19th Southeastern Symposium on System Theory

"THE APPLICATION OF TRANSMISSION LINE RESONATORS TO HIGH VOLTAGE RF POWER PROCESSING: HISTORY, ANALYSIS AND EXPERIMENT"

(an invited paper)

by

James F. Corum, Ph.D.
Department of Electrical and Computer Engineering
West Virginia University
Morgantown, W.V. 26506-6101

and

Kenneth L. Corum
CPG Communications, Inc.
158 Salem St.
Wilmington, Mass. 01887

ABSTRACT

In this paper we present a transmission line analysis of high power, high voltage RF transmission line resonators. This simple distributed circuit approach clearly demonstrates the inappropriateness of the lumped network analysis employed in the past. Voltage step up arises simply as a result of the transmission line VSWR, and the resulting model permits the tuned circuit representation on a standard Smith Chart. Specific historic examples are cited.

Introduction

With the advent of the Strategic Defense Initiative, there has emerged a renewed interest in the generation and processing of RF power. The specific advantage of employing RF techniques is that not only can high peak power be obtained, but energy conversion may be effected simultaneously at high average power as well. The latter is extremely important and is a serious deficiency inherent to low duty cycle pulse power techniques, where energy thruput may actually be quite slow. A recent review of contemporary pulse power technology is given in reference 1.

Interestingly, from an historical perspective, the problem of generating high energy at high average power rates was first addressed by the early inventors of spark gap wireless transmitters around the turn of the century. Thirty years later, in the pre World War II era, essentially the same techniques were employed (with the inclusion of vacuum tube technology) by the early pioneers of particle accelerator machines. These early accelerators employed devices called "resonance transformers" for the production of high voltages at radio frequencies. Quite surprisingly, their engineering analysis, over the years, has been performed entirely within the realm of "lumped circuit theory". [In fact, one famous physicist published the remarkable conclusion,". . . resonance transformers cannot be treated usefully by mathematics."(Ref.2)] This discrepancy between theory and analysis has been well known to many experimentalists.

In this paper, we report on our distributed circuit analysis of high voltage RF transmission line resonance transformers. Using a slow wave velocity factor, appropriate for helical transmission lines, and employing Schelkunoff's concept of "average characteristic impedance" for vertical monopole antennas, we find that high voltage resonance transformers may readily be analysed on a Smith chart. The technique is not unlike that developed by Siegel and Labus which has found extensive use in the broadcast antenna field for many years. (Ref.3)

Finally, we compare theoretical calculations with experimental results, some of which we have obtained and some of which were performed by early experimenters. The results are rather dramatic, and demonstrate the remarkable engineering utility of the Smith chart representation of high voltage RF resonance transformers.

Transmission Line Considerations

Consider the foreshortened coaxial resonator shown in figure 1(a). It's analysis is presented in many places. See, for example, reference 4. The center conductor of the resonator is terminated in the gap capacitance which we model as C. Adler, Chu and Fano point out that,

"With C in place, the line itself must store a little more magnetic than electric energy if the whole system is to have equal amounts of both types of energy. Thus the line length must be a little

March 15-17, 1987
Sirrine Hall
Clemson University
Clemson, South Carolina

less than an odd multiple of a quarter wavelength." (Ref.5)

This consideration will establish the criterion for resonance. A solution of the transmission line equations leads to an expression for the spatial voltage distribution as

$$(1)\quad V(z) = V_+ \, e^{-\gamma z} + V_- \, e^{+\gamma z}$$

where γ is the usual complex propagation constant $\alpha + j\beta$. The complex load reflection coefficient is defined at the load end (z=0) as

$$(2)\quad \Gamma_2 \triangleq \frac{V_-}{V_+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma_2| \; \underline{\big|\phi_2}$$

where the subscript 2 has been used to indicate that the reflected to forward wave ratio has been evaluated at the load end. At the input or low impedance end of the line, we have

$$(3)\quad \Gamma_1 \triangleq \frac{V_- \, e^{-\gamma \ell}}{V_+ \, e^{\gamma \ell}} = |\Gamma_2| \, e^{-2\alpha \ell} \quad \underline{\big|\phi_2 - 2\theta}$$

The VSWR follows from the definition

$$(4)\quad S \triangleq \frac{V_{max}}{V_{min}} = \frac{|V_+| + |V_-|}{|V_+| - |V_-|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

