Taka's memo

Helmholtz Decomposition is wrong
JULY 16, 2014

“True” actual components of a flow

We used to use isobaric charts for the analyses of weather phenomena.

For example, 200hPa charts show the situations on the plane of about 12000m height.

We can consider the wind data on 200hPa as a flow vectors in two-dimensions.

JMA (http://www.jma.go.jp/jma/indexe.html) and **NOAA (http://www.ncdc.noaa.gov/)** are making “**velocity potentials (http://en.wikipedia.org/wiki/Velocity_potential)**” and “**stream functions (http://en.wikipedia.org/wiki/Stream_function)**” from these wind data sets by applying “**Helmholtz Decomposition (http://en.wikipedia.org/wiki/Helmholtz_decomposition)**”.

But I think that Helmholtz Decomposition is wrong, so these velocity potentials and stream functions are made by their mistakes.

I want to show you that how these potentials are made.

1) On the Helmholtz Decomposition theorem

There is a theorem called **Helmholtz Decomposition** that says any flow can be separated into irrotational divergent flow and non-divergence rotational flow. And velocity potentials can be calculated from the irrotational flow and steam functions can be calculated from the non-divergence flow.

JMA and NOAA are publishing velocity potentials and stream functions on net.

Acording to Helmholtz Decomposition, two kinds of these potentials are independent from one another. So, if you wanted to analize the distributions of divergence in some layer, you could do it by just analyses of velocity potential map.

From Wikipedia, Helmholtz Decomposition is given as follows

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_\ell(\mathbf{r}) + \mathbf{F}_t(\mathbf{r})$$ **(https://taka19440606.files.wordpress.com/2014/07/equation1.png)** ………… 1)

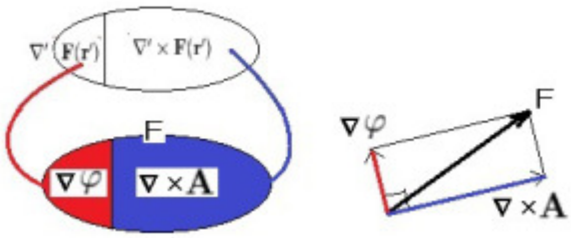
$$\mathbf{F}_l = -\nabla\Phi = -\frac{1}{4\pi}\nabla\int_V \frac{\nabla'\cdot\mathbf{F}}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V'$$ **(https://taka19440606.files.wordpress.com/2014/07/equation2.png)**……· 2)

$$\mathbf{F}_t = \nabla \times \mathbf{A} = \frac{1}{4\pi}\nabla \times \int_V \frac{\nabla' \times \mathbf{F}}{|\mathbf{r}-\mathbf{r}'|}dV'$$

(https://taka19440606.files.wordpress.com/2014/07/equation3.png)······ 3)

Here, **Fl** means irrotational divergent flow, and **Ft** means non-divergence rotational flow.

I show you Fig.1 to image the Helmholtz Decomposition..



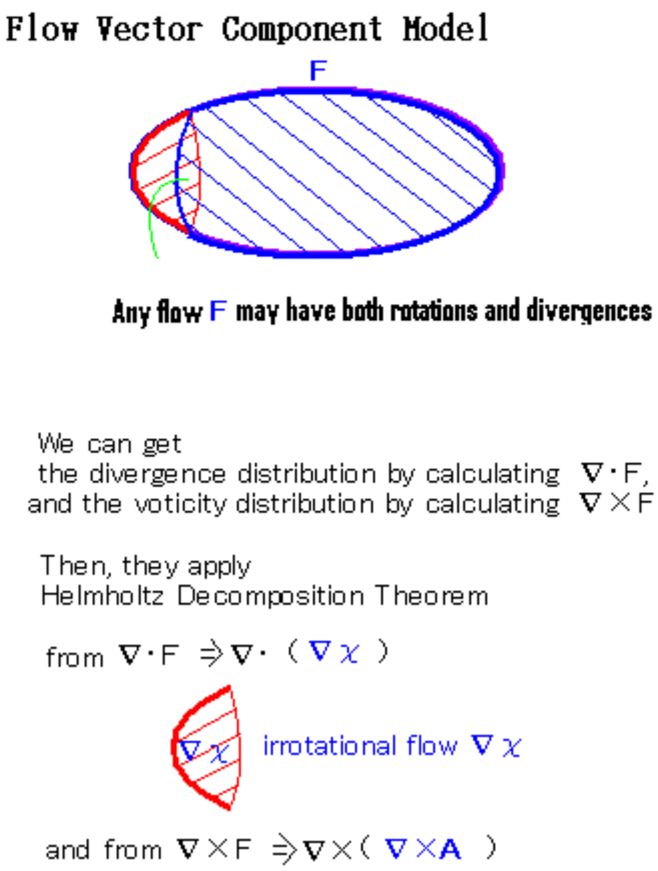
(https://taka19440606.files.wordpress.com/2014/07/fig01.jpg)
Fig.1 Illustration for Helmholtz Decomposition

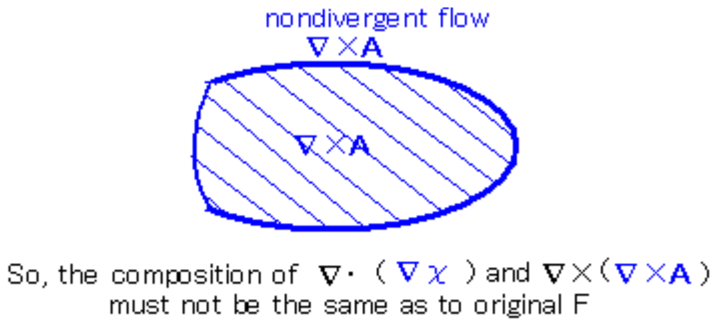
2) The third component

When we think about composition of vector, that is generically considered as projections of a vector which is given at one point onto the reference axes.

But the components in the Helmholtz Decomposition are given as roles which play as a flow in the set of neighbor flows. So, you should think about the component which play both role of curl and divergence, over and above the irrotational divergence component and the non-divergent rotational component.

If there is the third component which play both roles of curl and divergence in any flow, the components of any flow should be shown as Fig.2.





(https://taka19440606.files.wordpress.com/2014/07/fig02.gif) Fig.2 The components of any flow

Even if there is the third component, you can calculate ∇· **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)F** and ∇× ∇× **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)F** distributions, and therefor you can get velocity potential and stream function. And furthermore, you can get **Fl** and **Ft**.

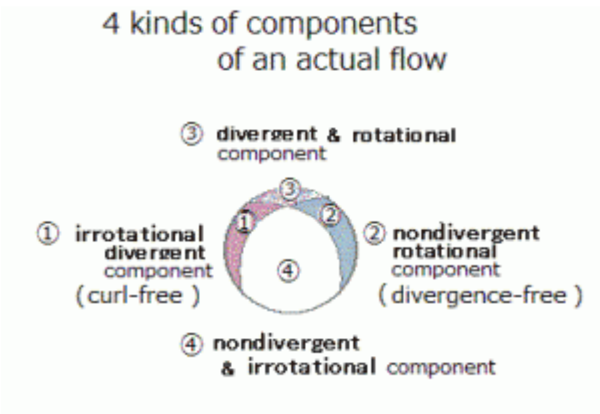
But as you can see inFig.2, the composition of these two components does not match the original flow.

So, I can say that Helmholtz Decomposition is wrong.

3) the components of a "true" actual flow

Actually, there is a fair percentage of non-divergence and irrotational component in actual flow.

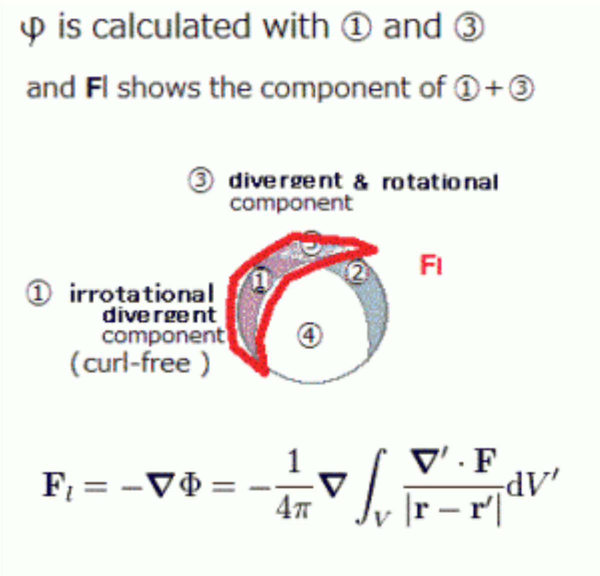
So, when you divide a flow into some components, you should think about the fourth component which has neither divergence nor curl(rotation).



(https://taka19440606.files.wordpress.com/2014/07/fig03.gif) Fig.3 the 4 components of a general flow

According to Equation 2), φ is calculated from the term of ∇· **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)F**. So, **Fl** should consist of the components ① **(https://taka19440606.files.wordpress.com/2014/07/1.png)**+ ③ **(https://taka19440606.files.wordpress.com/2014/07/3.png)**.

