Give up "point" particles for a **finite** solution of Maxwell's equations

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Abstract

"Point" leptons lead to difficulties. The following presents a simple, finite solution of Maxwell's equations that applies to the e, μ and τ leptons. With minor modification it also applies to all other leptons except photons and neutrinos.

A Finite Solution of Maxwell's Equations

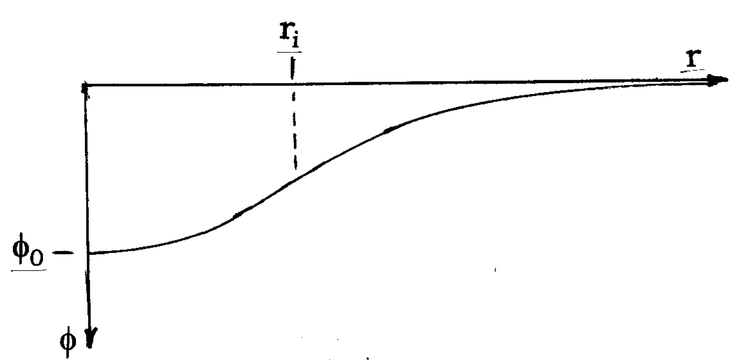
A solution of the scalar equation (Heaviside-Lorentz units),

$$\nabla^2\phi = -\rho \quad , \tag{1}$$

must be found that eliminates the infinities of the "point" charge. A simple, spherical *trial* solution is,

$$\phi = \phi_0(1 - \varepsilon^{-2r_i/r}) \quad ; \tag{2}$$

The Figure below indicates that this potential has only two significant features, the center value ϕ_0 (positive or negative) and the radius r_i of the inflection point.



The corresponding charge density distribution required to complete the solution is found by substituting the trial solution Eq.(2) in Eq.(1) to yield,

$$\rho = 4\frac{\phi_0 r_i^2}{r^4} e^{-2r_i/r} \quad , \tag{3}$$

a smooth shell of charge distortion that peaks at *half* r_i. This distribution is a reasonable one. Integrated *over all space*, the total charge is $q = 8\pi\phi_0 r_i$.

Similarly, the electric energy density distribution is found from,

$$\varepsilon_e = \tfrac{1}{2}(\nabla\phi)^2 = 2\frac{\phi_0^2 r_i^2}{r^4} e^{-4r_i/r} \quad , \tag{4}$$

a smooth shell of energy distortion *that peaks at the inflection radius* r_i. If ε_e is integrated *over all space*, the resulting finite energy is $E_0 = 2\pi\phi_0^2 r_i$.

Just to get some idea of the magnitudes involved, if the potential in Eq.(2) is assumed to represent an electron, then using $E_0 = 8.18711\times 10^{-7}$ ergs (0.511 MeV) and $q = -e = -1.7027\times 10^{-9}$ hlcoul (-1.6022×10^{-19} C), the center potential and inflection point radius are $\phi_0 = -1.9233\times 10^{3}$ hlvolts (approx. $-\ 2\times 10^{6}$ V) and $r_i = 3.522\times 10^{-14}$ cm.

It is important to notice that the expansion of the *gradient* of Eq.(2),

$$\frac{d\phi}{dr} = -2\frac{\phi_0 r_i}{r^2}\left(1 - 2\frac{r_i}{r} + 2\frac{r_i^2}{r^2} - \ldots\right) \qquad r > r_i$$

reduces, for the electron, to $d\phi/dr \cong e/4\pi r^2$ (for $r > 200 r_i$), the Coulomb field of the "point" charge. This explains why the well known collision experiments[1] that appear to support the "point" charge electron model are also in complete agreement with the present, finite solution. At low collision energies, the principal interaction is out in the Coulomb region.

1. D.P. Barker, et. al., Phys. Rev. Lett., **43**, 1915 (1979); Phys. Rev. Lett., **45**, 1904 (1980).

As the collision energy is increased, the Lorentz contraction of the gradient causes the inner, non-Coulomb volume to shrink, and *the interaction never catches up with that inner region.*

The Energy Compaction Relationship

Combining the rest energy E_0 and charge q found from Eqs.(4) and (3), for whole charge leptons e, μ and τ,

$$E_0 r_i = \frac{e^2}{32\pi} = 2.8838 \times 10^{-20} \text{erg} - \text{cm} \quad , \tag{5}$$

a relationship called the energy compaction equation. It indicates that the more energetic leptons are smaller. The true importance of Eq.(5) appears when analyzing the fractional charge leptons and composite particles such as the proton.

Lepton Size and Stability

Here, again, the leptons of interest will be limited to the series of whole charged particles, i.e. e, μ, τ, ..., that can exist alone and be observed for some finite time. Using the energy compaction relationship derived earlier, and the known values of E_0 for each of the leptons, the values for ϕ_0 and r_i are listed in the TABLE along with each particle's observed mean life.