The voltage maxima and minima are spatially separated by one quarter wavelength. On a loss less transmission line the magnitudes of gamma 1 and gamma 2 are equal, and the VSWR is equal to the maximum value which the normalized (WRT Zo) impedance takes on along the line. On a lossy line, the trajectories are not circles but spiral inward on the Smith chart from gamma 2, as one approaches the input end of the transmission line from the load end. Consequently the VSWRs at the load end 2 and input end 1 differ as

$$(5)\quad 1/S_1 = \tanh\left[\alpha \ell + \tanh^{-1}\left(\frac{1}{S_2}\right)\right]$$

When the standing wave ratio is high (i.e. S > 6:1) this expression may be approximated as

$$(6)\quad 1/S_1 \approx 1/S_2 + \alpha \ell$$

with less than 1% error. Since the foreshortened coaxial resonator has a capacitive load, S_{ℓ} tends to infinity and thus

$$(7)\quad S_1 \approx \frac{1}{\alpha \ell} \; .$$

If the line were a full quarter wavelength long, the voltage maximum, Vmax, would rise to SVmin and would occur at the top (load end), and Vmin would be at the base (input end). From equation (1) we may express the voltage at the base, or input end of the resonator, as

$$(8)\quad V_{base} = V_+ \left[e^{\alpha \ell} e^{j\theta} + |\Gamma_2| \, e^{j(\phi - \theta)} \right]$$

Further, the voltage at the top, or load end, may be expressed as:

$$(9)\quad V_{Top} = \frac{V_{base}[\,1 + \Gamma_2\,]}{[\,e^{\gamma \ell} + \Gamma_2 \, e^{-\gamma \ell}\,]}$$

Note that the quantities are complex. It should be obvious that high voltage resonators should be designed such that line VSWRs are extremely high and the denominator of equation (9) is minimized. The only parameters left to calculate the voltage step up are the attenuation constant α , and the phase constant β .

Transmission lines are commonly presented through the vehicle of distributed circuit theory. The series inductance per unit length is taken as L henries per meter; the series loss resistance as R ohms per meter; the shunt distributed capacitance as C farads per meter; and the shunt leakage conductance as G siemans per meter.

The propagation attenuation constant then follows as

$$(10)\quad \alpha = \left(\frac{R}{2Z_o} + \frac{GZ_o}{2} \right) \quad \text{Nepers/meter.}$$

For some transmission line resonators (in the absence of discharges) the shunt conductance G is negligible. Consequently, the attenuation constant is simply the first term, where R is the distributed copper loss. The parameter needed for equation (9) is $\alpha \ell$.

$$(11)\quad \alpha \ell = \frac{R\ell}{2Z_o} = \frac{R_{Loss}}{2Z_o}$$

where the wire loss resistance is explicit. The phase constant depends upon the velocity factor, Vf, for the line:

$$(12\text{ a,b})\quad \beta = 2\pi/\lambda_g \text{ where } \lambda_g = V_f \lambda_o \; .$$

Lastly, the input impedance, or impedance seen looking into the base of a driven resonator, is given by

$$(13)\quad Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} = Z_o \left[\frac{1 + \Gamma_2 \, e^{-2\gamma \ell}}{1 - \Gamma_2 \, e^{-2\gamma \ell}} \right]$$

where Zo is the characteristic impedance of the line.

The foreshortened, or end loaded, resonator is most clearly described by a Smith chart. The load impedance is simply the capacitive reactance. Since the magnitude of gamma 2 is unity, S_{ℓ} is infinite. If the line length and load capacitance have been chosen for a series resonance at the input end, then the impedance trajectory spirals in from 1 (where ϕ is some negative angle) to $|\Gamma| \angle{-\pi}$ and the input (base) impedance is purely resistive.

Knowing the size of the top loading capacitance, the velocity factor Vf, and Zo, the Smith chart immediately tells us how long to make the line to attain resonance. Alternatively, given the length of the line, the Smith chart tells us how large a capacitance is required to bring the system into resonance at the desired frequency. Furthermore, we can see the voltage rise from Vmin at the driving point to a relative voltage maximum at the load. Note that, because of the load capacitance, the actual line length is not one quarter wavelength. Consequently, one never gets to the absolute Vmax. One can get closer by reducing the value of the top capacitance (increasing the load reactance), however, from a physical point of view, as the load capacitance decreases so does its breakdown potential. Clearly a tradeoff should be made for the size of the machine and desired top electrode voltage.

