}$$

Having classified dyadics, it is now possible to describe an invariant that contributes to physical interpretation of various dyadic quantities. It is defined as,

$$\Psi_m = \sqrt{N\Psi : \Psi} \quad , \tag{C.4.5}$$

where N identifies the *class* by taking on the values 1 (linear), 1/2 (planar) and 1/3 (complete). Because Ψ cannot have all zero components unless the scalar Ψ_m is zero, Ψ_m is called the "scalar magnitude" of Ψ, although it can have no such clear meaning as the magnitudes of scalar and vector fields.

C.5 **Symmetric and Antisymmetric Dyadics**: Of the many possible dyadics, two very special kinds are used most often, symmetric and antisymmetric. A symmetric dyadic is one for which,

$$\mathbf{V}\bullet\Phi = \Phi\bullet\mathbf{V} \quad . \qquad \text{(symmetric)} \qquad \text{(C.5.1)}$$

An antisymetric dyadic is one for which,

$$\mathbf{V}\bullet\Omega = -\,\Omega\bullet\mathbf{V} \quad . \qquad \text{(antisymmetric)} \qquad \text{(C.5.2)}$$

These defining equations can hold only if the dyadics meet the conditions,

$$\Phi_c = \Phi \quad \text{(symm.)} \quad , \quad \Omega_c = -\,\Omega \quad . \quad \text{(antisymm.)} \qquad \text{(C.5.3)}$$

Therefore, symmetric dyadics are also called self-conjugate and antisymmetrics are called anti-self-conjugate.

The common use of these two restricted dyadic types results because *any dyadic* Ψ *can be expressed as the sum of a symmetric and an antisymmetric part*. This can be seen by separating Ψ into,

$$\Psi = \tfrac{1}{2}(\Psi + \Psi_c) + \tfrac{1}{2}(\Psi - \Psi_c) = \Phi + \Omega \quad . \qquad \text{(C.5.4)}$$

The symmetry of Φ and the antisymmetry of Ω is clear. In component form, Eq.(C.5.3) can be written,

$$\begin{aligned} \phi_{ji} &= \phi_{ij} \quad , \quad \phi^{ji} = \phi^{ij} \quad , \qquad \text{(symm.)} \\ \Omega_{ji} &= -\,\Omega_{ij} \quad , \quad \Omega^{ji} = -\,\Omega^{ij} \quad . \quad \text{(antisymm.)} \end{aligned} \qquad \text{(C.5.5)}$$

Expressed in full component notation, Φ takes the same form as Eq.(C.1.8) with the symmetric components of Eq.(C.5.5), and,

$$\begin{aligned} \Omega = \quad & \mathbf{k}_1\mathbf{k}_1\,0 + \mathbf{k}_1\mathbf{k}_2\Omega_{12} + \mathbf{k}_1\mathbf{k}_3\Omega_{13} \\ + \; & \mathbf{k}_2\mathbf{k}_1\,(-\Omega_{12}) + \mathbf{k}_2\mathbf{k}_2\,0 + \mathbf{k}_2\mathbf{k}_3\Omega_{23} \\ + \; & \mathbf{k}_3\mathbf{k}_1\,(-\Omega_{13}) + \mathbf{k}_3\mathbf{k}_2\,(-\Omega_{23}) + \mathbf{k}_3\mathbf{k}_3\,0 \qquad \text{(antisymm.)} \end{aligned} \qquad \text{(C.5.6)}$$

Any symmetric dyadic obeys the relationship,

$$(\Phi\bullet\Phi)_s = \Phi : \Phi \quad . \qquad \text{(symmetric)} \qquad \text{(C.5.7)}$$

The diagonal terms in the antisymmetric dyadic must always be zero, since they could not have any other value and still reverse the sign in the defining Eq.(C.5.2). In general, *six quantities are necessary to specify a symmetric dyadic*, whereas only *three will determine the antisymmetric type*. The latter is true because antisymetric dyadics are always planar and can be described completely by the vector normal to the plane at the point.

A fundamental expression for an antisymmetric dyadic is,

$$\Omega = -\tfrac{1}{2} I \times \Omega_v \quad . \qquad \text{(antisymm.)} \qquad \text{(C.5.8)}$$

At any point, if an arbitrary vector $\mathbf{r}$ is dotted into Ω,

$$\mathbf{r}\cdot\Omega = -\tfrac{1}{2}\mathbf{r}\cdot(I \times \Omega_v) \quad ,$$

and the resulting vector $\mathbf{P}$, found using Eq.(C.2.17), is,

$$\mathbf{P} = \mathbf{r}\cdot\Omega = -\tfrac{1}{2}\mathbf{r} \times \Omega_v \quad , \qquad \text{(C.5.9)}$$

which is always *perpendicular* to the vector Ω_v. Thus, $\mathbf{P}$ lies in a plane, and Ω is, therefore, a *planar dyadic*, completely specified by its vector Ω_v. The components of Ω_v can be found by using Eqs.(C.3.3) and (C.5.5) to form,

$$\begin{aligned}\Omega_v &= \mathbf{k}^1(2\sqrt{q}\Omega_{23}) + \mathbf{k}^2(-2\sqrt{q}\Omega_{13}) + \mathbf{k}^3(2\sqrt{q}\Omega_{12} \\ &= 2\sqrt{q}\left[\mathbf{k}^1\Omega_{23} + \mathbf{k}^2(-\Omega_{13}) + \mathbf{k}^3\Omega_{12}\right]\end{aligned} \quad . \qquad \text{(C.5.10)}$$

An antisymmetric dyadic first presented in Eq.(C.2.12) can be obtained by carrying out the cross product of I and a new vector $\mathbf{w}$ directly, i.e.,

$$I \times \mathbf{w} = \mathbf{w} \times I = (\mathbf{k}_1\mathbf{k}^1 + \mathbf{k}_2\mathbf{k}^2 + \mathbf{k}_3\mathbf{k}^3) \times (\mathbf{k}^1 w^1 + \mathbf{k}^2 w^2 + \mathbf{k}^3 w^3) \quad .$$

Carrying out the indicated steps,

$$I \times \mathbf{w} = \frac{1}{\sqrt{q}}\begin{bmatrix} \mathbf{k}_1\mathbf{k}_1 0 & +\mathbf{k}_1\mathbf{k}_2(-w^3) & +\mathbf{k}_1\mathbf{k}_3 w^2 \\ +\mathbf{k}_2\mathbf{k}_1 w^3 & +\mathbf{k}_2\mathbf{k}_2 0 & +\mathbf{k}_2\mathbf{k}_3(-w^1) \\ +\mathbf{k}_3\mathbf{k}_1(-w^2) & +\mathbf{k}_3\mathbf{k}_2 w^1 & +\mathbf{k}_3\mathbf{k}_3 0 \end{bmatrix} \quad . \qquad \text{(C.5.11)}$$

Often the identification of $\mathbf{w}$ and Ω_v is made so that $\mathbf{w} = \tfrac{1}{2}\Omega_v$ and,

$$w^1 = \sqrt{q}\ \Omega_{23} \quad , \quad w^2 = -\sqrt{q}\ \Omega_{13} \quad , \quad w^3 = \sqrt{q}\ \Omega_{12} \quad . \qquad \text{(C.5.12)}$$

Then,

$$\Omega = -\,I \times \mathbf{w} = -\,\mathbf{w} \times I \quad , \quad \mathbf{V}\cdot\Omega = \mathbf{w} \times \mathbf{V} \quad \text{and} \quad \Omega\cdot\mathbf{V} = \mathbf{V} \times \mathbf{w} \quad . \qquad \text{(C.5.13)}$$

Thus, any antisymmetric second order tensor can be written in the form of Eq.(C.5.!3), where $\mathbf{w}$ is a vector given by Eq.(C.5.12) and is *normal* to the plane of Ω. This is diagramed in Figure C.5.1.

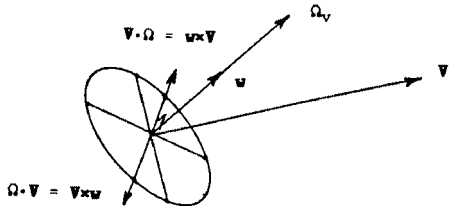


Figure C.5.1
W and the vector of plane dyadic Ω.

It should be emphasized that not all planar dyadics are antisymmetric. For example, $\Phi = \mathbf{k}_1\mathbf{k}_2 + \mathbf{k}_2\mathbf{k}_1$ is planar but also symmetric. Another concept that must be made clear is that, since the direction of $\mathbf{w}$ can vary from point to point, the antisymmetric dyadic has

different orientations at different points in space.

Going back to the general dyadic of Eq.(C.5.4), not only can it be separated into self-conjugate and anti-self-conjugate parts, but the latter is a planar dyadic specified by its vector according to Eq.(C.5.8). Since, as mentioned in connection with Eq.(C.3.3), any symmetric dyadic has a vector equal to zero, the vector of a general dyadic is equal to the vector of its antisymetric part, i.e.,

$$\Psi_v = \Phi_v + \Omega_v = \Omega_v \quad . \tag{C.5.14}$$

Thus, Eq.(C.5.4) becomes,

$$\Psi = \Phi - \tfrac{1}{2} I \times \Psi_v \quad \text{and} \quad \Psi_c = \Psi + I \times \Psi_v \quad . \tag{C.5.15}$$

A few more useful dyadic relationships are given in Table C.5.1.

TABLE C.5.1

For Φ symmetrical and Ω antisymmetrical.

$\hat{\mathbf{r}} \cdot \Omega \cdot \hat{\mathbf{r}} = 0$	$\Omega_s = 0$
$\Phi \cdot \Omega = -(\Omega \cdot \Phi)_c$	$\Phi : \Omega = 0$
$(\Omega \cdot \Omega)_s = -\Omega : \Omega$	$(\Phi \cdot \Omega)_s = 0$

C.6 **Principal Axes of a Symmetric Dyadic**: In the preceding section, the basic nature of an antisymmetric dyadic was shown to be *planar*. Dotting it with a unit vector $\hat{\mathbf{r}}$ of arbitrary orientation always results in an output vector confined to a particular plane in space.

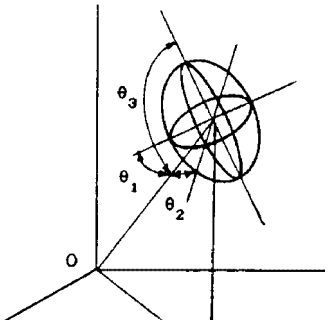


Figure C.6.1
The complete, symmetric dyadic ellipsoid.

If a complete, symmetric dyadic is dotted with a unit vector $\hat{\mathbf{r}}$ that takes all possible orientations at the point being studied, the output vector $\mathbf{R}$, given by $\mathbf{R} = \Phi \cdot \hat{\mathbf{r}}$, describes an ellipsoid as depicted in Figure C.6.1. Since the output vector $\mathbf{R}$, the dyadic Φ, and the input vector $\hat{\mathbf{r}}$ are independent of the coordinates used to describe them, the ellipsoid shape and size are invariant to space coordinate transformations, and *generally described by three numbers* λ_i *representing the lengths of the semi-axes*. Furthermore, the orientation of the ellipsoid with respect to a line drawn from some arbitrary point (e.g. the origin of the coordinates) to the point studied also must be independent of the coordinate system. So the three direction cosines of the axes of the ellipsoid relative to that arbitrarily drawn line must also be invariant (see Figure C.6.1). *Since*

the direction cosines are related through the geometry, they represent only two independent numbers. One other number represents the rotation of the ellipsoid around the line. *Thus, six quantities describe the ellipsoid.*

The expression for specifying the axes of the ellipsoid is,

$$\mathbf{R} = \Phi \bullet \hat{\mathbf{r}} = \lambda_i \hat{\mathbf{r}} \quad . \tag{C.6.1}$$

It says that for three specific directions of $\hat{\mathbf{r}}$, the output vector $\mathbf{R} = \Phi \bullet \hat{\mathbf{r}}$ has the same direction as the input vector $\hat{\mathbf{r}}$, and that given Φ, those three directions can be found by solving for the three particular values of $\hat{\mathbf{r}}$ for which this is true. *These three directions correspond to the principal axes of the dyadic* Φ. Eq.(C.6.1) is solved by substituting for Φ, in the form of Eq.(C.1.5), and $\hat{\mathbf{r}} = \mathbf{k}_1\alpha_1 + \mathbf{k}_2\alpha_2 + \mathbf{k}_3\alpha_3$ which, because the $\mathbf{k}_j$ are unit vectors, yields,

$$\begin{aligned} (\phi_1^1 - \lambda_i)\alpha_1 + \phi_1^2\alpha_2 + \phi_1^3\alpha_3 &= 0 \\ \phi_1^2\alpha_1 + (\phi_2^2 - \lambda_i)\alpha_2 + \phi_2^3\alpha_3 &= 0 \\ \phi_1^3\alpha_1 + \phi_2^3\alpha_2 + (\phi_3^3 - \lambda_i)\alpha_3 &= 0 \end{aligned} \quad . \tag{C.6.2}$$

These three linear, homogeneous equations for the direction cosines α_j have a unique solution only if the determinant of the coefficients of the α_j is zero. When the determinant is expanded, it gives a *cubic* equation for λ_i with coefficients that are found to be the three scalar invariants of Φ,

$$\lambda_i^3 - \Phi_1\lambda_i^2 - \Phi_2\lambda_i - \Phi_3 = 0 \quad . \tag{C.6.3}$$

Thus, if Φ is given, Φ_1, Φ_2 and Φ_3 can be determined (as in Section C.3), allowing the three λ_i to be found by solving Eq. (C.6.3). For each λ_i, the direction cosines α_i of the $\hat{\mathbf{r}}_i$ can be calculated using Eq.(C.6.2). The three $\hat{\mathbf{r}}_i$ give the directions of the principal axes of Φ. Since the λ_i are found directly from the scalar invariants, they too are invariant, showing that the principal axes are invariant as already pointed out. From the geometry of an ellipsoid, the three $\hat{\mathbf{r}}_i$ are orthogonal.

Because the principal axes of Φ are orthogonal, a useful simplification can result *when orthogonal coordinate systems are used.* In that case it is possible to *reorient* the coordinate axes to match the principal axes of Φ at the point under study. If this is done, the dyadic expressed in these *oriented* or *principal* coordinates has only three diagonal components given by,

$$\Phi = \mathbf{k}_1\mathbf{k}_1\lambda_1 + \mathbf{k}_2\mathbf{k}_2\lambda_2 + \mathbf{k}_3\mathbf{k}_3\lambda_3 \quad . \tag{C.6.4}$$

This diagonalizing rotation of orthogonal coordinates is often used. It cannot be done in non-orthogonal coordinates because the angle variables give off-diagonal components, since not all of the non-

orthogonal coordinate axes can be matched to the orthogonal principal axes of Φ. When the diagonalization is used, it signifies that the need for specifying the orientation of the dyadic has been eliminated, and only the three numbers needed to express the shape of the ellipsoid are involved in writing the dyadic.

The process of finding principal axes is not restricted to symmetrical dyadics. However, when it is applied to antisymmetric dyadics, they yield imaginary roots with no physical value, since a plane is more properly their physical representation. In general, it is more proper, in applying these mathematical tools to physics, to first separate the dyadic into its symmetric and antisymmetric parts and then apply the two physically different pictures to the respective parts. If an incomplete, *symmetric* dyadic is *planar*, then a similar kind of representation can be made in the plane, with two directions specifying the principal axes of an ellipse lying in the plane.

APPENDIX D

DIFFERENTIAL FORM OF THE FIELD FUNCTIONS

D.1 **Introduction**: Now that the concepts related to coordinate systems and dyadics have been set down, it is possible to return to the fundamental field functions such as the gradient, the divergence, the surrounding function, etc., and address them in terms of coordinates to facilitate the study of specific examples of physical fields. Generally, field functions are expressions representing changes in the fields over differentially small distances, or the integral of conditions in many differentially small regions. Thus, the need for differentials of dyadics, vectors, and scalars in *curvilinear* coordinates arises immediately and is discussed.

Next, the mathematical operator ∇ is introduced as a mechanical device to quickly calculate the component forms of the fundamental field functions in generalized coordinates.

Finally, these field functions are presented in the *simplified* form permitted by reduction, from the general, non-orthogonal coordinates to orthogonal and finally rectangular coordinates.

D.2 **First Partials of Unit Vectors and Christoffel Coefficients**: Differentiation of vectors was first introduced in Section B.2. The process of differentiating general products of scalars, vectors, and dyadics is carried out in a completely straightforward way according to the standard rule for differentiation of scalar products, with the addition of the requirement that the *order* of the factors be maintained. Examples of this are,

$$\begin{aligned}
&\frac{d(\phi\mathbf{V})}{dt} = \phi\frac{d\mathbf{V}}{dt} + \frac{d\phi}{dt}\mathbf{V} \\
&\frac{d(\phi\mathbf{Va})}{dt} = \phi\mathbf{V}\frac{d\mathbf{a}}{dt} + \phi\frac{d\mathbf{V}}{dt}\mathbf{a} + \frac{d\phi}{dt}\mathbf{Va} \quad . \qquad \text{(D.2.1)} \\
&\frac{\partial(\Psi\times\Phi)}{\partial s} = \Psi\times\frac{\partial\Phi}{\partial s} + \frac{\partial\Psi}{\partial s}\times\Phi
\end{aligned}$$

A typical problem that arises is to find some partial derivative of a dyadic such as $\Psi = \hat{\mathbf{r}}\hat{\boldsymbol{\Theta}}_c\psi_{12}$. One of these, written in cone coordinates, is,

$$\frac{\partial\Psi}{\partial\zeta} = \hat{\mathbf{r}}\hat{\boldsymbol{\Theta}}_c\frac{\partial\psi_{12}}{\partial\zeta} + \hat{\mathbf{r}}\frac{\partial\hat{\boldsymbol{\Theta}}_c}{\partial\zeta}\psi_{12} + \frac{\partial\hat{\mathbf{r}}}{\partial\zeta}\hat{\boldsymbol{\Theta}}_c\psi_{12} \quad .$$

In *curvilinear coordinates, unit vectors* like $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\Theta}}_c$, *vary in direction from point to point.* Therefore, to obtain the correct form for $\partial\Psi/\partial\zeta$,

the partial derivatives of the unit vectors must be found. Since the partials are also vectors, they must be found in terms of their vector components.

In general, the first partials of the unit vectors with respect to real length changes can be expressed in vector form as,

$$\frac{\partial \mathbf{k}_j}{\partial s_i} = C_{ijp}\mathbf{k}_p \quad ; \qquad (*p) \qquad (D.2.2)$$

and since there are three unit vectors $\mathbf{k}_j$ (j = l to 3), three differential distances ds_i (i = 1 to 3) and three components C_{ijp} for each partial (p = 1 to 3), there are altogether twenty-seven components at each point in space. The twenty-seven components C_{ijp} are called Christoffel coefficients of the second kind, and are to be found in terms of the coordinates, the angle variables q_{ij} and the scale factors h_i. From Eq.(B.5.2), the angle variable relationship $\mathbf{k}_i \bullet \mathbf{k}_j = q_{ij}$,

$$\frac{\partial \mathbf{k}_i}{\partial s_p} \bullet \mathbf{k}_j + \mathbf{k}_i \bullet \frac{\partial \mathbf{k}_j}{\partial s_p} = \frac{\partial q_{ij}}{\partial s_p} \quad .$$

Combining this with Eq.(D.2.2),

$$C_{pim}\mathbf{k}_m \bullet \mathbf{k}_j + \mathbf{k}_j \bullet C_{pjm}\mathbf{k}_m = C_{pim}q_{mj} + C_{pjm}q_{im} = \frac{\partial q_{ij}}{\partial s_p} \quad . \quad (*m) \qquad (D.2.3)$$

At this point a great simplification results if *a new set of quantities is defined*,

$$C_{pi}^{\ j} = C_{pim}q_{mj} \quad . \qquad (*m) \qquad (D.2.4)$$

These new quantities are called Christoffel coefficients of the first kind. In terms of these coefficients, Eq.(D.2.3) becomes,

$$C_{pi}^{\ j} + C_{pj}^{\ i} = \frac{\partial q_{ij}}{\partial s_p} \quad . \qquad (D.2.5)$$

If these equations are used to find the $C_{pi}^{\ j}$, *then the reciprocal relationship*,

$$C_{pik} = q^{kj}C_{pi}^{\ j} \quad , \qquad (*j) \qquad (D.2.6)$$

can be used to find the coefficients in the first partials of the unit vectors of Eq.(D.2.2). However, there are generally 27 $C_{pi}^{\ j}$ values, whereas *there are only 18 independent equations* represented by Eq.(D.2.5), since the q_{ij} are symmetric. *The 9 additional equations*

required can be obtained by first separating the coefficients from the $\mathbf{k}_p$ in Eq.(D.2.2) by the use of the reciprocal base vectors $\mathbf{k}^k$, so that,

$$C_{ijk} = \frac{\partial \mathbf{k}_j}{\partial s_i} \bullet \mathbf{k}^k \quad . \tag{D.2.7}$$

After considerable manipulation,[1]

$$C_{ij}^{\ m} - C_{ji}^{\ m} = \frac{q_{im}}{h_i}\frac{\partial h_i}{\partial s_j} - \frac{q_{jm}}{h_j}\frac{\partial h_j}{\partial s_i} \quad . \tag{D.2.8}$$

This relates the $C_{ij}^{\ k}$ and the *scale factors* h_i just as Eq.(D.2.5) related the coefficients to the *angle variables* q_{ij}. Eqs.(D.2.5) and (D.2.8) must now be solved for the 27 $C_{pi}^{\ j}$.