Fig.4 is calculated with the components of ① **(https://taka19440606.files.wordpress.com/2014/07/1.png)**+ ③ **(https://taka19440606.files.wordpress.com/2014/07/3.png)**

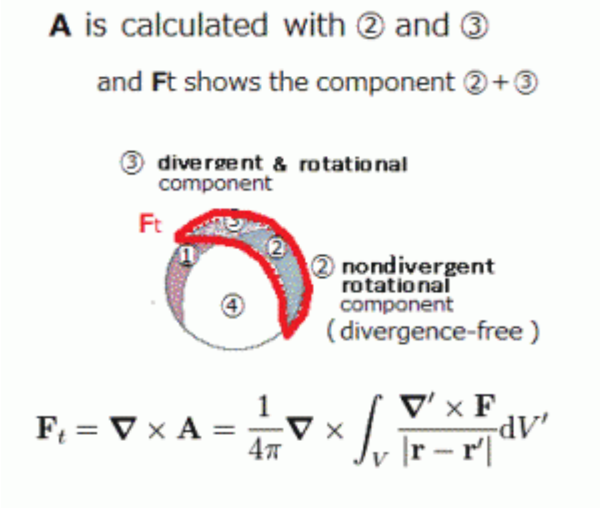


(https://taka19440606.files.wordpress.com/2014/07/fig041.gif)

Fig.4 **Fl** is calculated with the component of

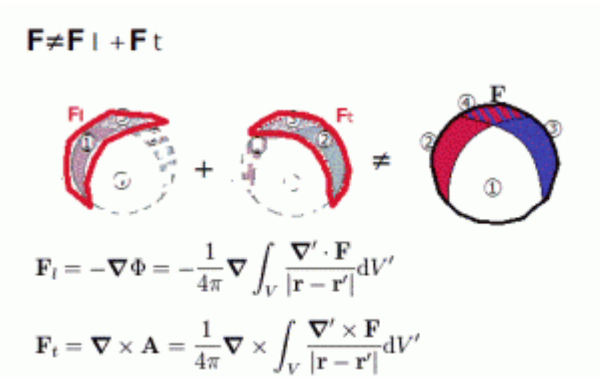
And, vector potential **A** is calculated with the term of ▽× **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)F**.

So **Ft** should consist of the components ②+③.



(https://taka19440606.files.wordpress.com/2014/07/fig051.gif) Fig.5 **Ft** is calculated with the components of ②+③

Therefore, Equation 1) is not correct. Therefore we should say that **Helmholtz Decomposition is wrong**.



(https://taka19440606.files.wordpress.com/2014/07/fig06.gif)
Fig.6 Helmholtz Decomposition is not correct

After publishing this article, I should edit or remove my latest blog "On the components in Helmholtz Decomposition Theorem", but I daringly keep it on the Net.

4) Another Decomposition

There is another way to decompose any wind into two components. You can decompose any wind into geostrophic wind component and ageostrophic wind component.

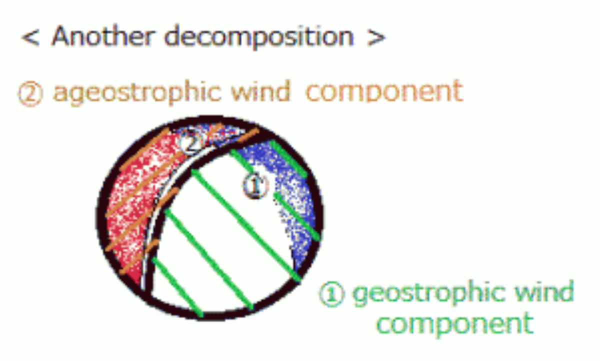
Geostrophic winds are theoretically given from the contours(heights of an isobaric surface). Geostrophic winds blow along contours in inverse proportion to the gap of contour lines. So, geostrophic winds are perfectly non-divergent wind.

And, because the natural winds blow as quasi-geostrophic winds, they mostly consist of geostrophic winds.

Ageostrophic winds are given as the difference calculated by subtracting geostrophic wind from the original(analyzed) winds.

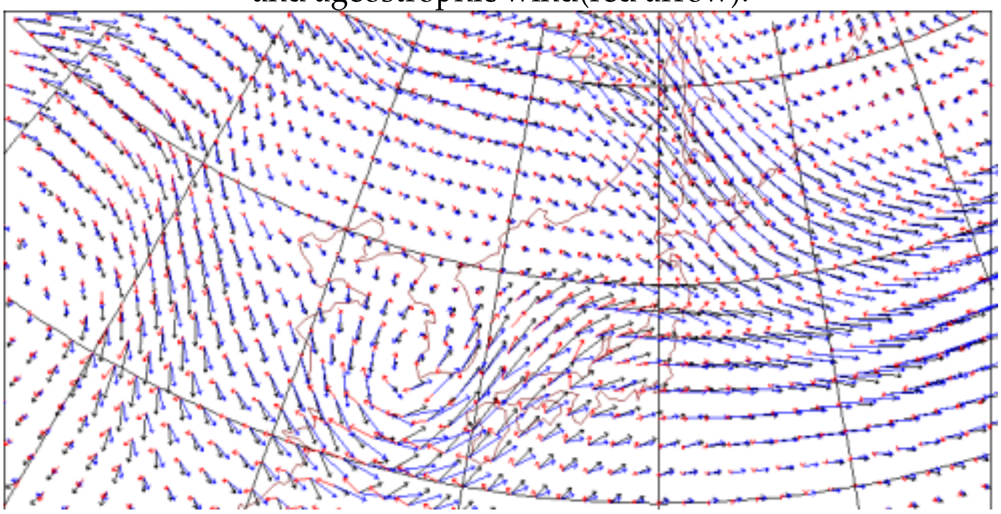
So, there is no doubt in this way to divide any flow into geostrophic wind and ageostrophic wind.

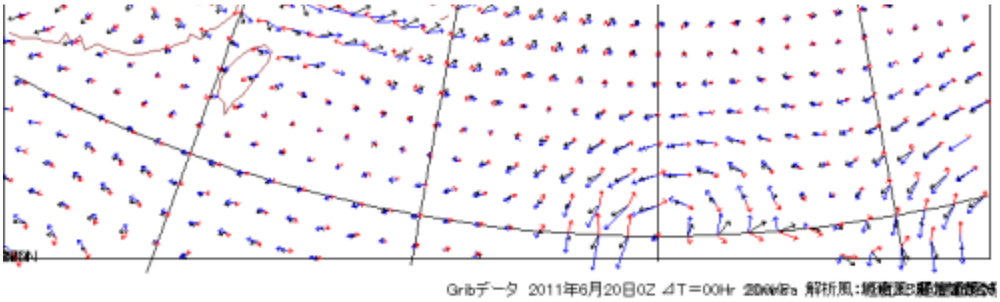
I show you a illustration to image the decomposition which make a flow divided into geostrophic wind and ageostrophic wind inFig.7.



(https://taka19440606.files.wordpress.com/2014/07/fig07.gif)
Fig.7 The Decomposition into Geostrophic wind and ageostrophic wind

And, Fig.8 is an example for analyzed(original) wind(black arrow), geostrophic wind(blue arrow) and ageostrophic wind(red arrow).





(https://taka19440606.files.wordpress.com/2014/07/fig08.png)
Fig.8 an example for geostrophic wind(blue), ageostrophic wind(red)

and analyzed wind(black) on 20th Jun 2011

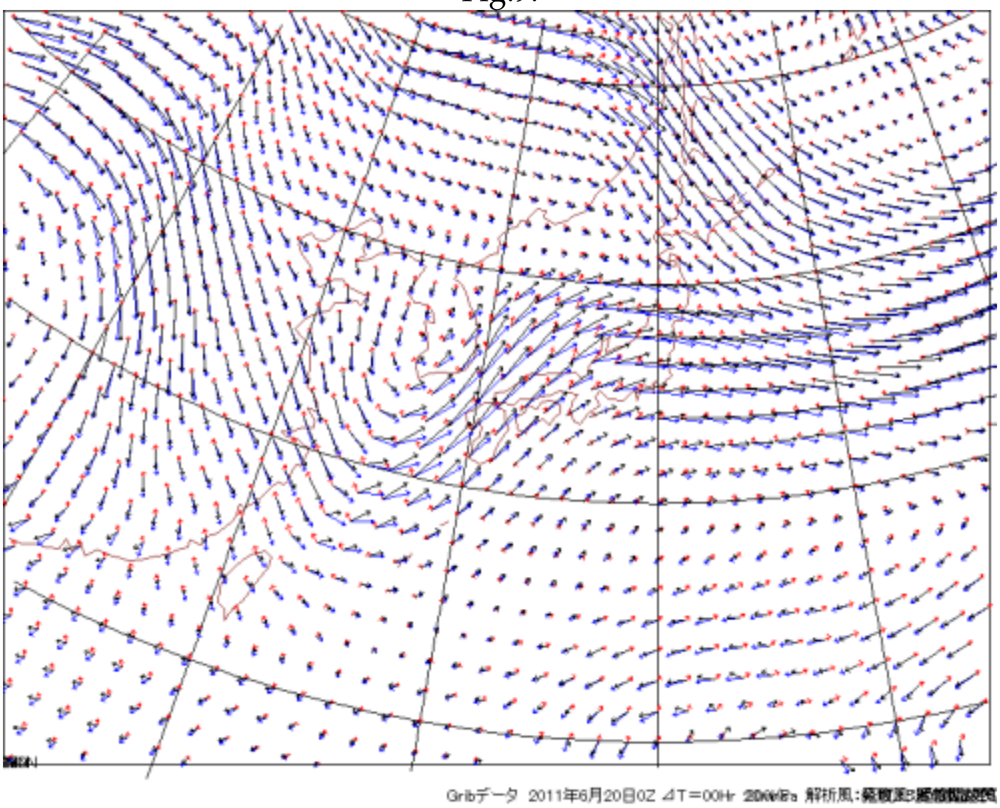
Fig.8 shows that the composition of geostrophic wind and ageostrophic wind is nearly equal to original analyzed wind. It might be expected.

We can’t say that Geostrophic wind (blue arrow) take out 100% of the non-divergent component from natural(analyzed) wind(black arrow). But it mostly consist of them.

Ageostrophic wind component(red arrow) is approximately compounded of divergence component which is shown (① **(https://taka19440606.files.wordpress.com/2014/07/1.png)**+ ③ **(https://taka19440606.files.wordpress.com/2014/07/3.png)**) in Fig.3.

Therefore, ageostrophic wind is nearly divergent wind which would be given from velocity potential.

Here I want to show the divergent wind and curl wind on the same day. The divergent winds were calculated from velocity potentials, and the curl winds were calculated from stream functions in Fig.9.



(https://taka19440606.files.wordpress.com/2014/07/fig09.png)

Fig.9 an example for curl wind(blue), divergent wind (red)

and analyzed wind(black) on 20th Jun 2011

After seeing Fig.9, I had been left speechless for a while. Because the composition of divergent wind and curl wind is nearly equal to the original(analyzed) wind.

