ELECTROMAGNETISM ©

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December 2007

INTRODUCTION

Almost all the shortcomings of classical E&M can be traced to a breakdown in its handling of particle structure. The combination of Dirac's equations and Maxwell's equations to form quantum electrodynamics has exactly the same problems. For about 102 years, these problems have been left uncorrected. The rush to quantum mechanics brought serious work on them to a crawl.

There were three things classical E&M did wrong:

1. It ended up with point particles
2. It defined electric and magnetic energy incorrectly (and often attributed a particle's kinetic energy to its magnetic field)
3. The Pointing theorem failed to correctly describe energy flow except for radiation

At the present state of our knowledge, all three of these problems have the rather simple solutions presented here.

EARLY PARTICLE MODELS

In the steady development of the understanding of electricity and magnetism, certain milestones stand out. In 1837, Faraday correctly defined static *electric* field energy density (in Heaviside-Lorentz Units) as,

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 \quad . \tag{1}$$

In 1853, Kelvin correctly defined static *magnetic* field energy density as (HLU),

$$\varepsilon_m = \tfrac{1}{2}\mathbf{B}^2 \quad . \tag{2}$$

Finally, in 1864, Maxwell combined the various definitions with Faraday's visualization of the fields to yield the **E** and **B** equations now known as Maxwell's equations.

Motivated by the advanced development of the atomic theory, speculation about particles became popular. In 1881, J. J. Thomson decided to apply Maxwell's theory to the motion of a charged particle. In the following, some of the details of the various models are slightly changed to better match the present state of knowledge and simplify the results. Roughly, Thomson assumed the particle to be a thin, hollow, rigid conducting sphere of radius r_s and charge q, as shown in Figure 1.

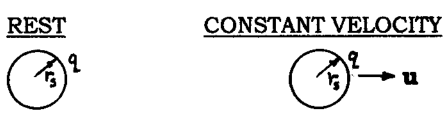

Figure 1. Hollow conducting sphere

He assumed that the charge was uniformly distributed over the surface so that inside the sphere the electric field was zero. Outside, the potential, electric energy density and total external field energy were (HLU),

$$\phi = \frac{q}{4\pi r} \quad , \quad \varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 \quad , \quad E_0 = \iint\int_{r_s}^{\infty} \varepsilon_e \, dvol = \frac{q^2}{8\pi r_s} \quad . \tag{3}$$

This was thought to be a reasonable result for a charged particle at rest.

More important was the calculation for the particle moving at constant velocity. It was assumed that at low velocity, $u/c_0 << 1$, the electric field would not change significantly, and that the magnetic field energy represented the kinetic energy to be added to E_0,

$$\varepsilon_m = \tfrac{1}{2}\mathbf{B}^2 \quad , \quad E_k = \iint\int_{r_s}^{\infty} \varepsilon_m \, dvol \quad , \quad E = E_0 + E_K = \gamma^{4/3} E_0 \tag{4}$$

where ,

$$\gamma = \frac{1}{\sqrt{1 - u^2/c_0^2}} \quad .$$

There was some dissention about this value for E because there were other reasons to believe that a moving field's energy should be $E = \gamma E_0$.

In 1884, Poynting derived his field energy flow theorem from Maxwell's equations, and thirteen years later J. J. Thomson was able to demonstrate that there were charged particles. They were ultimately named electrons. This stimulated more interest in charged particle theory.

Abraham, using the same rigid conducting sphere model that Thompson had used twenty two years before, adopted Thomson's rest energy calculation as correct; but in the moving charge case decided to apply the Pointing theorem for the total moving energy ($u/c_0 << 1$). The result was,

$$\mathbf{S} = c_0 \mathbf{E} \times \mathbf{B} \quad , \quad E\,\mathbf{u} = \iint\int_{r_s}^{\infty} \mathbf{S}\ \mathrm{dvol} = c_0 \iint\int_{r_s}^{\infty} \mathbf{E} \times \mathbf{B}\ \mathrm{dvol} = \tfrac{4}{3} E_0 \mathbf{u} \quad . \qquad (5)$$

Canceling the $\mathbf{u}$'s,

$$E = \tfrac{4}{3} E_0 \quad ,$$

a very unsatisfactory result.

One year later, in 1904, Lorentz repeated the derivation using the Lorentz Transformation and a *flexible* sphere. The result still had $4/3$ factors. For this reason, the preceding, incorrect derivations are often described as the $4/3$ problem.

When Einstein published his Special Relativity paper in 1905, it settled once and for all the need for the Lorentz Transformation and the constant velocity charge total energy as $E = \gamma E_0$, because the derivation did not require specifying the particle structure. This is essentially where classical particle structure development stopped.

Since quantum electrodynamics combines Dirac's equations of particle motion with Maxwell's potential field equations, it has all of the above $4/3$ problems. Also, both Dirac's equations and Lorentz' solution of Maxwell's equations require the use of *point* particles, with the attendant infinities.