When this is done, the general solution is found to be,

$$\begin{aligned} C_{ij}^{\ k} = \ & \frac{1}{2}\left[\frac{\partial q_{jk}}{\partial s_i} + \frac{\partial q_{ik}}{\partial s_j} - \frac{\partial q_{ij}}{\partial s_k}\right] - \frac{q_{jk}}{2}\left[\frac{1}{h_j}\frac{\partial h_j}{\partial s_i} - \frac{1}{h_k}\frac{\partial h_k}{\partial s_i}\right] \\ & + \frac{q_{ki}}{2}\left[\frac{1}{h_k}\frac{\partial h_k}{\partial s_j} + \frac{1}{h_i}\frac{\partial h_i}{\partial s_j}\right] - \frac{q_{ij}}{2}\left[\frac{1}{h_i}\frac{\partial h_i}{\partial s_k} + \frac{1}{h_j}\frac{\partial h_j}{\partial s_k}\right] \end{aligned} \quad . \tag{D.2.9}$$

Some reduction in complexity results from symmetries and repeated indices; so, in actual problem solving, it is much more convenient to calculate the $C_{ij}^{\ k}$ using the reduced forms in Table D.2.1.

It would be possible now to use the reciprocal relationship of Eq.(D.2.6) to obtain a general expression for the C_{pik} for use in Eq.(D.2. 2). This becomes very unwieldy, and generally it is better to use the equations in Table D.2.1 to find the actual $C_{ij}^{\ k}$ in a particular example and then to convert to the C_{pik}. To Illustrate the procedure, some of the coefficients will be obtained for cone coordinates where,

$$\begin{gathered} \mathbf{k}_1 = \hat{\mathbf{r}} \quad , \quad \mathbf{k}_2 = \hat{\boldsymbol{\Theta}}_c \quad , \quad \mathbf{k}_3 = \hat{\boldsymbol{\alpha}} \quad , \\ h_1 = \sec\theta \quad , \quad h_2 = \zeta \sec^2\theta \quad , \quad h_3 = \zeta \tan\theta \quad , \\ q_{12} = \hat{\mathbf{r}} \bullet \hat{\boldsymbol{\Theta}}_c = \sin\theta \quad , \quad q_{13} = \hat{\boldsymbol{\alpha}} \bullet \hat{\mathbf{r}} = 0 \quad , \quad q_{23} = \hat{\boldsymbol{\Theta}}_c \bullet \hat{\boldsymbol{\alpha}} = 0 \quad , \\ ds_1 = \sec\theta \, d\zeta \quad , \quad ds_2 = \zeta \sec^2\theta \, d\theta \quad , \quad ds_3 = \zeta \tan\theta \, d\alpha \quad . \end{gathered} \tag{D.2.10}$$

1. For the full details see R.H.Dishington, Physics, Chapter 5, Beak Publications, Pacific Palisades, CA (1989).

TABLE D.2.1

CHRISTOFFEL SYMBOLS OF THE FIRST KIND

$$C_{pi}^{i} = 0 \quad , \quad C_{ii}^{p} = \frac{\partial q_{ip}}{\partial s_i} - C_{ip}^{i} \quad , \quad C_{ip}^{i} = \frac{1}{h_i}\frac{\partial h_i}{\partial s_p} - \frac{q_{pi}}{h_p}\frac{\partial h_p}{\partial s_i}$$

$$C_{12}^{3} = \frac{1}{2}\left(\frac{\partial q_{23}}{\partial s_1} + \frac{\partial q_{13}}{\partial s_2} - \frac{\partial q_{12}}{\partial s_3}\right) - \frac{q_{23}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_1} - \frac{1}{h_3}\frac{\partial h_3}{\partial s_1}\right) + \frac{q_{13}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_2} + \frac{1}{h_1}\frac{\partial h_1}{\partial s_2}\right) - \frac{q_{12}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_3} + \frac{1}{h_2}\frac{\partial h_2}{\partial s_3}\right)$$

$$C_{13}^{2} = \frac{1}{2}\left(\frac{\partial q_{23}}{\partial s_1} + \frac{\partial q_{12}}{\partial s_3} - \frac{\partial q_{13}}{\partial s_2}\right) - \frac{q_{23}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_1} - \frac{1}{h_2}\frac{\partial h_2}{\partial s_1}\right) + \frac{q_{12}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_3} + \frac{1}{h_1}\frac{\partial h_1}{\partial s_3}\right) - \frac{q_{13}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_2} + \frac{1}{h_3}\frac{\partial h_3}{\partial s_2}\right)$$

$$C_{21}^{3} = \frac{1}{2}\left(\frac{\partial q_{13}}{\partial s_2} + \frac{\partial q_{23}}{\partial s_1} - \frac{\partial q_{12}}{\partial s_3}\right) - \frac{q_{13}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_2} - \frac{1}{h_3}\frac{\partial h_3}{\partial s_2}\right) + \frac{q_{23}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_1} + \frac{1}{h_2}\frac{\partial h_2}{\partial s_1}\right) - \frac{q_{12}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_3} + \frac{1}{h_1}\frac{\partial h_1}{\partial s_3}\right)$$

$$C_{23}^{1} = \frac{1}{2}\left(\frac{\partial q_{13}}{\partial s_2} + \frac{\partial q_{12}}{\partial s_3} - \frac{\partial q_{23}}{\partial s_1}\right) - \frac{q_{13}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_2} - \frac{1}{h_1}\frac{\partial h_1}{\partial s_2}\right) + \frac{q_{12}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_3} + \frac{1}{h_2}\frac{\partial h_2}{\partial s_3}\right) - \frac{q_{23}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_1} + \frac{1}{h_3}\frac{\partial h_3}{\partial s_1}\right)$$

$$C_{31}^{2} = \frac{1}{2}\left(\frac{\partial q_{12}}{\partial s_3} + \frac{\partial q_{23}}{\partial s_1} - \frac{\partial q_{13}}{\partial s_2}\right) - \frac{q_{12}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_3} - \frac{1}{h_2}\frac{\partial h_2}{\partial s_3}\right) + \frac{q_{23}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_1} + \frac{1}{h_3}\frac{\partial h_3}{\partial s_1}\right) - \frac{q_{13}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_2} + \frac{1}{h_1}\frac{\partial h_1}{\partial s_2}\right)$$

$$C_{32}^{1} = \frac{1}{2}\left(\frac{\partial q_{12}}{\partial s_3} + \frac{\partial q_{13}}{\partial s_2} - \frac{\partial q_{23}}{\partial s_1}\right) - \frac{q_{12}}{2}\left(\frac{1}{h_2}\frac{\partial h_2}{\partial s_3} - \frac{1}{h_1}\frac{\partial h_1}{\partial s_3}\right) + \frac{q_{13}}{2}\left(\frac{1}{h_1}\frac{\partial h_1}{\partial s_2} + \frac{1}{h_3}\frac{\partial h_3}{\partial s_2}\right) - \frac{q_{23}}{2}\left(\frac{1}{h_3}\frac{\partial h_3}{\partial s_1} + \frac{1}{h_2}\frac{\partial h_2}{\partial s_1}\right)$$

All others zero

Substituting the values from Eq.(D.2.10) into the equations of Table D.2.1,

$$C_{21}^{\ 2} = \frac{1}{\zeta \sec^2\theta} \quad , \quad C_{31}^{\ 3} = \frac{1}{\zeta \sec\theta} \quad , \quad C_{32}^{\ 3} = \frac{1}{\zeta \tan\theta}$$
$$C_{33}^{\ 1} = -\frac{1}{\zeta \sec\theta} \quad , \quad C_{33}^{\ 2} = -\frac{1}{\zeta \tan\theta} \quad , \quad \text{All others zero} \qquad . \qquad \text{(D.2.11)}$$

Now, with the aid of the reciprocal relationship $C_{ijn} = q^{nk} C_{ij}^{\ k}$, (*k),

$$C_{211} = -\frac{\tan\theta}{\zeta \sec^2\theta} \quad , \quad C_{212} = \frac{1}{\zeta \sec\theta} \quad , \quad C_{313} = \frac{1}{\zeta \sec\theta}$$
$$C_{323} = \frac{1}{\zeta \tan\theta} \quad , \quad C_{332} = -\frac{1}{\zeta \tan\theta} \quad , \quad \text{All others zero} \qquad . \qquad \text{(D.2.12)}$$

Finally, from Eq.(D.2.2),

$$\frac{\partial \hat{\mathbf{r}}}{\partial s_1} = 0 \quad , \quad \frac{\partial \hat{\boldsymbol{\Theta}}_c}{\partial s_1} = 0 \quad , \quad \frac{\partial \hat{\boldsymbol{\alpha}}}{\partial s_1} = 0$$
$$\frac{\partial \hat{\mathbf{r}}}{\partial s_2} = -\hat{\mathbf{r}} \frac{\tan\theta}{\zeta \sec^2\theta} + \hat{\boldsymbol{\Theta}}_c \frac{1}{\zeta \sec\theta} \quad , \quad \frac{\partial \hat{\boldsymbol{\Theta}}_c}{\partial s_2} = 0 \quad , \quad \frac{\partial \hat{\boldsymbol{\alpha}}}{\partial s_2} = 0 \qquad \text{(D.2.13)}$$
$$\frac{\partial \hat{\mathbf{r}}}{\partial s_3} = \hat{\boldsymbol{\alpha}} \frac{1}{\zeta \sec\theta} \quad , \quad \frac{\partial \hat{\boldsymbol{\Theta}}_c}{\partial s_3} = \hat{\boldsymbol{\alpha}} \frac{1}{\zeta \tan\theta} \quad , \quad \frac{\partial \hat{\boldsymbol{\alpha}}}{\partial s_3} = -\hat{\boldsymbol{\Theta}}_c \frac{1}{\zeta \tan\theta}$$

Before leaving this subject, it should be mentioned that in co-contra, unitary systems, where the angle variables q_{ij} and the scale factors h_i are combined into the metric coefficients g_{ij}, the Christoffel coefficients are represented by even simpler and more symmetrical equations. For example, the defining equations,

$$\frac{\partial \mathbf{b}_u}{\partial \xi^s} = \Gamma_{su}^{\ e} \mathbf{b}_e \quad , \qquad \text{(D.2.14)}$$

have coefficients of the second kind given by the reciprocal relationship,

$$\Gamma_{su}^{\ e} = \{su, e\} = g^{en}[su, n] = g^{en}\Gamma_{sun} \quad , \qquad \text{(D.2.15)}$$

where Γ_{sun} is the coefficient of the first kind written as,

$$\Gamma_{sun} = \tfrac{1}{2}\left[\frac{\partial g_{un}}{\partial \xi^s} + \frac{\partial g_{sn}}{\partial \xi^u} - \frac{\partial g_{su}}{\partial \xi^n}\right] \quad . \qquad \text{(D.2.16)}$$

Admittedly more simple, these forms do not allow the intuitive understanding of the roles of the *angle variables* and the *scale factors*. Once the real-length, unit vector Christoffel coefficients are obtained,

they are often simpler than those found from Eqs.(D.2.15) and (D.2.16), and are no more complicated.

D.3 **Derivatives of Vectors and Dyadics**: The total derivative of the vector **A** with respect to distance is,

$$d\mathbf{A} = \frac{\partial \mathbf{A}}{\partial s_1} ds_1 + \frac{\partial \mathbf{A}}{\partial s_2} ds_2 + \frac{\partial \mathbf{A}}{\partial s_3} ds_3 \quad , \qquad (D.3.1)$$

and it is instructive to work out the partials using the formulas developed in the preceding section. To show the facility of using the index notation in this type of derivation, the general case will be obtained. Writing,

$$\mathbf{A} = A_j \mathbf{k}_j \quad , \qquad (*j)$$

the general partial is,

$$\frac{\partial \mathbf{A}}{\partial s_i} = \frac{\partial A_j}{\partial s_i} \mathbf{k}_j + A_j \frac{\partial \mathbf{k}_j}{\partial s_i} \quad , \qquad (*j) \quad (D.3.2)$$

which says that in addition to the changes in the magnitudes of the components A_j, further changes in the vector **A** occur due to the changes in directions of the unit vectors. Now, substituting from Eq.(D.2.2),

$$\frac{\partial \mathbf{A}}{\partial s_i} = \frac{\partial A_j}{\partial s_i} \mathbf{k}_j + A_j C_{ijp} \mathbf{k}_p \quad . \qquad (*j,p) \quad (D.3.3)$$

In any summation, such as that over j or p in this equation, *any pair of matched indices can be replaced by any other pair*, and the summation remains the same. These *repeated* letters are called "dummy-indices". It helps in the regrouping of terms to replace j by m and p by j in the second term of Eq.D.3.3). They could have been replaced by any other two letters, but it is desired to combine the $\mathbf{k}_j$ terms, so the equation is rewritten,

$$\frac{\partial \mathbf{A}}{\partial s_i} = \frac{\partial A_j}{\partial s_i} \mathbf{k}_j + A_m C_{imj} \mathbf{k}_j \quad . \qquad (*j,m)$$

Now, factoring the $\mathbf{k}_j$,

$$\frac{\partial \mathbf{A}}{\partial s_i} = \left(\frac{\partial A_j}{\partial s_i} + A_m C_{imj} \right) \mathbf{k}_j \quad , \qquad (*j,m) \quad (D.3.4)$$

which is the general case for vectors. The process can be repeated to give higher derivatives.

The saving of steps using the index notation is even more spectacular when the general expression for the partial of a dyadic is desired. Writing,

$$\Psi = \psi_{ij} \mathbf{k}_i \mathbf{k}_j \quad , \qquad (*i,j) \quad (D.3.5)$$

the general partial is,

$$\frac{\partial \Psi}{\partial s_m} = \frac{\partial \psi_{ij}}{\partial s_m}\mathbf{k}_i\mathbf{k}_j + \psi_{ij}\frac{\partial \mathbf{k}_i}{\partial s_m}\mathbf{k}_j + \psi_{ij}\mathbf{k}_i\frac{\partial \mathbf{k}_j}{\partial s_m} \quad . \qquad (*i,j)$$

Following the same steps as above,

$$\frac{\partial \Psi}{\partial s_m} = \left(\frac{\partial \psi_{ij}}{\partial s_m} + \psi_{pj}C_{mpi} + \psi_{ip}C_{mpj}\right)\mathbf{k}_i\mathbf{k}_j \quad , \quad (*i,j,p) \quad (D.3.6)$$

which is the general case for dyadics.

For a mixed dyadic such as $\Psi = \psi^i{}_j\mathbf{k}^i\mathbf{k}_j$, (*i,j), the general partial is,

$$\frac{\partial \Psi}{\partial s_m} = \left(\frac{\partial \psi^i{}_j}{\partial s_m} - \psi^p{}_jC_{mip} + \psi^i{}_pC_{mpj}\right)\mathbf{k}^i\mathbf{k}_j \quad . \quad (*i,j,p) \quad (D.3.7)$$

Generally one extra term correcting for unit vector variations is added or subtracted for each increase in the number of unit vectors involved.

It takes little convincing to use the component notation when working out a general derivation of this kind, because the labor involved in the long-hand approach is appalling. It is equally bad to insist on using the component notation when working with the fields and field operators introduced in earlier sections. This division of usefulness will be observed carefully in this work, and component notation will be used only in those examples where required to increase the clarity or significantly reduce the labor.

One word of caution is needed in taking higher order derivatives. In such a case, it is often useful to interchange the order of differentiation. However, Eqs.(B.4.2), (B.4.4) and (D.3.1), representing total derivatives written in terms of changes in the real distances ds_i, *may not be integrable*. In such a case, the order of $\partial^2\Psi / \partial s_i \partial s_j$ cannot be interchanged. Instead, the ds_i and ds_j must be replaced through Eq.(B.4.7), and then the differentiation can be carried out. Higher order differentiation with respect to coordinate variables is interchangeable.

D.4 **The Field Operator ∇**: In Appendix A all of the important field functions were defined as field entities independent of coordinates. Each of these functions represents a clearly defined, intuitively understandable property of fields, and in developing the theory of fields, each will be used in that way. Nevertheless, in solving field problems, coordinates and a representation of those field functions in terms of the coordinates are required. The expressions which give the components of the field functions will be examined next.

The simplest way to begin is to combine Eqs.(A.4.1) and (A.4.5) with the result,

$$d\phi = d\mathbf{s}\bullet\nabla\phi \quad . \tag{D.4.1}$$

Compare this with the definition of the total derivative,

$$d\phi = \frac{\partial\phi}{\partial s_1}\,ds_1 + \frac{\partial\phi}{\partial s_2}\,ds_2 + \frac{\partial\phi}{\partial s_3}\,ds_3 \quad . \tag{D.4.2}$$

Equating these two forms of $d\phi$ gives,

$$d\mathbf{s}\bullet\nabla\phi = ds_1\,\frac{\partial\phi}{\partial s_1} + ds_2\,\frac{\partial\phi}{\partial s_2} + ds_3\,\frac{\partial\phi}{\partial s_3} \quad , \tag{D.4.3}$$

and since both $d\mathbf{s}$ and $\nabla\phi$ are vectors, Eqs.(D.4.3), (B.7.1) and (B.8.7) suggest that they can be written, $d\mathbf{s} = \mathbf{k}_1 ds_1 + \mathbf{k}_2 ds_2 + \mathbf{k}_3 ds_3$, and,

$$\nabla\phi = \mathbf{k}^1\,\frac{\partial\phi}{\partial s_1} + \mathbf{k}^2\,\frac{\partial\phi}{\partial s_2} + \mathbf{k}^3\,\frac{\partial\phi}{\partial s_3} \quad . \tag{D.4.4}$$

Eq.(D.4.4) *will be taken as the basic form for obtaining the components of the gradient of any scalar field.*

The components $\partial\phi/\partial s_i$ are *not the real length components* of the vector $\nabla\phi$ (which are symbolized as $(\nabla\phi)_i$ and are used with unit vectors $\mathbf{k}_i$). Instead, they are the inverse components $(\nabla\phi)^i$. Using Eq.(B.8.25), $\nabla\phi$ can also be written in terms of the $\mathbf{k}_i$ components, but Eq.(D.4.4) gives the <u>simplest</u> form, with the components being equal in magnitude to the changes in ϕ with respect to changes in distance (taken along the $\mathbf{k}_i$ directions as specified by ds_1, ds_2 and ds_3).