Open Helical Resonators

To this point the analysis has been fairly conventional. However, the application which we propose is to a class of devices which have traditionally been discussed within the framework of lumped coupled coils. Perhaps the most well recognized of the common resonance transformers is the classic "Tesla coil". In figure 1 we show how, conceptionally, one may pass from the foreshortened coaxial resonator to a slow wave resonator, to an open helical resonator. [It should parenthetically be remarked that, first, what some authors have called "Tesla coils" in the past (as for example Sloan in reference 1) are simply lumped tuned coupled coils. Tesla was using these prior to 1892. See Ref.6. The best engineering analysis which we have seen of this configuration is given in reference 7. Secondly, the "Tesla coil" so commonly seen today is in fact a link-coupled distributed tuned resonance transformer. It is easily documented that Tesla was using the latter prior to 1898. As Sloan observes, the lumped analysis of this configuration totally fails. Thirdly, Tesla's most famous high voltage RF experiments, the photographs of which the public at large is so familiar with, employed what he called his "extra coil". From his recently published Colorado Springs diary of 1899, it is clear that this structure is actually the slow wave helical transmission line resonator of figure 1(d). The structure was excited at its base by a relatively narrow band RF signal generator.]

In order to apply the above distributed circuit theory, we need the appropriate velocity factor, wire losses and transmission line characteristic impedance for a helix operated against ground as a resonator. Fortunately these quantities are available. Many years ago, Kandoian and Sichak analyzed the helix and obtained engineering formulas of great practical value. See references 9 and 10. In summary

$$V_f = \frac{v}{c} = \left[1 + 20 \left(\frac{D}{s}\right)^{2.5} \left(\frac{D}{\lambda_o}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}} \qquad (14)$$

where λ_o = free space wavelength
D = helix diameter
s = turn-to-turn spacing
(all in the same units)

$$\alpha \ell = \frac{7.8125 \left(\frac{H}{D}\right)^{1/5}}{d_w Z_o \sqrt{f_{MHz}}} \qquad (15)$$

where d_w = wire diameter in inches.

$$Z_o = \frac{60}{V_f}\left[\ln\left(\frac{4H}{D}\right) - 1\right] \qquad (16)$$

The latter formula is Schelkunoff's transmission line "effective characteristic impedance". For those unfamiliar with the term, the concept is developed in reference 3. These formulas, along with the transmission line theory reviewed above, permit one to predict the behavior of high voltage RF resonators with surprising accuracy.

Examples of Extra Coil Design

As an example of some historical interest, consider the extra coil physical parameters recently disclosed in Tesla's diary. See reference 8 under the November 1,1899 entrance.

D = coil diameter = 8.25 ft
s = turn to turn spacing = 1 inch
N = number of turns = 106
d_W = wire diameter = .162 in. (#6)

L = 0.020 henries
C_T = 92 pf

H = coil height = 106 in.
f = 94 KHz
V_{base} = 25Ø KV

These lead to the calculated values
Vf = .00428
Zo = 6380
$Z_L = -j\ 18.8\ K\Omega$
$\Gamma_2 = 1 \angle{-38^\circ}$
$\theta = \beta \ell$ = 71 degrees
$\Gamma_1 = 0.951 \angle{0^\circ}$

$$S = 40$$

On a Smith chart (see figure 2), we enter at the normalized load reactance

$$z_2 = \frac{-jX_c}{Z_o} = -j2.95$$

and advance

$$2\theta = 2\beta H = 142.1^\circ$$

toward the generator, spiraling inward by $\exp(-2\alpha\ell)$ to the point

$$Z_1 = .025\ Zo = 159.5 + j\ 0 \text{ ohms.}$$

Finally, the predicted load voltage is 9.5 Megavolts. This, and the predicted helix current of 1100 amps RMS (at the crest of an exponentially decaying envelope), are in pleasant agreement with Tesla's public disclosures.

The remarkable distinction between lumped circuit and distributed circuit (or VSWR) voltage rise seems to have first been recognized by Tesla on July 10,1899,"I expect that this method of raising the emf with an open coil will be recognized later as a material and beautiful advance in the art." (Ref.8) Unfortunately, theory lagged far behind art.

