On Superluminal Propagation of Electromagnetic Wave in Nondispersive Media

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Abstract

Much experimental evidence of superluminal phenomena has been available by electromagnetic wave propagation experiments in both dispersive media and open space. Based on the first principles, we study the superluminal propagation in nondispersive medium and obtain the following results: 1) the group velocity v_g and the instantaneous energy velocity u_e of the near-zone fields of source can be not only $+\infty > v_g, u_e > c$, but also $0 > v_g, u_e > -\infty$; 2) these superluminal phenomena have theoretical foundation in quantum field theory, and do not violate causality principle; 3) furthermore, we propose that the superluminal propagation of evanescent waves, which is also related to the one of the near-zone fields, correspond to the spacelike virtual photon process; 4) the experimental results themselves by Mugnai et al (PRE 48, 1453; PRE 54, 5692) are *qualitatively* tenable, though their theoretical interpretations are problematic.

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1. Introduction

A series of recent experiments revealed that electromagnetic wave was able to travel at a group velocity $v_g > c$ or $v_g < 0$ (c is the velocity of light in vacuum) in dispersive media [1-4], in electronic circuits [5], and in evanescent wave cases [6-13]. In fact, over the last decade, the discussion of the tunneling time [14-15] problem has experienced a new stimulus by the results of analogous experiments with evanescent electromagnetic wave packets [16-17], related to which the superluminal effects of evanescent waves have been revealed in photonic tunneling experiments in both the optical domain and the microwave range [6-13]. All the experimental results have shown that the earlier quantum mechanics calculations for phase time do describe the barrier traversal time [13,18].

Furthermore, A. Ranfagni and D. Mugnai *et al* have reported another kind of experiment in which microwave packets appear to travel at a superluminal group velocity *in open space* [19-21], which is seemingly independent of tunneling process and is perplexing. This kind of experiment is more interesting because, according to the traditional theory, the group velocity in nondispersive medium has a physical meaning. They interpreted the effect by assuming the existence of a special kind of evanescent wave (leaky waves) [19] or complex waves [20]. Another interpretation for these experimental results has also been given in [22-23], i.e. the so-called *superluminal electromagnetic X-waves* interpretation. However, all these interpretations are problematic [24-26] (the criticisms in Ref. [24-26] hold true also for Ref. [19-20] because of the improper assumption $\Delta\omega/\Delta k = \omega/k$). In fact, even if in the free space, we have $d\omega/dk_z \neq \omega/k_z$ (or $\Delta\omega/\Delta k_z \neq \omega/k_z$), provided that we choose $z-axis$ as the observe direction that not parallel to the total wavevector $\vec{k}$ (at this moment ω/k_z is called as *the apparent phase velocity*), where

k_z is the wavenumber along the z direction, ω the frequency (see Fig. 1). In addition, in Ref. [19-20], the authors extended the limits of a geometric angle variable from real number to complex number, which is devoid of physical foundation. As for Ref. [21], the error analysis of the data and the potential accuracy of the apparatus are subject to question [25-26]; whereas as for Ref. [19-20], according to our discussion, the experimental results themselves are *qualitatively* tenable (but not *quantitatively*), though the authors' theoretical interpretations are problematic.

In this article, by a general investigation for the superluminal propagation of electromagnetic wave in nondispersive open space, we obtain some new interesting results. Furthermore, we give a unified discussion for the superluminal phenomena related to the evanescent waves and the near-zone fields, from the viewpoint of both classical and quantum field theories.