Starting with Eqs.(A.6.1) and (A.6.2),

$$d\mathbf{V} = d\mathbf{s}\bullet\nabla\mathbf{V} \quad , \tag{D.4.5}$$

and repeating the same steps as above, the component equation for $\nabla\mathbf{V}$ becomes,

$$\nabla\mathbf{V} = \mathbf{k}^1\,\frac{\partial\mathbf{V}}{\partial s_1} + \mathbf{k}^2\,\frac{\partial\mathbf{V}}{\partial s_2} + \mathbf{k}^3\,\frac{\partial\mathbf{V}}{\partial s_3} \quad , \tag{D.4.6}$$

which has a form almost identical to that of Eq.(D.4.4). These two equations suggest that, for finding field quantities in component form the vector operator,

$$\nabla = \mathbf{k}^1\,\frac{\partial}{\partial s_1} + \mathbf{k}^2\,\frac{\partial}{\partial s_2} + \mathbf{k}^3\,\frac{\partial}{\partial s_3} \quad , \tag{D.4.7}$$

might offer a simple, mechanical technique. Expanding expressions for the divergence, curl, etc. verifies this. For example, using Eq.(D.3.4),

$$\text{div } \mathbf{V} = \mathbf{k}^1 \bullet \frac{\partial \mathbf{V}}{\partial s_1} + \mathbf{k}^2 \bullet \frac{\partial \mathbf{V}}{\partial s_2} + \mathbf{k}^3 \bullet \frac{\partial \mathbf{V}}{\partial s_3} \quad , \tag{D.4.8}$$

which clearly can be written,

$$\text{div } \mathbf{V} = \nabla \bullet \mathbf{V} \quad . \tag{D.4.9}$$

In a similar fashion, it can be shown *that every differential field is given by the operation indicated between* ∇ *and the scalar or vector field*. The curl of $\mathbf{F}$, for example, is given by,

$$\nabla \times \mathbf{F} = \mathbf{k}^1 \times \frac{\partial \mathbf{F}}{\partial s_1} + \mathbf{k}^2 \times \frac{\partial \mathbf{F}}{\partial s_2} + \mathbf{k}^3 \times \frac{\partial \mathbf{F}}{\partial s_3} \quad , \tag{D.4.10}$$

which is nothing more than the vector of $\nabla\mathbf{F}$ of Eq(D.4.6), and the surrounding function of ϕ is,

$$\nabla^2 \phi = \nabla \bullet \nabla \phi = \mathbf{k}^1 \bullet \frac{\partial \nabla \phi}{\partial s_1} + \mathbf{k}^2 \bullet \frac{\partial \nabla \phi}{\partial s_2} + \mathbf{k}^3 \bullet \frac{\partial \nabla \phi}{\partial s_3} \quad . \tag{D.4.11}$$

No difficulty arises, and all of the standard rules of vector and dyadic products apply, as long as each scalar, unit vector, reciprocal base vector, etc. that follows ∇ is operated on by the partials according to those rules *without changing the order* of any vectors or dyadics. Equations found using the ∇ operator as a vector, such as Eqs.(D.4.4), (D.4.6), (D.4.8), (D.4.10) and (D.4.11), apply in any coordinate system.

Many authors make an issue of the fact that the component forms resulting from ∇ operations in different coordinate systems are not the same. It would be surprising if they were. Instead of pointing out this expected difference, it is most important to emphasize that *the vector operator* ∇ *of* Eq.(D.4.7) *can be applied to any field in any operation such as* $\bullet$ *or* $\times$ *and in any number of orders of* ∇, *and the resulting entity, independent of coordinates and coordinate systems, will be correct for all coordinate systems, and when expanded out will yield the correct components in those systems.*

Next, a few special cases will be inspected to show further the power of the mechanical operator ∇. Just as $\nabla\mathbf{V}$ can be expressed by Eq.(D.4.6), another useful dyadic is,

$$\mathbf{V}\nabla = \frac{\partial \mathbf{V}}{\partial s_1}\mathbf{k}^1 + \frac{\partial \mathbf{V}}{\partial s_2}\mathbf{k}^2 + \frac{\partial \mathbf{V}}{\partial s_3}\mathbf{k}^3 \quad . \tag{D.4.12}$$

Notice that the order of the vectors is preserved and the operator partials are applied to the total vector $\mathbf{V}$. If the component form of $\mathbf{V}$ is now substituted and the partials are carried out, the nine components of the dyadic are obtained.

Other forms obtained in a similar way are found for $(\mathbf{V}\bullet\nabla)$ and $(\mathbf{V}\times\nabla)$. Again, replacing the vector with a dyadic, $(\Psi\bullet\nabla)$ and $(\Psi\times\nabla)$ have essentially the same form. The parentheses are used to eliminate possible confusion with another type of field operation, for example $\mathbf{V}\bullet\nabla\phi$, where ∇ operates on ϕ but not on $\mathbf{V}$.

The field vector of position $\mathbf{r}$ provides several useful examples of the operations described in this section. The results are:

$$\nabla\mathbf{r} = I \quad , \quad \nabla\bullet\mathbf{r} = 3 \quad , \quad \nabla\times\mathbf{r} = 0 \quad , \quad \nabla\bullet\hat{\mathbf{r}} = \frac{2}{r}$$
$$\nabla r = \hat{\mathbf{r}} \quad , \quad \nabla\frac{1}{r} = -\frac{\hat{\mathbf{r}}}{r^2} \quad , \quad \nabla\bullet I = 0 \quad , \quad \nabla\times I = 0 \qquad . \quad \text{(D.4.13)}$$

In a few cases involving singularities, the operator ∇ can lead to indeterminate forms where the actual physical problem is determinate. In these circumstances, going back to the integral definitions gives the correct answers; because they are fundamental. It is a deplorable trend of modern texts to present the operator forms (e.g. Eqs.D.4.8, D.4.10 and D.4.11) as *definitions* of the all important physical field functions.

D.5 **Orthogonal Coordinates**: Whenever possible, field physics problems should be solved in the simplest coordinates. Often this implies the use of an <u>orthogonal</u> system. The advantages are numerous. By definition, an orthogonal system is one in which the three coordinate curves at each point are mutually perpendicular, and the cosines of the angles between them are, thus, zero. The angle variables q_{ij} from Eq.(B.5.3) reduce to,

$$q_{ij} = q^{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad \left(\begin{smallmatrix}\text{orthogonal}\\ \text{coordinates}\end{smallmatrix}\right) \qquad \text{(D.5.1)}$$

and the only variables having an important effect in determining the geometry of the system are the scale factors h_i. One simplification this produces is that the line element becomes,

$$ds^2 = ds_1^2 + ds_2^2 + ds_3^2 = (h_1 d\overset{1}{\xi})^2 + (h_2 d\overset{2}{\xi})^2 + (h_3 d\overset{3}{\xi})^2 \quad . \qquad \text{(D.5.2)}$$

Another significant consequence is that the unit vectors are all mutually perpendicular so that $q = (\mathbf{k}_1\bullet\mathbf{k}_2\times\mathbf{k}_3)^2 = 1$, and when Eq. (D.5.1) is used with Eqs.(B.8.24) and (B.8.25), it is clear that,

$$\mathbf{k}_i = \mathbf{k}^i \quad , \quad V_i = V^i \quad . \qquad \text{(D.5.3)}$$

This means that there is no difference between the unit and reciprocal base vectors, and that the perpendicular and parallelogram components of vectors are identical. Again, all dyadics of the type in

Eq.(C.1.5) reduce to Eq.(C.1.8), since $\psi_{ij} = \psi^{j}_{i} = \psi^{ij}$, and the first three idemfactors of Eq.(C.1.26) reduce to,

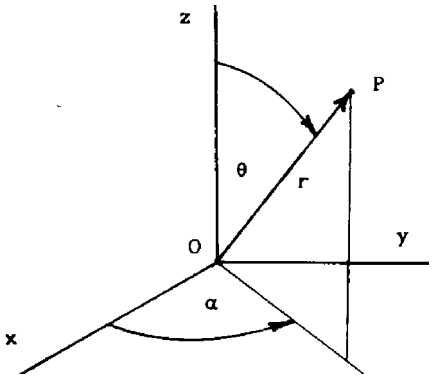
$$\mathbf{I} = \mathbf{k}_1\mathbf{k}_1 + \mathbf{k}_2\mathbf{k}_2 + \mathbf{k}_3\mathbf{k}_3 \quad . \tag{D.5.4}$$

From the point of view of reducing labor, the greatest simplification appears in the Christoffel coefficients. In *orthogonal systems*, the coefficients of the first and second kinds become identical,

$$C_{ijk} = C^{k}_{ij} \quad , \tag{D.5.5}$$

and since the angle variables are effectively eliminated,

$$C_{ipi} = \frac{1}{h_i}\frac{\partial h_i}{\partial s_p} \quad , \quad C_{iip} = -\frac{1}{h_i}\frac{\partial h_i}{\partial s_p} \quad . \quad (^{\text{all others}}_{\text{zero}}) \tag{D.5.6}$$



Based on all these changes, Tables D.5.1 and D.5.2 give the most often used field functions in terms of spherical and cylindrical coordinates, since these systems are commonly involved in the fundamental examples. Figure D.5.1 shows the spherical coordinates.

⇐ Figure D.5.1

TABLE D.5.1

FIELD FUNCTIONS IN SPHERICAL COORDINATES

$$h_1 = 1 \quad , \quad h_2 = r \quad , \quad h_3 = r\sin\theta$$

$$ds_1 = dr \quad , \quad ds_2 = r\,d\theta \quad , \quad ds_3 = r\sin\theta\,d\alpha$$

$$C_{212} = \frac{1}{r} \quad , \quad C_{313} = \frac{1}{r} \quad , \quad C_{323} = \frac{\cos\theta}{r\sin\theta}$$

$$C_{221} = -\frac{1}{r} \quad , \quad C_{331} = -\frac{1}{r} \quad , \quad C_{332} = -\frac{\cos\theta}{r\sin\theta} \qquad \left(^{\text{all others}}_{\text{zero}}\right)$$

$$\nabla\phi = \hat{\mathbf{r}}\frac{\partial\phi}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial\phi}{\partial\theta} + \frac{\hat{\boldsymbol{\alpha}}}{r\sin\theta}\frac{\partial\phi}{\partial\alpha}$$

$$\begin{aligned}\nabla\mathbf{V} = {} & \hat{\mathbf{r}}\hat{\mathbf{r}}\frac{\partial V_r}{\partial r} + \hat{\mathbf{r}}\hat{\boldsymbol{\theta}}\frac{\partial V_\theta}{\partial r} + \hat{\mathbf{r}}\hat{\boldsymbol{\alpha}}\frac{\partial V_\alpha}{\partial r} + \hat{\boldsymbol{\theta}}\hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta}{r}\right) \\ & + \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}\left(\frac{1}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{V_r}{r}\right) + \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\alpha}}\frac{1}{r}\frac{\partial V_\alpha}{\partial\theta} + \hat{\boldsymbol{\alpha}}\hat{\mathbf{r}}\left(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\alpha} - \frac{V_\alpha}{r}\right) \\ & + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\theta}}\left(\frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\alpha} - \frac{V_\alpha}{r}\frac{\cos\theta}{\sin\theta}\right) + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left(\frac{1}{r\sin\theta}\frac{\partial V_\alpha}{\partial\alpha} + \frac{V_r}{r} + \frac{V_\theta}{r}\frac{\cos\theta}{\sin\theta}\right)\end{aligned}$$

$$\nabla\cdot\mathbf{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\alpha}{\partial\alpha}$$

TABLE D.5.1 Continued

$$\nabla\times\mathbf{V} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta V_\alpha) - \frac{\partial V_\theta}{\partial\alpha}\right) + \frac{\hat{\boldsymbol{\theta}}}{r}\left(\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\alpha} - \frac{\partial}{\partial r}(rV_\alpha)\right) + \frac{\hat{\boldsymbol{\alpha}}}{r}\left(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right)$$

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\alpha^2}$$

$$\nabla^2\mathbf{V} = \hat{\mathbf{r}}\left(\nabla^2 V_r - \frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) - \frac{2}{r^2\sin\theta}\frac{\partial V_\alpha}{\partial\alpha} - \frac{2}{r^2}V_r\right)$$

$$\hat{\boldsymbol{\theta}}\left(\nabla^2 V_\theta + \frac{2}{r^2}\frac{\partial V_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial V_\alpha}{\partial\alpha} - \frac{V_\theta}{r^2\sin^2\theta}\right)$$

$$\hat{\boldsymbol{\alpha}}\left(\nabla^2 V_\alpha + \frac{2}{r^2\sin\theta}\frac{\partial V_r}{\partial\alpha} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial V_\theta}{\partial\alpha} - \frac{V_\alpha}{r^2\sin^2\theta}\right)$$

$$\Phi = \frac{1}{2}(\nabla\mathbf{V} + \mathbf{V}\nabla)$$

$$= \hat{\mathbf{r}}\hat{\mathbf{r}}\frac{\partial V_r}{\partial r} + \hat{\mathbf{r}}\hat{\boldsymbol{\theta}}\left[\frac{1}{2}\left(\frac{\partial V_\theta}{\partial r} + \frac{1}{r}\frac{\partial V_r}{\partial\theta}\right) - \frac{V_\theta}{2r}\right] + \hat{\mathbf{r}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\alpha}\right) - \frac{V_\alpha}{2r}\right]$$

$$+ \hat{\boldsymbol{\theta}}\hat{\mathbf{r}}\left[\frac{1}{2}\left(\frac{\partial V_\theta}{\partial r} + \frac{1}{r}\frac{\partial V_r}{\partial\theta}\right) - \frac{V_\theta}{2r}\right] + \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}\left[\frac{1}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{V_r}{r}\right]$$

$$+ \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial V_\alpha}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\alpha}\right) - \frac{\cos\theta}{2r\sin\theta}V_\alpha\right] + \hat{\boldsymbol{\alpha}}\hat{\mathbf{r}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\alpha}\right) - \frac{V_\alpha}{2r}\right]$$

$$+ \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\theta}}\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial V_\alpha}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\alpha}\right) - \frac{\cos\theta}{2r\sin\theta}V_\alpha\right] + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{r\sin\theta}\frac{\partial V_\alpha}{\partial\alpha} + \frac{V_r}{r} + \frac{\cos\theta}{r\sin\theta}V_\theta\right]$$

$$\Omega = \frac{1}{2}(\nabla\mathbf{V} - \mathbf{V}\nabla)$$

$$= \hat{\mathbf{r}}\hat{\mathbf{r}}\left[0\right] + \hat{\mathbf{r}}\hat{\boldsymbol{\theta}}\left[\frac{1}{2}\left(\frac{\partial V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial\theta}\right) + \frac{V_\theta}{2r}\right] + \hat{\mathbf{r}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial r} - \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\alpha}\right) + \frac{V_\alpha}{2r}\right]$$

$$+ \hat{\boldsymbol{\theta}}\hat{\mathbf{r}}\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial V_r}{\partial\theta} - \frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{2r}\right] + \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}\left[0\right]$$

$$+ \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial V_\alpha}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\alpha}\right) + \frac{\cos\theta}{2r\sin\theta}V_\alpha\right] + \hat{\boldsymbol{\alpha}}\hat{\mathbf{r}}\left[\frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\alpha} - \frac{\partial V_\alpha}{\partial r}\right) - \frac{V_\alpha}{2r}\right]$$

$$+ \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\theta}}\left[\frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\alpha} - \frac{1}{r}\frac{\partial V_\alpha}{\partial\theta}\right) - \frac{\cos\theta}{2r\sin\theta}V_\alpha\right] + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left[0\right]$$

TABLE D.5.2

FIELD FUNCTIONS IN CYLINDRICAL COORDINATES

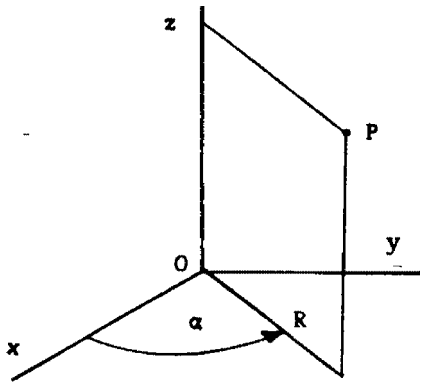


Figure D.5.2 shows the cylindrical coordinates used in the equations.

⇐ Figure D.5.2

$$h_1 = 1 \quad , \quad h_2 = R \quad , \quad h_3 = 1$$

$$ds_1 = dR \quad , \quad ds_2 = Rd\alpha \quad , \quad ds_3 = dz$$

$$C_{212} = \frac{1}{R} \quad , \quad C_{221} = -\frac{1}{R} \quad , \quad \left(\text{all others zero}\right)$$

$$\nabla\phi = \hat{\mathbf{R}}\frac{\partial\phi}{\partial R} + \hat{\boldsymbol{\alpha}}\frac{1}{R}\frac{\partial\phi}{\partial\alpha} + \mathbf{k}\frac{\partial\phi}{\partial z}$$

$$\nabla\mathbf{V} = \hat{\mathbf{R}}\hat{\mathbf{R}}\frac{\partial V_R}{\partial R} + \hat{\mathbf{R}}\hat{\boldsymbol{\alpha}}\frac{\partial V_\alpha}{\partial R} + \hat{\mathbf{R}}\mathbf{k}\frac{\partial V_z}{\partial R} + \hat{\boldsymbol{\alpha}}\hat{\mathbf{R}}\left(\frac{1}{R}\frac{\partial V_R}{\partial\alpha} - \frac{V_\alpha}{R}\right) + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left(\frac{1}{R}\frac{\partial V_\alpha}{\partial\alpha} + \frac{V_R}{R}\right)$$
$$+ \hat{\boldsymbol{\alpha}}\mathbf{k}\frac{1}{R}\frac{\partial V_z}{\partial\alpha} + \mathbf{k}\hat{\mathbf{R}}\frac{\partial V_R}{\partial z} + \mathbf{k}\hat{\boldsymbol{\alpha}}\frac{\partial V_\alpha}{\partial z} + \mathbf{k}\mathbf{k}\frac{\partial V_z}{\partial z}$$

$$\nabla\cdot\mathbf{V} = \frac{1}{R}\frac{\partial}{\partial R}(RV_R) + \frac{1}{R}\frac{\partial V_\alpha}{\partial\alpha} + \frac{\partial V_z}{\partial z}$$

$$\nabla\times\mathbf{V} = \hat{\mathbf{R}}\left(\frac{1}{R}\frac{\partial V_z}{\partial\alpha} - \frac{\partial V_\alpha}{\partial z}\right) + \hat{\boldsymbol{\alpha}}\left(\frac{\partial V_R}{\partial z} - \frac{\partial V_z}{\partial R}\right) + \mathbf{k}\left(\frac{1}{R}\frac{\partial}{\partial R}(RV_\alpha) - \frac{1}{R}\frac{\partial V_R}{\partial\alpha}\right)$$

$$\nabla^2\phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\phi}{\partial\alpha^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla^2\mathbf{V} = \hat{\mathbf{R}}\left(\nabla^2 V_R - \frac{2}{R^2}\frac{\partial V_\alpha}{\partial\alpha} - \frac{V_R}{R^2}\right) + \hat{\boldsymbol{\alpha}}\left(\nabla^2 V_\alpha - \frac{2}{R^2}\frac{\partial V_R}{\partial\alpha} - \frac{V_\alpha}{R^2}\right) + \mathbf{k}\,\nabla^2 V_z$$

$$\Phi = \hat{\mathbf{R}}\hat{\mathbf{R}}\frac{\partial V_R}{\partial R} + \hat{\mathbf{R}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial R} + \frac{1}{R}\frac{\partial V_R}{\partial\alpha} - \frac{V_\alpha}{R}\right)\right] + \hat{\mathbf{R}}\mathbf{k}\left[\frac{1}{2}\left(\frac{\partial V_z}{\partial R} + \frac{\partial V_R}{\partial z}\right)\right]$$
$$+ \hat{\boldsymbol{\alpha}}\hat{\mathbf{R}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial R} + \frac{1}{R}\frac{\partial V_R}{\partial\alpha} - \frac{V_\alpha}{R}\right)\right] + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{R}\frac{\partial V_\alpha}{\partial\alpha} + \frac{V_R}{R}\right] + \hat{\boldsymbol{\alpha}}\mathbf{k}\left[\frac{1}{2}\left(\frac{1}{R}\frac{\partial V_z}{\partial\alpha} + \frac{\partial V_\alpha}{\partial z}\right)\right]$$
$$+ \mathbf{k}\hat{\mathbf{R}}\left[\frac{1}{2}\left(\frac{\partial V_z}{\partial R} + \frac{\partial V_R}{\partial z}\right)\right] + \mathbf{k}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{1}{R}\frac{\partial V_z}{\partial\alpha} + \frac{\partial V_\alpha}{\partial z}\right)\right] + \mathbf{k}\mathbf{k}\frac{\partial V_z}{\partial z}$$

TABLE D.5.2 Continued

$$\Omega = \hat{\mathbf{R}}\hat{\mathbf{R}}\left[0\right] + \hat{\mathbf{R}}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial R} - \frac{1}{R}\frac{\partial V_R}{\partial \alpha} + \frac{V_\alpha}{R}\right)\right] + \hat{\mathbf{R}}\mathbf{k}\left[\frac{1}{2}\left(\frac{\partial V_z}{\partial R} - \frac{\partial V_R}{\partial z}\right)\right]$$
$$+ \hat{\boldsymbol{\alpha}}\hat{\mathbf{R}}\left[\frac{1}{2}\left(\frac{1}{R}\frac{\partial V_R}{\partial \alpha} - \frac{\partial V_\alpha}{\partial R} - \frac{V_\alpha}{R}\right)\right] + \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}\left[0\right] + \hat{\boldsymbol{\alpha}}\mathbf{k}\left[\frac{1}{2}\left(\frac{1}{R}\frac{\partial V_z}{\partial \alpha} - \frac{\partial V_\alpha}{\partial z}\right)\right]$$
$$+ \mathbf{k}\hat{\mathbf{R}}\left[\frac{1}{2}\left(\frac{\partial V_R}{\partial z} - \frac{\partial V_z}{\partial R}\right)\right] + \mathbf{k}\hat{\boldsymbol{\alpha}}\left[\frac{1}{2}\left(\frac{\partial V_\alpha}{\partial z} - \frac{1}{R}\frac{\partial V_z}{\partial \alpha}\right)\right] + \mathbf{k}\mathbf{k}\left[0\right]$$

D.6 **Cartesian Coordinates and Field Function Relationships**: The most elementary coordinates, called rectangular or Cartesian, are the special case where the scale factors are not only constant throughout space but equal to unity, i.e., $h_i = 1$, which is equivalent to,

$$ds_i = d\overset{i}{\xi} \quad . \tag{D.6.1}$$

All of the equations for orthogonal systems apply, but some can be simplified considerably. The line element, for example, is,

$$ds^2 = ds_1^2 + ds_2^2 + ds_3^2 = dx^2 + dy^2 + dz^2 \quad . \tag{D.6.2}$$

The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are everywhere constant, so the partials of the unit vectors are all zero. Furthermore, $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = 0$ and $\mathbf{i}\times\mathbf{j} = \mathbf{k}$, $\mathbf{j}\times\mathbf{k} = \mathbf{i}$ and $\mathbf{k}\times\mathbf{i} = \mathbf{j}$. As in all orthogonal systems,

$$d\mathbf{r} = \mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz \quad , \tag{D.6.3}$$

but unlike any other system, the radius vector is given by,

$$\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z \quad . \tag{D.6.4}$$

In spherical coordinates, for example, $\mathbf{r} = \hat{\mathbf{r}}r$. All of the equations applicable to dyadics and their invariants in orthogonal systems apply directly, and the idemfactor is,

$$I = \mathbf{ii} + \mathbf{jj} + \mathbf{kk} \quad . \tag{D.6.5}$$

The most significant simplification is that all of the Christoffel coefficients C_{ijk} are zero, i.e., the unit vectors do not vary, so that,

$$\frac{\partial \mathbf{A}}{\partial s} = \mathbf{i}\frac{\partial A_x}{\partial s} + \mathbf{j}\frac{\partial A_y}{\partial s} + \mathbf{k}\frac{\partial A_z}{\partial s} \quad . \tag{D.6.6}$$

The field functions are also simplified, as can be seen in Table D.6.1.