Had I mistook in former article related Fig.6?

Please put it aside for a while, and confirm that the divergent winds are fairly equal to ageostrophic winds.

This is an example for that **Fl** in Fig.4 is nearly equal to the component of ② **(https://taka19440606.files.wordpress.com/2014/07/2.png)** in Fig.7. We can say that ageostrophic winds are nearly equal to divergent winds.

5) about stream function

Comparing Fig.9 to Fig.8, curl winds **Ft** calculated from stream functions nearly equal to geostrophic winds.

And we have confirmed that the composition of divergent wind **Fl** and curl wind **Ft** is nearly equal to the original(analyzed) wind. This can be a proof for that Helmholtz Decomposition is crrect.

Here, I doubt if these stream function was truly calculated by using equation 3).

Please look at Fig.5 again.

Stream function must be driven from a vector potential expressed as below.

$$\mathbf{F}_t = \nabla \times \mathbf{A} = \frac{1}{4\pi}\nabla \times \int_V \frac{\nabla' \times \mathbf{F}}{|\mathbf{r}-\mathbf{r}'|}dV'$$

(https://taka19440606.files.wordpress.com/2014/07/equation3.png)

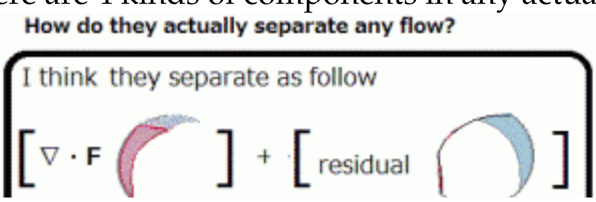
Therefore, the components of just ② **(https://taka19440606.files.wordpress.com/2014/07/2.png)** and ③ **(https://taka19440606.files.wordpress.com/2014/07/3.png)** in Fig.5 is useful to calculate $\nabla\times$ **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)F**. Because, even if the component ① **(https://taka19440606.files.wordpress.com/2014/07/1.png)** and ④ **(https://taka19440606.files.wordpress.com/2014/07/4.png)** were used, they came to 0 as a consequence. So, **Ft** definitely not be nearly equal to geostrophic wind. **Ft** should be fairly small than geostrophic wind.

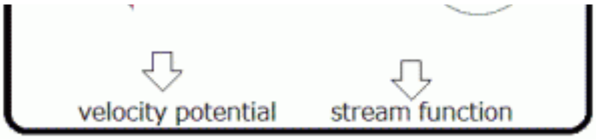
According to equation 2) and 3), the composition of **Fl** and **Ft** should be smaller than original analyzed wind.

There is a way to make these stream functions published from JMA or NOAA.

If you priliminaly beleaved Helmholtz Decomposition is right(Fig.1), you could get stream function from the difference calculated by subtracting divergent wind from the original(analyzed) wind.

But, there are 4 kinds of components in any actual wind.





(https://taka19440606.files.wordpress.com/2014/07/fig10.gif)
Fig.10 Actual way to get "Stream function"

It is all right to get **Fl** from the equation 2). But **Ft** must be calculated as the differences calculated by subtracting **Fl** from original wind, for getting equation 1). In any another way, **Fl** + **Ft** would not be equal to original **F**.

To take this way is definitely distinct from Helmholtz Decomposition. This way is the same way to separate a wind into geostrophic wind and ageostrophic wind.

Here, I must confess that I don't know exactly how to make stream function. Please ask some person who know how to calculate the stream function, if you know. And ask him to publish the way how to calculate the stream function. I think it have been top-secret among them.

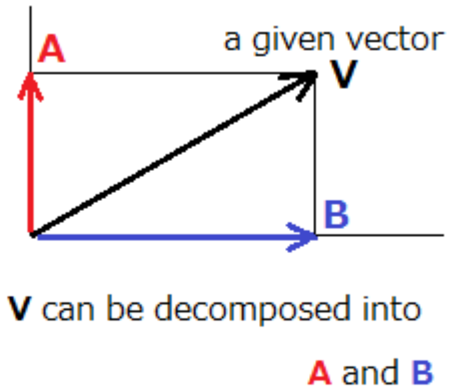
Leave a Comment
JUNE 19, 2014

On the components in HelmholtzDecomposition Theorem

There are two kinds of vector decompositions. One is a normal vector decomposition, and the second appears in Helmholtz Decomposition.

1. The normal decomposition of vector
Talking of the decomposition, you generally think about projected components of a given vector on the reference axes or rectangular coordinates.

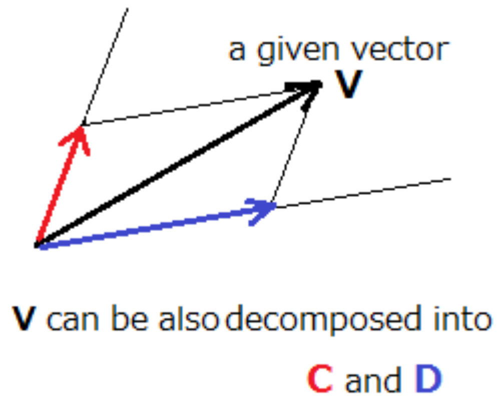
And the vector projected on the reference axes is called the components of a given vector.



(https://taka19440606.files.wordpress.com/2014/06/normal-decomposition.png)
Fig.1 a normal decomposition and components

But you can also think about a decomposition into non orthogonal directions.

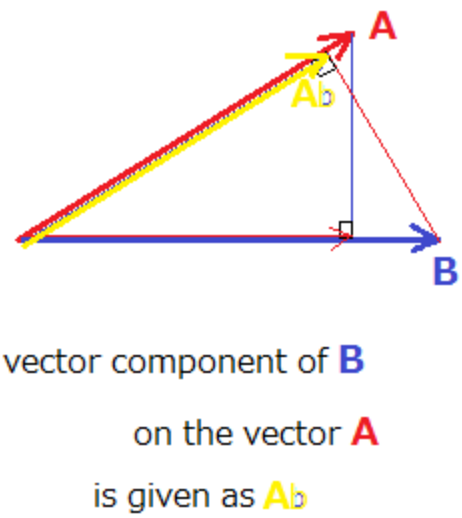
If a given vector is equal to the vector sum of other two vectors, these two vectors are called as components of a given vector.



(https://taka19440606.files.wordpress.com/2014/06/abnormal-decomposition.png)

Fig.2 a decomposition into non-right angle components

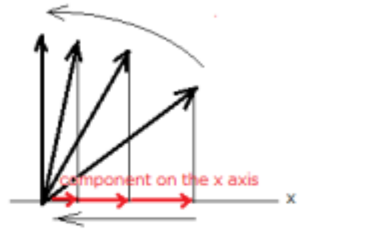
The basic concept of vector is that two vectors which are not perpendicular to each other have the components of each other. The dimension of it is given by the length from the foot of perpendicular to the point of intersection of two vectors.



(https://taka19440606.files.wordpress.com/2014/06/the-component-of-another-vector.png)

Fig.3 a vector component of a vector on another vector direction

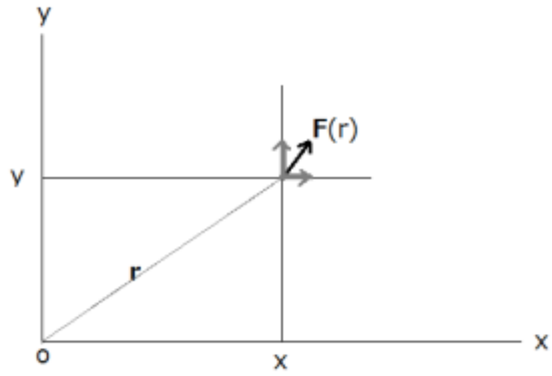
When the directions of two vectors are perpendicular to each other, the foot of perpendicular to other axis is on the foot of other vectors. So, in these situations, you can say that these two vectors are independent of each other.



(https://taka19440606.files.wordpress.com/2014/06/component-in-the-orthogonal-direction.png)

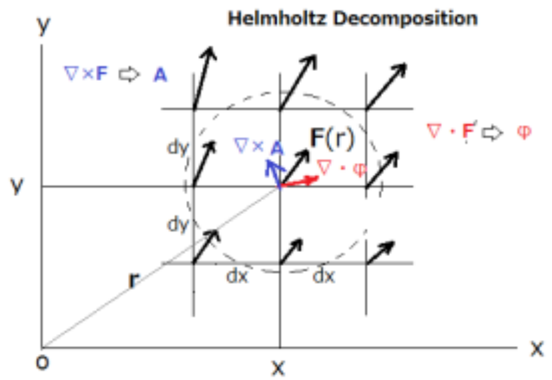
Fig.4 the component on orthogonal axis

In other words, when two vectors are perpendicular to each other, one does not influence the other. The normal vector decomposition is thinkable at a point.



(https://taka19440606.files.wordpress.com/2014/06/normal-decomposition-is-given-at-apoint.png)

2. The decomposition in Helmholtz Decomposition theorem.
Generally you can't think about the components of vectors which can't be decomposed.
But the components that come up by the decomposition of Helmholtz Decomposition are not simply the projections of the flow vectors. They should be called "Roles". They are a curl-free **divergence (http://en.wikipedia.org/wiki/Divergence)** component and a divergence-free **curl (//en.wikipedia.org/wiki/Curl_(mathematics))** component. Those components are given as "Roles" of a flow at a point in the set of adjacent flows.
These components are not thinkable at just a point. It is given in a region.



(https://taka19440606.files.wordpress.com/2014/06/helmholtz-decomposition-is-given-

in-a-region.png)

3. It is not needed to get a curl distribution and divergence

Helmholtz Decomposition is not needed to get distributions of curl and divergence.
You can calculate “a distribution of curl” and “a distribution of divergence” by calculating $\nabla\times$ **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)**F and $\nabla\cdot$ **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)**F respectively.