2. The general expressions of phase velocity and group velocity

In the most general case, we can write a field quantity $\Phi(\vec{r},t)$ as ($\vec{r}=(x,y,z)$, $\vec{k}=(k_x,k_y,k_z)$):

$$\Phi(\vec{r},t)=\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\Psi(\vec{r},t,\vec{k},\omega)\exp[j\Theta(\vec{r},t,\vec{k},\omega)]d^3k \qquad (1)$$

where $j=\sqrt{-1}$, $d^3k=dk_xdk_ydk_z$, $\omega=\omega(k)$ and $\Psi(\vec{r},t,\vec{k},\omega)$ is the amplitude of the k-th component of $\Phi(\vec{r},t)$. Let $r=|\vec{r}|$, $k=|\vec{k}|$ and so on. As for the k-th component of $\Phi(\vec{r},t)$, the phase velocity $\vec{v}_p$ is the velocity of a point of constant phase on the k-th monochromatic wave. Then let $\Theta(\vec{r},t,\vec{k},\omega)=C$ (C is a constant) we have $d\Theta/dt=(\partial\Theta/\partial r)(dr/dt)+\partial\Theta/\partial t=0$, i.e. ($\vec{a}_r=\vec{r}/r$)

$$\vec{v}_p = -\vec{a}_r (\frac{\partial \Theta}{\partial t}) / (\frac{\partial \Theta}{\partial r}) \qquad (2)$$

On the other hand, the group velocity $\vec{v}_g$ can be taken as the move velocity of the center of wave packet, where the center location r_c of the wave packet is given by $\partial \Theta / \partial k = 0$ (consider that the superposition of different frequency components of $\Phi(\vec{r}, t)$ gives an extremum at r_c). Then the group velocity is $\vec{v}_g = \vec{a}_r dr_c / dt$. Let $\Delta \equiv \partial \Theta / \partial k = 0$, and consider that $d\Delta / dt = 0 = (\partial \Delta / \partial r)(dr / dt) + \partial \Delta / \partial t$, then

$$\vec{v}_g = -\vec{a}_r (\frac{\partial^2 \Theta}{\partial t \partial k}) / (\frac{\partial^2 \Theta}{\partial r \partial k}) \qquad (3)$$

Equ. 2 and Equ. 3 are the general expressions for calculating the phase velocity and group velocity, respectively. For example, let $\Theta = \omega t - \vec{k} \cdot \vec{r}$, one have $v_p = \omega / k$ and $v_g = \partial \omega / \partial k$. However, if our observe direction is parallel to $z-axis$ for example, in the same way one can obtain the phase velocity and the group velocity along the observe direction $\vec{a}_z = \vec{z} / z$ are,

$$\vec{v}_p = -\vec{a}_z (\frac{\partial \Theta}{\partial t}) / (\frac{\partial \Theta}{\partial z}), \quad \vec{v}_g = -\vec{a}_z (\frac{\partial^2 \Theta}{\partial t \partial k_z}) / (\frac{\partial^2 \Theta}{\partial z \partial k_z}), \qquad (4)$$

respectively, provided that Θ is the function with the variables k, $\vec{r}$ in the form of $\vec{k} \cdot \vec{r}$. In view of the fact that the direction of $\vec{r}$ is one of the observe directions as long as $\vec{r} = (0, 0, z)$ (as for the electromagnetic waves inside a waveguide, the observe direction is the propagation direction along the waveguide), Equ. 4 is the most general expression for calculating the phase velocity and group velocity.

3. The essential characteristic of the fields of horn antenna

We might as well take horn antenna as the source of electromagnetic wave (see Fig. 2). Traditionally, we only study the radiant fields of horn antenna, in spite of the total fields including near-zone fields (in this paper, the near-zone fields of an antenna include the static-zone fields and the induction-zone fields). In fact, when electromagnetic waves are incident on a metal surface, the free electrons nearby will be forced to vibrate and excite electromagnetic waves in return, which form the reflective waves. Therefore, when the electromagnetic waves propagate along a metal waveguide and arrive at a metal horn antenna, every location on the metal surface where reflects electromagnetic waves is equivalent to a source or antenna generating electromagnetic waves. Particularly, the inner surface of a metal horn antenna is equivalent to a set of elemental-area antennas. As is known to all, an elemental-area antenna is equivalent to a pair of mutually vertical electric dipole and magnetic dipole antennas. On the other hand, according to the electricity-magnetism duality principle, the fields of a magnetic dipole antenna can be obtained from the ones of an electric dipole antenna. Consequently, we can grasp the essential characteristic of the fields of a horn antenna only by considering the fields of the electric dipole antenna.