THE LAST 102 YEARS (1905 - 2007)

In the models discussed previously, the problems can be summarized under three headings:

1. "Point" particle infinity
2. Incorrect definitions of energy densities, including the assumption that E_k , the particle's kinetic energy, is magnetic
3. Correction of the Poynting Theorem

These three problems have been unsolved for the last 102 years. In the following, a simple solution to each of them is presented.

SOLUTION OF THE "POINT" ELECTRON PROBLEM

It is desired to find a finite solution of Maxwell's Equations, as they are used in quantum electrodynamics (Heaviside-Lorentz Units),

$$\rho = -\left(\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2}\right) \quad , \quad \rho\frac{\mathbf{u}}{c_0} = -\left(\nabla^2\mathbf{A} - \frac{1}{c_0^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right) \quad , \quad \nabla\bullet\mathbf{A} = -\frac{1}{c_0}\frac{\partial\phi}{\partial t} \quad . \quad (6)$$

For the particle at rest, a solution of the scalar equation,

$$\nabla^2\phi = -\rho \quad , \qquad (7)$$

must be found that eliminates the infinities of the "point" charge. A simple, spherical *trial* solution is,

$$\phi = \phi_0(1 - \varepsilon^{-2r_i/r}) \quad ; \qquad (8)$$

Figure 2 below indicates that this potential has only two significant features, the center value ϕ_0 (positive or negative) and the radius r_i of the inflection point.

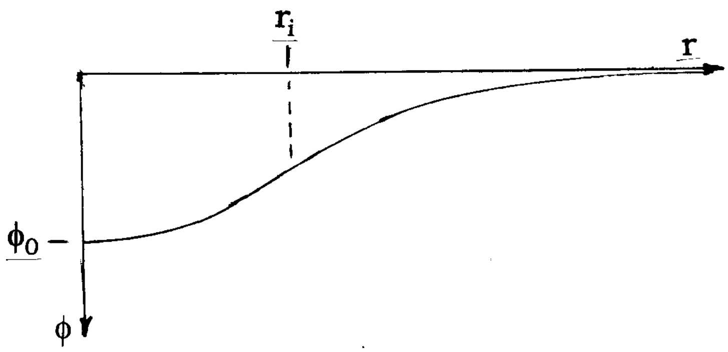

Figure 2. Finite electron solution

The corresponding charge density distribution required to complete the solution is found by substituting the trial solution Eq.(8) in Eq.(7) to yield,

$$\rho = 4\frac{\phi_0 r_i^2}{r^4} e^{-2r_i/r} \quad , \tag{9}$$

a smooth shell of charge distortion that peaks at *half* r_i. This distribution is a reasonable one. Integrated *over all space*, the total charge is $q = 8\pi\phi_0 r_i$.

Similarly, the electric energy density distribution is found from,

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 = 2\frac{\phi_0^2 r_i^2}{r^4} e^{-4r_i/r} = \frac{q^2}{32\pi^2 r^4} e^{-4r_i/r} \quad , \tag{10}$$

a smooth shell of energy distortion *that peaks at the inflection radius* r_i. If ε_e is integrated *over all space*, the resulting finite energy is $E_0 = 2\pi\phi_0^2 r_i$.

Just to get some idea of the magnitudes involved, if the potential in Eq.(8) is assumed to represent an electron, then using $E_0 = 8.18711\times10^{-7}$ ergs (0.511 MeV) and $q = -e = -1.7027\times10^{-9}$ hlcoul (-1.6022×10^{-19} C), the center potential and inflection point radius are $\phi_0 = -1.9233\times10^{3}$ hlvolts (approx. $-\ 2\times10^{6}$ V) and $r_i = 3.522\times10^{-14}$ cm. These are not unreasonable numbers for the electron.

THE EXPERIMENTAL "PROOF" FOR THE "POINT" ELECTRON

What about the experiments that "prove" the electron is a "point" particle? It is important to notice that the expansion of the *gradient* of Eq.(8),

$$\frac{d\phi}{dr} = -2\frac{\phi_0 r_i}{r^2}\left(1 - 2\frac{r_i}{r} + 2\frac{r_i^2}{r^2} - \dots\right) \qquad r > r_i$$

reduces to $d\phi/dr \cong e/4\pi r^2$ (for $r > 200 r_i$), the Coulomb field of the "point" charge. *This explains why the well known collision experiments*[1] that appear to support the "point" charge electron model *are also in complete agreement with the present, finite solution.*

At low collision energies, the principal interaction is out in the Coulomb region. As the collision energy is increased, the Lorentz contraction of the gradient causes the inner, non-Coulomb volume to shrink, and *the interaction never catches up with that inner region.*

THE EXTENDED ELECTRON IN MOTION

So far, the extended electron model appears to be satisfactory; but until it is shown to give the correct constant velocity total energy γE_0, it is incomplete. The calculation begins by going back to Eq.(6) and looking for a finite solution of the changing field scalar equation,

$$\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} = -\rho \quad . \tag{11}$$