It is not necessary to use Helmholtz Decomposition theorem to get them. Therefor to be able to uniquely obtain these distributions can not be the proof of Helmholtz Decomposition.

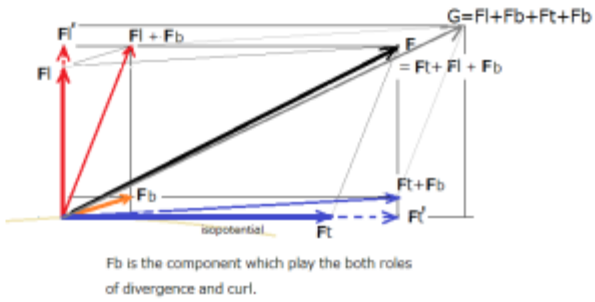
You just be able to calculate these distributions, so you should not think that any flow is decomposed into two flows.

4. The component which plays the both roles of curl and divergence

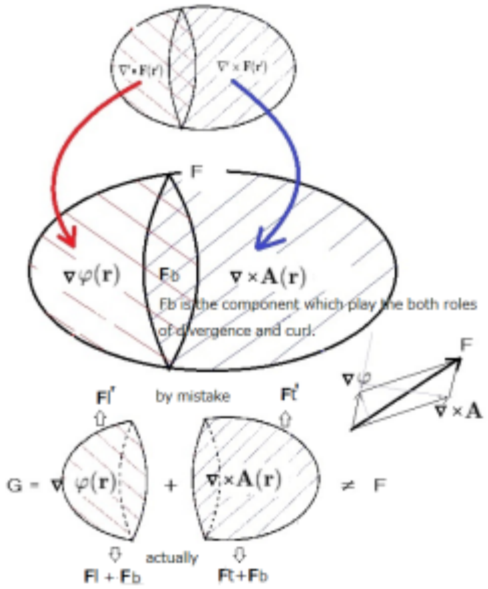
Because a decomposition in Helmholtz Decomposition can be thought as roles of any flow at a point in the neighborhood, you should think about a component which plays both roles of curl and divergence.

If you don’t think of it, you preliminary use the Helmholtz Decomposition theorem without the proof.
If you would prove that there is no component which plays the both role of curl and divergence in any flows, you can say that Helmholtz Decomposition is correct.

At first you should think about a component which plays the both roles of curl and divergence.



(https://taka19440606.files.wordpress.com/2014/06/illust-for-the-third-composition1.png)



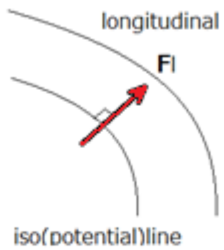
(https://taka19440606.files.wordpress.com/2014/06/illust-for-the-third-composition2.png)

Fig.5 If any flow has the component which play the both roles

In Fig.5, **F** is any flow which has both of curl and divergence. And **F**l shows a divergence component as a curl-free component, and **F**t shows a curl component as a divergence-free component, and **F**b shows a component which plays the both roles of curl and divergence.

These **F**l and **F**t are borroeing character from the article of ”**Longitudinal and transverse fields (http://en.wikipedia.org/wiki/Helmholtz_decomposition)**” in Wikipedia.

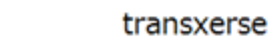
The substitute character “l” means as shown in Fig.6,

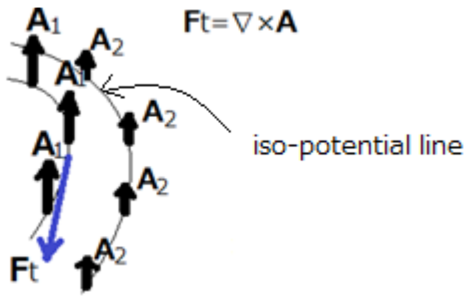


(https://taka19440606.files.wordpress.com/2014/06/fl_illust.png)

Fig.6 the longitudinal component

and “t” means as shown inFig.7,





(https://taka19440606.files.wordpress.com/2014/06/ft_illust.png)

Fig.7 the transversal component

I might have mistook "**k**" what he/she wrote, but I beleave that "k" should be the gradient directin of potential.

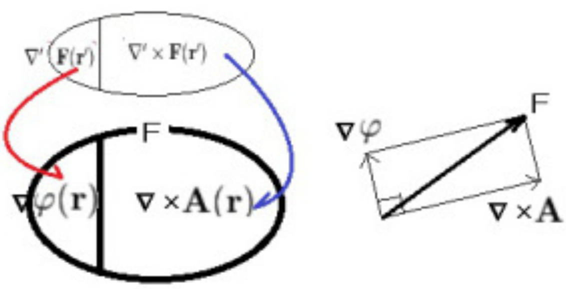
These illustrations are made by myself, so if there is any fault in these illustrations, it is my fault. As we know, they can calculate "a distribution of curl" and "a distribution of divergence" by calculate ∇× **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)F** and ∇・ **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)F** .

Here, if they had mistook a flow had been decomposed into a curl-free flow and a divergence-free flow, they could calculate a velocity potential from the distribution of divergence, and a stream function from distribution of curl.

But actually they just calculated the distributions of divergence by ∇・ **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)**(Fl+Fb）, not by ∇・ **(https://taka19440606.files.wordpress.com/2014/06/nabra_dot.png)**（Fl'）, and the distributions of curl by ∇× **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)**(Ft+Fb）, not by ∇× **(https://taka19440606.files.wordpress.com/2014/06/nabra_product.png)**（Ft'）.

Therefore the composition **G** of these two flows does not match the original flow **F**.

If there is no component which plays the both roles of curl and divergence , Helmholtz Decomposition theorem is correct as showing in Fig.8.

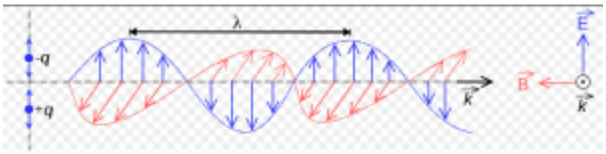


(https://taka19440606.files.wordpress.com/2014/06/if-hd-is-correct.jpg)

Fig.8 If any flow has not the component which play the both roles

In fact, there is an example in real world. I may say that an electromagnetic wave has an electric field as a scalar potential φ, and a magnetic field as a vector potential **A** as shown in Fig.9. But I have to say that I don't know exactly the electromagnetic wave.

Fig.9 is drawn from **Wiki (http://en.wikipedia.org/wiki/Electromagnetic_radiation)**.



The electromagnetic waves that compose electromagnetic radiation can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. This diagram shows a plane linearly polarized EMR wave propagating from left to right. The electric field is in a vertical plane and the magnetic field in a horizontal plane. The two types of fields in EMR waves are always in phase with each other with a fixed ratio of electric to magnetic field intensity.

(https://taka19440606.files.wordpress.com/2014/06/light-wave.png)

Fig.9 The electromagnetic wave

In the article of Wikipedia, an electromagnetic wave has an electric field as a scalar potential and a magnetic field as a vector potential, and they are perpendicular to each other.

In this case, the force given in an electric field and the force in a magnetic field are perpendicular to each other.

In electromagnetic wave case, you can confirm to exist a set of a scalar potential and a vector potential. But in fluid case, we can not confirm to exist these potential.

5. How to verify the Helmholtz Decomposition

As I said before, if Helmholtz Decomposition is right, there is not a component which plays the both roles of curl and divergence.

Then
1) The composition of a curl component and a divergence component is equal to the original flow(vector)
2) The two of components, a curl component flow and a divergence flow are perpendicular to each other.

And a curl component flow is directed along the line of stream functions, and a divergence flow is directed toward perpendicular to iso-velocity potential lines.

Therefore at any point, the lines of stream function and the line of iso-velocity potential line are on a parallel with each other.(Please see Fig.6 and Fig.7)

Stream function is expressed by the dimension of “vector potentials” in the horizontal plane as vector potential. The direction of the stream function as a “vector potential” stands up perpendicular to the plane.(Please see Fig.7)

6. Current status

I will show you some current status of “the velocity potential” and “stream function”.

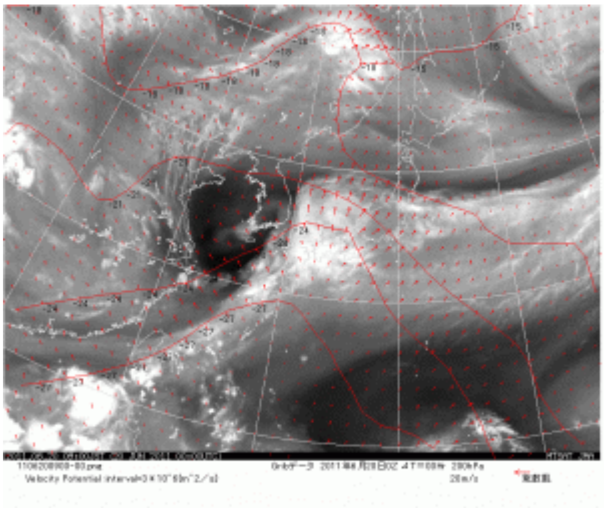
The natural wind in our atmosphere generally blow horizontally, because the air is generally in the condition of hydrostatic equilibrium. So we can think the natural wind as a flow in a horizontal plane.

NOAA(National Oceanic and Atmospheric Administration) and JMA(Japan Meteorological Agency) etc. beleave Helmholtz Decomposition is correct, and they publish their data of “the velocity potential” and “the stream function”

6.1) Do the compositions of “the divergent flow” and “curl flow” match the original winds?

JMA had been publishing their data on the net until October 2011.