4. The superluminal velocity behavior in open space

According to the simplified consideration mentioned above, in the *local* spherical coordinate system (r,θ,φ), the non-zero field components at every position of the inner surface of horn antenna can be written as $H_\varphi = \sum_k H_\varphi(k)$, $E_r = \sum_k E_r(k)$, $E_\theta = \sum_k E_\theta(k)$, where $\sum_k$ stands for the *continuous* sum with respect to k, $H_\varphi(k)$, $E_r(k)$ and $E_\theta(k)$, as the k-th components, correspond to the fields of an electric dipole antenna with the frequency $\omega(k)$, and they are[27]:

$$H_\varphi = H_0(1+\frac{1}{jkr})\exp[j(\omega t - kr)] \equiv |H_\varphi|\exp(j\Theta_1)$$

$$E_r = E_1(1+\frac{1}{jkr})\exp[j(\omega t - kr)] \equiv |E_r|\exp(j\Theta_2) \qquad (5)$$

$$E_\theta = E_2(1+\frac{1}{jkr}-\frac{1}{k^2r^2})\exp[j(\omega t - kr)] \equiv |E_\theta|\exp(j\Theta_3)$$

where H_0, E_1, E_2, as the functions of r and θ, are the purely real or purely imaginary numbers, and $k = |\vec{k}|$, $\vec{k}$ the total wavevector. The phase factors are (up to a minor constant that has no effect on calculating wave velocity later):

$$H_\varphi, E_r: \quad \Theta_1 = \Theta_2 = \omega t - kr + arctg(\frac{-1}{kr})$$

$$E_\theta: \quad \Theta_3 = \omega t - kr + arctg(\frac{kr}{1-k^2r^2}) \qquad (6)$$

where $arctg(x)$ is the inverse tangent function of x. In view of the directivity of horn antenna (the intensity of the fields is maximum in the direction along the center axis of horn antenna), we consider Equ. 5 only in the case of $\vec{k}$ parallel to the center axis of horn antenna, so that we can study the primary feature of the fields of horn antenna by the fields of electric dipole antenna. Meanwhile, we assume that the angle included between $\vec{k}$ and the observe direction (say, the z direction) is β ($0 \le |\beta| < \pi/2$) (here we adopt the denotation β rather than θ because of Equ. 5 being expressed in the local coordinate system (r, θ, φ)). Let k_z and $k_\perp$ stand for the wavenumbres along the directions that are parallel to and perpendicular to the observe direction, respectively, we have

$$k = \sqrt{k_z^2 + k_\perp^2}, \quad \frac{k}{k_z} = \frac{1}{\cos\beta}, \quad \frac{\partial k}{\partial k_z} = \cos\beta \qquad (7)$$

Noting that $\omega / k = \partial\omega / \partial k = c$ in vacuum medium as well as $k_z z = kr$ with respect to a given wavefront (reference Fig. 1 with the replacement $\theta \to \beta$), and using Equ. 2-7, we obtain the phase velocity v_p and group velocity v_g along the observe direction as follows:

$$H_\varphi, E_r: \ v_p = [1 + \frac{1}{k^2 z^2 (\cos\beta)^2}] \frac{c}{\cos\beta} \tag{8}$$

$$v_g = \frac{[k^2 z^2 (\cos\beta)^2 + 1]^2 c \cos\beta}{k^4 z^4 (\cos\beta)^4 + 3k^2 z^2 (\cos\beta)^2} \tag{9}$$

$$E_\theta: \ v_p = [\frac{k^4 z^4 (\cos\beta)^4 - k^2 z^2 (\cos\beta)^2 + 1}{k^4 z^4 (\cos\beta)^4 - 2k^2 z^2 (\cos\beta)^2}] \frac{c}{\cos\beta} \tag{10}$$

$$v_g = \frac{[k^4 z^4 (\cos\beta)^4 - k^2 z^2 (\cos\beta)^2 + 1]^2 c \cos\beta}{k^8 z^8 (\cos\beta)^8 - k^6 z^6 (\cos\beta)^6 + 7k^4 z^4 (\cos\beta)^4 - 6k^2 z^2 (\cos\beta)^2} \tag{11}$$

where we express these wave velocities in term of $k = |\vec{k}|$ (rather than k_z) and z so that we can show the their dependence on the observe angle β and distance z in the observe direction.