TABLE D.6.1

FIELD FUCTIONS IN RECTANGULAR COORDINATES

$$\nabla\phi = \mathbf{i}\frac{\partial\phi}{\partial x} + \mathbf{j}\frac{\partial\phi}{\partial y} + \mathbf{k}\frac{\partial\phi}{\partial z}$$

$$\begin{aligned}\nabla\mathbf{V} = {} & \mathbf{ii}\,\frac{\partial V_x}{\partial x} + \mathbf{ij}\,\frac{\partial V_y}{\partial x} + \mathbf{ik}\,\frac{\partial V_z}{\partial x} \\ & + \mathbf{ji}\,\frac{\partial V_x}{\partial y} + \mathbf{jj}\,\frac{\partial V_y}{\partial y} + \mathbf{jk}\,\frac{\partial V_z}{\partial y} \\ & + \mathbf{ki}\,\frac{\partial V_x}{\partial z} + \mathbf{kj}\,\frac{\partial V_y}{\partial z} + \mathbf{kk}\,\frac{\partial V_z}{\partial z}\end{aligned}$$

$$\nabla\cdot\mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\nabla\times\mathbf{V} = \mathbf{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla^2\mathbf{V} = \mathbf{i}\nabla^2 V_x + \mathbf{j}\nabla^2 V_y + \mathbf{k}\nabla^2 V_z$$

$$\Phi = \frac{1}{2}(\nabla\mathbf{V} + \mathbf{V}\nabla) = \mathbf{k}_i\mathbf{k}_j\frac{1}{2}\left(\frac{\partial V_j}{\partial x_i} + \frac{\partial V_i}{\partial x_j}\right) \quad , \quad \Omega = \frac{1}{2}(\nabla\mathbf{V} - \mathbf{V}\nabla) = \mathbf{k}_i\mathbf{k}_j\frac{1}{2}\left(\frac{\partial V_j}{\partial x_i} - \frac{\partial V_i}{\partial x_j}\right)$$

$$\underline{\qquad\qquad (*i,j) \qquad\qquad}$$

Rectangular coordinates are the natural coordinates to use in working out general relationships of the type presented in Table D.6.2. Once these relationships are obtained, they hold in any coordinate system.

D.7 **Coordinate Transformations**: Sometimes a problem is most easily solved in one particular coordinate system, but some of the quantities are given in terms of another coordinate system. To write all elements of the problem in the preferred system, a *space coordinate transformation* is used. It is assumed that both systems are fixed in space and therefore fixed relative to each other.

Scalars, vectors, and dyadics are invariant to *space* transformations, and look the same to any observer using any coordinate system. Thus, if $\phi(x, y, z)$ represents the value of the scalar field ϕ at the point (x,y,z) at time t, as seen by an observer using Cartesian coordinates, another observer, using spherical coordinates will see the same ϕ at time t, if he measures it at a point (r, θ, α) that corresponds to the original point. The same thing is true of the vector

TABLE D.6.2

FIELD FUNCTION RELATIONSHIPS

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \qquad \nabla\cdot(\phi\mathbf{V}) = \nabla\phi\cdot\mathbf{V} + \phi\nabla\cdot\mathbf{V}$$
$$\nabla(\phi\mathbf{V}) = \nabla\phi\ \mathbf{V} + \phi\ \nabla\mathbf{V} \qquad \nabla\times(\phi\mathbf{V}) = \nabla\phi\times\mathbf{V} + \phi\nabla\times\mathbf{V}$$
$$\nabla(\phi\Psi) = \phi\nabla\Psi + \nabla\phi\ \Psi$$
$$\nabla\cdot(\phi\mathbf{I}) = \nabla\cdot(\mathbf{I}\phi) = \nabla\phi \qquad \nabla\cdot(\phi\Psi) = \nabla\cdot(\Psi\phi) = \nabla\phi\cdot\Psi + \phi\nabla\cdot\Psi$$
$$\nabla\times(\phi\mathbf{I}) = \nabla\times(\mathbf{I}\phi) = \nabla\phi\times\mathbf{I} \qquad \nabla\times(\phi\Psi) = \nabla\phi\times\Psi + \phi\nabla\times\Psi$$
$$(\phi\Psi)\times\nabla = \phi(\Psi\times\nabla) + \Psi\times(\nabla\phi) \qquad \nabla\times\phi\mathbf{I}\times\nabla = \nabla\nabla\phi - \nabla^2\phi\ \mathbf{I}$$

$$\nabla\cdot(\mathbf{uv}) = \nabla\cdot\mathbf{u}\ \mathbf{v} + \mathbf{u}\cdot\nabla\mathbf{v} \qquad \nabla\times(\mathbf{uv}) = \nabla\times\mathbf{u}\ \mathbf{v} - \mathbf{u}\times\nabla\mathbf{v}$$
$$\nabla(\mathbf{u}\cdot\mathbf{v}) = \nabla\mathbf{u}\cdot\mathbf{v} + \nabla\mathbf{v}\cdot\mathbf{u} = \mathbf{v}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{v} + \mathbf{v}\times\nabla\times\mathbf{u} + \mathbf{u}\times\nabla\times\mathbf{v}$$
$$\nabla(\mathbf{u}\times\mathbf{v}) = \nabla\mathbf{u}\times\mathbf{v} - \nabla\mathbf{v}\times\mathbf{u} \qquad \nabla\cdot(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\cdot\nabla\times\mathbf{u} - \mathbf{u}\cdot\nabla\times\mathbf{v}$$
$$\nabla\times(\mathbf{u}\times\mathbf{v}) = \nabla\cdot(\mathbf{vu} - \mathbf{uv}) = \mathbf{v}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{v} + \mathbf{u}\nabla\cdot\mathbf{v} - \mathbf{v}\nabla\cdot\mathbf{u}$$

$$\nabla\cdot(\mathbf{V}\Psi) = \nabla\cdot\mathbf{V}\ \Psi + \mathbf{V}\cdot\nabla\Psi \qquad \nabla\cdot(\Psi\mathbf{V}) = \nabla\cdot\Psi\ \mathbf{V} + \Psi_c\cdot\nabla\mathbf{V}$$
$$\nabla(\mathbf{V}\cdot\Psi) = \nabla\mathbf{V}\cdot\Psi + \nabla\Psi_c\cdot\mathbf{V} \qquad \nabla(\Psi\cdot\mathbf{V}) = \nabla\Psi\cdot\mathbf{V} + \nabla\mathbf{V}\cdot\Psi_c$$
$$\nabla\cdot(\mathbf{V}\cdot\Psi) = \nabla\mathbf{V}:\Psi_c + (\nabla\cdot\Psi_c)\cdot\mathbf{V} \qquad \nabla\cdot(\Psi\cdot\mathbf{V}) = (\nabla\cdot\Psi)\cdot\mathbf{V} + \Psi:\nabla\mathbf{V}$$
$$\nabla\cdot(\mathbf{V}\cdot\nabla\mathbf{V}) = \mathbf{V}\cdot\nabla(\nabla\cdot\mathbf{V}) + (\nabla\mathbf{V}\cdot\nabla\mathbf{V})_s$$

$$\nabla(\mathbf{I}\times\mathbf{V}) = \nabla\mathbf{V}\times\mathbf{I} \qquad \nabla\cdot(\mathbf{I}\times\mathbf{V}) = \nabla\times\mathbf{V}$$
$$\nabla\times(\mathbf{I}\times\mathbf{V}) = \nabla\mathbf{V} - \mathbf{I}\ \nabla\cdot\mathbf{V} \qquad (\mathbf{I}\times\mathbf{V})\nabla = \nabla\mathbf{V} - \mathbf{I}\ \nabla\cdot\mathbf{V}$$

$$\nabla\times\nabla\times\mathbf{V} = \nabla\nabla\cdot\mathbf{V} - \nabla^2\mathbf{V} \qquad \nabla\times\nabla\times\Psi = \nabla\nabla\cdot\Psi - \nabla^2\Psi$$
$$\nabla\cdot\nabla\mathbf{V} = \nabla^2\mathbf{V} \qquad \nabla\cdot(\mathbf{V}\nabla) = \nabla(\nabla\cdot\mathbf{V})$$

$$\nabla\cdot\nabla\times\mathbf{V} \equiv 0 \qquad \nabla\times\nabla\phi \equiv 0 \qquad \nabla\cdot\nabla\times\Psi \equiv 0$$
$$\nabla\times\nabla\mathbf{V} \equiv 0 \qquad (\mathbf{V}\nabla)\times\nabla \equiv 0$$
$$\nabla\cdot(\mathbf{I}\times\nabla\phi) \equiv 0 \qquad \nabla\times(\mathbf{I}\cdot\nabla\phi) \equiv 0 \qquad \nabla\cdot(\nabla\cdot\Psi_{as}) \equiv 0$$

$$\mathbf{u}\cdot\nabla(\nabla\mathbf{V}) = \nabla(\mathbf{V}\nabla)\cdot\mathbf{u} \qquad \mathbf{u}\cdot\nabla(\nabla\cdot\mathbf{V}) = \left(\mathbf{u}\cdot\nabla(\nabla\mathbf{V})\right)_s$$
$$(\mathbf{u}\nabla)\cdot\mathbf{V} = \mathbf{V}\cdot(\nabla\mathbf{u}) \qquad \nabla(\mathbf{V}\cdot\nabla\mathbf{V}) = \nabla\mathbf{V}\cdot\nabla\mathbf{V} + \mathbf{V}\cdot\nabla(\nabla\mathbf{V})$$

Volume $\longrightarrow$ Closed surface

$$\int\nabla\phi\ dv = \int\mathbf{n}\phi\ dS \quad , \quad \int\nabla\mathbf{V}\ dv = \int\mathbf{n}\mathbf{V}\ dS \quad , \quad \int\nabla\cdot\mathbf{V}\ dv = \int\mathbf{n}\cdot\mathbf{V}\ dS$$
$$\int\nabla\cdot\Psi\ dv = \int\mathbf{n}\cdot\Psi\ dS \quad , \quad \int\nabla\times\mathbf{V}\ dv = \int\mathbf{n}\times\mathbf{V}\ dS \quad , \quad \int\nabla\times\Psi\ dv = \int\mathbf{n}\times\Psi\ dS$$

Open surface $\longrightarrow$ Closed curve

$$\int\mathbf{n}\times\nabla\phi\ dS = \oint d\mathbf{r}\ \phi \qquad \int\mathbf{n}\times\nabla\mathbf{V}\ dS = \int d\mathbf{r}\ \mathbf{V}$$
$$\int\mathbf{n}\cdot\nabla\times\mathbf{V}\ dS = \oint d\mathbf{r}\cdot\mathbf{V} \qquad \int\mathbf{n}\cdot\nabla\times\Psi\ dS = \oint d\mathbf{r}\cdot\Psi$$

$\mathbf{V}(x,y,z)$ and the dyadic $\Psi(x,y,z)$. The general coordinate transformation equations are,

$$\begin{aligned} \overset{1}{\eta} &= \overset{1}{\eta}\left(\overset{1}{\bar{\eta}},\overset{2}{\bar{\eta}},\overset{3}{\bar{\eta}}\right) \quad , \quad \overset{2}{\eta} = \overset{2}{\eta}\left(\overset{1}{\bar{\eta}},\overset{2}{\bar{\eta}},\overset{3}{\bar{\eta}}\right) \quad , \quad \overset{3}{\eta} = \overset{3}{\eta}\left(\overset{1}{\bar{\eta}},\overset{2}{\bar{\eta}},\overset{3}{\bar{\eta}}\right) \\ \overset{1}{\bar{\eta}} &= \overset{1}{\bar{\eta}}\left(\overset{1}{\eta},\overset{2}{\eta},\overset{3}{\eta}\right) \quad , \quad \overset{2}{\bar{\eta}} = \overset{2}{\bar{\eta}}\left(\overset{1}{\eta},\overset{2}{\eta},\overset{3}{\eta}\right) \quad , \quad \overset{3}{\bar{\eta}} = \overset{3}{\bar{\eta}}\left(\overset{1}{\eta},\overset{2}{\eta},\overset{3}{\eta}\right) \end{aligned} \quad , \qquad \text{(D.7.1)}$$

which in the case of spherical coordinates become,

$$\begin{aligned} &x = r\sin\theta\cos\alpha \quad , \quad y = r\sin\theta\sin\alpha \quad , \quad z = r\cos\theta \\ &r = \sqrt{x^2+y^2+z^2} \quad , \quad \theta = \arctan\frac{\sqrt{x^2+y^2}}{z} \quad , \quad \alpha = \arctan\frac{y}{x} \end{aligned} \quad . \qquad \text{(D.7.2)}$$

Given the coordinates of any point in one system, these equations locate the same point in the other system.

To illustrate some of these ideas, consider two Cartesian systems S and $\bar{S}$, tilted and displaced in some way with respect to each other, and each using the same absolute time t. The partial derivative of $\mathbf{V}$ with respect to x is,

$$\frac{\partial \mathbf{V}}{\partial x} = \frac{\partial \mathbf{V}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial x} + \frac{\partial \mathbf{V}}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial x} + \frac{\partial \mathbf{V}}{\partial \bar{z}}\frac{\partial \bar{z}}{\partial x} \quad ,$$

and the y and z derivatives have a similar form, so that, $\nabla\mathbf{V} = \nabla\bar{\mathbf{r}}\bullet\bar{\nabla}\mathbf{V}$. However, according to Eq.(D.4.13), $\nabla\bar{\mathbf{r}} = I$, so,

$$\nabla\mathbf{V} = \bar{\nabla}\mathbf{V} \quad . \qquad \text{(D.7.3)}$$

Taking the vector and scalar of both sides,

$$\nabla\bullet\mathbf{V} = \bar{\nabla}\bullet\mathbf{V} \quad , \quad \nabla\times\mathbf{V} = \bar{\nabla}\times\mathbf{V} \quad . \qquad \text{(D.7.4)}$$

Clearly the bars are superfluous. Knowing that dyadics, vectors, and scalars are invariant to space coordinate transformations, the equivalence in these three relationships could have been predicted, since $\nabla\mathbf{V}$, $\nabla\times\mathbf{V}$, and $\nabla\bullet\mathbf{V}$ are a dyadic, a vector, and a scalar respectively.

The true usefulness of transformation theory is in problem solving where components and unit vectors are needed; because, although the invariance of vectors and dyadics is clear from their physical nature, it is equally clear that their components and associated unit vectors can vary widely from system to system. A series of simple operations can be used to obtain the required transformation equations. All of the frequently used transformations are listed in Table D.7.1. Table D.7.2 gives the specific transformations relating the orthogonal spherical, cylindrical and rectangular coordinate systems.

TABLE D.7.1

TRANSFORMATION EQUATIONS (GENERAL)

$$d\eta^i = \frac{\partial \eta^i}{\partial \bar{\eta}^j}\, d\bar{\eta}^j \quad , \quad d\bar{\eta}^i = \frac{\partial \bar{\eta}^i}{\partial \eta^j}\, d\eta^j \quad , \quad d\eta_i = \frac{\partial \bar{\eta}^j}{\partial \eta^i}\, d\bar{\eta}_j \quad , \quad d\bar{\eta}_i = \frac{\partial \eta^j}{\partial \bar{\eta}^i}\, d\eta_j \quad (*j)$$

$$\mathbf{k}^i = \frac{h_i}{\bar{h}_j}\frac{\partial \eta^i}{\partial \bar{\eta}^j}\,\bar{\mathbf{k}}^j \quad , \quad \bar{\mathbf{k}}^i = \frac{\bar{h}_i}{h_j}\frac{\partial \bar{\eta}^i}{\partial \eta^j}\,\mathbf{k}^j \quad , \quad \mathbf{k}_i = \frac{\bar{h}_j}{h_i}\frac{\partial \bar{\eta}^j}{\partial \eta^i}\,\bar{\mathbf{k}}_j \quad , \quad \bar{\mathbf{k}}_i = \frac{h_j}{\bar{h}_i}\frac{\partial \eta^j}{\partial \bar{\eta}^i}\,\mathbf{k}_j \quad (*j)$$

$$v_i = \frac{h_i}{\bar{h}_j}\frac{\partial \eta^i}{\partial \bar{\eta}^j}\,\bar{v}_j \quad , \quad \bar{v}_i = \frac{\bar{h}_i}{h_j}\frac{\partial \bar{\eta}^i}{\partial \eta^j}\,v_j \quad , \quad v^i = \frac{\bar{h}_j}{h_i}\frac{\partial \bar{\eta}^j}{\partial \eta^i}\,\bar{v}^j \quad , \quad \bar{v}^i = \frac{h_j}{\bar{h}_i}\frac{\partial \eta^j}{\partial \bar{\eta}^i}\,v^j \quad (*j)$$

$$\psi_{ij} = \frac{h_i h_j}{\bar{h}_l \bar{h}_m}\frac{\partial \eta^i}{\partial \bar{\eta}^l}\frac{\partial \eta^j}{\partial \bar{\eta}^m}\,\bar{\psi}_{lm} \quad , \quad \bar{\psi}_{ij} = \frac{\bar{h}_i \bar{h}_j}{h_l h_m}\frac{\partial \bar{\eta}^i}{\partial \eta^l}\frac{\partial \bar{\eta}^j}{\partial \eta^m}\,\psi_{lm} \quad , \quad \psi_i^{\ j} = \frac{h_i \bar{h}_m}{h_j \bar{h}_l}\frac{\partial \eta^i}{\partial \bar{\eta}^l}\frac{\partial \bar{\eta}^m}{\partial \eta^j}\,\bar{\psi}_l^{\ m} \quad (*l,m)$$

$$\bar{\psi}_i^{\ j} = \frac{\bar{h}_i h_m}{\bar{h}_j h_l}\frac{\partial \bar{\eta}^i}{\partial \eta^l}\frac{\partial \eta^m}{\partial \bar{\eta}^j}\,\psi_l^{\ m} \quad , \quad \psi^{ij} = \frac{\bar{h}_l \bar{h}_m}{h_i h_j}\frac{\partial \bar{\eta}^l}{\partial \eta^i}\frac{\partial \bar{\eta}^m}{\partial \eta^j}\,\bar{\psi}^{lm} \quad , \quad \bar{\psi}^{ij} = \frac{h_l h_m}{\bar{h}_i \bar{h}_j}\frac{\partial \eta^l}{\partial \bar{\eta}^i}\frac{\partial \eta^m}{\partial \bar{\eta}^j}\,\psi^{lm} \quad (*l,m)$$

for q_{ij} these are reversed, i.e.