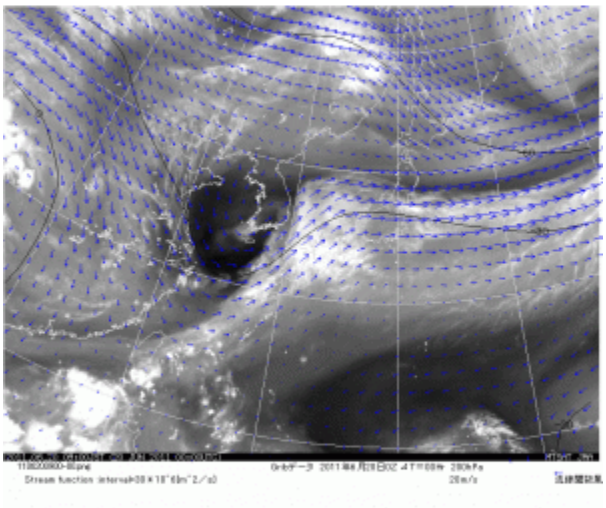
By using those data, I will show you how “the velocity potentials” and “the stream functions” are going. The following examples are about on 20 Jun 2011.
Fig.10 shows the distributions of the velocity potentials and divergent winds on the water vapor imagery.



(https://taka19440606.files.wordpress.com/2014/06/img_2.gif)

Fig.10 distribution of the velocity potentials and divergent wind

And Fig.11 shows the distribution of stream functions and curl winds.

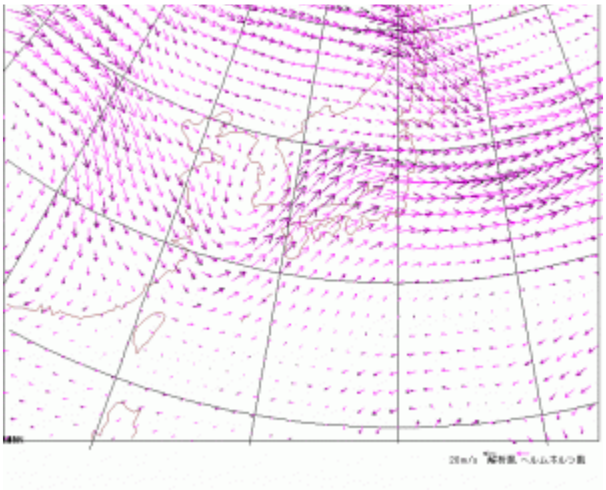


(https://taka19440606.files.wordpress.com/2014/06/img_3.gif)

Fig.11 distribution of stream functions and curl winds

If Helmholtz Decomposition theorem is correct, the vector sum of these divergent winds and curl winds should match the original winds.

Fig.12 shows the original winds(black) and the vector sums of two kinds of components(purple).



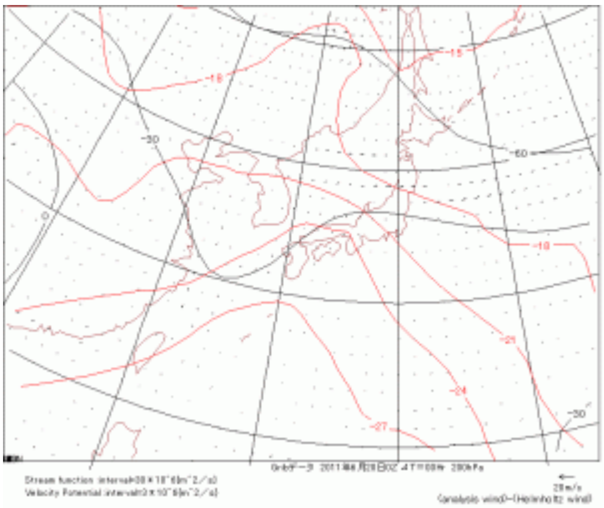
(https://taka19440606.files.wordpress.com/2014/06/img_5.gif)

Fig.12 The comparison between analyzed winds(black) and composed winds

I first drew analyzed wind with black arrows, and after that I drew composed wind with purple arrows. So if the composed winds perfectly match the analyzed winds, all of arrows should be purple.

You may think that analyzed winds(original winds) match the vector sum of divergent winds and curl winds. But I can see the difference between these two kinds of winds.

Fig.13 shows the differences between original winds and the vector sum of divergent winds and curl winds.



(https://taka19440606.files.wordpress.com/2014/06/img_6.gif)

Fig.13 the differences between analyzed winds and vector sum of two components

There are some differences about 5m/sec in some area.

Here, what do you think with Fig.13. Permissible? or Impermissible?

I do not know how to make these potentials, but I want to applaud the efforts of them. Good jobs!

6.2) Are "the divergent wind" and "the curl wind" perpendicular to each other?

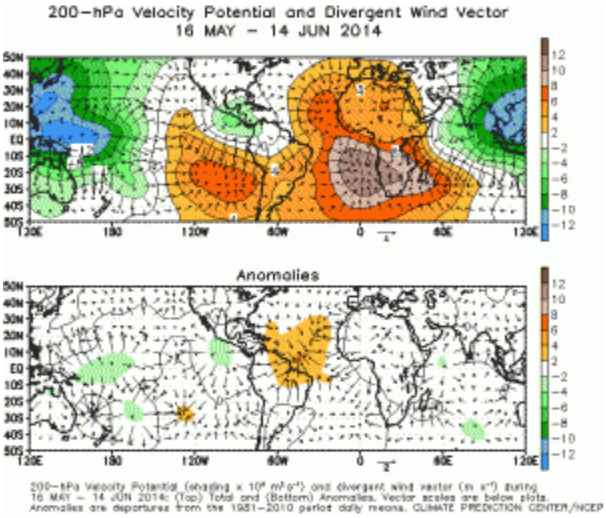
If “Helmholtz Decomposition” is correct, “the divergent wind” and “the curl wind” should be perpendicular to each other.

But these two kinds of winds are not perpendicular to each other as we can see in Fig.10 and Fig.11.

And if “Helmholtz Decomposition” is correct, two kinds of isolines of “the velocity potential” and “the stream function” should be parallel to each other as we have seen in Fig.6 and Fig.7.

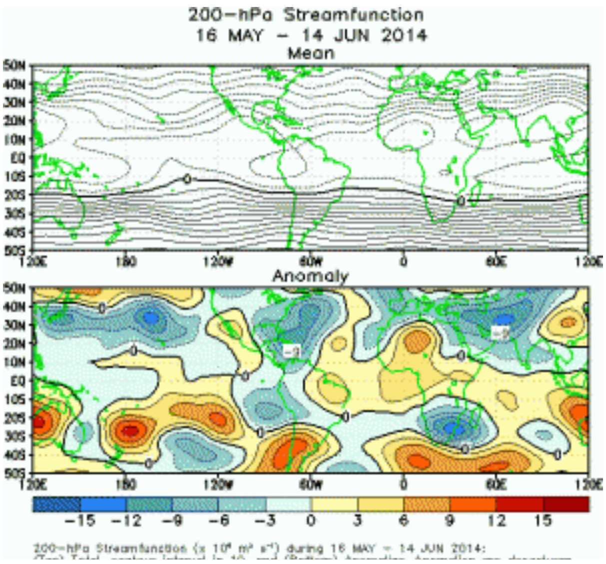
But these two kinds of iso-lines are not parallel to each other as we can see in Fig.13.

<by NOAA data>
We can also see the same consequences in NOAA data. Fig.14 and Fig.15 show the velocity potential and stream function respectively. They were published in NOAA home page(**http://www.cpc.ncep.noaa.gov/products/hurricane/ (http://www.cpc.ncep.noaa.gov/products/hurricane/)**).



(https://taka19440606.files.wordpress.com/2014/06/vpot_200_30d.gif)

Fig.14 The example of Velocity potential distribution
(30days mean from 16 May to 14 June in 2014)



(https://taka19440606.files.wordpress.com/2014/06/strmfn_200_30d.gif)

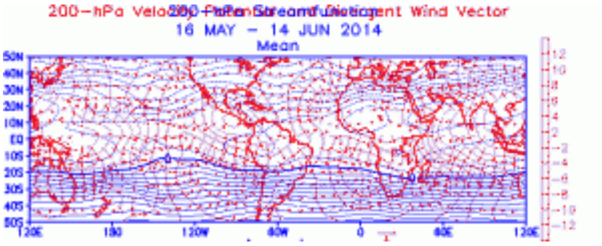
Fig.15 The example of Stream function distribution
(30days mean from 16 May to 14 June in 2014)

Fig.14 shows 30days means of 200hPa Velocity potential and divergent wind from 16 May to 14 June in 2014(top of them).

Fig.15 shows 30days mean of 200hPa stream function as the same term as Fig.14. The bottoms of them show anomalies of velocity potential, but now it is out of our argument.

To see how to go on the two potentials, I draw stream function on velocity potential in Fig.16. In Fig.16, blue lines show stream functions, and red lines show velocity potential. And red arrows show divergent wind.

Fig.16 is made by Fig.14 and Fig.15.



(https://taka19440606.files.wordpress.com/2014/06/streaf_on_vpot.gif)

Fig.16 velocity potential on stream function at 200hPa as same term as Fig.14,15

The curl wind(=divergence-free wind) blow parallel to blue line(stream function). So we can see that divergent wind and curl wind are not orthogonal to each other.

As far as has been hitherto seen, we can say Helmholtz Decomposition is not correct.

I think Helmholtz Decomposition has been preventing (http://blogs.yahoo.co.jp/taka19440606/36692433.html)the progress of Meteorology.

Leave a Comment
JULY 9, 2013

Velocity potential and Ageostrophic wind

Velocity potential and Ageostrophic wind

If we lost Helmholtz Decomposition theorem, does it become problems?

No, not at all.

If you want to know the distributions of divergence of flow **F**, you can get them from just original flows by $\nabla \cdot \mathbf{F}$ **(https://taka19440606.files.wordpress.com/2013/04/nabladotf.png)**.

You don't need to use velocity potential.

You usually use weather map to do meteorological analysis. That is an analysis in the plane. And if you want to analyze a distribution of divergence, you regularly use the velocity potential.

But, I have already proved that Helmholtz Decomposition is mathematically wrong. You should not use Helmholtz Decomposition, and therefor velocity potentials.

There is another simple way to decompose any flow into two kinds of flows. They are **geostrophic winds (http://nsidc.org/arcticmet/glossary/geostrophic_winds.html)**and **ageostrophic winds (http://en.wikipedia.org/wiki/Geostrophic_wind)**.