In the open space only the group velocity has real physical significance. We give the diagrammatic curves and curved surface (Fig. 3-Fig. 6) related to the ratio v_g / c instead of the group velocity v_g below ($0 \le \beta < 0.49\pi$):

5. On the anomalous propagation experiment in open space

In the previous sections, the essential physical features of the fields of horn antenna have been obtained by analyzing the fields of electric dipole antenna. Following Equ.8-11 or Fig. 3-6 we summarize the main characters of the fields of horn antenna as follows (note that $k\cos\beta = k_z$ is the wavenumber component in the observe direction z):

(A) For the far-zone fields of horn antenna ($kz\cos\beta \to \infty$), their group velocities are $c\cos\beta$ in the observe direction (while the phase velocities are $c/\cos\beta$);

(B) For the near-zone fields but $kz \ge 1$, the group velocities of all the non-zero field components are c (or c approximatively) as $\beta = 0$; while as $\beta \neq 0$, in the observe direction the group velocities may become superluminal.

(C) Moreover, as $kz\cos\beta \to 0$, the group velocities for some field components change from positive to negative infinity, while for the other components their group velocities change from c to positive infinity (see Fig. 3-6).

Obviously, judged by the statements (A) and (B), the experimental results obtained in Ref. [19-20] are *qualitatively* reasonable, which are described as follows (sometimes we take the air medium as the vacuum one, which has no effect on what we discuss): by using launcher and receiver horn antennas, A. Ranfagni and D. Mugnai *et al* have made measures of pulse delay in microwave propagation in open space and for short distances. When the horn antennas are facing each other (corresponds to $\beta = 0$) they observe a delay time corresponding to a speed equal to c. However, if the receiver horn is shifted or tilted with respect to the launcher horn (corresponds to $\beta \neq 0$), the delay time decreases, showing a superluminal behavior, which disappears for longer distances (corresponds to the statement (A)). As far as the experiment in Ref. [21] is concerned, though the error analysis of the data (or the statistical significance of the data) and the potential accuracy of the apparatus are subject to question [25,26], there is a case that may have nothing to do with this experiment: if we take the two opposite surfaces of the launcher horn and the circular mirror as the two end surfaces of a cylinder, the electromagnetic fields between the two surfaces would become *evanescent* modes provided that the interval between them is small enough (in this case the electromagnetic fields are described by the Bessel function of *imaginary* argument), and then the superluminal propagation (related to the photon tunneling) along the radial direction of the cylinder would appear.

However, these experimental results in Ref. [19-20] do not agree with statements (A) and (B) *quantitatively* (e.g., in our case, the group velocities become superluminal as $kz\cos\beta < 1$; while in Ref.[19-20] the group velocities may become superluminal even as $kz\cos\beta > 10$). After all, the error analysis of the data and the accuracy of the apparatus in Ref. [19-20] are subject to question; on the other hand, the actual fields of horn antenna correspond to the superposition of the fields produced by a numbers of the electric and the magnetic dipole antennas, quadrupole antennas, etc., which must be reconsidered when one try to give a quantitative discussion..