$$q_{ij} = \frac{\bar{h}_l \bar{h}_m}{h_i h_j}\frac{\partial \bar{\eta}^l}{\partial \eta^i}\frac{\partial \bar{\eta}^m}{\partial \eta^j}\,\bar{q}_{lm} \quad (*l,m)$$

TABLE D.7.2

SPHERICAL/RECTANGULAR COORDINATES

Coordinate Transformation

$$x = r\sin\theta\cos\alpha \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r\sin\theta\sin\alpha \qquad \theta = \arctan\frac{\sqrt{x^2 + y^2}}{z}$$

$$z = r\cos\theta \qquad \alpha = \arctan\frac{y}{x}$$

Unit Vector Tansformations

$$\begin{aligned} \mathbf{i} &= \hat{\mathbf{r}}\sin\theta\cos\alpha + \hat{\boldsymbol{\theta}}\cos\theta\cos\alpha - \hat{\boldsymbol{\alpha}}\sin\alpha \\ \mathbf{j} &= \hat{\mathbf{r}}\sin\theta\sin\alpha + \hat{\boldsymbol{\theta}}\cos\theta\sin\alpha + \hat{\boldsymbol{\alpha}}\cos\alpha \\ \mathbf{k} &= \hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{r}} &= \mathbf{i}\sin\theta\cos\alpha + \mathbf{j}\sin\theta\sin\alpha + \mathbf{k}\cos\theta \\ \hat{\boldsymbol{\theta}} &= \mathbf{i}\cos\theta\cos\alpha + \mathbf{j}\cos\theta\sin\alpha - \mathbf{k}\sin\theta \\ \hat{\boldsymbol{\alpha}} &= -\mathbf{i}\sin\alpha + \mathbf{j}\cos\alpha \end{aligned}$$

TABLE D.7.2 Continued

CYLINDRICAL/RECTANGULAR COORDINATES

Coordinate Transformation

$$x = R\cos\alpha \qquad R = \sqrt{x^2 + y^2}$$

$$y = R\sin\alpha \qquad \alpha = \arctan\frac{y}{x}$$

$$z = z \qquad z = z$$

Unit Vector Transformation

$$\mathbf{i} = \hat{\mathbf{R}}\cos\alpha - \hat{\boldsymbol{\alpha}}\sin\alpha \qquad \hat{\mathbf{R}} = \mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha$$

$$\mathbf{j} = \hat{\mathbf{R}}\sin\alpha + \hat{\boldsymbol{\alpha}}\cos\alpha \qquad \hat{\boldsymbol{\alpha}} = -\mathbf{i}\sin\alpha + \mathbf{j}\cos\alpha$$

$$\mathbf{k} = \mathbf{k} \qquad \mathbf{k} = \mathbf{k}$$

Mathematically, vectors and tensors can be defined in a way quite different from the physical approach.[1] The idea is based on transformation theory and applies even in coordinate systems for which there is no metric defined (i.e., abstract "spaces" where no spatial relationship exists between the coordinates.). First a "space" is chosen. In the preceding sections, the assumption was made that the coordinates were defined in a three dimensional, galilean space, and the transformations were galilean. The special theory of relativity defines a mathematical, 4 dimensional "space" with one imaginary coordinate, and the transformations are Lorentzian. Mathematically, other coordinate transformations can be postulated including those for even higher dimensional abstract "spaces". Having specified the "space", and enumerated the various coordinate systems, and coordinate transformations relating them, i.e. relationships like Eq.(D.7.1) relating $S, \bar{S}, \bar{\bar{S}}, \ldots\ldots, \bar{S}^n$, the mathematician now defines quantities, such as (A_1, A_2, A_3) or (F_1, F_2, F_3, F_4), etc., sets of numbers, at a given point. However, they are not defined arbitrarily, but according to a certain *transformation* rule. The transformation is not one of the coordinate transformations already defined, but another process that, for a given set of numbers (A_1, A_2, A_3), for example, at a given point in S, produces another set of numbers $(\bar{A}_1, \bar{A}_2, \bar{A}_3)$ at the

1. A.S.Eddington, The Mathematical Theory of Relativity, p. 44 ff, Cambridge U. Press, (1952).

corresponding point in $\bar{S}$. The only transformations of this type to be admitted are those that have the property that when applied first to (A_1, A_2, A_3) going from S to $\bar{S}$ and then to $(\bar{A}_1, \bar{A}_2, \bar{A}_3)$ going from $\bar{S}$ to $\bar{\bar{S}}$ (always at the same space point) the result will be identical to that found when the transformation is applied to (A_1, A_2, A_3) going directly from S to $\bar{\bar{S}}$. It is found, that *only three transformations have this property*; those that exhibit invariance (as discussed in connection with Eqs. D.7.3 and D.7.4) and those that exhibit contravariance and covariance as shown for the first two orders in Table D.7.1. Further, only those quantities that obey one of these three transformation laws, are defined as scalars, vectors, and tensors. Scalars, vectors, and tensors defined this way are said to have the *group* property. If the coordinate transformation is galilean, the corresponding tensors are of the galilean group. If the coordinate transformation is Lorentzian, the tensors are of the Lorentz group, etc.

With the advent of relativity theory, with its heavy dependence on coordinate transformations and multiple observers, it has become popular to insist that the preceding mathematical definition of tensors is physical and fundamental. There is no denying that it is illuminating to the physicist. However, the advantage to be gained by understanding the group nature of vectors and tensors is in helping to keep in mental order the various coordinate transformations and operations involved. Actually, all physical vectors and tensors automatically have the mathematical properties described, so if the physicist had no confusion in his physical picture, and a proper mathematical way to express the phenomena, there would be no need to use transformation theory to help identify the nature of a quantity. In fact, the idea that this mathematical approach to tensors is fundamental to a physicist is false. *Physical vectors and tensors have other properties as well as those implied in this mathematical definition*. In particular, all of physics can be discovered and the world can be described using *one* observer system, without component transformations. Here, transformation theory will be relegated to the position of a sometimes useful tool, no more. Physics is field theory, not transformation theory.

APPENDIX E

KINEMATICS OF A PERFECT FLUID

E.1 **Introduction**: No fluid composed of *particles* is a perfect fluid, i.e. a continuum (see Section 2.3). Nevertheless, field equations can give a good approximation to imperfect fluid flow. On the other hand, the ether appears to be a perfect fluid, so here it will be treated as a continuum. Kinematic relationships derived here apply to fluids in general, so far as their properties approach those of the continuum.

Kinematic variables are position, length, area, volume, velocity, acceleration, and "density". The latter is a scalar ϕ or ϕ_a, that represents the amount of *fluid per unit volume*, and is not, in the case of the ether, related to mass as it is generally understood. Contrary to the ease with which the velocity and acceleration of a mathematical point can be grasped intuitively, in a fluid these require some additional abstraction. Beginning with some portion of the fluid contained in an arbitrarily chosen small volume, an imaginary closed surface is visualized which separates that fluid from all the rest and moves along with the fluid. If the volume of this element is chosen smaller and smaller till it approaches a differential limit, then it takes on the properties of a mathematical point necessary to express its position, velocity and acceleration. General fluid motion is then described by dividing the total fluid up into elemental volumes whose size approaches a point, whose number approaches infinity, and which move so that no fluid crosses any of the surfaces. In the limit, the motion of these point-like elements can then be represented as a velocity vector field $\mathbf{V}$ with an acceleration vector field $\mathbf{a}$.

In Section 2.5, the difference between kinematic and dynamic equations of fluid motion was indicated. *Kinematic relations are the same for all fluids*, dynamic relations are specific to each fluid type. Thus, relationships between the space and time derivatives of ϕ_a, $\mathbf{V}$ and $\mathbf{a}$ that are the same for all fluids constitute fluid kinematics.

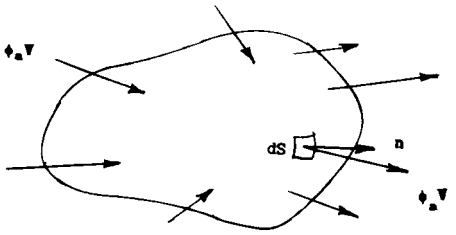


Figure E.2.1
Fluid through a small, fixed, closed volume.

E.2 **The Continuity Equation**: To derive the equation of continuity, imagine a small closed surface, of arbitrary shape, fixed relative to the observer. Assume a general and arbitrary flow to be passing through this surface as set forth in Figure E.2.1. Defining the net rate of

outflow of fluid as,

$$\text{net rate of outflow} = \int \phi_a \mathbf{V}\cdot\mathbf{n}\, dS \quad , \qquad \text{(E.2.1)}$$

and the total fluid inside the surface as,

$$\text{total fluid inside} = \int \phi_a \, dv \quad , \qquad \text{(E.2.2)}$$

then, *in a region where no fluid is being created or destroyed*, the two must be related by,

$$\frac{\partial}{\partial t}(\text{total fluid inside}) = -\ \text{net rate of outflow} \quad .$$

Combining Eqs.(E.2.1) and (E.2.2),

$$\frac{\partial}{\partial t}\int \phi_a \, dv = -\int \phi_a \mathbf{V}\cdot\mathbf{n}\, dS \quad .$$

Since the volume is fixed in the observer's space, the partial can be taken under the integral sign. Therefore, using Gauss' divergence theorem (see Table D.6.2),

$$\int \frac{\partial \phi_a}{\partial t}\, dv = -\int \nabla\cdot\phi_a \mathbf{V} dv \quad .$$

No restriction has been set on the size of the volume, so if it is reduced to differential size,

$$\frac{\partial \phi_a}{\partial t} = -\nabla\cdot \phi_a \mathbf{V} \quad , \qquad \text{(E.2.3)}$$

which is the continuity equation. It says that the time rate of increase of density at a fixed point is equal to the negative of the divergence of the flow vector $\phi_a \mathbf{V}$ at that point.

E.3 **Moving Coordinate Systems**: To measure the motion of a fluid, an observer usually sets up a coordinate system fixed with respect to himself. He then examines the details of the fluid motion in the immediate neighborhood of *a point fixed in his system*. Customarily this gives him a partial differential equation applicable at each and every point in space, as in the derivation of the continuity equation in the preceding section. There is one other vantage point which often is better for describing certain properties of the motion, and that is in a system of coordinates locked to and moving with some particular element of the fluid. The problem of relating physical phenomena seen in this moving system to those seen by the general observer will now be discussed.

Consider the two Cartesian systems illustrated in Figure E.3.1. S is the general observer, and $\bar{S}$ is the moving observer. The origin of $\bar{S}$ is transporting in some *known* manner, and the frame of $\bar{S}$ rotates with a *known* angular velocity represented by $\mathbf{w}_0$ (as seen by S).

Furthermore, $\mathbf{w}_0$ may be changing in a *known* way as a function of time. The position of $\bar{0}$ relative to 0 is given by $\mathbf{r}_0$, and the position of the moving point P is given by $\mathbf{r}$ as seen by S and by $\bar{\mathbf{r}}$ as seen by $\bar{S}$. The relationships between the velocities and accelerations of P as seen by the two observers will now be found.

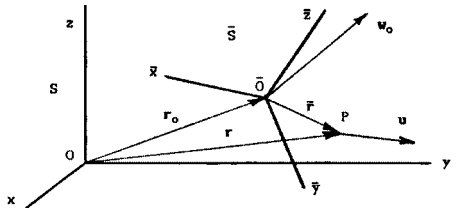


Figure E.3.1
Point P as seen by observers S and $\bar{S}$.

Because $\bar{S}$ *moves* relative to S, the coordinate transformations given in Eq.(D.7.1) must be modified to include the time,

$$x_i = x_i(\bar{x}, \bar{y}, \bar{z}, \bar{t}) \quad , \quad \bar{x}_i = \bar{x}_i(x, y, z, t) \quad , \qquad \text{(E.3.1)}$$

where it is assumed $t = \bar{t}$. A more convenient form of this transformation is,

$$\mathbf{r} = \mathbf{r}_0 + \bar{\mathbf{r}} \quad , \quad \bar{\mathbf{r}} = \mathbf{r} - \mathbf{r}_0 \quad , \qquad \text{(E.3.2)}$$

where,

$$\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z \quad , \quad \bar{\mathbf{r}} = \bar{\mathbf{i}}\bar{x} + \bar{\mathbf{j}}\bar{y} + \bar{\mathbf{k}}\bar{z}$$
$$\mathbf{r}_0 = \mathbf{i}x_0 + \mathbf{j}y_0 + \mathbf{k}z_0 \qquad \text{(E.3.3)}$$

and all can be functions of time. In Eq.(E.3.3), $\mathbf{r}$ is expressed in terms of its components as measured by S; but it can also be written in terms of $\bar{S}$ components. At this point it is necessary to clearly distinguish between the *components* of a vector and the *coordinates* of a point. In writing $\mathbf{r}$ (the distance and direction of the point P relative to the origin of S) in terms of $\bar{S}$ coordinates, although there might be a temptation to write it as $\bar{\mathbf{i}}\bar{x} + \bar{\mathbf{j}}\bar{y} + \bar{\mathbf{k}}\bar{z}$, this cannot be done, since in Eq.(E.3.3) $\bar{x}$, $\bar{y}$, and $\bar{z}$ were taken as the "coordinates" of P as observed by $\bar{S}$, and these coordinates are <u>not</u> the *components* of $\mathbf{r}$ in $\bar{S}$, but of $\bar{\mathbf{r}}$. Thus, some other designation must be used for the $\bar{S}$ <u>components</u> of the vector $\mathbf{r}$. The following notation will be used,

$$\mathbf{r} = \overbrace{\mathbf{i}x + \mathbf{j}y + \mathbf{k}z}^{S} = \overbrace{\bar{\mathbf{i}}r_{\bar{x}} + \bar{\mathbf{j}}r_{\bar{y}} + \bar{\mathbf{k}}r_{\bar{z}}}^{\bar{S}} \quad . \qquad \text{(E.3.4)}$$

The system of base vectors used determines which observer is describing a vector. S uses $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and $\bar{S}$ uses $\bar{\mathbf{i}}$, $\bar{\mathbf{j}}$, $\bar{\mathbf{k}}$. The *components* of the vectors are *magnitudes measured in the directions of the base vectors*. The time rate of change of the vector $\mathbf{r}$ as seen by S is defined as,

$$\frac{d\mathbf{r}}{dt} = \mathbf{i}\frac{dx}{dr} + \mathbf{j}\frac{dy}{dt} + \mathbf{k}\frac{dz}{dt} \quad , \qquad \text{(E.3.5)}$$

and the meaning of "as seen by S" is that $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are fixed relative to S during the change of $\mathbf{r}$. The time rate of change of $\mathbf{r}$, "as seen by $\bar{S}$" is symbolized by a primed derivative $d'\mathbf{r}/dt$, which means that relative to $\bar{S}$, the observer expressing the time derivative, $\bar{\mathbf{i}}$, $\bar{\mathbf{j}}$, $\bar{\mathbf{k}}$ are fixed, giving,

$$\frac{d'\mathbf{r}}{dt} = \bar{\mathbf{i}}\frac{dr_{\bar{x}}}{dt} + \bar{\mathbf{j}}\frac{dr_{\bar{y}}}{dt} + \bar{\mathbf{k}}\frac{dr_{\bar{z}}}{dt} \quad . \tag{E.3.6}$$

Notice that the derivatives of the components are written without primes, and dt is used rather than $d\bar{t}$. That is because first, t and $\bar{t}$ are assumed to be the same, and second, the time rate of change of any scalar field (which each component $r_{\bar{x}}$, etc., can be considered to be) at a particular point is the same regardless of who measures it. To relate the two time derivatives, write the derivative of both sides of Eq.(E.3.4) as seen by S, meaning that $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are constant and $\bar{\mathbf{i}}$, $\bar{\mathbf{j}}$, $\bar{\mathbf{k}}$ vary, so that,

$$\mathbf{i}\frac{dx}{dr} + \mathbf{j}\frac{dy}{dt} + \mathbf{k}\frac{dz}{dt} = \bar{\mathbf{i}}\frac{dr_{\bar{x}}}{dt} + \bar{\mathbf{j}}\frac{dr_{\bar{y}}}{dt} + \bar{\mathbf{k}}\frac{dr_{\bar{z}}}{dt} + \frac{\partial\bar{\mathbf{i}}}{\partial t}r_{\bar{x}} + \frac{\partial\bar{\mathbf{j}}}{\partial t}r_{\bar{y}} + \frac{\partial\bar{\mathbf{k}}}{\partial t}r_{\bar{z}} \quad .$$

Substituting Eqs.(E.3.5) and (E.3.6)

$$\frac{d\mathbf{r}}{dt} = \frac{d'\mathbf{r}}{dt} + \left(\frac{\partial\bar{\mathbf{i}}}{\partial t}r_{\bar{x}} + \frac{\partial\bar{\mathbf{j}}}{\partial t}r_{\bar{y}} + \frac{\partial\bar{\mathbf{k}}}{\partial t}r_{\bar{z}}\right) \quad . \tag{E.3.7}$$

To reduce the bracketed term, the time derivatives of the $\bar{S}$ unit vectors, as seen by S, must be found. This can be done most easily by realizing that the frame $\bar{S}$ *moves as a rigid body* and recalling that the expression for the velocity of any point in a rigid body with an instantaneous angular velocity of rotation $\mathbf{w}$ is,

$$\frac{d\mathbf{R}}{dt} = \mathbf{w}\times\mathbf{R} \quad , \tag{E.3.8}$$

where $\mathbf{R}$ is the position vector of the point referred to the origin through which $\mathbf{w}$ is measured. Applying Eq.(E.3.8) to the unit vectors of $\bar{S}$,

$$\frac{d\bar{\mathbf{i}}}{dt} = w_z\bar{\mathbf{j}} - w_y\bar{\mathbf{k}} \quad , \quad \frac{d\bar{\mathbf{j}}}{dt} = w_x\bar{\mathbf{k}} - w_z\bar{\mathbf{i}} \quad , \quad \frac{d\bar{\mathbf{k}}}{dt} = w_y\bar{\mathbf{i}} - w_x\bar{\mathbf{j}} \quad . \tag{E.3.9}$$

When Eqs.(E.3.9) and (E.3.7) are combined, the result is,

$$\frac{d\mathbf{r}}{dt} = \frac{d'\mathbf{r}}{dt} + \mathbf{w}_0\times\mathbf{r} \quad , \tag{E.3.10}$$

which is the time rate of change of the position vector $\mathbf{r}$ of an arbitrarily moving point, as seen by S, in terms of the time rate of

change of the same vector $\mathbf{r}$ as seen by $\bar{S}$. In Eq.(E.3.10), $\mathbf{w}_0$ is measured by S.

Taking the S derivative of both sides of Eq.(E.3.10),

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d}{dt}\left(\frac{d'\mathbf{r}}{dt}\right) + \frac{d\mathbf{w}_0}{dt}\times\mathbf{r} + \mathbf{w}_0 \times \frac{d\mathbf{r}}{dt} \quad ,$$

which upon repeated use of Eq.(E.3.10) yields,

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d'^2\mathbf{r}}{dt^2} + 2\mathbf{w}_0 \times \frac{d'\mathbf{r}}{dt} + \mathbf{w}_0 \times (\mathbf{w}_0 \times \mathbf{r}) + \frac{d\mathbf{w}_0}{dt}\times\mathbf{r} \quad , \qquad \text{(E.3.11)}$$

the transformation of the second time rate of change of the vector $\mathbf{r}$ in $\bar{S}$ to the second time rate of change of the same vector $\mathbf{r}$ in S. It is expressed in terms of the first time rate of change of $\mathbf{r}$ in $\bar{S}$ and $\mathbf{w}_0$ and $d\mathbf{w}_0/dt$ as seen by S.