Geostrophic winds are calculated from heights, and actual winds consist largely of these geostrophic winds, and these are perfectly non-divergent wind component.

And ageostrophic wind is calculated as a vector difference of actual wind and a geostrophic wind.

Although, it may have a very little part of non-divergent component, you can use this ageostrophic wind instead of divergent wind from velocity potential.

The worst effect of "Helmholtz Decomposition" is that you think that you can analyze the distributions of the flows by only "velocity potential".

You can decompose any flow into geostrophic wind and ageostrophic wind. And at some instance, geostrophic wind is a solenoidal flow. But in the next instance, it may influence ageostrophic wind. I might want to say that there is no solenoidal flows.

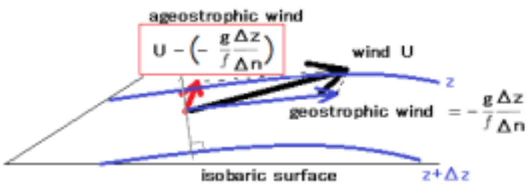
Any way, it is wrong to consider that you can find the cause of convergence in upper layer from only velocity potential.

Ageostrophic winds bring very similar consequents to the wind from velocity potential which are driven from "Helmholtz Decomposition" for the analysis of divergence distributions.

This decomposition which decompose any flows into geostrophic winds and ageostrophic winds has not any problems at all, because it is just applied to basic vector difference.

Below are qaoted from (**http://nsidc.org/arcticmet/glossary/geostrophic_winds.html (http://nsidc.org/arcticmet/glossary/geostrophic_winds.html)**) for geostrophic wind.

> *Geostrophic wind Theoretical wind which results from the equilibrium between horizontal components of the pressure gradient force and the Coriolis force (deviating force) above the friction layer. Only these two forces (no frictional force) are supposed to act on the moving air. It blows parallel to straight isobars or contours*

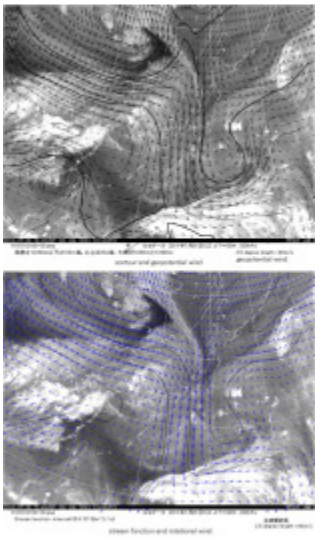


(https://taka19440606.files.wordpress.com/2013/07/another_decomposition.png)
Fig5.3 Another Decompositon

And according to a definition, geostrophic winds blow in a parallel direction with a inversely proportional to interval of contours. The contour of geopotential are supposed to be continuous. So geostrophic winds are supposed to be continuous, and solenoidal winds.

Contours of geopotential looks like stream function from Helmholtz Decomposition theorem.

Actually, we can see that the contour are similar to stream function. For example, I show the weather map in the Asia area at 12Z on July 31 in 2011 inFig.5.4.



(https://taka19440606.files.wordpress.com/2013/07/contour-and-stream-function.jpg)

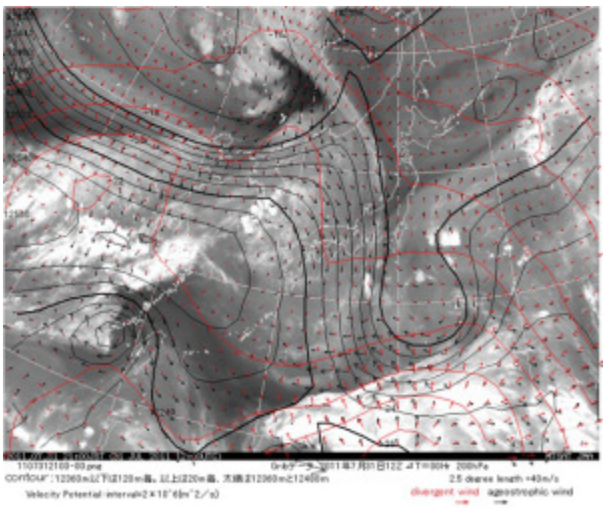
Fig5.4 Similarities between geostrophic wind and the wind driven from stream function

Meanwhile, ageostrophic wind is the vector which is the rest after substructing geostrophic wind from the actual wind. And actual wind blows nearly geostrophic motion.

So, ageostrophic wind is generally small, but it has all divergent component of the actual wind.

So, ageostrophic wind is similar to divergent wind from Helmholtz Decomposition theorem.
But, ageostrophic wind has not potentials like velocity potentials.

Fig5.6 shows the similarities between ageostrophic wind and divergent wind driven from velocity potential in the vicinty of large clouds seen in the south of Japen.

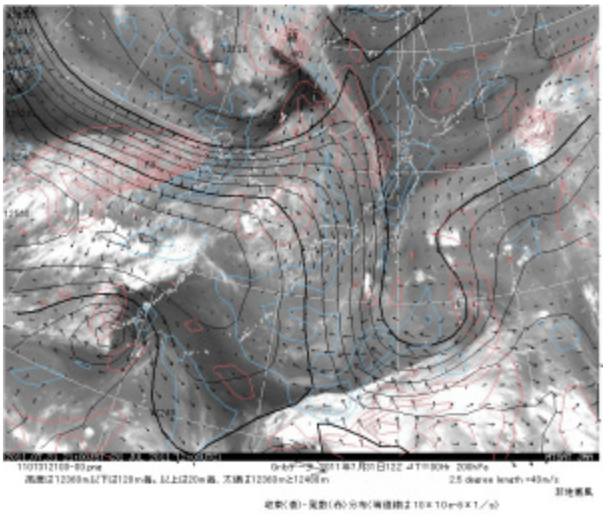


(https://taka19440606.files.wordpress.com/2013/07/ageostrophic-wind-and-divergence-wind.jpg)

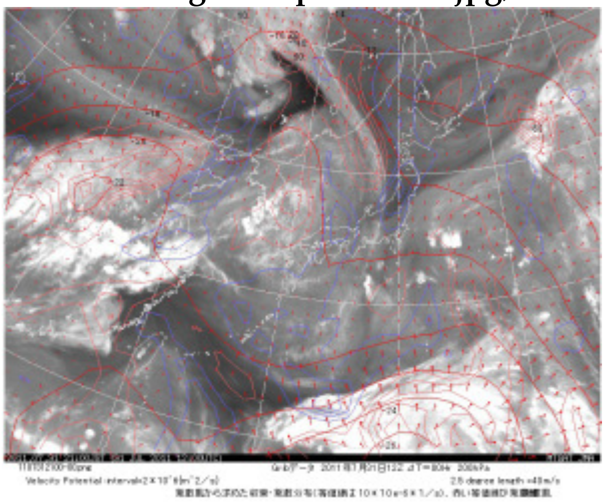
Fig5.5 similarities between ageostrophic wind and divergent wind driven from velocity potential

By this decomposition, ageostrophic wind has all of divergence component of actual(or analysis) wind. And divergent wind is supposed to have all of divergence too.

So, The distributions of divergence from both ageostrophic winds and divergent winds from velocity potentials are supposed to be almost the same.



(https://taka19440606.files.wordpress.com/2013/07/2011073112-00-20distribution-of-divergence-from-ageostrophic-wind.jpg)



(https://taka19440606.files.wordpress.com/2013/07/2011073112-00-20distribution-of-divergence-from-divergent-wind.jpg)

Fig5.6 Conparison of distributions of divergences by using two types of winds
upper: from ageostrophinc wind,
lower: from divergent wind from “Helmholtz Decomposition”

These divergent distributions are drawn on the water vapor imagery. The plus divergence of the upper layer are closely-linked to clouds, and minus divergence(convergence) are closely-linked to black area.

Whichever wind you choose to calculate the distribution of divergent, you can get almost the same consequence.

But if you choose the divergent wind from Helmholtz Decomposition, it is the end.

If you choose ageostrophic wind, you can go **more (http://blogs.yahoo.co.jp/taka19440606/36780217.html)**.

Leave a Comment

APRIL 30, 2013

The New Model on the Hadley Circulation

Summary

The formation of the subtropical jet ,the subtropical heigh and the **Hadley circulation (http://en.wikipedia.org/wiki/Atmospheric_circulation)** must be explained without contradiction. By introducing the theory of ageostrophic wind into the general circulation, I would like to propose the new Hadley circulation model.

In the modern meteorology air motion is thought to be the one in which the geostrophic approximation holds good because air undergoes the geostrophic adjustment. However, the result of the objective analysis in the present numerical forecasting model shows ageostrophic wind element clearly. The present numerical forecasting uses the primitive model. Judging from its accuracy, I can say that the primitive model represents the real air motion almost exactly. The analyzed wind in that model has clear ageostrophic wind element. So it is impossible to think of the real air as the one in which the geostrophic approximation holds.

And in the present general circulation model the relations between the Hadley circulation and the subtropical jet contradict each other. But taking ageostrophic motion into account, I have found the Hadley circulation model which includes the subtropical jet.

Some meteorologists insist that there is no contradiction between them. The reason is that both the descending branch of the Hadley circulation and the Ferrel circulation in the westerly wind belt cause the concentration of heat and then form the subtropical jet. According to them, air which causes the thermal concentration just releases the heat and then descends. Their theory is as old as the idea of thermal element at the beginning of the 19th century and cannot explain heat conduction reasonably. In the real meridional cross section, you often find the subtropical jet which cannot be explained by the theory of thermal wind.

By introducing ageostrophic motion, I would like to propose the new general circulation model in which the formation of the subtropical jet and the Hadley circulation can be explained reasonably. First, I will talk about ageostrophic motion.

＜1. Ageostrophic Motion＞

Suppose you put an air parcel quietly on a surface which has the pressure gradient and the air parcel has the same density as the surrounding air at that height. How does this air parcel behave? To make things easier, I assume that the surrounding air never changes its pressure gradient and the air parcel never mingles with the surrounding air.