6. Further consideration for the superluminal phenomena in open space

Furthermore, one can easily obtain the instantaneous energy velocity vector $\vec{u}_e$ of the fields expressed by Equ. 5 (i.e. the k - th components) as follows:

1) In the far-zone, $\vec{u}_e = (u_r, u_\theta, u_\varphi) = (c, 0, 0)$;

2) In the near-zone, $\vec{u}_e = (u_r, u_\theta, u_\varphi)$, where $u_\varphi = 0$, and

$$
\begin{aligned}
u_r &= \frac{2ckr(\sin\theta)^2}{[1+3(\cos\theta)^2]} ctg(\omega t - kr) \\
u_\theta &= -\frac{2ckr\sin 2\theta}{[1+3(\cos\theta)^2]} ctg(\omega t - kr)
\end{aligned}
\qquad (12)
$$

In fact, when our observation is performed in the near-zone, what we are dealing with is instantaneous velocity instead of the average one, owing to the fact that as $kr \to 0$ (or $k_z z \to 0$), the corresponding observation time $\Delta t \to 0$. As it is given in the statement (C) and Equ. 12, the group velocity v_g and the instantaneous energy velocity u_e of the near-zone fields can be not only $+\infty > v_g, u_e > c$, but also $0 > v_g, u_e > -\infty$.

Further experiments are needed to confirm all these results. Surely, when $k_z z \to 0$, the corresponding experiment becomes difficult and even can hardly implemented.

7. The physical foundation for the superluminal phenomena in open space

In undersize waveguide propagation case, the superluminal behavior is related to the evanescent waves, while in open-space transmission case, just as the experimental results in Ref. [19-21] and our theoretical analysis have shown, the corresponding superluminal behavior is related to the near-zone fields. However, there exists a similarity between the evanescent waves and the near-zone fields:

a) Within an undersize waveguide, the evanescent waves attenuate exponentially along the direction of propagation, and the average power flow will not exist in the waveguide because the impedance is purely capacitive (for the TM mode) or inductive (for the TE mode), causing only a reactive power or energy storage in the waveguide. As for the energy storage in the undersize waveguide, the electric energy is more than magnetic energy for the TM mode and on the contrary for the TE mode.

b) Similarly, the near-zone fields of an antenna are also sharply attenuated as compared with the far-zone fields of the antenna, and the average power flow of the near-zone fields does not exist because the impedance is purely capacitive (for the electric dipole antenna) or inductive (for the magnetic dipole antenna), causing only a reactive power or energy storage. As for the energy storage in the near-zone of an antenna, the electric energy is more than magnetic energy for the electric dipole antenna and on the contrary for the magnetic dipole antenna.

In fact, the similarity between the evanescent waves and the near-zone fields is not accidental. As for a waveguide, When its cross-section size is gradually reduced and becomes an cutoff one, the wavelength of the electromagnetic waves propagating along the waveguide approaches the cutoff wavelength, the wavenumber component k_z along the direction of waveguide approaches zero, then $k_z z < 1$ for any given position z, that is, the electromagnetic waves in the direction of waveguide correspond to the near-zone fields. The superluminal tunneling of photon in the undersize waveguide owes to the propagation law of near-zone fields, or equivalently, to the quantum effects (i.e. the position (z)-momentum (k_z) or time-energy uncertainty) operating remarkably within the range $k_z z < 1$. In the case of frustrated total internal reflection [11], a similar but more complicated analysis can be provided. That is to say, the superluminal phenomena of evanescent waves are related to the ones of the near-zone fields. Sometimes people call the near-zone fields evanescent ones directly [28].

Moreover, we propose that these superluminal phenomena have a common theoretical foundation in quantum field theory and do not violate causality principle. As for our scenario, the electromagnetic fields can be taken as the Proca fields with the mass $m \to 0$ [29]. Let $\hat{A}^{\mu}(x)$ ($\mu = 0,1,2,3$) denote the field operator of the Proca fields, $|0\rangle$ denote the vacuum state, then the probability amplitude for a particle to propagation from y to x with the polarization indices changing from ν to μ is $D^{\mu\nu}(x-y) \equiv \langle 0|\hat{A}^{\mu}(x)\hat{A}^{\nu}(y)|0\rangle$ $(\mu,\nu = 0,1,2,3)$. When the 4-dimensional interval $(x-y)$ is spacelike, following the discussion in Ref. [30] ($\hbar = c = 1$), one can find $D^{\mu\nu}(x-y) \neq 0$, and the conclusion is valid as the mass $m \to 0$ too. That is, the superluminal propagation for photons is possible. To really discuss causality, however, we should ask whether a measurement performed at one point could affect another measurement at a point separated from the first with a spacelike interval [30], rather than whether particles can propagation over such a spacelike interval. Therefore, we should compute the commutator

