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Faraday and Maxwell didn't know about electrons. Maxwell had a vague idea that electric current in conductors was a fluid. In his treatise of 1873, he gave two sets of equations for the electromagnetic field. One set assumed the *fundamental* field quantities were "forces" that would act on minute test charges if placed in the field, and the other set was thought to be a simplifying *mathematical* manipulation in terms of a scalar and a vector potential. The remaining errors in classical E&M, still ignored or repeated in most textbooks, can best be understood and corrected by looking at Maxwell's equations for "matter free" space.

The "force" equations are written (Heaviside-Lorentz units):

$$\nabla\bullet\mathbf{E} = \rho \quad , \quad \nabla\times\mathbf{E} + \frac{1}{c_0}\frac{\partial\mathbf{B}}{\partial t} = 0 \quad , \quad \nabla\bullet\mathbf{B} = 0 \quad , \quad \nabla\times\mathbf{B} - \frac{\partial\mathbf{E}}{\partial t} = \frac{\mathbf{J}}{c_0} \quad . \tag{1}$$

The potential equations are written (Heaviside-Lorentz units):

$$\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} = -\rho \quad , \quad \nabla^2\mathbf{A} - \frac{1}{c_0^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{\mathbf{J}}{c_0} \quad , \quad \nabla\bullet\mathbf{A} = -\frac{1}{c_0}\frac{\partial\phi}{\partial t} \quad , \tag{2}$$

Any ϕ and $\mathbf{A}$ found from Eqs.(2) give the proper values of $\mathbf{E}$ and $\mathbf{B}$ through the connecting equations:

$$\mathbf{E} = -\left(\nabla\phi + \frac{1}{c_0}\frac{\partial\mathbf{A}}{\partial t}\right) \quad , \quad \mathbf{B} = \nabla\times\mathbf{A} \quad , \tag{3}$$

To find $\mathbf{E}$ and $\mathbf{B}$ from Eqs.(1), four equations must be solved; from Eqs.(2), essentially half that number.

Most modern textbooks label $\mathbf{E}$ and $\mathbf{B}$ as the *fundamental* fields, $\mathbf{E}$ is called the "electric" field and $\mathbf{B}$ is called the "magnetic" field. These definitions lead to the major source of error in classical E&M. A significant amount of experimental and theoretical work makes it obvious that the *fundamental* fields are ϕ and $\mathbf{A}$; so that, from Eqs.(3), $\mathbf{E}$ is a *mixture* of the electric field ϕ and the magnetic field $\mathbf{A}$, and $\mathbf{B}$ is an *incomplete* part of the magnetic field $\mathbf{A}$.

Most of the difficulties appear in the microscopic regime of particle structure. At present, the most commonly used scalar potential in Eqs.(2) for a spherically symmetrical, charged particle at rest is the "point" charge which, with its infinities, is not a "physical" solution. As explained in the Short Book, a much simpler and easier to work with *finite* form is,

$$\phi = \phi_0(1 - e^{-2r_i/r}) \quad . \tag{4}$$

This potential has only two significant features, the center value ϕ_0 (positive or negative) and the radius r_i of the inflection point (Figure 1).

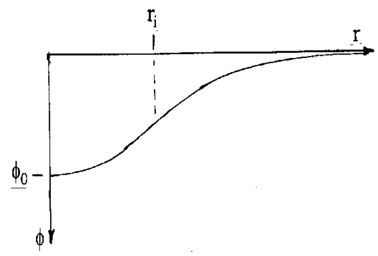

Figure 1

The corresponding charge density distribution,

$$\rho = -\nabla^2(\phi) \quad , \tag{5}$$

is a smooth shell of charge distortion that peaks at half r_i. Integrated over all space, the total charge is $\boxed{q = 8\pi\phi_0 r_i}$.

Again, the at rest *electric* energy density distribution,

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 \quad , \tag{6}$$

is a smooth shell of energy distortion that peaks at the inflection radius r_i. Integrated over all space, the resulting finite energy is $\boxed{E_0 = 2\pi\phi_0^2 r_i}$.

So far, the finite particle solution is satisfactory; but it still must be shown to give the *constant velocity* total energy γE_0, where u is the velocity and γ is defined as,

$$\gamma = 1 \Big/ \sqrt{1 - \frac{u^2}{c_0^2}} \quad . \tag{7}$$

The calculation begins by going back to Eqs.(2) and looking for a finite potential of the full *changing* field scalar equation,

$$\rho = -\left\{ \nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} \right\} \quad . \tag{8}$$

Paralleling Eq.(4), the potential of the *constant velocity* field, moving in the x direction at velocity u, is,

$$\phi = \gamma\phi_0(1 - \varepsilon^{-2 r_i / r'}) \quad , \tag{9}$$

where, $r' = \sqrt{\gamma^2 x^2 + R^2}$ in cylindrical coordinates. Eq.(9) differs from the spherical case of Eq.(4) mainly in that the equipotentials are oblate spheroids; *not because of any <u>longitudinal</u> contraction*, but because *the potential ϕ expands <u>laterally</u>.* The longitudinal contraction of **E** is always emphasized, but *the lateral expansion of ϕ is more significant in relation to energy and charge.*

The potential in Eq.(9) can be checked by using a Lorentz transformation on the rest potential of Eq.(4). If the charge density ρ found from Eqs.(8) and (9) is *integrated over all space*, the total moving electron charge is found to be $q = 8\pi\phi_0 r_i$, *the same as for the charge at rest*, a well established fact.