Paralleling the derivation of Eqs.(8) and (9), the potential of the constant velocity field, moving in the x direction, is,

$$\phi = \gamma\phi_0(1 - \varepsilon^{-2r_i/r'}) \quad , \tag{12}$$

1. D.P. Barker, et. al., Phys. Rev. Lett., 43, 1915 (1979); Phys. Rev. Lett., 45, 1904 (1980).

where, $r' = \sqrt{\gamma^2 x^2 + R^2}$ in cylindrical coordinates, and γ is defined in Eq.(4). This differs from the spherical case of Eq.(8) mainly in that the equipotentials are oblate spheroids; *not because of any longitudinal contraction*, but because *the potential ϕ expands laterally*. The longitudinal contraction of **E** is always emphasized, but *the lateral expansion of ϕ is more significant in relation to energy and charge*.[2]

The corresponding charge density distribution is,

$$\rho = 4\gamma \frac{\phi_0 r_i^2}{r'^4} e^{-2r_i / r'} \quad . \tag{13}$$

The solution of Eq.(12) can be checked by using a Lorentz transformation on the rest solution of Eq.(8). Furthermore, if Eq.(13) is integrated over all space, the total moving electron charge is found to be $q = 8\pi\phi_0 r_i$, *the same as for the charge at rest*, a well established fact.

THE ELECTRIC ENERGY DENSITY CORRECTION

This is the second point at which the classical E&M theory of particle structure breaks down. Conventionally the expression for *electric energy density* is commonly written,

$$\varepsilon_e = \tfrac{1}{2}\mathbf{E}^2 = \tfrac{1}{2}\left(-\nabla\phi - \frac{1}{c_0}\frac{\partial \mathbf{A}}{\partial t}\right)^2 \quad . \tag{14}$$

This form works for radiation propagation (where $\nabla\phi$ is zero), but in other applications (in association with the Poynting theorem) it has led to a long, confusing literature of strange paradoxes and suggested alternatives[3]. If Eq.(14) is integrated over all space, it fails to give a total energy γE_0. Combinations of Eq.(14) and the **B** field also fail.

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2. P. Lorrain, D. R. Carson, Electromagnetic Fields and Waves, 2nd Ed., W. H. Freeman and Company, San Francisco, p.266, (1970).
3. J.W. Butler, Amer. J. Phys., **36**, 936 (1968); **37**, 1258 (1969).

There are several hints as to why this is so. For example, the conventional definition of magnetic energy density is,

$$\varepsilon_m = \tfrac{1}{2}\mathbf{B}^2 \quad , \tag{15}$$

where **B** is *defined as the magnetic field*,

$$\mathbf{B} = \nabla \times \mathbf{A} \quad . \tag{16}$$

Both the definition of **B** as the magnetic field and $\mathbf{B}^2/2$ as the magnetic energy density have problems similar to those of **E** and ε_e. For example, the **E** field of Eq.(14) and the **B** field of Eq.(16) are so called "force" fields, because, in principle they are measured by inserting a "test" charge at any space point and observing the charge's behavior, i.e. the "force" on it. But, the vector **B** does not point in the direction of either the test charge reaction or that of the current source of the field, but instead points in a non-physically motivated direction that is determined by several conventions. On the other hand, **A** always points in the general direction of the motion of the sources of the field.

Several subtleties appear in the process of defining a magnetic field. Usually the **B** field is regarded as basic, but the Aharonov-Bohm experiment[4] clearly indicates that, even in some situations where **B** is zero, an **A** field can produce magnetic effects on charged particles. Thus, it makes sense to *define the presence of* **A** *as the magnetic field*. Here ϕ and **A** are considered to be the *fundamental* fields. This leads to an important observation related to Eq.(6). The equations for ϕ and **A** are completely separate. The only connection between them is in the divergence equation, which represents fields where magnetic energy is changing into electric energy or vice versa. So Eq.(14) fails because it *mixes* electric and magnetic effects.

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4. Y. Aharonov, D. Bohm, Phys. Rev. **115**, 485 (1959). R. G. Chambers, Phys. Rev. Lett. **5**, 3 (1960). G. Moellenstedt, W. Bayh, Naturwiss. **45**, 81 (1962).

Another hint as to the failure of Eq.(14) relates to the *success* of Eq.(11) in defining moving microscopic charge density, for there is an alternative picture of particle structure that gives *insight into the basic nature of microscopic charge and electric energy density*. If it is assumed that *the potential* ϕ *is the only physical entity in the electric field*, then the construct in Figure 2 is *the total essence of the electron's bulk nature*, i.e. some kind of distortion in the vacuum. Conventionally, in the "point charge" model, charge is felt to be "something" *at the point* that produces the field. Electric energy density is even more evanescent.[5] In the point charge model, the conventional electric energy density ε_e is correctly used only outside some radius far from the point. However, the fundamental nature of ϕ in the preceding allows a different approach. The microscopic Eqs.(1) and (7) can be considered to *define two secondary implicit distortions,* $\frac{1}{2}(\nabla\phi)^2$ and $-\nabla^2\phi$, *automatically present if* ϕ *is present. They do* ***not*** *cause the field, they are the result of it.*

An assumption, adopted almost unanimously around 1900 and still held today, is *that, in the microscopic case, the elements of distributed charge* inside *a single electron, for example, individually obey Coulomb's law just as whole charged particles do in the macroscopic case*. Lorentz had doubts,[6] but they did not prevail. However, there is no direct experiment to support this assumption, and electrons do not fly apart. Thus, *microscopically, there is no reason to expect the distributed "elements" of the* ϕ *field to produce distant actions on each other such as the Coulomb force, which, macroscopically, results from two* ***whole particle*** *fields interacting.* That Eq.(11) gives the correct moving

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5. R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics I*, (Addison -Wesley, Reading, MA 1963) p. 4-1, 4-2 (last paragraph).
6. H. A. Lorentz, *The theory of Electrons*, 2nd Ed. (Dover Publications, Inc., New York 1952) p.215.

microscopic charge density bears this out.