The forces which act on the air parcel are the pressure gradient and Coriolis forces. At first the Coriolis force doesn't act on it, because the speed of the air parcel is zero. Only the pressure gradient force acts on it. So the air parcel starts to sink perpendicular to contours of height.

The following equation of motion can be obtained:

$$\partial \mathbf{V} / \partial t = -f \cdot \mathbf{V} - g \cdot \nabla h$$

where **V** is the air parcel's velocity after t time and f, g and ∇h denote the Coriolis parameter, the acceleration of gravity and the gradient of the geopotential height, respectively.

Geostrophic wind at this moment Vg gives

$\partial \mathbf{Vg} / \partial t = 0 = -f \cdot \mathbf{Vg} - g \cdot \nabla h$

Subtracting the latter from the former leads to

$\partial(\mathbf{V} - \mathbf{Vg}) / \partial t = -f \cdot (\mathbf{V} - \mathbf{Vg})$

This equation reduces to

$\partial \mathbf{A} / \partial t = -f \cdot \mathbf{A}$ where a vector **A** keeps on rotating with a frequency f. Its period T is

$2 \cdot \pi / f = 2 \cdot \pi / 2 \cdot \omega \cdot \sin\varphi \fallingdotseq 12 / \sin\varphi$

where φ means latitude and ω denotes the rotating angular velocity of the earth, which is equal to $2 \cdot \pi / 24$ hours. So at 30°N the rotation has a 24-hour period.

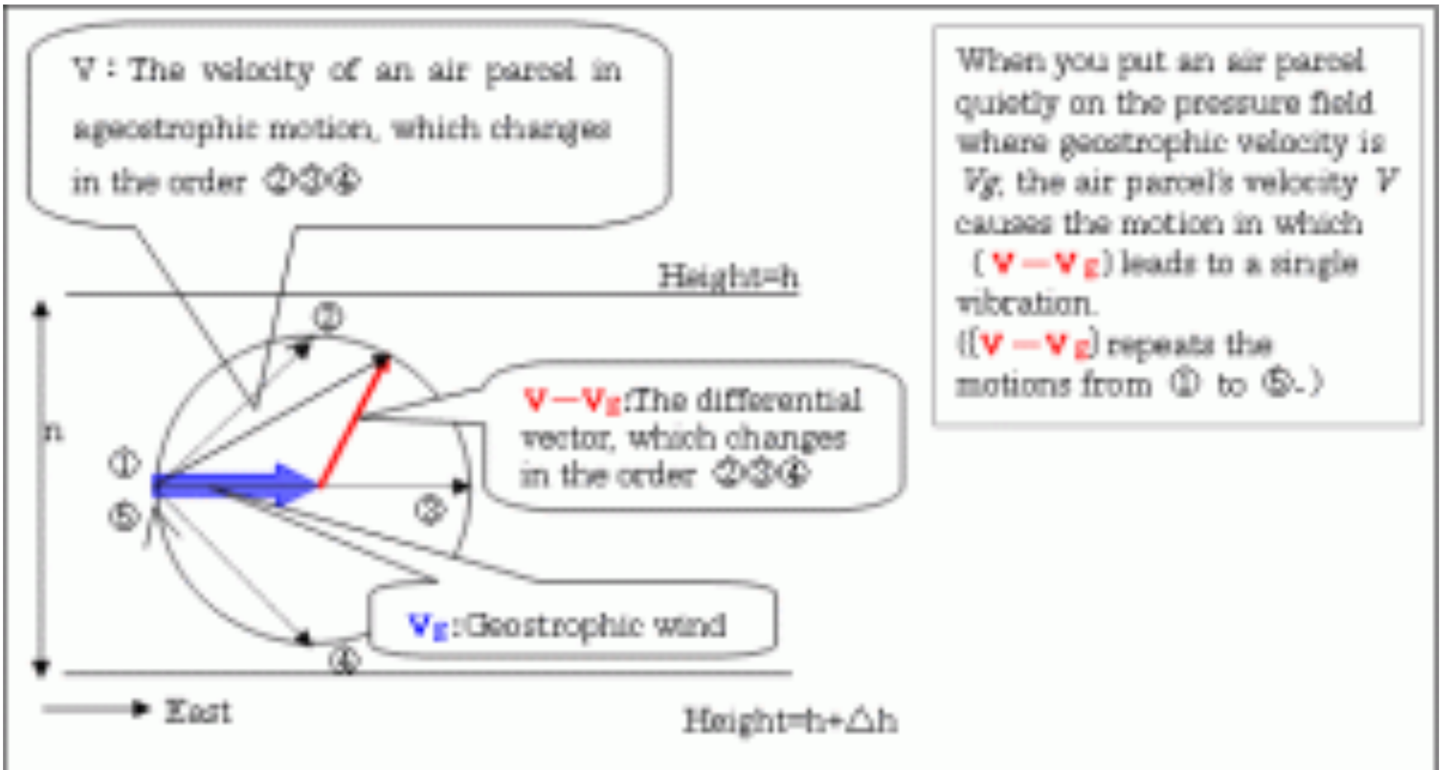


Figure 1. The differential vector which causes a single vibration

(https://taka19440606.files.wordpress.com/2013/04/fig11.gif)

Fig. 1 shows what the above equations mean.

1. ① : An air parcel is at rest on the pressure field.
2. ② : First, it starts to move toward low height perpendicular to height contours in response to the pressure gradient force, but as soon as motion develops, the Coriolis force also acts on it. So the parcel moves acceleratingly with both forces acting on it. The pressure gradient force remains unchanged, whereas the Coriolis force acts deflecting the parcel's motion toward the right in proportion to its speed. So descending along contours of isobaric height, the parcel gradually accelerates parallel to contours of height.
3. ③ : Eventually the parcel's motion becomes parallel to height contours and has the same direction as geostrophic wind's. Converting potential energy into kinetic energy, the parcel moves downward along the slope of the pressure surface and its speed becomes twice as high as

geostrophic wind's. The Coriolis force also becomes twice as strong as the one needed for geostrophic balance and acts on the parcel the way it makes the parcel move upward perpendicular to contours of height, just as strongly as the force acting on the initial motionless air parcel, but reversely.

4. ④ : The parcel's direction is gradually deflected to the right and the parcel decelerates upward across contours of isobaric height.
5. ⑤ : Getting to its original height, the parcel is again at rest for a moment. Then the parcel repeats the motions from ① to ⑤.Fig. 2 shows one period of ageostrophic motion and the forces acting on it. There contours of isobaric height are not drawn in straight lines but in curved lines.Fig. 2 shows one period of ageostrophic motion and the forces acting on it. There contours of isobaric height are not drawn in straight lines but in curved lines.

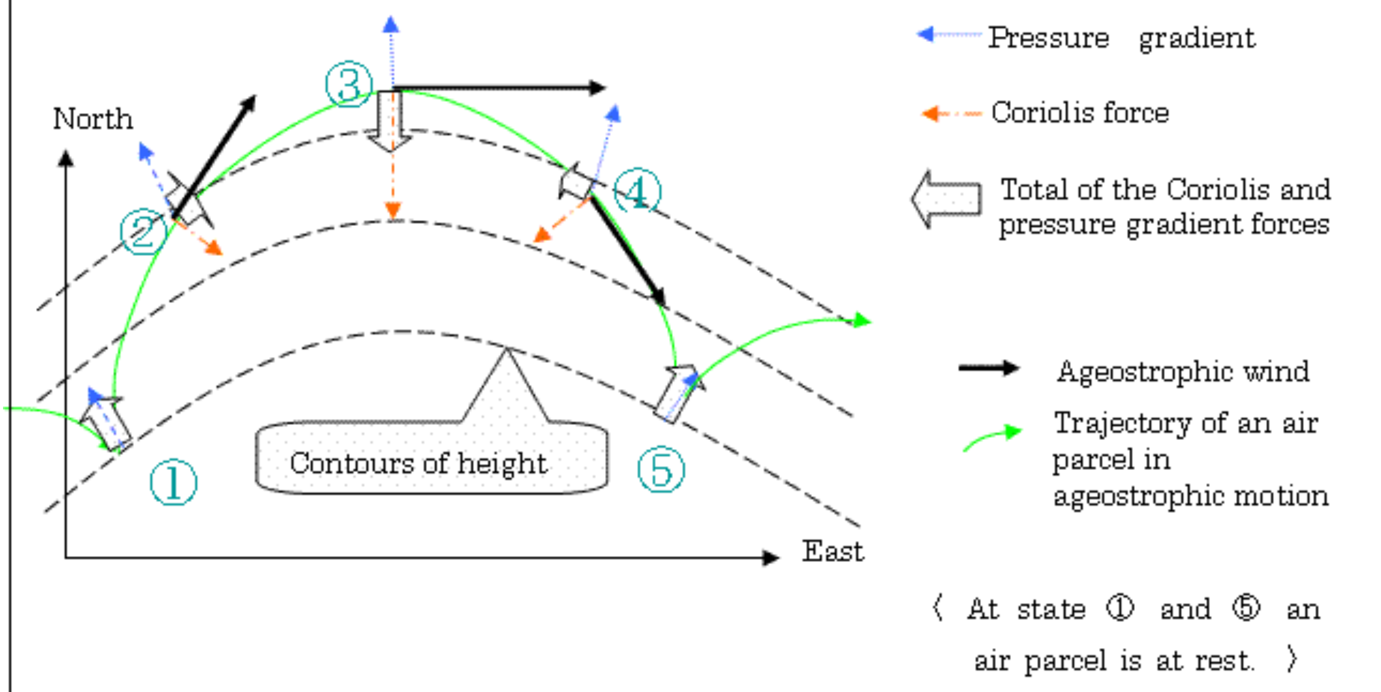


Figure 2. Forces acting on an air parcel in ageostrophic motion.

Fig. 3 shows ageostrophic wind, geostrophic wind and their differential vectors.