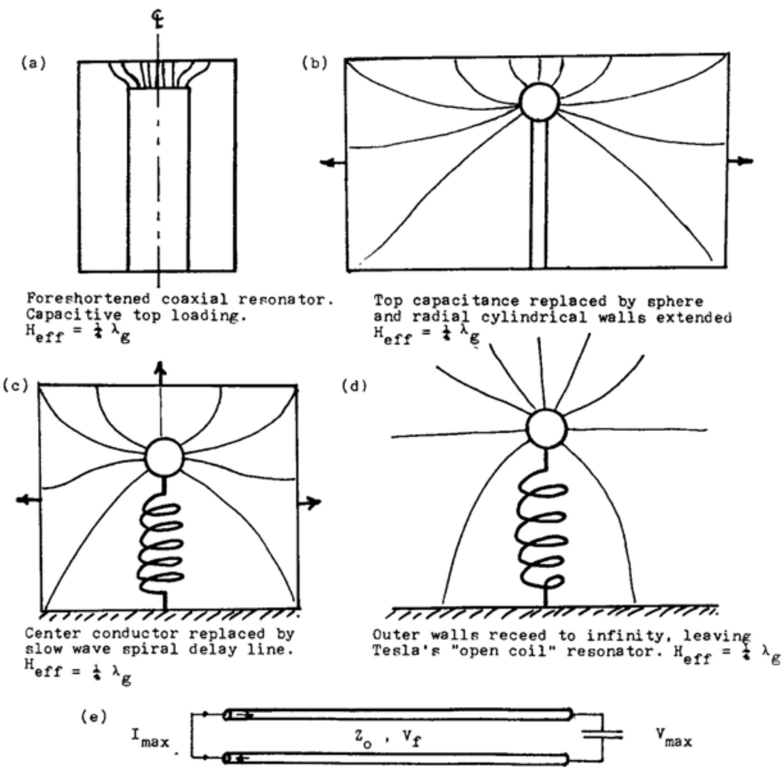


Figure 1. Conceptual development of Tesla's Extra Coil. (a,b) The capacitive loaded foreshortened coaxial resonator.(c) The slow wave top loaded resonator. (d) The final open coil resonator used at Colorado Springs. (e) Equivalent slow wave transmission line resonator.

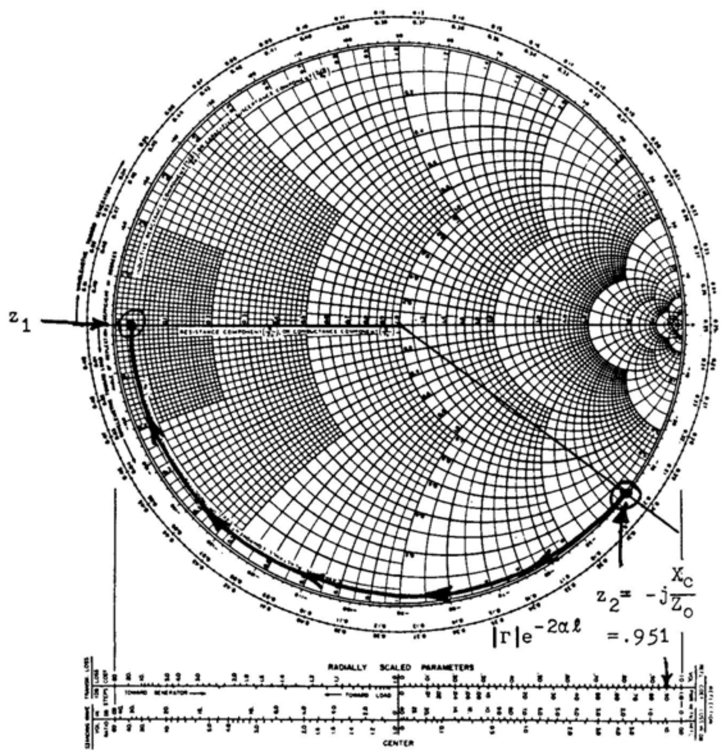


Figure 2. Smith chart representation of Tesla's November 1,1899 Extra Coil.

References

1. "Dielectric Properties of Water and Water/Ethylene Glycol Mixture for Use in Pulsed Power System Design", by M. Zahn, et.al., Proc. IEEE, Vol. 74, No. 9, Sept. 1986, PP. 1182-1221.

2."A Radio Frequency High-Voltage Generator". by D.H. Sloan, Physical Review, Vol. 47, Jan. 1, 1935, PP. 62-70.

3. Electromagnetic Waves and Radiating Systems, by E.C. Jordan and K.G. Balmain, Prentice-Hall, 2nd Edition, 1968, PP. 385-396.

4. Fields and Waves in Communication Electronics, by S. Ramo, J.R. Whinnery and T. Van Duzer, John Wiley & Sons, 2nd Edition, 1984, PP. 504-505.