$[\hat{A}^{\mu}(x), \hat{A}^{\nu}(y)] = \langle 0 | [\hat{A}^{\mu}(x), \hat{A}^{\nu}(y)] | 0 \rangle$ ($= D^{\mu\nu}(x-y) - D^{\nu\mu}(y-x)$). Since this commutator vanishes provided that $(x-y)$ is spacelike, causality is preserved. Likewise, the conclusion is valid as the mass $m \to 0$. In quantum mechanics language, because of the quantum effects, e.g. the position-momentum uncertainty, superluminal process can exist without violating the principle of causality (or the *micro-causality*).

On the one hand, according to the quantum field theory, the spacelike process mentioned above corresponds, in fact, to the virtual particle (the off-shell particle) process, which, as we have known, corresponds to the longitudinal polarization or scalar photons. On the other hand, the near-zone fields of source and the evanescent waves (also as near-zone fields) involve the longitudinal polarization fields, and, as $kr \to 0$ (i.e. $\exp(-jkr) \to 1$), the fields given in Equ. 5 are the same as those produced by a static electric dipole and hence correspond to the virtual photon fields. That is to say, the *quasi-static* electric fields in the near-zone of source correspond to the spacelike virtual photon fields, or equivalently, owing to the quantum effects (i.e. the position (z)-momentum (k_z) or time-energy uncertainty) operating remarkably within the range $kz\cos\beta = k_z z < 1$ of source, the spacelike virtual-photon processes would take place. In conclusion, we propose that, the superluminal phenomena that are related to the near-zone fields and the evanescent waves (also as the near-zone fields), correspond to the spacelike virtual-photon processes, consequently the superluminal phenomena in nondispersive media have theoretical foundation in QFT (in our next paper we shall give a specific discussion in QFT level).

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Figure Captions

Figure 1 (Color online): In the observe direction z the wavelength $\lambda_g = \lambda / \cos\theta$, the group velocity $v_g = c\cos\theta$, the phase velocity $v_p = c/\cos\theta$

Figure 2: The sketch map of a pyramidal horn antenna

Figure 3 (Color online): The group velocity of E_r and H_φ as the function of kz and β