Now that the physical nature of ρ and ε_e as *secondary* implicit distortions *dependent* upon ϕ, rather than as sources of ϕ, has been indicated, the path to the correct form of *moving* electric energy density ε_e is clear. It should be formulated in exactly the same way that *moving* charge density ρ was.

In going from the rest Eq.(7) to the moving Eq.(11), because of the finite rate of propagation, *the charge density in time variable fields is assumed to change* as,

$$\rho = -\nabla^2\phi \qquad \rightarrow \qquad \rho = -\Box^2\phi = -\left(\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2}\right) \quad . \tag{17}$$

That this is true is a well verified fact. Considering the *similar* natures of ρ and ε_e as auxiliary distortions implicit in the shape of ϕ, it would be surprising if electric energy density did not have the simple definition, *parallel to Eq.(17),*

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 \qquad \rightarrow \qquad \varepsilon_e = \tfrac{1}{2}(\Box\phi)^2 = \tfrac{1}{2}\left((\nabla\phi)^2 - \frac{1}{c_0^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right) \quad , \tag{18}$$

for changing fields. Thus,

$$\boxed{\varepsilon_e = \tfrac{1}{2}\left((\nabla\phi)^2 - \frac{1}{c_0^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right)}$$

is regarded as the ***complete*** *definition of electric energy density.* It deserves serious attention, because it not only resolves the many paradoxes, but is also Lorentz covariant like Eq.(17). Its success in providing the correct energy of the constant velocity electron warrants its adoption. This can be seen as follows: the implication is that, in addition to spreading out laterally, at each point in the moving field the rest electric energy density distortion found from Eq.(10) has increased to,

$$\varepsilon_e = 2\gamma^2 \frac{\phi_0^2 r_i^2}{r'^4} e^{-4 r_i / r'} \quad , \tag{19}$$

and when integrated over all space gives a *total* electric energy γE_0, a well established fact. Thus, a reasonable finite electron description has been demonstrated, and the correct form of the moving electric energy density has been derived.

ENERGYLESS MAGNETIC FIELDS

The correction to the Poynting Theorem can now be developed. It involves some surprising insights into the nature of magnetic fields. Even to this day, the incorrect form of $\varepsilon_e = \frac{1}{2}\mathbf{E}^2$ and the idea that *all* magnetic fields have energy density $\varepsilon_m = \frac{1}{2}\mathbf{B}^2$ are used to imply that the magnetic energy in the moving electron is in some way responsible for its kinetic energy; but this is not the case. *The electron's kinetic energy is due entirely to the increase of the electric distortion in the laterally expanded* ϕ *field*, as borne out by the total energy γE_0 integrated above. *The constant velocity electron carries no magnetic energy due to its* **A** *field*. This is similar to the **A** field due to the electron's spin, which is an energy-less magnetic field, i.e. no energy can be added to or removed from it.

The electron's spin and magnetic moment are established when the electron is formed (e.g. in pair production) and are *intrinsic* properties that never change until the electron is annihilated. Because the spin field cannot take on or give off energy, it is essentially an energy-less field. It is true that an electron placed in an external magnetic field can be torqued, and the combined fields will store the interaction energy; but that energy also can be recovered. Neither the electron's spin nor magnetic moment changes during the torqueing process, so the stored interaction energy cannot be regarded as part of the spin field energy. The constant velocity electron, then, has two energy-less magnetic fields; its spin field and the one generated by its motion. These can be ignored in many energy flow calculations, although they can still exert forces on other charged particles, and then the interaction energies (torques, etc.) must be considered.

Because the total moving electron energy γE_0 is 100% electric (i.e. produced only by ϕ), any energy density associated with the quantity $\frac{1}{2}(\partial \mathbf{A}/\partial t)/c_0$ that appears in Eq.(14) must be considered as part of the *magnetic* energy density ε_m. To emphasize this, the Lorentz force equation is written,

$$\mathbf{F}_L = q\left[-\nabla\phi + \left(\frac{1}{c_0}\mathbf{u}\times\nabla\times\mathbf{A} - \frac{1}{c_0}\frac{\partial \mathbf{A}}{\partial t}\right)\right] \quad , \tag{20}$$

where the first RHS term is the *electric* force component and the second and third terms are the *magnetic* force component. This complete separation of electric and magnetic effects is to be expected from Maxwell's Eqs.(6), which show the separation clearly. In light of Eq.(20), the most reasonable way to define magnetic energy density is to assume that each term involves a separate form of energy storage that is not necessarily influenced by the presence of the other forms, so that ε_m can be defined as,

$$\varepsilon_m = \varepsilon_v + \varepsilon_t \quad , \tag{21}$$

where ε_v and ε_t are called the *vortex* and *transformer* components of ε_m, represented by,

$$\varepsilon_v = \frac{1}{2}(\nabla\times\mathbf{A})^2 \quad \text{and} \quad \varepsilon_t = \frac{1}{2c_0^2}\left(\frac{\partial \mathbf{A}}{\partial t}\right)^2 \quad . \tag{22}$$