5. Electromagnetic Energy, Transmission and Radiation, by R. Adler, L. Chu, and R. Fano, John Wiley & Sons, 1960, Pg. 266.

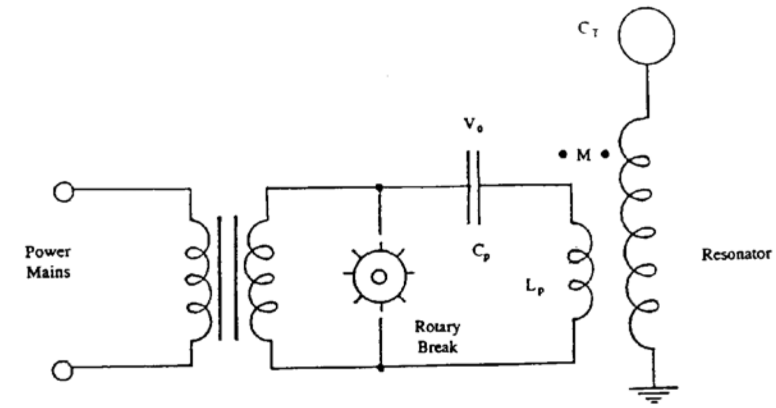
6. "A Technical Analysis of the Extra Coil as a Slow Wave Helical Resonator", by J.F. Corum and K.L. Corum, Proceedings of the 2nd International Tesla Symposium, Colorado Springs, 1986.

7. Static and Dynamic Electricity, by W.R. Smythe, McGraw-Hill, 2nd Edition, 1950, PP. 340-346. (Note the error in the constant in Equation (14) on page 345 where C2,1 should read C2,2.)

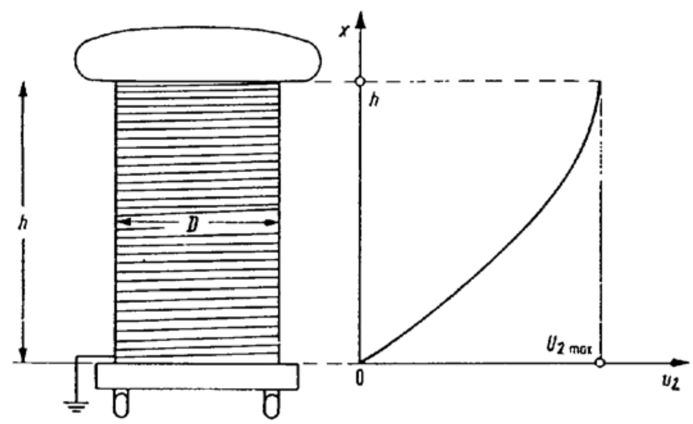
8. Colorado Springs Notes, 1899-1900 by Nikola Tesla, Nolit, Beograd, Yugoslavia, 1978, PP. 79, 103, 180, 345.

9. "Wide Frequency Range Tuned Helical Antennas and Circuits", by A.G. Kandoian and W. Sichak, IRE Convention Record, 1953, Part 2-Antennas and Communications, PP. 42-47. Reprinted in Electrical Communications, Vol. 30, December, 1953, PP. 294-299.

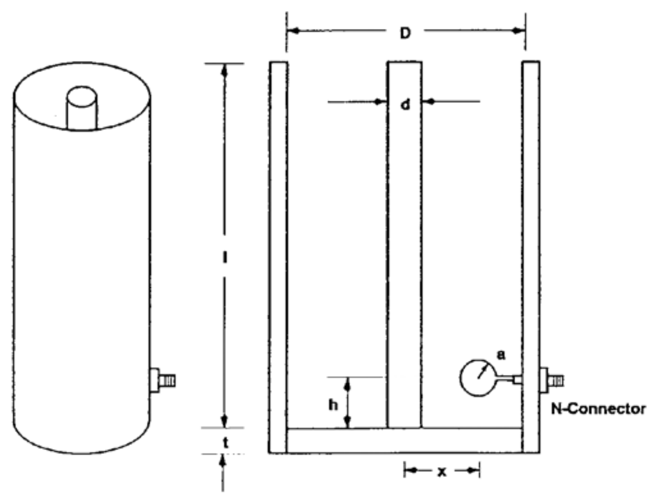
10. Reference Data For Radio Engineers, 4th Edition, 1956, PP. 682-686.



C T
V o
M
Power
Mains
C p
L p
Rotary
Break
Resonator



x
h
D
U 2 max
0
u 2



D
d
l
a
h
t
x
N-Connector