To find the total field energy of the moving particle, the correct energy density must be integrated over all space. *This is the second point at which the classical E&M theory of particle structure breaks down.*

In most modern textbooks the expression for *electric energy density* is commonly written,

$$\varepsilon_e = \tfrac{1}{2}\mathbf{E}^2 = \tfrac{1}{2}\left(-\nabla\phi - \frac{1}{c_0}\frac{\partial \mathbf{A}}{\partial t}\right)^2 \quad . \quad \text{WRONG} \qquad (10)$$

In a way, this form works for radiation propagation (where $\nabla\phi$ is zero), but in other applications (in association with the Poynting theorem) it has led to a long, confusing literature of strange paradoxes and suggested alternatives. If Eq.(10) is integrated over all space, it fails to give a total energy γE_0. Combinations of Eq.(10) and the **B** field also fail (consult the Short Book).

One hint as to the failure of Eq.(10) relates to the *success* of Eq.(8) in defining moving microscopic charge density, for there is an alternative picture of elementary particle structure that gives *insight into the basic nature of microscopic charge and electric energy densities*. If it is assumed that *the potential* ϕ *is the only physical entity in the electric field*, then the construct in Figure 1 is *the total essence of an elementary particle's bulk nature*, i.e. a specific distortion. In the "point charge" model, charge is "something" *at the point producing* the field. Electric energy density is even more evanescent. However, the nature of ϕ in the preceding allows a different approach. The microscopic Eqs.(5) and (6) can be considered to *define two secondary implicit distortions*, $\tfrac{1}{2}(\nabla\phi)^2$ and $-\nabla^2\phi$, *automatically present if* ϕ *is present. They do* ***not*** *cause the field, they are the result of it.*

An *erroneous* assumption, adopted almost unanimously around 1900 and still held today, is *that, in the microscopic case, the elements of distributed charge* ρ inside *a single particle, for example, individually obey Coulomb's law just as whole charged particles do in the macroscopic case.* Lorentz had doubts, but they did not prevail. However, there is no direct experiment to support this assumption, and electrons, for example, do not fly apart. Thus, *microscopically, there is no reason to expect the distributed "elements" of the* ϕ *field to produce distant actions on each other such as the Coulomb force, which, macroscopically, results from two* ***whole particle*** *fields interacting.* That Eq.(8) gives the correct moving microscopic charge density bears this out. Now that the physical nature of ρ and ε_e as *secondary* implicit distortions *dependent* upon ϕ, rather than as sources of ϕ, has been indicated, the path to the correct form of *moving* electric energy density ε_e is clear. *It should be formulated in exactly the same way that moving charge density* ρ *was.*

In going from the rest Eq.(5) to the moving Eq.(8), because of the finite rate of propagation, *the charge density in time variable fields is assumed to change* as,

$$\rho = -\nabla^2\phi \qquad \rightarrow \qquad \rho = -\Box^2\phi = -\left(\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2}\right) \quad . \tag{11}$$

That this is true is a well verified fact. Considering the *similar* natures of ρ and ε_e as auxiliary distortions implicit in the shape of ϕ, it would be surprising if electric energy density did not have the simple definition, *parallel to Eq.(11)*,

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 \qquad \rightarrow \qquad \varepsilon_e = \tfrac{1}{2}(\Box\phi)^2 = \tfrac{1}{2}\left((\nabla\phi)^2 - \frac{1}{c_0^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right) \quad , \tag{12}$$

for changing fields. Thus,

$$\boxed{\varepsilon_e = \tfrac{1}{2}\left((\nabla\phi)^2 - \frac{1}{c_0^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right)}$$

is offered here as the correct, ***complete*** *definition of electric energy density*. It deserves serious attention, because it not only resolves the many paradoxes, but also leads to Lorentz invariance like Eq.(11). Its success in providing the correct energy of the constant velocity particle warrants its adoption. This can be seen as follows: the implication is that, in addition to spreading out laterally, at each point in the moving field the *rest* electric energy density distortion found from Eq.(6) has increased, and when integrated over all space gives a *total* electric energy γE_0, a well established fact. Thus, a reasonable finite charged particle description has been demonstrated, and the correct form of the moving electric energy density has been derived.

The third major breakdown in classical E&M relates to the description of energy flow in the fields using the Poynting theorem. The old Poynting theorem leads to weird and erroneous visualizations of field energy flow, because **E** and **B** are not the true electric and magnetic fields. A new Poynting theorem must be derived from the ϕ and **A** equations. This leads to a simple visualization of energy flow with no apparent paradoxes. Consult the Short Book and PHYSICS 2001Rev.