LEPTONS, THE "PREFERRED" VACUUM STATES

	E_0 (ergs)	r_i (cm)	ϕ_0 (hlvolts)	mean life (s)
e	8.1871×10^{-7}	$r_1 = 3.5224 \times 10^{-14}$	1.9233×10^{3}	Stable
μ	1.6929×10^{-4}	$r_2 = 1.7035 \times 10^{-16}$	3.9768×10^{5}	2.1970×10^{-6}
τ	2.8472×10^{-3}	$r_3 = 1.0129 \times 10^{-17}$	6.6886×10^{6}	2.9100×10^{-13}

The interesting features of the TABLE are that, first, although each of these leptons has the same charge $\pm e$, the more energetic particles have higher potentials; and, their energy being packed into a smaller volume correlates with their being less stable. Second, it appears that the lepton sequence is a set of *preferred* states that can exist as "stable" particles because of some fundamental property of the vacuum.

UNITS

To obtain the quantity in HLU, multiply the MKS quantity by the factor given. To go from HLU to MKS, divide.

	HLU	MKS
Electric Potential	$\bar{\bar{\phi}}$	9.40967×10^{-4} Volts
Magnetic Vector Potential	$\mathbf{A}$	2.82095×10^{5}
Energy	$\mathcal{E}$	10^{7} Joules
Energy Density	ε	10 Joules
Charge	q	1.06274×10^{10} Coulombs
Charge Density	ρ	1.06274×10^{4} Coulombs / m^3
Current	i	1.06274×10^{10} Amperes
Resistance	$\mathbb{R}$	8.85419×10^{-14} Ohms
Capacitance		1.12941×10^{13} Farads
Inductance	$\mathbb{L}$	8.85419×10^{-14} Henrys
Electric Intensity	$\mathbf{E}$	9.40967×10^{-6} Volts/m
Magnetic Induction	$\mathbf{B}$	2.82095×10^{3} Teslas
Electric Displacement	$\mathbf{D}$	1.06274×10^{6}
Magnetic intensity	$\mathbf{H}$	$3.54491 \times 10^{-3} \frac{\text{Amp Turns}}{\text{m}}$

UNITS

Starred quantities are Gaussian. Listed quantities are substituted directly. Quantities along rows are equal.

	HLU	MKS	EMU	ESU
Electric Potential	$\overset{=}{\phi}$	$\frac{10^8}{c_0\sqrt{4\pi}}\phi_{mks}$	$\frac{1}{c_0\sqrt{4\pi}}\phi_m$	$\frac{1}{\sqrt{4\pi}}\phi_s$ *
Magnetic Vector Potential	$\mathbf{A}$	$\frac{10^6}{\sqrt{4\pi}}\mathbf{A}_{mks}$		$\frac{1}{\sqrt{4\pi}}\mathbf{A}_s$ *
Charge	q	$\frac{c_0\sqrt{4\pi}}{10}q_{mks}$	$c_0\sqrt{4\pi}\ q_m$	$\sqrt{4\pi}\ q_s$ *
Current	i	$\frac{c_0\sqrt{4\pi}}{10}i_{mks}$	$c_0\sqrt{4\pi}\ i_m$	$\sqrt{4\pi}\ i_s$ *
Electric Intensity	$\mathbf{E}$	$\frac{10^6}{c_0\sqrt{4\pi}}\mathbf{E}_{mks}$	$\frac{1}{c_0\sqrt{4\pi}}\mathbf{E}_m$	$\frac{1}{\sqrt{4\pi}}\mathbf{E}_s$ *
Magnetic Intensity	$\mathbf{H}$	$\sqrt{4\pi}10^{-3}\mathbf{H}_{mks}$ (A.T./m)	$\frac{1}{\sqrt{4\pi}}\mathbf{H}_m$ *	$\frac{1}{c_0\sqrt{4\pi}}\mathbf{H}_s$
Electric Displacement	$\mathbf{D}$	$\sqrt{4\pi}10^{-5}c_0\mathbf{D}_{mks}$	$\frac{c_0}{\sqrt{4\pi}}\mathbf{D}_m$	$\frac{1}{\sqrt{4\pi}}\mathbf{D}_s$ *
Magnetic Induction	$\mathbf{B}$	$\frac{10^4}{\sqrt{4\pi}}\mathbf{B}_{mks}$ (Teslas)	$\frac{1}{\sqrt{4\pi}}\mathbf{B}_m$ *	$\frac{c_0}{\sqrt{4\pi}}\mathbf{B}_s$
Magnetic Moment	μ	$10^3\sqrt{4\pi}\ \mu_{mks}$	$\sqrt{4\pi}\ \mu_m$	$\frac{\sqrt{4\pi}}{c_0}\ \mu_s$ *
Conductivity	σ	$\frac{4\pi c_0^2}{10^9}\sigma_{mks}$	$4\pi c_0^2\ \sigma_m$	$4\pi\sigma_s$ *
Resistance	$\mathbb{R}$	$\frac{10^9}{4\pi c_0^2}\mathbb{R}_{mks}$	$\frac{1}{4\pi c_0^2}\mathbb{R}_m$	$\frac{1}{4\pi}\mathbb{R}_s$ *
Capacitance	$\mathbb{C}$	$\frac{4\pi c_0^2}{10^9}\mathbb{C}_{mks}$	$4\pi c_0^2\ \mathbb{C}_m$	$4\pi\mathbb{C}_s$ *
Inductance	$\mathbb{L}$	$\frac{10^9}{4\pi c_0^2}\mathbb{L}_{mks}$	$\frac{1}{4\pi c_0^2}\mathbb{L}_m$	$\frac{1}{4\pi}\mathbb{L}_s$ *

TRUNCATIION INTEGRALS

1. $\int_0^x \varepsilon^{-1/y} dy = T(x)$ The truncation integral.