The time derivative transformations of Eqs.(E.3.10) and (E.3.11) relate the rates of change of the same vector $\mathbf{r}$ as seen by S and $\bar{S}$. Often, what is desired is a transformation of the same kind of vector rather than the same vector. The simplest example of this is the transformation from $\bar{\mathbf{r}}$ in $\bar{S}$ to $\mathbf{r}$ in S. As is clear from Figure E.3.1, $\mathbf{r}$ and $\bar{\mathbf{r}}$ are not the same vector, but each represents the same kind of thing to S and $\bar{S}$ respectively. To $\bar{S}$, $\bar{\mathbf{r}}$ is the position vector of the point P, and to S, $\mathbf{r}$ is the position vector of the same point P. Similarly, $\mathbf{u}$ and $\bar{\mathbf{u}}$ represent the velocity of the same point P to the different observers, and surely $\mathbf{u}$ and $\bar{\mathbf{u}}$ are not the same vector though they are the same kind of vector. Completely different transformations are required to relate the same vector as compared with relating two vectors which are the same kind. To emphasize this concept, the velocities $\mathbf{u}$ and $\bar{\mathbf{u}}$ of an arbitrarily moving point P can be related by taking the S derivative of Eq.(E.3.2),

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}_0}{dt} + \frac{d\bar{\mathbf{r}}}{dt} \quad . \qquad \text{(E.3.12)}$$

Eq.(E.3.12) says that the velocity of an arbitrarily moving point relative to S is equal to the sum of the velocity of the origin of $\bar{S}$ measured by S and the velocity $d\bar{\mathbf{r}}/dt$ of the point P relative to a third set of coordinates always parallel to S but with origin at $\mathbf{r}_0$. If $\bar{\mathbf{r}}$ is thought of as transposed to the origin of S, then from Eq.(E.3.10),

$$\frac{d\bar{\mathbf{r}}}{dt} = \frac{d'\bar{\mathbf{r}}}{dt} + \mathbf{w}_0 \times \bar{\mathbf{r}} \quad , \qquad \text{(E.3.13)}$$

where $d'\bar{\mathbf{r}}/dt$ is the velocity of the point P relative to $\bar{S}$. This combined with Eq.(E.3.12) leads to,

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}_0}{dt} + \frac{d'\bar{\mathbf{r}}}{dt} + \mathbf{w}_0 \times \bar{\mathbf{r}} \quad , \qquad \text{(E.3.14)}$$

the velocity transformation,

$$\mathbf{u} = \mathbf{u}_0 + \bar{\mathbf{u}} + \mathbf{w}_0 \times \bar{\mathbf{r}} \quad . \tag{E.3.15}$$

Comparing Eqs.(E.3.10) and (E.3.14), the former transforms derivatives of the same vector $\mathbf{r}$ while the latter transforms derivatives of $\bar{\mathbf{r}}$ to derivatives of $\mathbf{r}$. Equation (E.3.15), says that the velocity as seen by S is the sum of the velocity of the origin $\bar{0}$, the velocity relative to $\bar{0}$ as seen by $\bar{S}$ in his coordinates, *and an extra velocity resulting from the rotation of the* $\bar{S}$ *coordinates*.

Another example of transformation of the same kind of vector is that relating the acceleration of an arbitrarily moving point as seen by the two observers. Taking the derivative of Eq.(E.3.14) and combining it with Eqs.(E.3.10) and (E.3.13) provides,

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d^2\mathbf{r}_0}{dt^2} + \frac{d'^2\bar{\mathbf{r}}}{dt^2} + 2\mathbf{w}_0 \times \frac{d'\bar{\mathbf{r}}}{dt} + \mathbf{w}_0 \times (\mathbf{w}_0 \times \bar{\mathbf{r}}) + \frac{d\mathbf{w}_0}{dt} \times \bar{\mathbf{r}} \quad , \tag{E.3.16}$$

the acceleration transformation,

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}_0 + \dot{\bar{\mathbf{u}}} + 2\mathbf{w}_0 \times \bar{\mathbf{u}} + \mathbf{w}_0 \times (\mathbf{w}_0 \times \bar{\mathbf{r}}) + \frac{d\mathbf{w}_0}{dr} \times \bar{\mathbf{r}} \quad . \tag{E.3.17}$$

This equation differs from Eq.(E.3.11) in that the latter transformed derivatives of the same vector $\mathbf{r}$ from $\bar{S}$ to S, whereas Eq.(E.3.16) relates the derivatives of $\bar{\mathbf{r}}$ in $\bar{S}$ to those of $\mathbf{r}$ in S, where $\bar{\mathbf{r}}$ and $\mathbf{r}$ are the same kind of vector. Eq(E.3.16) says that the acceleration $d^2\mathbf{r}/dt^2$ of an arbitrarily moving point as measured by S is composed of the acceleration $d^2\mathbf{r}_0/dt^2$ of the origin of $\bar{S}$ as seen by S plus the acceleration $d'^2\bar{\mathbf{r}}/dt^2$ of P relative to $\bar{S}$, the acceleration of Coriolis, the centripetal acceleration due to the rotation of $\bar{S}$, and an acceleration resulting from the *angular* acceleration of $\bar{S}$. The Coriolis acceleration results from the curvature of the path of any point *fixed* in $\bar{S}$ and from the motion of the point inward or outward to a place where the linear velocity of a point in $\bar{S}$ due to its rotation is smaller or larger.

The final topic of this section relates to the transformation from $\bar{S}$ to S of time derivatives of fields. For example, the time derivative of a scalar field $\phi(x, y, z, t)$ as observed at a *moving point* is the same to both S and $\bar{S}$ as long as each observes the field at the same moving point. Mathematically this is expressed as,

$$\frac{d\phi}{dt.} = \frac{d'\phi}{dt.} \quad . \tag{E.3.18}$$

The meaning of the *dot subscript* is discussed in the next section. Extension of this idea to the time derivative of a field vector $\mathbf{F}$

observed at a *moving point* is implemented by expressing $\mathbf{F}$ as the difference between two position vectors $\mathbf{r}_2$ and $\mathbf{r}_1$. Then,

$$\frac{d\mathbf{F}}{dt}_{\cdot} = \frac{d\mathbf{r}_2}{dt} - \frac{d\mathbf{r}_1}{dt} \quad ,$$

and applying the position vector transformation, Eq.(E.3.10),

$$\frac{d\mathbf{F}}{dt}_{\cdot} = \frac{d'\mathbf{r}_2}{dt} + \mathbf{w}_0 \times \mathbf{r}_2 - \frac{d'\mathbf{r}_1}{dt} - \mathbf{w}_0 \times \mathbf{r}_1 \quad .$$

Combining terms,

$$\frac{d\mathbf{F}}{dt}_{\cdot} = \frac{d'\mathbf{F}}{dt}_{\cdot} + \mathbf{w}_0 \times \mathbf{F} \quad . \qquad \text{(E.3.19)}$$

The same principles can be used to find the transformations of a second rank tensor or dyadic,

$$\frac{d\Psi}{dt}_{\cdot} = \frac{d'\Psi}{dt}_{\cdot} + \mathbf{w}_0 \times \Psi - \Psi \times \mathbf{w}_0 \quad . \qquad \text{(E.3.20)}$$

As before, repeated use of these forms leads to the transformations of the second time derivatives,

$$\frac{d^2\mathbf{F}}{dt^2}_{\cdot} = \frac{d'^2\mathbf{F}}{dt^2}_{\cdot} + 2\mathbf{w}_0 \times \frac{d'\mathbf{F}}{dt}_{\cdot} + \mathbf{w}_0 \times (\mathbf{w}_0 \times \mathbf{F}) + \frac{d\mathbf{w}_0}{dt} \times \mathbf{F} \quad , \quad \text{(E.3.21)}$$

and,

$$\frac{d^2\Psi}{dt^2}_{\cdot} = \frac{d'^2\Psi}{dt^2}_{\cdot} + 2\mathbf{w}_0 \times \frac{d'\Psi}{dt}_{\cdot} - 2\frac{d'\Psi}{dt}_{\cdot} \times \mathbf{w}_0 + \mathbf{w}_0 \times (\mathbf{w}_0 \times \Psi) + \frac{d\mathbf{w}_0}{dt} \times \Psi - \Psi \times \frac{d\mathbf{w}_0}{dt} - 2\mathbf{w}_0 \times \Psi \times \mathbf{w}_0 + (\Psi \times \mathbf{w}_0) \times \mathbf{w}_0 \quad . \qquad \text{(E.3.22)}$$

At the beginning of this section, the moving system $\bar{S}$ was introduced because having coordinates that move along locked to an element of the medium is useful in the general field theory. However, the *relationships derived in this section are not restricted to a moving system locked to the medium.* They apply to any system $\bar{S}$ moving relative to S in "free space". *When $\bar{S}$ does move locked to a fluid medium, $\bar{S}$ will be designated as the "fluid coordinate" system.*

E.4 **Moving Point Derivatives**: The previous section presented methods of transforming time derivatives of field quantities observed at a moving point as seen by $\bar{S}$ to time derivatives of field quantities observed at the same moving point as seen by S, assuming the $\bar{S}$ time derivatives were already known. The present section defines and presents methods for obtaining these moving point derivatives.

The simplest m.p. (moving point) derivative is that of a scalar field such as $\phi = \phi(x, y, z, t)$. Any change in ϕ is described by the total

derivative,

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz + \frac{\partial\phi}{\partial t}dt \quad , \qquad (E.4.1)$$

which says that, given any small change in position (dx,dy,dz) of the point P at which ϕ is observed, or of time (dt) at that point, ϕ changes by $d\phi$. The total change of ϕ in a time dt can be written,

$$\frac{d\phi}{dt} = \frac{dx}{dt}\frac{\partial\phi}{\partial x} + \frac{dy}{dt}\frac{\partial\phi}{\partial y} + \frac{dz}{dt}\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial t} \quad . \qquad (E.4.2)$$

This says that the time rate of change of ϕ *moving along with the point* P depends on the velocity components of P (dx/dt, dy/dt, dz/dt), the way ϕ changes in space ($\partial\phi/\partial x$, etc.) and also on the way ϕ would change with time *if the point P were not moving* ($\partial\phi/\partial t$). Eq.(E.4.2) can be written in the form,

$$\frac{d\phi}{dt_{\cdot}} = \mathbf{u}\cdot\nabla\phi + \frac{\partial\phi}{\partial t} \quad , \qquad (E.4.3)$$

where the dot in the denominator of the LHS is used as a reminder that *unless the location and velocity of the m.p.* (moving point) *is specified, the moving point derivative cannot be found*. Similarly, the m.p. derivatives of a vector field $\mathbf{F}$ and of a dyadic field Ψ are,

$$\frac{d\mathbf{F}}{dt_{\cdot}} = \mathbf{u}\cdot\nabla\mathbf{F} + \frac{\partial\mathbf{F}}{\partial t} \quad \text{and} \quad \frac{d\Psi}{dt_{\cdot}} = \mathbf{u}\cdot\nabla\Psi + \frac{\partial\Psi}{\partial t} \quad . \qquad (E.4.4)$$

In all of these expressions, $\mathbf{u}$ is the *arbitrary* velocity of the moving point at which the quantity is being observed, and is usually written by S as,

$$\mathbf{u} = \mathbf{i}\frac{dx}{dt} + \mathbf{j}\frac{dy}{dt} + \mathbf{k}\frac{dz}{dt} \quad . \qquad (E.4.5)$$

When $\mathbf{u} = 0$, the total rate of change of any of the field quantities is given by the partial, $\partial/\partial t$. $\bar{S}$ would express Eq.(E.4.3) as,

$$\frac{d'\phi}{dt_{\cdot}} = \bar{\mathbf{u}}\cdot\nabla\phi + \frac{\partial'\phi}{\partial t} \quad , \qquad (E.4.6)$$

where,

$$\bar{\mathbf{u}} = \bar{\mathbf{i}}\frac{d\bar{x}}{dt} + \bar{\mathbf{j}}\frac{d\bar{y}}{dt} + \bar{\mathbf{k}}\frac{d\bar{z}}{dt} \quad . \qquad (E.4.7)$$

Notice that the space derivatives in Eq.(E.4.6) are the same for S and $\bar{S}$ (see Section D.7).

So far, all of the equations in this section have been written for a single observer. This will be true throughout most of the remainder of this section. Generally, they will be written as seen by S, but can be converted easily (as in Eq.E.4.6) to $\bar{S}$ equations.

One special case of the m.p. derivative is that where the moving

point follows right along with some particular fluid element, in which case the velocity becomes $\mathbf{u} = \mathbf{V}$, where $\mathbf{V}$ is the fluid velocity field vector, and the derivatives become,

$$\frac{d\phi}{dt} = \mathbf{V}\cdot\nabla\phi + \frac{\partial\phi}{\partial t} \quad , \quad \frac{d\mathbf{F}}{dt} = \mathbf{V}\cdot\nabla\mathbf{F} + \frac{\partial\mathbf{F}}{\partial t} \quad , \quad \frac{d\Psi}{dt} = \mathbf{V}\cdot\nabla\Psi + \frac{\partial\Psi}{\partial t} \quad . \qquad \text{(E.4.8)}$$

These are called the "fluid" derivatives. Here, by convention, a distinguishing dot is not used in the LHS denominators. It is important, in using other works, to ascertain which type of derivative is being used, since the dot is not used by other authors to identify the general m.p. derivative. Even in the following, there are derivatives such as $d\mathbf{r}/dt$ that do not have the dot when they should in the strictest sense. The dot notation is more a convenience than a necessity, but helps considerably in some cases.

The m.p. and "fluid" derivatives can be related by first adding and subtracting $\mathbf{V}\cdot\nabla\phi$ on the RHS of Eq.(E.4.3),

$$\frac{d\phi}{dt_{\cdot}} = \mathbf{V}\cdot\nabla\phi + \frac{\partial\phi}{\partial t} + (\mathbf{u} - \mathbf{V})\cdot\nabla\phi \quad ,$$

and then recognizing the fluid derivative of Eq.(E.4.8), ending with,

$$\frac{d\phi}{dt_{\cdot}} = \frac{d\phi}{dt} + (\mathbf{u} - \mathbf{V})\cdot\nabla\phi \quad . \qquad \text{(E.4.9)}$$

A vector or dyadic can replace ϕ in this equation. Physically, this equation says that the general variation of ϕ (or $\mathbf{F}$, or Ψ) at any arbitrarily moving point is composed of the fluid derivative of ϕ, i.e. the variation of ϕ seen by S at the same point but moving along with the fluid element instantaneously located there, plus another term resulting from the space variation of ϕ and the motion of the m.p. relative to the fluid element instantaneously located there.

A few words about the use of "fluid" coordinates can serve here to clarify some of the questions that probably have arisen. Earlier, the statement was made that transformation theory, as it is popularly practiced today, is not fundamental to physics. Yet, this and the preceding section have entered heavily into a kind of transformation theory. The principal difference between the two approaches is this. In popular transformation theory, the transformation almost replaces the physics, certainly it dominates it, and it is used to first show the equivalence of all observers, and then to replace an intuitive understanding of space and time with a metaphysical ritual called space-time. The important feature of popular transformation theory is the absence of a preferred system. The transformations being presented here, however, are used quite differently.

Using the ether theory, the equations of physics are written in the coordinates of S, the general observer because S is the one who wishes to solve some specific problem. But, nature is so constructed that the ether is the basic constituent of all things, and the laws of its motion and motion of "particles" made of clumps of it, relative to the general unclumped ether flow field are laws expressing interaction between contiguous parts of it. Thus, there is a *preferred* vantage point from which the laws governing the ether can be viewed: that is locked to the fluid, transporting and rotating with the particular element for which the laws are written. In these "fluid" coordinates, the laws have the simplest form. The need for the type of transformation that has been presented is now clear. It allows conversion of the laws in the preferred system (different coordinates at each different point, if the ether flow is space variable) to their equivalent form in the general system S. Consequently, although the transformations presented just previously apply to any two systems in relative motion, from this point on, unless otherwise specified, $\bar{S}$ refers to the "fluid" coordinates locked to a fluid element.

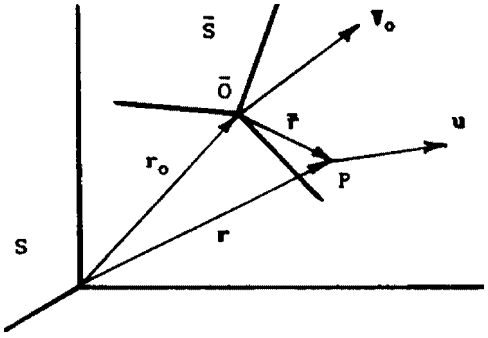


Figure E.4.1
$\bar{S}$ as the fluid coordinate system.

Keeping this in mind, an equation of considerable significance will now be derived. It was first obtained in an essentially equivalent form by Kirkwood,[1] and gives the transformation of an m.p.'s acceleration relative to a fluid field. Figure E.4.1 portrays the configuration of two observers S and $\bar{S}$, where S sees a general velocity field **V** and $\bar{S}$ moves so that the origin $\bar{0}$ is locked to the fluid, and the axes of $\bar{S}$ rotate with the fluid at $\bar{0}$. An arbitrarily moving point has a velocity relative to the fluid element at $\bar{0}$, of $d'\bar{\mathbf{r}}/dt$, as seen by $\bar{S}$. The acceleration of P or time rate of change of the velocity $d'\bar{\mathbf{r}}/dt$ as seen by $\bar{S}$ will be expressed in terms of m.p. derivatives as seen by S.

The standard transformation of the acceleration of the point P is given in Eq.(E.3.16), where $\mathbf{w}_0$ is the vorticity of the fluid at $\bar{0}$ (see Section E.6) and is also the rotation rate of the axes of $\bar{S}$. It will be sufficient to obtain the acceleration for a *point just passing through* $\bar{0}$,

1. R.L.Kirkwood, Phys.Rev., **92**, 1557 (1953).

so that instantaneously, $\bar{\mathbf{r}} = 0$, and substituting that $\bar{\mathbf{r}}$ into Eq.(E.3.14),

$$\frac{d'\bar{\mathbf{r}}}{dt} = \frac{d\mathbf{r}}{dt} - \frac{d\mathbf{r}_0}{dr} = \mathbf{u} - \mathbf{V}_0 \quad , \tag{E.4.10}$$

Furthermore, no confusion will arise if the subscript $_0$ is now omitted. Transposing Eq.(E.3.16), and using $\bar{\mathbf{r}} = 0$ and Eq.(E.4.10),

$$\frac{d'^2\bar{\mathbf{r}}}{dt^2} = \frac{d\mathbf{u}}{dt_.} - \frac{d\mathbf{V}}{dt} - 2\mathbf{w}\times(\mathbf{u}-\mathbf{V}) \quad , \tag{E.4.11}$$

where $d\mathbf{u}/dt_.$ is the m.p. derivative of the point's velocity, $d\mathbf{V}/dt$ is the "fluid" derivative of $\mathbf{V}$ taken moving with the medium at $\bar{0}$, and all quantities are measured at $\bar{0}$.

Since it is desired to express $d'^2\bar{\mathbf{r}}/dt^2$ as seen by $\bar{S}$, in terms of $d(\mathbf{u}-\mathbf{V})/dt_.$ as seen by S, use must be made of the m.p. derivative of $\mathbf{V}$ given in Eq.(E.4.9) as,

$$\frac{d\mathbf{V}}{dt_.} = \frac{d\mathbf{V}}{dt} + (\mathbf{u}-\mathbf{V})\bullet\nabla\mathbf{V} \quad . \tag{E.4.12}$$

Substituting $d\mathbf{V}/dt$ from Eq.(E.4.12) into Eq.(E.4.11),

$$\frac{d'^2\bar{\mathbf{r}}}{dt^2} = \frac{d}{dt_.}(\mathbf{u}-\mathbf{V}) + (\mathbf{u}-\mathbf{V})\bullet\nabla\mathbf{V} - 2\mathbf{w}\times(\mathbf{u}-\mathbf{V}) \quad . \tag{E.4.13}$$

The combination of $d(\mathbf{u}-\mathbf{V})/dt_.$ is not the way terms of this type customarily have been written in the past. To see what this means, rearrange Eq.(E.4.13) to give,

$$\frac{d}{dt_.}(\mathbf{u}-\mathbf{V}) = \frac{d'^2\bar{\mathbf{r}}}{dt^2} - (\mathbf{u}-\mathbf{V})\bullet\nabla\mathbf{V} + 2\mathbf{w}\times(\mathbf{u}-\mathbf{V}) \quad . \tag{E.4.14}$$

which says that the m.p. time rate of change of the velocity of a point relative to the medium at that point is the sum of first, the actual acceleration of the point relative to $\bar{0}$ in $\bar{S}$, second, the change due to the space variation of $\mathbf{V}$ giving a new $\mathbf{V}$ as the point moves, and third, the Coriolis acceleration as the point moves relative to the center of circulation.