Written out in full,

$$\varepsilon_m = \frac{1}{2}\left((\nabla \times \mathbf{A})^2 + \left(\frac{1}{c_0} \frac{\partial \mathbf{A}}{\partial t} \right)^2 \right)_r \quad , \tag{23}$$

where the subscript r indicates that there are serious restrictions in applying Eq.(23). These are the result of there being *two different types of* **A** *fields, one that stores energy and one that is energy-less.* Eq.(23) *does not apply to energy-less* **A** *fields, even though* **A** *is not zero.*

PROPAGATING TRANSVERSE WAVES

Although layered particles involve only *electric* energy, propagating antenna radiation involves only *magnetic* energy. Radiation comes in two forms, antenna and photon radiation. The latter is not, as yet, understood; but antenna radiation is well described. Except in rare cases, a system of charges and currents, varying in time and confined to a region of dimensions d < < λ, radiate energy which, at distance r > > λ, is essentially plane wave. Figure 3 shows this transverse radiation. The

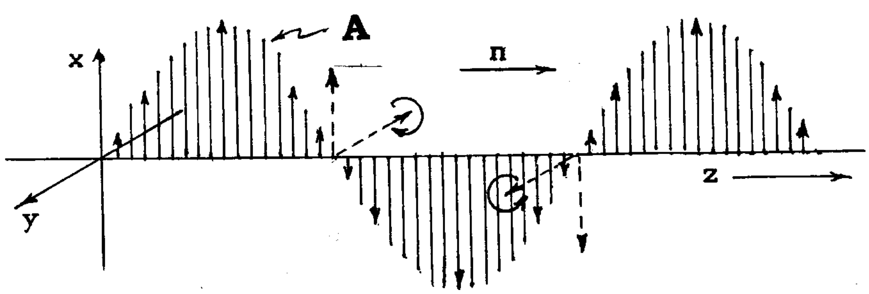

Figure 3. The field pattern of plane wave radiation.

wave propagates in the z direction with velocity $\mathbf{u} = c_0\mathbf{n}$, **n** being a unit vector. The vector **A** is constant over any x,y plane, and varies sinusoidally along the axis of propagation. Where **A** is maximum, there is no energy density; but ε_m increases towards the null regions, where the vortex and transformer energy densities are maximum. At each plane along the wave, the energy is half vortex and half transformer. It also is possible to generate waves that corkscrew circularly polarized.

This picture of wave propagation differs from the conventional, because the position is taken here that *antenna radiation is solely a magnetic phenomenon*, requiring only one magnetic vector field $\mathbf{A}$ to describe it. The scalar potential ϕ is zero, and the above description says that the amplitudes of the vortex and transformer components of the wave are equal, i.e.,

$$(\nabla\times\mathbf{A})_a = \left(\frac{1}{c_0}\frac{\partial\mathbf{A}}{\partial t}\right)_a \quad , \tag{24}$$

so that, from Eq.(23),

$$\varepsilon_m = (\nabla\times\mathbf{A})^2 \quad . \tag{25}$$

The two components are also perpendicular to each other and to $\mathbf{n}$, so that,

$$(\nabla\times\mathbf{A})\times\frac{1}{c_0}\frac{\partial\mathbf{A}}{\partial t} = (\nabla\times\mathbf{A})^2\,\mathbf{n} \quad . \tag{26}$$

In Figure 3 the energy in both components is maximum at the nulls of the $\mathbf{A}$ wave, and zero at the peaks.

THE OLD POYNTING THEOREM

The conventional Poynting theorem is written,

$$\frac{\partial\varepsilon}{\partial t} = -\nabla\bullet\mathbf{S}_o - \rho\mathbf{u}\bullet\mathbf{E} \quad , \tag{27}$$

where,

$$\mathbf{S}_o = c_0\mathbf{E}\times\mathbf{B} \quad , \quad \text{and} \quad \varepsilon = \tfrac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) \quad .$$

Eq.(27) purports to describe the change in energy density at each point in the field as a result of the energy flow away from the point and the work done by the field on the free charge at that point. $\mathbf{S}_o$ is the old

Poynting vector. *Eq.(27) represents a rigorously correct, macroscopic identity derived directly from Maxwell's equations*. However, only in cases of transverse wave radiation propagation do $\mathbf{S}_o$ and ε actually represent energy flow and density.[3] In such cases, $\phi = 0$, and the Poynting vector and energy density are,

$$\mathbf{S} = -\frac{\partial \mathbf{A}}{\partial t} \times (\nabla \times \mathbf{A}) \quad , \quad \varepsilon_m = \frac{1}{2}\left((\nabla \times \mathbf{A})^2 + \left(\frac{1}{c_0} \frac{\partial \mathbf{A}}{\partial t} \right)^2 \right)_r \quad . \qquad (28)$$

A MODIFIED POYNTING THEOREM

In Eq.(28), ε_m is not the conventional magnetic energy density but the one newly defined. Then the question arises immediately as to whether there might be another equation, rigorously derivable from Maxwell's Eqs.(6), that would replace the old Poynting theorem with a new magnetic one that works in all cases. Such an equation will be presented here for energy carrying magnetic fields described by Eq.(23).