2. $\int_0^x \varepsilon^{-a/y} dy = a\,T\left(\frac{x}{a}\right)$

3. $\int_0^x y\,\varepsilon^{-a/y} dy = \frac{x^2}{2}\varepsilon^{-a/x} - \frac{a^2}{2}T\left(\frac{x}{a}\right)$

4. $\int_0^x y^2 \varepsilon^{-a/y} dy = \left(\frac{x^3}{3} - \frac{ax^2}{2\bullet 3}\right)\varepsilon^{-a/x} + \frac{a^3}{2\bullet 3}T\left(\frac{x}{a}\right)$

5. $\int_0^x y^3 \varepsilon^{-a/y} dy = \left(\frac{x^4}{4} - \frac{ax^3}{3\bullet 4} + \frac{a^2x^2}{2\bullet 3\bullet 4}\right)\varepsilon^{-a/x} - \frac{a^4}{2\bullet 3\bullet 4}T\left(\frac{x}{a}\right)$

6. $\int_0^x y^n \varepsilon^{-a/y} dy = \left(\frac{x^{n+1}}{n+1} - \frac{ax^n}{n(n+1)} + \frac{a^2x^{n-1}}{(n-1)n(n+1)} - \ldots\ldots\right.$

$\left.\ldots\ldots\ldots\ldots \pm \frac{a^{n-1}x^2}{(n+1)!}\right)\varepsilon^{-a/x} \mp \frac{a^{n+1}}{(n+1)!}T\left(\frac{x}{a}\right)$

7. $\int_0^x \frac{\varepsilon^{-a/y}}{y^2} dy = \frac{\varepsilon^{-a/x}}{a}$

8. $\int_0^x \frac{\varepsilon^{-a/y}}{y^3} dy = \frac{\varepsilon^{-a/x}}{a^2}\left(1 + \frac{a}{x}\right)$

9. $\int_0^x \frac{\varepsilon^{-a/y}}{y^4} dy = \frac{2\varepsilon^{-a/x}}{a^3}\left(1 + \frac{a}{x} + \frac{a^2}{2x^2}\right)$

10. $\int_0^x \frac{\varepsilon^{-a/y}}{y^n} dy = \frac{(n-2)!\varepsilon^{-a/x}}{a^{n-1}}\left(1 + \frac{a}{x} + \frac{a^2}{2!x^2} + \frac{a^3}{3!x^3} + \ldots\ldots + \frac{a^{n-2}}{(n-2)!x^{n-2}}\right)$

11. $Q(x) = \varepsilon^{1/x}T(x)$, $T(x) = \varepsilon^{-1/x}Q(x)$

12. $\frac{dQ(x)}{dx} = 1 - \frac{1}{x^2}Q(x)$

x	T(x)	x	T(x)
0.05	4.7024×10^{-12}	7.00	4.5615
0.10	3.8302×10^{-7}	7.50	4.9971
0.15	2.2539×10^{-5}	8.00	5.4365
0.20	1.9929×10^{-4}	8.50	5.8794
0.25	7.9955×10^{-4}	9.00	6.3254
0.30	2.1277×10^{-3}	9.50	6.7742
0.35	4.4403×10^{-3}	10.0	7.2254
0.40	7.9190×10^{-3}	11.0	8.1345
0.45	1.2674×10^{-2}	12.0	9.0512
0.50	$1..8767 \times 10^{-2}$	13.0	9.9743
0.55	2.6207×10^{-2}	14.0	10.9029
0.60	3.4990×10^{-2}	15.0	11.8362
0.65	0.04508	16.0	12.7737
0.70	0.05645	17.0	13.7149
0.75	0.06903	18.0	14.6593
0.80	0.08279	19.0	15.6067
0.85	0.09766	20.0	16.5567
0.90	0.11361	25.0	21.3385
0.95	0.13057	30.0	26.1594
1.00	0.14850	35.0	31.0076
1.20	0.2288	40.0	35.8759
1.40	0.3214	45.0	40.7595
1.60	0.4241	50.0	45.6552
1.80	0.5351	55.0	50.5608
2.00	0.6532	60.0	55.4746
2.50	0.9734	65.0	60.3952
3.00	1.3207	70.0	65.3216
3.50	1.6881	75.0	70.2531
4.00	2.0709	80.0	75.1890
4.50	2.4660	85.0	80.1287
5.00	2.8710	90.0	85.0719
5.50	3.2842	95.0	90.0181
6.00	3.7044	100.0	94.9671
6.50	4.1304	$x \to \infty$,	$T(x) \to x - \log_e x$