Using an identity first stated by Kirkwood,[1]

$$2\mathbf{w}\times\mathbf{F} = \mathbf{F}\bullet\nabla\mathbf{V} - (\nabla\mathbf{V})\bullet\mathbf{F} \quad , \tag{E.4.15}$$

where $\mathbf{F}$ is an arbitrary vector, and Eq.(E.4.13) becomes,

$$\frac{d'^2\bar{\mathbf{r}}}{dt^2} = \frac{d}{dt_.}(\mathbf{u}-\mathbf{V}) + (\nabla\mathbf{V})\bullet(\mathbf{u}-\mathbf{V}) \quad . \tag{E.4.16}$$

This transformation enables the general observer S to predict, from

1. R.L.Kirkwood, loc. cit.

his observation of **u** and **V** at a given point, what acceleration of the m.p. $\bar{S}$ is observing in his system. Equation (E.4.10) re-written without the subscript 0,

$$\frac{d'\mathbf{r}}{dt} = \mathbf{u} - \mathbf{V} \quad , \qquad \text{(E.4.17)}$$

allows S to predict the velocity of the m.p. seen by $\bar{S}$. Clearly, if the m.p. velocity relative to the medium, $\mathbf{u} - \mathbf{V}$, is zero, $\bar{S}$ sees no acceleration of the point. The reader should be cautioned that *all the equations following* Eq.(E.4.9) *embody the very special restriction that the m.p. is just passing through the origin of* $\bar{S}$. *Equations already under this restriction should not be used in derivations where further differentiation is involved*. If such further differentiation is necessary, the more general form developed before the restriction was imposed should be used. After all differentiations have been carried out, the restriction can again be imposed. Once final forms of such equations as (E.4.14) or (E.4.16) are written, they apply to any point in S at which an m.p. is instantaneously located, while moving about, as long as the values of **V**, **w**, and $\nabla\mathbf{V}$ are those observed at that point at that instant, and a different differentially small $\bar{S}$ is visualized at each point.

E.5 **Intrinsic and Invariant Properties of Fields**: Because scalars, vectors, and tensors are independent of the coordinates used to express them in mathematical form, and because their space derivatives in the form of the gradient, curl, divergence, etc. are also independent of the coordinates used (see Section D.7), they are called *invariant* to space coordinate transformations (time held fixed). In the four dimensional space-time of special relativity, time has been included and space-time invariance is used to identify special quantities in a certain class of systems. Invariance plays a major role in the transformation theory approach to physics popular today. However, *there is another characteristic that appears in fluid motion, which is far more important than invariance*, and this importance carries over into the physics of fields because of the basic role the ether plays. Fluid properties that have this characteristic are called intrinsic. An intrinsic property is one that is completely determined by what goes on at and within a particular fluid element, uninfluenced in any way by anything outside the immediate neighborhood of that element. More specifically, *a property is intrinsic if it depends only on the relative motion of adjoining parts of the fluid, and is independent of the motion of the observer.* Dynamic properties as well as kinematic can be intrinsic.

To clearly distinguish between the concepts of invariant and

intrinsic properties, some simple examples are useful. The velocity field $\mathbf{V}$, because it is a vector, is clearly invariant, but it is definitely not *intrinsic*, since a set of observers moving differently will each see a different velocity field for the same flow pattern, because the velocity is expressed in terms of an arbitrary set of coordinates external to each fluid element observed, and therefore is not measured relative to the adjoining fluid at each point. On the other hand, a good example of an intrinsic property of fluid motion can be illustrated using the continuity equation of Section E.2. Expanding Eq.(E.2.3),

$$\frac{\partial \phi_a}{\partial t} = - \mathbf{V} \cdot \nabla \phi_a - \phi_a \nabla \cdot \mathbf{V} \quad , \tag{E.5.1}$$

which transposed, gives the fluid derivative,

$$\frac{d \phi_a}{dt} = - \phi_a \nabla \cdot \mathbf{V} \quad , \tag{E.5.2}$$

that can be written,

$$\frac{1}{\phi_a} \frac{d \phi_a}{dt} = - \nabla \cdot \mathbf{V} \quad . \tag{E.5.3}$$

Assuming no creation or destruction of fluid, if the quantity and density of fluid are related to the volume occupied by $\phi_a = \delta Q / \delta V$, then,

$$\frac{d \phi_a}{\phi_a} = - \frac{d \delta V}{\delta V} \quad . \tag{E.5.4}$$

Combining Eqs.(E.5.3) and (E.5.4),

$$\nabla \cdot \mathbf{V} = \frac{1}{\delta V} \frac{d \delta V}{dt} \quad , \tag{E.5.5}$$

where it is understood that the volume is differentially small. Equation (E.5.5) gives a physical interpretation of the divergence of the velocity of a fluid as the time rate of change of volume per unit volume of a fluid element as it moves along, clearly an intrinsic property dependent only on the relative motion of the adjoining parts of the fluid as it expands or contracts.

These examples illustrate the fact that not all invariant properties are intrinsic but all intrinsic properties (because they are scalars, vectors, or tensors) are invariant. However, intrinsic properties have a much more powerful invariance characteristic than space coordinate transformation invariance. In the case of the non intrinsic velocity field, for example, by moving along with any fluid element, its velocity can be made to go to zero. The curl of $\mathbf{V}$ is another example of a field that is not intrinsic. By a suitable rotation, following any element, $\nabla \times \mathbf{V}$ at that element can be reduced to zero, which could not happen if it depended only on the relative motion of adjoining parts of the fluid. Clearly no intrinsic quantity, dependent only on relative motion, can be eliminated by changing the motion of the observer's coordinate

system. In fact, intrinsic *properties of a fluid are completely independent of the motion of the observer.*

Another way to better understand the idea of an intrinsic quantity is to consider the question raised in Section E.3 regarding transforming the same entity as compared with transforming the same kind of entity. A vector, for example, is called invariant because it appears identical to different observers. Each observer examines the same vector. When different observers look at the same kind of vector instead, eg., the position vector ($\mathbf{r}$ for S, $\bar{\mathbf{r}}$ for $\bar{S}$) or velocity ($\mathbf{V}$ for S, $\bar{\mathbf{V}}$ for $\bar{S}$), they generally see *different* things. However, when every different observer performs the same operation to observe the same kind of entity, and when these entities all appear identical, then they describe an intrinsic property of the fluid.

A most obvious set of intrinsic quantities is that given by the fluid derivative as seen by $\bar{S}$ whose coordinates are moving locked to the fluid and whose measurements are restricted to a differentially small region about $\bar{0}$. Thus, the d'/dt derivatives given earlier are called intrinsic derivatives when $\bar{S}$ uses fluid coordinates and measures inside a differentially small region about $\bar{0}$. In fact, *any fluid property* seen by $\bar{S}_0$ (i.e. at $\bar{0}$ by $\bar{S}$) is intrinsic, and when it is expressed in terms of any other observer's coordinates (i.e. S) it is still intrinsic.

E.6 **Motion of a Fluid Element**: A fluid continuum can be described by a vector point function, indicating the position of every element α at time t, $\mathbf{r} = \mathbf{r}(\alpha)$, where each different value of α represents a different element of the fluid. At time t', or $t + dt$, after deforming, the continuum can be represented by a vector point function,

$$\mathbf{r}' = \mathbf{r}'(\alpha) \quad , \qquad \text{(E.6.1)}$$

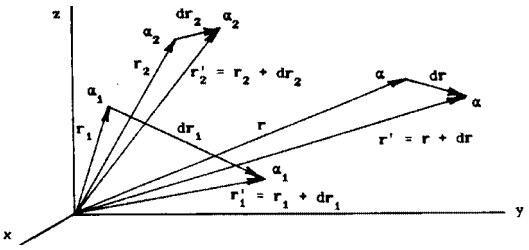


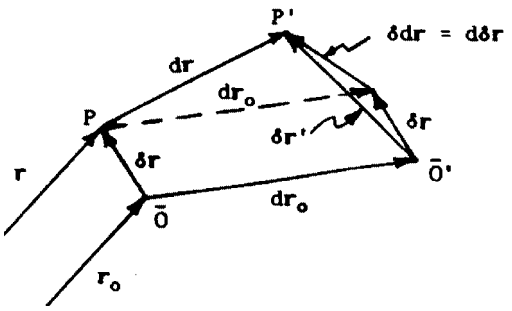
Figure E.6.1
Changes in fluid element positions in time dt.

such that, for the same element α, Eq.(E.6.1) gives a new position, as exhibited in Figure E.6.1. The components of $\mathbf{r}(\alpha)$ at time t are $x(\alpha)$, $y(\alpha)$, $z(\alpha)$, and are related to the $t + dt$ components $x'(\alpha)$, $y'(\alpha)$, $z'(\alpha)$ of the position $\mathbf{r}'(\alpha)$ of the same element. The differential displacement of each element is represented by the vector function, $d\mathbf{r} = \mathbf{r}'(\alpha) - \mathbf{r}(\alpha)$. "Strain" is the name given to this displacement field. Visualizing the

$d\mathbf{r}$ field frozen in time (at t, $d\mathbf{r}$ shows to where each element will go; at $t + dt$, $d\mathbf{r}$ shows from where each element has come), the change in $d\mathbf{r}$ in going a differential distance $\delta\mathbf{r}$ is given by the total differential,

$$\delta d\mathbf{r} = \delta\mathbf{r} \cdot \nabla(d\mathbf{r}) \quad , \tag{E.6.2}$$

where δ is used to indicate a variation produced by moving from one space point to another in the frozen field picture (at either t or $t + dt$), i.e. $\delta\mathbf{r}$ is the initial and $\delta\mathbf{r}'$ the final spacing of two fluid elements being observed.



$\delta d\mathbf{r}$ = vector dist. P moves rel. to $\bar{0}$ in dt. $(d\mathbf{r} - d\mathbf{r}_0)$

$d\delta\mathbf{r}$ = change in vector position of P rel. to $\bar{0}$ in dt. $(\delta\mathbf{r}' - \delta\mathbf{r})$

Figure E.6.2
Change in displacement between two fluid elements

Now, consider an element displaced from $\bar{0}$ to $\bar{0}'$ as depicted in Figure E.6.2. For another element P located $\delta\mathbf{r}$ from $\bar{0}$ at time t, the displacement to a point P′ located $\delta\mathbf{r}'$ from $\bar{0}'$ at $t + dt$ is,

$$d\mathbf{r} = d\mathbf{r}_0 + \delta\mathbf{r}' - \delta\mathbf{r} = d\mathbf{r}_0 + \delta d\mathbf{r} \quad . \tag{E.6.3}$$

Combining Eqs.(E.6.2) and (E.6.3),

$$d\mathbf{r} = d\mathbf{r}_0 + \delta\mathbf{r} \cdot \nabla(d\mathbf{r}) \quad . \tag{E.6.4}$$

Thus, for every $\delta\mathbf{r}$ around $\bar{0}$ at time t, Eq.(E.6.4) gives the displacement of the fluid at that point in terms of the displacement of the fluid at $\bar{0}$, and this permits a very simple description of fluid motion, as will now be seen.

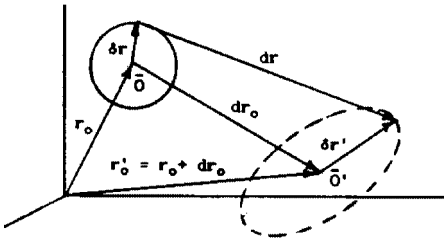


Figure E.6.3
Motion of a spherical element of fluid

With the help of Figure E.6.3, consider a small element of fluid in the form of a sphere at time t. Here the word element is being used in a somewhat different way than before, but no confusion results from this. Each part of the sphere of fluid moves a distance $d\mathbf{r}$ during dt, and again Figure E.6.3 shows the final position and configuration of the element. First, notice that the center of the

sphere originally located at $\bar{0}$ has been translated to $\bar{0}'$, a distance $d\mathbf{r}_0$, as predicted by Eq.(E.6.4) when the point is identified with $\delta\mathbf{r} = 0$. Next, consider the fluid originally surrounding $\bar{0}$. In general, if observed for a long time, the final form of the fluid originally inside the spherical volume could be distorted into shapes that defy mathematical representation; but for a short time dt, the flow is quite regular, always producing a final configuration of the original fluid that is an ellipsoid. Each different spherical element produces an ellipsoid of a different size and shape, unless the flow is specialized in some way.

The preceding statements become clearer when Eq.(E.6.4) is analyzed in more detail. Since the displacement of every part of the fluid is composed of a component $d\mathbf{r}_0$ plus $\delta\mathbf{r}\cdot\nabla(d\mathbf{r})$, then if an observer translates his position by $d\mathbf{r}_0$, he observes only $\delta\mathbf{r}\cdot\nabla(d\mathbf{r})$. Thus $d\mathbf{r}_0$ represents a translation of the original fluid essentially as a rigid body. Any turning or deforming of the fluid about $\bar{0}$ must be inherent in the term $\delta\mathbf{r}\cdot\nabla(d\mathbf{r})$, which contains the strain dyadic $\nabla(d\mathbf{r})$. Any dyadic can be separated into two parts (see section C.5), one symmetric and one antisymmetric. When this is done, Eq.(E.6.4) becomes,

$$d\mathbf{r} - d\mathbf{r}_0 = \delta\mathbf{r}\cdot S + \delta\mathbf{r}\cdot\Theta \quad , \tag{E.6.5}$$

where S and Θ are the symmetric and antisymmetric components of $\nabla(d\mathbf{r})$. S is often called the "pure-strain" tensor, but here it will be called the strain tensor. Confusion seldom results, even though the same name is applied to $\nabla(d\mathbf{r})$. According to Eq.(C.5.9), $\delta\mathbf{r}\cdot\Theta = \frac{1}{2}\Theta_v \times \delta\mathbf{r}$. But, Θ_v can be related back to $\nabla(d\mathbf{r})$ using Eq.(C.3.3) to give, $\Theta_v = [\nabla(d\mathbf{r})]_v = \nabla \times d\mathbf{r}$. Identifying,

$$\boldsymbol{\Theta} = \tfrac{1}{2}\nabla \times d\mathbf{r} \quad , \tag{E.6.6}$$

as a vector representing *rotation* in the plane of the antisymmetric tensor Θ, then the $\delta\mathbf{r}\cdot\Theta$ term in Eq.(E.6.5) is seen to represent the equivalent of a rigid body rotation of the fluid originally contained in the sphere about $\bar{0}$.

Evidently the term $\delta\mathbf{r}\cdot S$ represents the *distortion* of the original spherical element after the *translation* $d\mathbf{r}_0$, and the *rotation* $\delta\mathbf{r}\cdot\Theta$ are removed. According to the discussion in Section C.6, since S is symmetric, the result of dotting it with the vector $\delta\mathbf{r}$ taken in all possible directions about the point $\bar{0}$ is that the spherical element takes the shape of an *ellipsoid*. Thus, the strain always produces an ellipsoid, with its principal axes rotated through an angle $\boldsymbol{\Theta}$, and its center transported to a new position $d\mathbf{r}_0$, from its starting point.

The shape of the ellipsoid is given by the strain along the principal axes.

Inasmuch as the strain takes place in a differential time dt, the *strain* field $d\mathbf{r}$ is related to the *velocity* field by,

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} \quad . \tag{E.6.7}$$

Having frozen the field, the change in $\mathbf{V}$ in going a differential distance $\delta\mathbf{r}$ is, dividing Eq.(E.6.2) by the fixed value dt, $\delta\mathbf{V} = \delta\mathbf{r}\bullet\nabla\mathbf{V}$. The equivalence of the *velocity* field and the *rate of strain* field leads on to the parallel of Eq.(E.6.4),

$$\mathbf{V} = \mathbf{V}_0 + \delta\mathbf{r}\bullet\nabla\mathbf{V} \quad , \tag{E.6.8}$$

and of Eq.(E.6.5),

$$\mathbf{V} = \mathbf{V}_0 + \delta\mathbf{r}\bullet\Phi + \delta\mathbf{r}\bullet\Omega \quad , \tag{E.6.9}$$

where Φ and Ω are the symmetric and antisymmetric parts of $\nabla\mathbf{V}$ respectively, i.e.,

$$\nabla\mathbf{V} = \Phi + \Omega = \Phi - I \times \mathbf{w} \quad . \tag{E.6.10}$$

Eq. (E.6.9) says that the general motion of a fluid element has three components, a rate of transport $\mathbf{V}_0$ of the element as a rigid body, a rate of distortion into an ellipsoid $\delta\mathbf{r}\bullet\Phi$, and a rate of rotation as a rigid body of $\delta\mathbf{r}\bullet\Omega$. The rate of rotation is found from $\delta\mathbf{r}\bullet\Omega = \mathbf{w} \times \delta\mathbf{r}$, where,

$$\mathbf{w} = \tfrac{1}{2}\Omega_v = \tfrac{1}{2}(\nabla\mathbf{V})_v = \tfrac{1}{2}\nabla \times \mathbf{V} \quad . \tag{E.6.11}$$

The latter angular *rate of rotation* vector corresponds to the *vorticity* of the fluid at the point at the center of the small, spherical element.

The rate of strain tensor Φ and the rate of rotation tensor Ω are fundamental to a description of the relative motion of ether elements in various flow fields. Other basic quantities such as the invariants of Φ, the rate of surface strain tensor ${}^{s}\Phi$, and certain transport theorems play an important role in visualizing the fluid field. These are discussed elsewhere in greater detail.[1]

1. R.H.Dishington, Physics, Beak Publications, Pacific Palisades,CA (1989).

APPENDIX F

FIELD EQUATIONS AND THEIR SOLUTION

F.1 **Introduction**: In attempting to establish the dynamic field equations in any specific physical situation, several questions arise immediately. One relates to the number of dependent variables necessary, another to the best of possible alternative forms of the field functions used, and a third to the more subtle possible existence of implicit field equations in addition to those specified. These and other similar questions are dealt with in the earlier version of this work.[1] There it was shown that many functions represent constrained quantities that require auxiliary relationships to specify a field completely, but the Laplacian gives a complete description requiring no augmentation.

It was also shown that, on the basis of Poisson's integral equation, the Laplacian gives the most basic representation of any field, and some conditions pertaining to the solution of the related equations were discussed there.

F.2 **The Bulk Equations**: On the basis of that earlier discussion, it should not be surprising that the bulk equations of Section 11.2 involve the Laplacian in the form of the wave equation. As far as can be determined at this time, those bulk equations are complete, and although there are some "intractable" problems that should have closed form solutions to be really useful, other miscellaneous and numerical methods are available and generously applied.

F.3 **The ℓ-Wave Equations**: Here again the Laplacian/equilibrium type equations of Section 2.12 prevail. However, although solutions of these equations have been found using the try and see approach, no direct solution for $V_\bullet$ or $\phi_\bullet$ is available; because the equations involve both $V_\bullet$ and the time average $\overline{\overline{\mathbf{V}_\bullet^2}}$. This points up the need for another more complete set of ℓ-wave equations that do not involve the time averages. The writer has come close to finding these, but not close enough.

The one great hope for obtaining the advanced ℓ-wave equations is by way of the four-tensor approach pursued by Kirkwood.[2] This is not a return to the weird space-time geometry of the relativists, but just the recognition that the formalisms of ordinary space, time and ether are kept in rather neat form by nature.

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1. R.H.Dishington, loc. cit.
2. See references in Chapter 14.

The whole structure of the present book has been devised to separate the physics, with its visualizations, from the formal geometry and kinematics of the Appendices, with their non-dynamic rigor. It is this separation of, mathematics and physics that has led successfully to the picture presented so far.

APPENDIX G

UNITS

Table G.1

To obtain the quantity in HLU, multiply the MKS quantity by the factor given. To go from HLU to MKS, divide.

	HLU	MKS
Electric Potential	$\bar{\bar{\phi}}$	9.40967×10^{-4} Volts
Magnetic Vector-Potential	$\mathbf{A}$	2.82095×10^{5} Webers/m
Energy	$\mathcal{E}$	10^{7} Joules
Energy Density	ε	10 Joules
Charge	q	1.06274×10^{10} Coulombs
Charge Density	ρ	1.06274×10^{4} Coulombs/m
Current	i	1.06274×10^{10} Amperes
Resistance	$\mathbb{R}$	8.85419×10^{-14} Ohms
Capacitance	$\mathbb{C}$	1.12941×10^{13} Farads
Inductance	$\mathbb{L}$	8.85419×10^{-14} Henrys
Electric Intensity	$\mathbf{E}$	9.40967×10^{-6} Volts/m
Magnetic Induction	$\mathbf{B}$	2.82095×10^{3} Teslas
Electric Displacement	$\mathbf{D}$	1.06274×10^{6}
Magnetic intensity	$\mathbf{H}$	$3.54491 \times 10^{-3} \frac{\text{Amp Turns}}{\text{m}}$

Table G.2

Electric Energy Density	ε_e (ergs/cm^3) $= 10^{-1} \varepsilon_{mks}$ (Joules/m^3)
Energy	$\mathcal{E}$ (ergs) $= 10^{-7} \mathcal{E}_{mks}$ (Joules)

Table G.3

Starred quantities are Gaussian. Listed quantities are substituted directly. Quantities along rows are equal.