Figure 4 (Color online): The group velocity of E_θ as the function of kz and β

Figure 5 (Color online): The relation between the group velocities and β

Figure 6 (Color online): The relation between the group velocities and kz

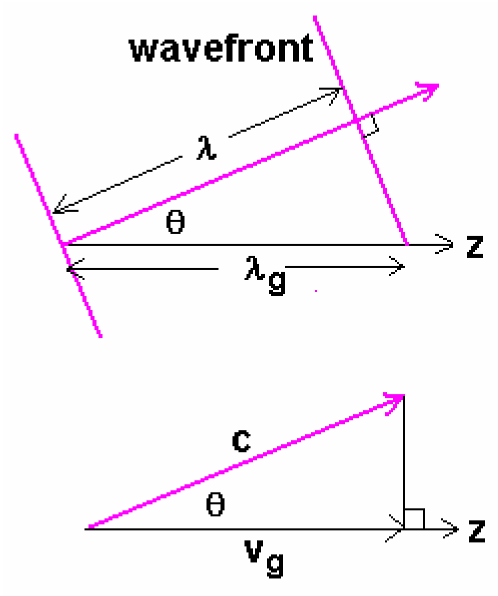

Figure 1

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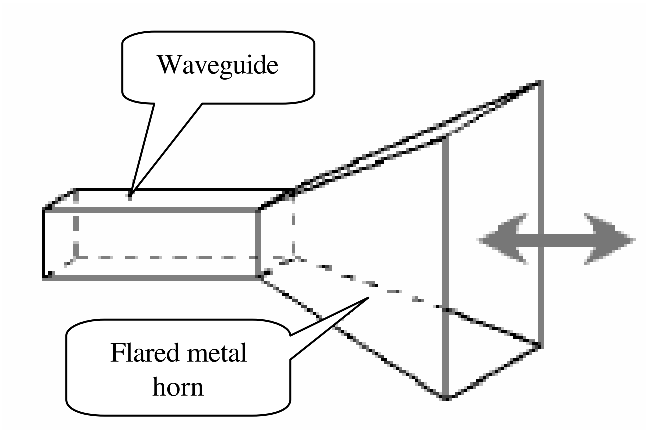

Figure 2

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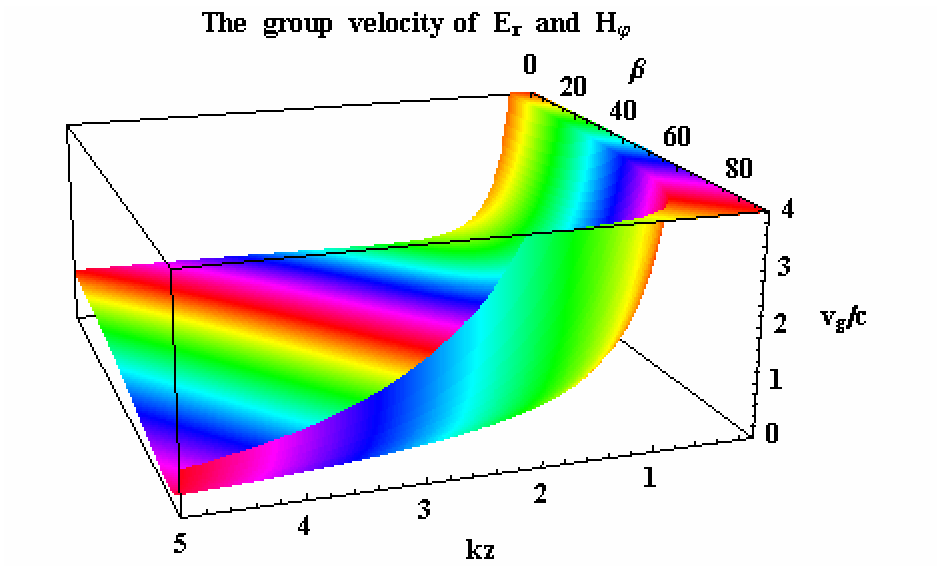

Figure 3

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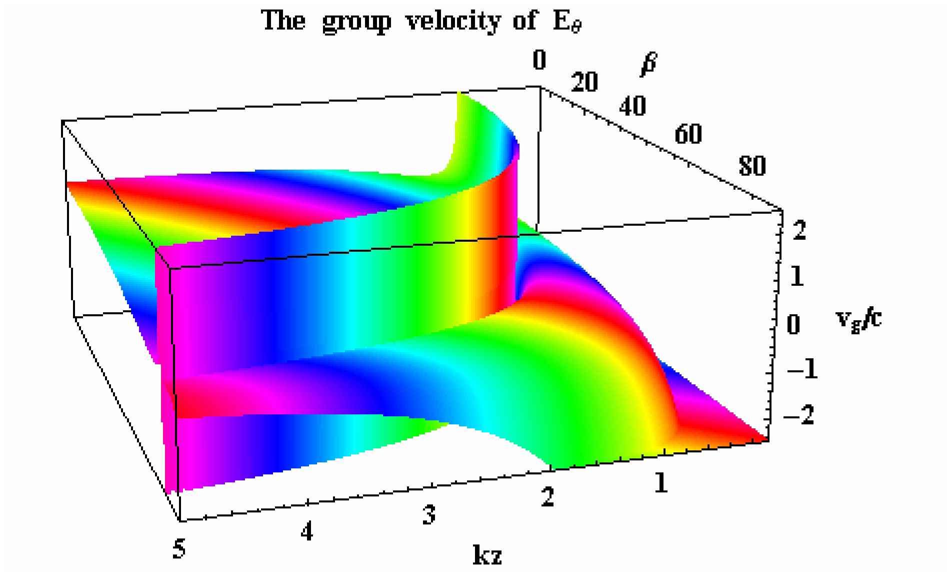