The rate of change of ε_m is found from Eq.(23) to be,

$$\frac{\partial \varepsilon_m}{\partial t} = (\nabla \times \mathbf{A}) \bullet \frac{\partial (\nabla \times \mathbf{A})}{\partial t} + \frac{\partial \mathbf{A}}{\partial t} \bullet \left(\frac{1}{c_0^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) \quad .$$

Substituting from Maxwell's vector Eq.(6) this becomes,

$$\frac{\partial \varepsilon_m}{\partial t} = (\nabla \times \mathbf{A}) \bullet \frac{\partial (\nabla \times \mathbf{A})}{\partial t} + \frac{\partial \mathbf{A}}{\partial t} \bullet \left(\nabla^2 \mathbf{A} + \frac{\rho \mathbf{u}}{c_0} \right) \quad .$$

Replacing $\nabla^2 \mathbf{A}$ with an identity,

$$\frac{\partial \varepsilon_m}{\partial t} = (\nabla \times \mathbf{A}) \bullet \frac{\partial (\nabla \times \mathbf{A})}{\partial t} + \frac{\partial \mathbf{A}}{\partial t} \bullet \left(-\nabla \times \nabla \times \mathbf{A} + \nabla \nabla \mathbf{A} \right) + \frac{\rho \mathbf{u}}{c_0} \bullet \frac{\partial \mathbf{A}}{\partial t} \quad .$$

Again referring to Maxwell's Eqs.(6), the divergence equation converts the third RHS term above to give,

$$\frac{\partial \varepsilon_m}{\partial t} = (\nabla \times \mathbf{A}) \bullet \frac{\partial(\nabla \times \mathbf{A})}{\partial t} - \frac{\partial \mathbf{A}}{\partial t} \bullet \left(\nabla \times \nabla \times \mathbf{A}\right) - \frac{1}{c_0}\frac{\partial \mathbf{A}}{\partial t} \bullet \frac{\partial \nabla \phi}{\partial t} + \frac{\rho \mathbf{u}}{c_0} \bullet \frac{\partial \mathbf{A}}{\partial t} \quad .$$

From here it is easy to show that,

$$\boxed{\frac{\partial \varepsilon_m}{\partial t} = -\nabla \bullet \mathbf{S}_m - \frac{1}{c_0}\frac{\partial \mathbf{A}}{\partial t} \bullet \frac{\partial \nabla \phi}{\partial t} + \frac{\rho \mathbf{u}}{c_0} \bullet \frac{\partial \mathbf{A}}{\partial t}} \quad , \qquad (29)$$

where,

$$\mathbf{S}_m = -\frac{\partial \mathbf{A}}{\partial t} \times (\nabla \times \mathbf{A}) \quad . \qquad (30)$$

Eq.(29) is a new Poynting theorem that is rigorously derivable from Maxwell's Eqs.(6) and appears to correctly describe *magnetic* energy flow. Not too surprisingly, the new flow vector $\mathbf{S}_m$ is exactly the same as the one that has always worked correctly for radiation propagation, i.e. Eq.(28). However, the magnetic energy density ε_m is quite different from the one in the old theorem.

In Eq.(29), the last RHS term describes the work done by the magnetic field **A** on the free charge at each point. A commonly found statement in present textbooks is that the magnetic field does no work on free charges (only turning their paths), but the inclusion of the transformer **A** field as a *magnetic* field changes that. Only the vortex field does no work on free charges.

The second RHS term in Eq.(29) represents the transfer of energy from the magnetic field to an electric field (or radiation).

The new Poynting theorem appears to give the correct picture of energy flow in all cases so far examined. It does, however, require a significant shift of viewpoint. The fact that *no electric energy appears anywhere, except in the field structure of particles*, is hard to reconcile with already learned conventional thinking. The fact that *all antenna radiation is purely magnetic*, and has no *electric* component is also difficult to become accustomed to. Even more bothersome is the idea of *two kinds of magnetic fields*, energy carrying and energy-less; although the electron's spin field has been known for over 80 years.

The best way to overcome these prejudices is to look at a few examples.