	HLU	MKS	EMU	ESU
Electric Potential	$\underline{\underline{\phi}}$	$\frac{10^8}{c_0\sqrt{4\pi}}\phi_{mks}$	$\frac{1}{c_0\sqrt{4\pi}}\phi_m$	$\frac{1}{\sqrt{4\pi}}\phi_s$ *
Magnetic Vector Potential	$\mathbf{A}$	$\frac{10^6}{\sqrt{4\pi}}\mathbf{A}_{mks}$		$\frac{1}{\sqrt{4\pi}}\mathbf{A}_s$ *
Charge	q	$\frac{c_0\sqrt{4\pi}}{10}q_{mks}$	$c_0\sqrt{4\pi}\ q_m$	$\sqrt{4\pi}\ q_s$ *
Current	i	$\frac{c_0\sqrt{4\pi}}{10}i_{mks}$	$c_0\sqrt{4\pi}\ i_m$	$\sqrt{4\pi}\ i_s$ *
Electric Intensity	$\mathbf{E}$	$\frac{10^6}{c_0\sqrt{4\pi}}\mathbf{E}_{mks}$	$\frac{1}{c_0\sqrt{4\pi}}\mathbf{E}_m$	$\frac{1}{\sqrt{4\pi}}\mathbf{E}_s$ *
Magnetic Intensity	$\mathbf{H}$	$\sqrt{4\pi}10^{-3}\mathbf{H}_{mks}$ (A.T./m)	$\frac{1}{\sqrt{4\pi}}\mathbf{H}_m$ *	$\frac{1}{c_0\sqrt{4\pi}}\mathbf{H}_s$
Electric Displacement	$\mathbf{D}$	$\sqrt{4\pi}10^{-5}\mathbf{D}_{mks}$	$\frac{c_0}{\sqrt{4\pi}}\mathbf{D}_m$	$\frac{1}{\sqrt{4\pi}}\mathbf{D}_s$ *
Magnetic Induction	$\mathbf{B}$	$\frac{10^4}{\sqrt{4\pi}}\mathbf{B}_{mks}$ (Teslas)	$\frac{1}{\sqrt{4\pi}}\mathbf{B}_m$ *	$\frac{c_0}{\sqrt{4\pi}}\mathbf{B}_s$
Magnetic Moment	μ	$10^3\sqrt{4\pi}\ \mu_{mks}$	$\sqrt{4\pi}\ \mu_m$ *	$\frac{\sqrt{4\pi}}{c_0}\ \mu_s$
Conductivity	σ	$\frac{4\pi c_0^2}{10^9}\sigma_{mks}$	$4\pi c_0^2\ \sigma_m$	$4\pi\sigma_s$ *
Resistance	$\mathbb{R}$	$\frac{10^9}{4\pi c_0^2}\mathbb{R}_{mks}$	$\frac{1}{4\pi c_0^2}\mathbb{R}_m$	$\frac{1}{4\pi}\mathbb{R}_s$ *
Capacitance	$\mathbb{C}$	$\frac{4\pi c_0^2}{10^9}\mathbb{C}_{mks}$	$4\pi c_0^2\ \mathbb{C}_m$	$4\pi\mathbb{C}_s$ *
Inductance	$\mathbb{L}$	$\frac{10^9}{4\pi c_0^2}\mathbb{L}_{mks}$	$\frac{1}{4\pi c_0^2}\mathbb{L}_m$	$\frac{1}{4\pi}\mathbb{L}_s$ *

APPENDIX H

TRUNCATION INTEGRALS

1. $\int_0^x \varepsilon^{-1/y}\,dy = T(x)$ The truncation integral.

2. $\int_0^x \varepsilon^{-a/y}\,dy = a\,T\left(\frac{x}{a}\right)$

3. $\int_0^x y\,\varepsilon^{-a/y}\,dy = \frac{x^2}{2}\varepsilon^{-a/x} - \frac{a^2}{2}T\left(\frac{x}{a}\right)$

4. $\int_0^x y^2\varepsilon^{-a/y}\,dy = \left(\frac{x^3}{3} - \frac{ax^2}{2\bullet 3}\right)\varepsilon^{-a/x} + \frac{a^3}{2\bullet 3}T\left(\frac{x}{a}\right)$

5. $\int_0^x y^3\varepsilon^{-a/y}\,dy = \left(\frac{x^4}{4} - \frac{ax^3}{3\bullet 4} + \frac{a^2x^2}{2\bullet 3\bullet 4}\right)\varepsilon^{-a/x} - \frac{a^4}{2\bullet 3\bullet 4}T\left(\frac{x}{a}\right)$

6. $\int_0^x y^n\varepsilon^{-a/y}\,dy = \left(\frac{x^{n+1}}{n+1} - \frac{ax^n}{n(n+1)} + \frac{a^2x^{n-1}}{(n-1)n(n+1)} - \ldots\ldots\right.$

$$\left. \ldots\ldots\ldots\ldots \pm \frac{a^{n-1}x^2}{(n+1)!}\right)\varepsilon^{-a/x} \mp \frac{a^{n+1}}{(n+1)!}T\left(\frac{x}{a}\right)$$

7. $\int_0^x \frac{\varepsilon^{-a/y}}{y^2}\,dy = \frac{\varepsilon^{-a/x}}{a}$

8. $\int_0^x \frac{\varepsilon^{-a/y}}{y^3}\,dy = \frac{\varepsilon^{-a/x}}{a^2}\left(1 + \frac{a}{x}\right)$

9. $\int_0^x \frac{\varepsilon^{-a/y}}{y^4}\,dy = \frac{2\varepsilon^{-a/x}}{a^3}\left(1 + \frac{a}{x} + \frac{a^2}{2x^2}\right)$

10. $\int_0^x \frac{\varepsilon^{-a/y}}{y^n}\,dy = \frac{(n\text{-}2)!\varepsilon^{-a/x}}{a^{n-1}}\left(1 + \frac{a}{x} + \frac{a^2}{2!\,x^2} + \frac{a^3}{3!\,x^3} + \ldots\ldots + \frac{a^{n-2}}{(n-2)!\,x^{n-2}}\right)$

11. $Q(x) = \varepsilon^{1/x}T(x)$, $T(x) = \varepsilon^{-1/x}Q(x)$

12. $\frac{dQ(x)}{dx} = 1 - \frac{1}{x^2}Q(x)$

x	T(x)	x	T(x)
0.05	4.7024×10^{-12}	7.00	4.5615
0.10	3.8302×10^{-7}	7.50	4.9971
0.15	2.2539×10^{-5}	8.00	5.4365
0.20	1.9929×10^{-4}	8.50	5.8794
0.25	7.9955×10^{-4}	9.00	6.3254
0.30	2.1277×10^{-3}	9.50	6.7742
0.35	4.4403×10^{-3}	10.0	7.2254
0.40	7.9190×10^{-3}	11.0	8.1345
0.45	1.2674×10^{-2}	12.0	9.0512
0.50	$1..8767 \times 10^{-2}$	13.0	9.9743
0.55	2.6207×10^{-2}	14.0	10.9029
0.60	3.4990×10^{-2}	15.0	11.8362
0.65	0.04508	16.0	12.7737
0.70	0.05645	17.0	13.7149
0.75	0.06903	18.0	14.6593
0.80	0.08279	19.0	15.6067
0.85	0.09766	20.0	16.5567
0.90	0.11361	25.0	21.3385
0.95	0.13057	30.0	26.1594
1.00	0.14850	35.0	31.0076
1.20	0.2288	40.0	35.8759
1.40	0.3214	45.0	40.7595
1.60	0.4241	50.0	45.6552
1.80	0.5351	55.0	50.5608
2.00	0.6532	60.0	55.4746
2.50	0.9734	65.0	60.3952
3.00	1.3207	70.0	65.3216
3.50	1.6881	75.0	70.2531
4.00	2.0709	80.0	75.1890
4.50	2.4660	85.0	80.1287
5.00	2.8710	90.0	85.0719
5.50	3.2842	95.0	90.0181
6.00	3.7044	100.0	94.9671
6.50	4.1304	$x \to \infty$, $T(x) \to x - \log_e x$	

APPENDIX I

$I_i(\infty)$

ω	I_1	I_2	I_3	I_4	I_5
4×10^{20}	0.60777	0.73200	0.73200	0.73200	0.73200
5	0.57591	↓	↓	↓	↓
6	0.54641	↓	↓	↓	↓
8	0.49422	↓	↓	↓	↓
1×10^{21}	0.45003	↓	↓	↓	↓
1.5	0.36663	↓	↓	↓	↓
2×10^{21}	0.30643	↓	↓	↓	↓
8×10^{22}	9.7043×10^{-3}	0.61211	0.73009	0.73200	0.73200
1×10^{23}	7.7634	0.58096	0.72897	0.73043	↓
1.5	5.1756	0.51265	0.72612	0.73094	↓
2	3.8817	0.45678	0.72275	0.73050	0.73143
3	2.5878	0.37263	0.71478	0.72933	0.73094
4	1.9409	0.31314	0.70603	0.72763	0.73050
5	1.5527	0.26921	0.69653	0.72701	0.72970
6	1.2939×10^{-3}	0.23560	0.68662	0.72344	0.72897
8	9.7043×10^{-4}	0.18777	0.66642	0.71952	0.72701
1×10^{24}	7.7634	0.15556	0.64615	0.71334	0.72473
1.5	5.1756	0.10809	0.59754	0.69882	0.71847
2	3.8817	0.08239	0.55353	0.68314	0.71130
3	2.5878	0.05548	0.47958	0.65098	0.69526
4	1.9409	0.04169	0.42122	0.61946	0.67825
5	1.5527	0.03329	0.37453	0.60973	0.66088
6	1.2939×10^{-4}	0.02769	0.33654	0.56160	0.64361
8	9.7043×10^{-5}	0.02069	0.27880	0.51150	0.61006
1×10^{25}	7.7634	0.01649	0.23721	0.46847	0.57872
1.5	5.1756	0.01070	0.17153	0.38486	0.50972
2×10^{25}	3.8817×10^{-5}	0.00803	0.13357	0.32501	0.45357

$J_i(\infty)$

ω	J_1	J_2	J_3	J_4	J_5
4×10^{20}	0.26460	0.27145	0.27145	0.27145	0.27145
5	0.26119	↓	↓	↓	↓
6	0.25736	↓	↓	↓	↓
8	0.24891	↓	↓	↓	↓
1×10^{21}	0.23992	↓	↓	↓	↓
1.5	0.21778	↓	↓	↓	↓
2×10^{21}	0.19699	0.27145	0.27145	0.27145	0.27145
8×10^{22}	9.70431×10^{-3}	0.26501	0.27143	0.27145	0.27145
1×10^{23}	7.76343	0.26178	0.27142	↓	↓
1.5	5.17564	0.25215	0.27137	0.27144	↓
2	3.88172	0.24141	0.27134	↓	↓
3	2.58781	0.21961	0.27112	0.27142	0.27144
4	1.94086	0.19953	0.27082	0.27140	↓
5	1.55269	0.18185	0.27053	0.27139	0.27143
6	1.29391	0.16650	0.27011	0.27132	0.27142
8	9.70431×10^{-4}	0.14168	0.26908	0.27124	0.27139
1×10^{24}	7.76343	0.12278	0.26461	0.27108	0.27135
1.5	5.17564	0.09133	0.26358	0.27061	0.27121
2	3.88172	0.07224	0.25835	0.26995	0.27103
3	2.58781	0.05073	0.24613	0.26812	0.27048
4	1.94086	0.03892	0.23308	0.26568	0.26972
5	1.55269	0.03152	0.22019	0.26479	0.24075
6	1.29391	0.02645	0.20794	0.25941	0.26761
8	9.70431×10^{-5}	0.01999	0.18593	0.25196	0.26482
1×10^{25}	7.76344	0.01604	0.16728	0.24389	0.26152
1.5	5.17563	0.01070	0.13238	0.22325	0.25166
2×10^{25}	3.88172×10^{-5}	8.02633×10^{-3}	0.10877	0.20388	0.24071

$J_{ij}(\infty)$

ω	J_{1j}	J_{2j}	J_{34}	J_{35}	J_{45}
4×10^{20}	0.26757	0.27145	0.27145	0.27145	0.27145
5	0.26540	↓	↓	↓	↓
6	0.26284	↓	↓	↓	↓
8	0.25673	↓	↓	↓	↓
1×10^{21}	0.24967	↓	↓	↓	↓
1.5	0.23035	↓	↓	↓	↓
2×10^{21}	0.21029	0.27145	0.27145	0.27145	0.27145
8×10^{22}	9.76974×10^{-3}	0.26782	0.27144	0.27144	0.27145
1×10^{23}	7.80543	0.26578	0.27143	↓	↓
1.5	5.19427	0.25913	0.27141	0.27141	↓
2	3.89219	0.25087	0.27137	0.27138	0.27144
3	2.59246	0.23203	0.27127	0.27128	0.27143
4	1.94347	0.21281	0.27113	0.27115	↓
5	1.55436	0.19483	0.27108	0.27097	0.27141
6	1.29507	0.17864	0.27070	0.27074	0.27137
8	9.71084×10^{-4}	0.15172	0.27103	0.27017	0.27131
1×10^{24}	7.76761	0.13092	0.26931	0.26942	0.27121
1.5	5.17749	0.09627	0.26665	0.26942	0.27091
2	3.88276	0.07555	0.26308	0.26393	0.27048
3	2.58781	0.05239	0.25399	0.25444	0.26924
4	1.94112	0.03992	0.24328	0.24380	0.26754
5	1.55286	0.03218	0.23209	0.23241	0.26663
6	1.29402	0.02692	0.22045	0.22091	0.26299
8	9.70496×10^{-5}	0.02026	0.19864	0.19897	0.25725
1×10^{25}	7.76386	0.01622	0.17922	0.17942	0.25010
1.5	5.17581	0.01214	0.14144	0.14150	0.23287
2×10^{25}	3.88182×10^{-5}	8.0712×10^{-3}	0.11544	0.11545	0.21483

$L_i(\infty)$

ω	L_1	L_2	L_3	L_4	L_5
4×10^{20}	0.07977	0.09354	0.09354	0.09354	0.09354
5	0.07494	↓	↓	↓	↓
6	0.07023	↓	↓	↓	↓
8	0.06153	↓	↓	↓	↓
1×10^{21}	0.05396	↓	↓	↓	↓
1.5	0.03973	↓	↓	↓	↓
2×10^{21}	0.02990	0.09354	0.09354	0.09354	0.09354
8×10^{22}	4.7087×10^{-5}	0.08040	0.09347	0.09354	0.09354
1×10^{23}	3.0136	0.07572	0.09342	0.09352	↓
1.5	1.3394	0.06464	0.09328	0.09351	↓
2	7.5339×10^{-6}	0.05512	0.09307	0.09349	0.09352
3	3.3484	0.04074	0.09251	0.09344	0.09351
4	1.8835	0.03097	0.09180	0.09336	0.09349
5	1.2054	0.02419	0.09092	0.09332	0.09346
6	8.3710×10^{-7}	0.01934	0.08992	0.09312	0.09342
8	4.7087	0.01308	0.08764	0.09286	0.09332
1×10^{24}	3.0136	9.385×10^{-3}	0.08510	0.09240	0.09320
1.5	1.3394	4.855	0.07825	0.09114	0.09279
2	7.5339×10^{-8}	2.935	0.07138	0.08955	0.09224
3	3.3484	1.392×10^{-3}	0.05903	0.08573	0.09080
4	1.8835	$8.052\times10^{=4}$	0.04901	0.08146	0.08901
5	1.2054	5.215	0.04106	0.08005	0.08697
6	8.371×10^{-9}	3.647	0.03474	0.07268	0.08477
8	4.709	2.064	0.02563	0.06445	0.08010
1×10^{25}	3.014	1.324×10^{-4}	0.01956	0.05713	0.07537
1.5	1.339×10^{-9}	5.726×10^{-5}	0.01116	0.04280	0.06415
2×10^{25}	7.534×10^{-10}	3.221×10^{-5}	0.00714	0.03287	0.05457

$M_i(\infty)$

ω	M_1	M_2	M_3	M_4	M_5
4×10^{20}	0.30391	0.36602	0.36602	0.36602	0.36602
5	0.28798	↓	↓	↓	↓
6	0.27323	↓	↓	↓	↓
8	0.24713	↓	↓	↓	↓
1×10^{21}	0.22504	↓	↓	↓	↓
1.5	0.18334	↓	↓	↓	↓
2×10^{21}	0.15324	0.36602	0. 36602	0.36602	0.36602
8×10^{22}	4.8522×10^{-3}	0.30608	0.36507	0.36602	0.36602
1×10^{23}	3.8817	0.29051	0.36451	0.36574	↓
1.5	2.5878	0.25635	0.36308	0.36550	↓
2	1.9409	0.22841	0.36140	0.36528	0.36574
3	1.2939	0.18634	0.35742	0.36469	0.36550
4	9.7043×10^{-4}	0.15659	0.35304	0.36384	0.36528
5	7.7635	0.13463	0.34829	0.36353	0.36488
6	6.4696	0.11782	0.34334	0.36175	0.36451
8	4.8522	0.09391	0.33324	0.35978	0.36353
1×10^{24}	3.8817	0.07781	0.32310	0.35670	0.36239
1.5	2.5878	0.05407	0.29880	0.34944	0.35926
2	1.9409	0.04122	0.27679	0.34160	0.35567
3	1.2939	0.02777	0.23982	0.32552	0.34765
4	9.7043×10^{-5}	0.02087	0.21064	0.30976	0.33915
5	7.7635	0.01667	0.18729	0.30489	0.39047
6	6.4696	0.01387	0.16830	0.28082	0.32183
8	4.8522	0.01037	0.13942	0.25578	0.30506
1×10^{25}	3.8817	0.00827	0.11863	0.23426	0.28939
1.5	2.5878	0.00535	0.08579	0.19246	0.25489
2×10^{25}	1.9409×10^{-5}	0.00401	0.06681	0.16253	0.22681

APPENDIX J

2nd LAYER SELF CONSISTENT INTEGRALS

ω	I_2	J_2	J_{2j}	L_2	M_2
4×10^{20}	0.73200	0.27145	0.27145	0.09354	0.36602
5	↓	↓	↓	↓	↓
6	0.73200	↓	↓	↓	0.36602
8	0.73194	↓	↓	↓	0.36599
1×10^{21}	0.73188	↓	↓	0.09354	0.36596
1.5	0.73175	↓	↓	0.09353	0.36590
2×10^{21}	0.73153	0.27145	0.27145	0.09353	0.36579
8×10^{22}	0.61405	0.26519	0.26792	0.08068	0.30705
1×10^{23}	0.58323	0.26204	0.26594	0.07607	0.29164
1.5	0.51545	0.25262	0.25946	0.06511	0.25775
2	0.45983	0.24207	0.25139	0.05565	0.22994
3	0.37580	0.22058	0.23290	0.04127	0.18793
4	0.31621	0.20067	0.21394	0.03146	0.15813
5	0.27210	0.18309	0.19618	0.02462	0.13608
6	0.23830	0.16780	0.18002	0.01971	0.11917
8	0.19012	0.14298	0.15315	0.01337	0.09508
1×10^{24}	0.12306	0.10174	0.13284	0.00615	0.06156
1.5	0.10962	0.09241	0.09746	0.00498	0.05483
2	0.08359	0.07323	0.07655	0.00302	0.04182
3	0.05632	0.05143	0.05313	0.00143	0.02818
4	0.04230	0.03948	0.04050	8.279×10^{-4}	0.02118
5	0.03381	0.03198	0.03266	5.371×10^{-4}	0.01693
6	0.02812	0.02684	0.02733	3.758×10^{-4}	0.01409
8	0.02109	0.02029	0.02056	2.127×10^{-4}	0.01053
1×10^{25}	0.01675	0.01628	0.01646	1.364×10^{-4}	0.00840
1.5	0.01086	0.01086	0.01095	5.902×10^{-5}	0.00543
2×10^{25}	8.149×10^{-3}	8.149×10^{-3}	8.195×10^{-3}	3.320×10^{-5}	0.00407

Get J_{12} from Appendix I.

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