Figure 4

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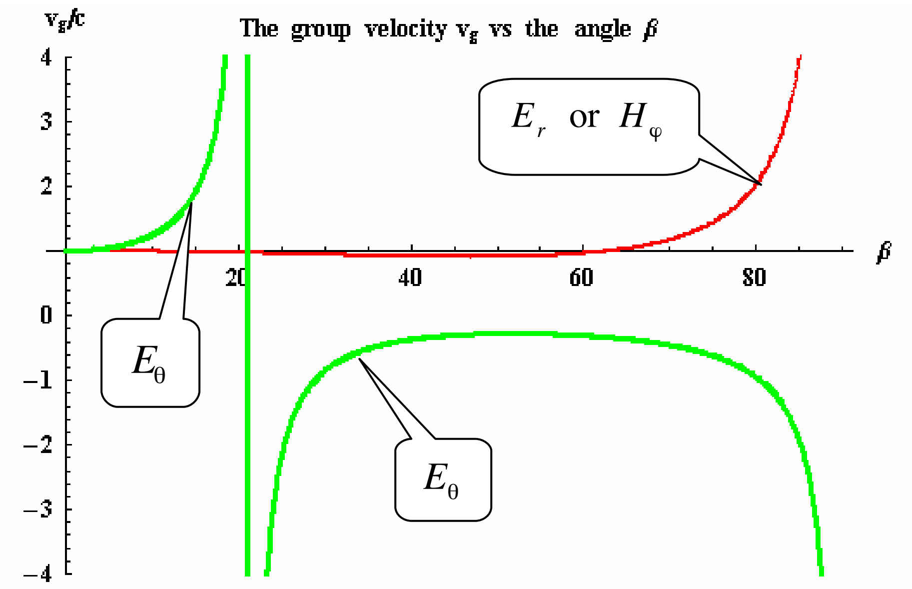

Figure 5

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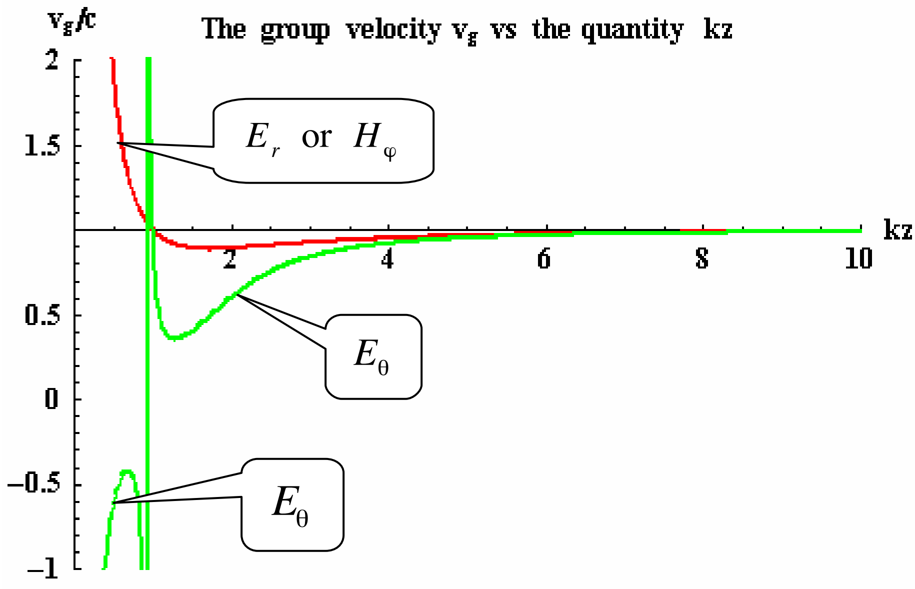

Figure 6

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