The Charging Capacitor: A simple example of *electric* energy transport is shown in Figure 4. It consists of a capacitor connected to a battery by a twisted pair of wires. Initially, the capacitor is uncharged so the voltage across it is zero. At the instant the wires are connected to the battery, all their conduction electrons start to drift away from the positive plate and toward the negative plate of the capacitor at an average speed as low as a fraction of a centimeter per second. For each electron that pops onto the (-) plate, another electron leaves a nucleus on the (+) plate and enters the wire. All of the voltage appears as an IR drop on the wires, so the first electron that enters the negative plate requires little energy to arrive. To put some numbers in an almost real example, let the capacitor be two parallel disks 16 cm in diameter and a distance $d = 0.05$ cm apart. In Heaviside-Lorentz units, $C = 4021$ hlf and the battery voltage is $V = 4.705 \times 10^{-2}$ hlV.

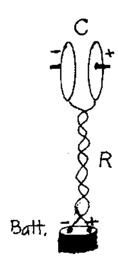

Figure 4. Charging capacitor

The first electron that enters the (-) plate teams, across the space between the plates, with the positive nucleus left on the (+) plate, and the pair has an interaction energy $E_{int} = e^2 / 4\pi d = 4.614 \times 10^{-18}$ ergs which is stored in the space between them. This establishes a voltage across C of $V_1 = e / C = 4.234 \times 10^{-13}$ hlV.

The second electron to enter the (-) plate must team with the two positive nuclei left on the (+) plate and both interaction energies are stored in the intervening space. Each subsequently arriving electron must store one more packet of interaction energy than the previous electron.

As the process continues, the voltage across the capacitor increases until it reaches V, the battery voltage. Then the current stops, and the full charge on C is $Q = VC = 189.2$ hlC. Ignoring signs, the gradient between the plates is $\nabla\phi = V / d = 0.941$ hlV/cm, corresponding to a stored *electric* energy density $\varepsilon_e = (\nabla\phi)^2 / 2 = .443$ ergs/cm^3, or a total stored energy $E_c = 4.450$ ergs.

The total number of electrons moved to the (-) plate is $N_e = Q / e = 1.11 \times 10^{11}$, so the total rest energy of all those electrons is $E_{0t} = N_e E_0 = 9.096 \times 10^4$ ergs. The excess, recoverable stored interaction energy is only a small fraction of that rest energy $E_c / N_e E_0 = 0.00499$ %.

Clearly, the stored energy came through the wires with the electrons. It did not come in through the field outside the capacitor as the old Poynting theorem suggests. The use of a twisted pair of wires eliminates any *magnetic* effect due to the changing current, so the experiment is totally *electric* in nature.

<u>The Long Straight Conductor:</u> A simple example discussed in the literature is that of a long, straight conducting wire through which current is steadily driven by a battery (see Fig. 5). If the leads connecting the wire to the voltage source have negligible resistance, then the potential, neglecting end effects, is uniform across the wire's interior and the electric field inside and just outside the wire is uniform, with the equipotentials perpendicular to the axis of the wire.

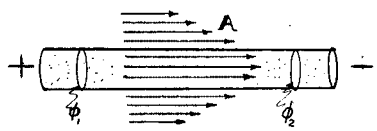

Figure 5. Long, straight conductor

As long as the current is *steady state*, the magnetic **A** field inside and outside the wire and the electric field ϕ are constant with time, so no energy from the battery goes into the magnetic or electric fields after they are established. The energy that goes into heat (collisions of conduction electrons with atoms) is carried by the electrons as their individual electric kinetic energies. It is stored in a sphere about each electron that is less than 10^{-11} cm in radius and *none of it gets near the region outside the wire*. Clearly, in this case, the old Poynting theorem makes no sense; since it indicates that the steady state energy from the battery doesn't go directly into the wire with the electrons, but leaves the source and travels through the space around the wire, entering it radially through its long cylindrical surface.

In its most rigorous form, this example is pathological, because the conductor length must be infinite and there is no return loop. In that case the **A** field would be completely energy-less, because constant velocity electrons have energy-less **A** fields. Only when a conductor makes a complete loop does the interaction of the electrons in the loop create an energy-carrying vortex **A** field centered on the loop. Nevertheless, by making a finite loop large enough, the vortex energy of the loop can be made so small that the **A** field is essentially energy-less. In spite of the energy-less nature of the **A** field, outside the conductor, that field can deflect a free electron; but no energy can be exchanged.

Solenoid With Decaying Current: If a long solenoid coil, wound with high resistance wire, has a D.C. voltage applied to the ends of the coil with connecting leads of very low resistance, the steady state physics is similar to that of the long straight wire, except that vortex energy is stored inside the coil. A sudden short circuiting of the coil produces a result similar to the straight wire case. The potential gradient between the ends of the coil is reduced essentially to zero and remains that way. In a somewhat oversimplified visualization, the only physical actions on the electrons now are their stopping upon colliding with atoms, and the decreasing $\mathbf{A}$ in the vortex field. When an electron passes its kinetic energy to an atom, and stops, it accelerates and gains kinetic energy again. The process repeats for all conduction electrons until the vortex is gone. If the vortex energy is to be considered actually localized, then there will be a flow of magnetic energy outward to the electrons in the coil, but the velocity $\mathbf{u}$ is not determined easily.

Using Maxwell's Eqs.(6), the transient decay solution for the $\mathbf{A}$ field is found in terms of Bessel functions to be,

$$\mathbf{A} = \hat{\alpha}\,\mathbf{A}_0 \frac{I_1(\not{Z})}{I_1(\not{Z}_0)}\,\varepsilon^{-t/\tau} \quad , \qquad (31)$$

where $\not{Z} = R / c_0\tau$, $\not{Z}_0 = R_0 / c_0\tau$, R_0 is the coil radius and τ is the time constant of the decay ($\tau = \mathbb{L}/\mathbb{R}$, where $\mathbb{L}$ is the coil inductance and $\mathbb{R}$ the coil resistance, both per unit coil length). The $\mathbf{A}$ field is zero at the coil axis and $\mathbf{A}_0$ at R_0.

From Eq.(31),

$$\varepsilon_v = \frac{\mathbf{A}_0^2}{2c_0^2\tau^2}\frac{\left[I_0(\not{Z})\right]^2}{\left[I_1(\not{Z}_0)\right]^2}\varepsilon^{-2t/\tau} \quad , \quad \text{and} \quad , \quad \varepsilon_t = \frac{\mathbf{A}_0^2}{2c_0^2\tau^2}\frac{\left[I_1(\not{Z})\right]^2}{\left[I_1(\not{Z}_0)\right]^2}\varepsilon^{-2t/\tau} \quad . \qquad (32)$$

The ratio of transformer energy density to vortex energy density is,

$$\frac{\varepsilon_t}{\varepsilon_v} = \frac{\left[I_1(\not{Z})\right]^2}{\left[I_0(\not{Z})\right]^2} \quad . \tag{33}$$

For a wide range of coil sizes, $\not{Z} = R / c_0 \tau$ is of the order of 10^{-4} or smaller, so the ratio in Eq.(33) is less than 10^{-7}. Thus, neglecting small terms,

$$\varepsilon_m \cong \varepsilon_v \cong \frac{2A_0^2}{R_0^2} \varepsilon^{-2t/\tau} \quad . \tag{34}$$

The picture that unfolds is this. The steady state solution has no transformer energy, only vortex; but when the coil is shorted and the electric field collapses, a small residual transformer energy is produced as the decay starts. According to Eq.(33), ε_t remains extremely small relative to ε_v, which is quite different from the radiation case where ε_t and ε_v were equal. This is a good indication that the new Poynting theorem is extremely close to being a magnetic energy conservation law in this case; because, if any radiation takes place during the decay, the vortex part of the radiation will be of the same order as ε_t, which makes the radiation essentially negligible.

It is possible to say, in this case, that $\mathbf{S} = \varepsilon_m \mathbf{u}$, where ε_m includes both the coil energy and the radiation energy. In that case, the determination of $\mathbf{u}$ is not obvious, because the radiation component velocity is c_0, but the coil vortex component would have a lower speed. Where the radiation is negligible, $\mathbf{u}$ can be calculated with the result,

$$\mathbf{u} \cong \hat{\mathbf{R}} \frac{R}{\tau} \quad . \tag{34}$$

In some ways this is satisfying, but it is not intuitive. The velocity is zero at $R = 0$, and increases linearly out to R_0, which is not surprising, but it is not a function of time. To understand this, it is necessary to examine the physical nature of $\mathbf{A}$ to a deeper level of abstraction.

There are numerous other examples, such as a fixed charged particle outside a fixed permanent magnet around which the old Poynting theorem predicts a circulating energy flow. They are easily explained by the new Poynting theoerm, often just by inspection.

CONCLUSION

Simple solutions to the three standing problems with classical particle structure have been presented. There is no reason for avoiding their use.

ACKNOWLEDGMENT

The author wishes to acknowledge the considerable help on these problems rendered by R. L. Kirkwood, L. O. Heflinger, B. H. Mueller, D. J. Margaziotis, T. Hudspeth , R. S. Margulies and G. Ialongo.

UNITS

To obtain the quantity in HLU, multiply the MKS quantity by the factor given. To go from HLU to MKS, divide.

	HLU	MKS
Electric Potential	$\overline{\overline{\phi}}$	9.40967×10^{-4} Volts
Magnetic Vector Potential	$\mathbf{A}$	2.82095×10^{5}
Energy	$\mathcal{E}$	10^{7} Joules
Energy Density	ε	10 Joules
Charge	q	1.06274×10^{10} Coulombs
Charge Density	ρ	1.06274×10^{4} Coulombs / m
Current	i	1.06274×10^{10} Amperes
Resistance	$\mathbb{R}$	8.85419×10^{-14} Ohms
Capacitance	$\mathbb{C}$	1.12941×10^{13} Farads
Inductance	$\mathbb{L}$	8.85419×10^{-14} Henrys
Electric Intensity	$\mathbf{E}$	9.40967×10^{-6} Volts / m
Magnetic Induction	$\mathbf{B}$	2.82095×10^{3} Teslas
Electric Displacement	$\mathbf{D}$	1.06274×10^{6}
Magnetic intensity	$\mathbf{H}$	$3.54491 \times 10^{-3} \frac{\text{Amp Turns}}{\text{m}}$