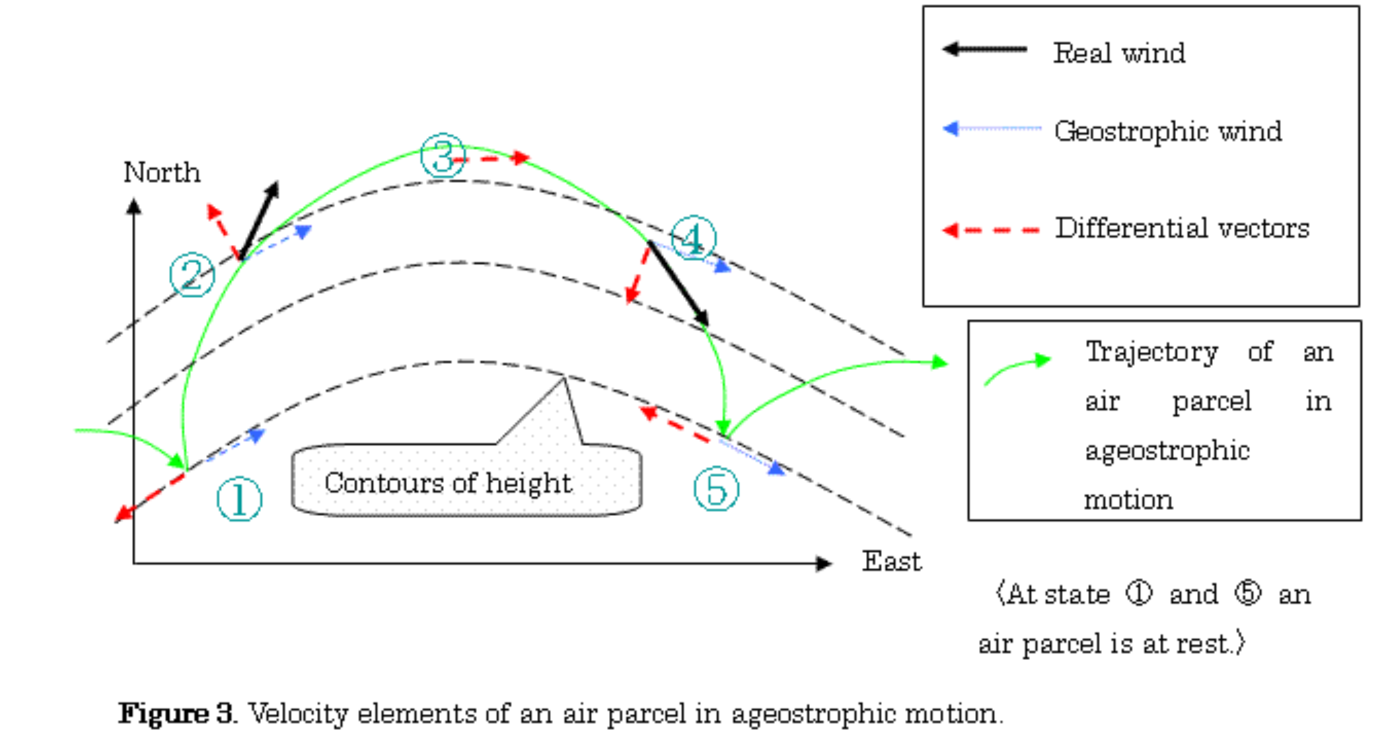


Figure 3. Velocity elements of an air parcel in ageostrophic motion.

The motion illustrated with green arrows in Fig. 3 is characteristic of ageostrophic motion. When isobaric surfaces surround the Northern Hemisphere high in the south and low in the north, the motion follows the trajectory in Fig. 4.

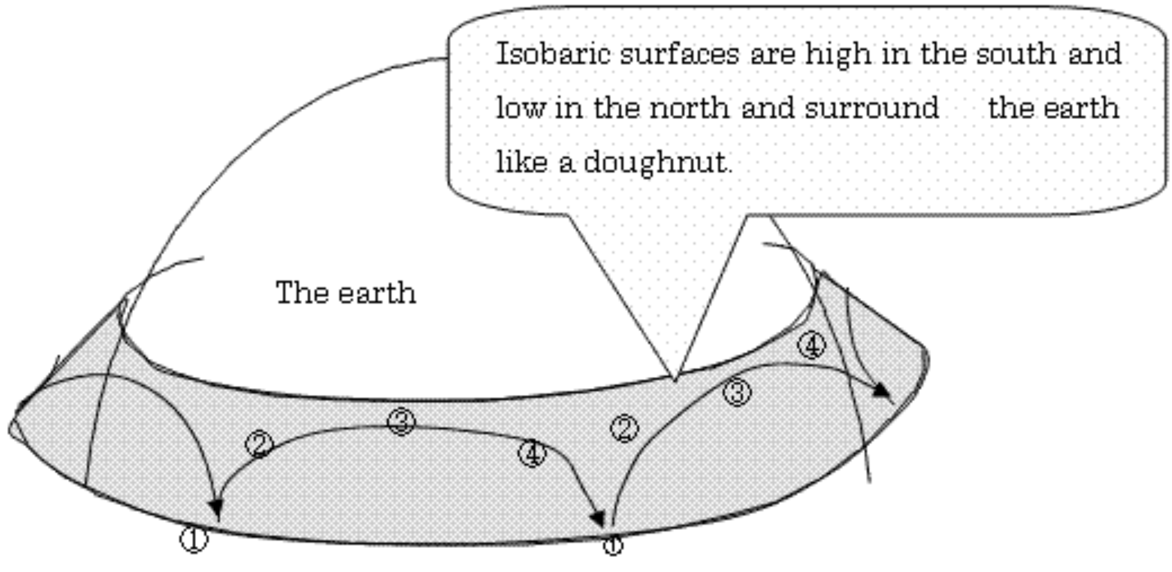


Figure 4. Trajectories of an air parcel which is put quietly on the height field whose meridional slope is fixed.

Now I would like to calculate how much kinetic energy an air parcel obtains when evolving from its motionless state ① to state ③ where the speed reaches maximum. The force a unit volume of air parcel undergoes and the distance it covers constitute work. The energy of this work is converted into kinetic energy. Of the two forces the Coriolis force is perpendicular to the direction of the motion and does not contribute to the work. (refer to Fig. 5)

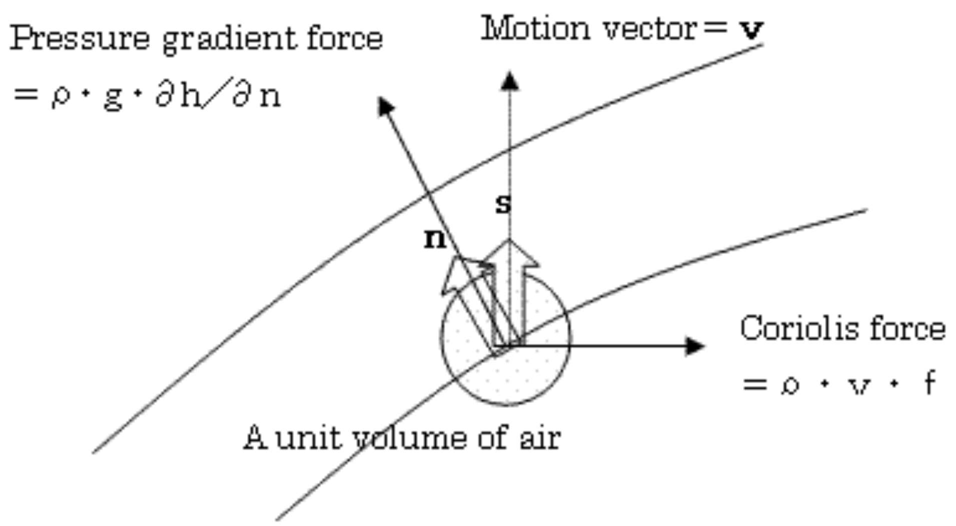


Figure 5. *Work* provided to the pressure gradient force during ageostrophic motion.

Therefore, kinetic energy obtained by a unit volume of air parcel during this motion is provided only by the pressure gradient force. The pressure gradient force is always perpendicular to contours of height. So the total amount of the work during this motion is obtained by integrating from ① to ③ (in Fig.4) the inner product of the pressure gradient force and the line segment along the path of the air parcel.

The amount of work given is

$$\int -\rho \cdot g \cdot \partial h / \partial n \cdot ds \quad = \rho \cdot g(\text{height ①} - \text{height ③})$$

where ρ is the density of air, h is the height of isobaric surface and n and s denote the unit vector directed to the steepest slope of isobaric surface and the unit vector directed to the path of the parcel, respectively.

This equation means that the energy produced during the parcel's transferring from ① to ③ is equal to the lost amount of potential energy. This amount of work becomes kinetic energy, which is $1/2 \cdot \rho \cdot v2$

That means an air parcel in ageostrophic motion obtains kinetic energy by being compressed by the pressure gradient force, and this kinetic energy is equal to the potential energy which is lost while moving down the isobaric surface. In other words, an air parcel obtains the velocity by moving on isobaric surface and lowering its height.

<2. Ageostrophic Wind in the Real Atmosphere>

The characteristic of the ageostrophic motion is explained to some extent in the former section but that explanation is based on the unreal assumption. Through observing the behavior of the real atmosphere I would like to examine how much near?ageostrophic winds are blowing. The materials I use are the followings.

Grid point data on the Internet which are based on the materials of numerical forecasting by Japan Meteorological Agency. (**http://ddb.kishou.go.jp/gpvftp.html (http://ddb.kishou.go.jp/gpvftp.html)**)
Global cloud image published on the Internet by WSI. Co., U.S.A. (**http://www.intellicast.com/LocalWeather/World/United (http://www.intellicast.com/LocalWeather/World/United)** States/World/)

Fig. 6 illustrates the analyzed local wind velocity **V**, the geostrophic wind **V**g, and the difference between **V** and Vg , that is, **V** − **V**g , on the 200 − hPa isobaric surface on June 5, 2001.

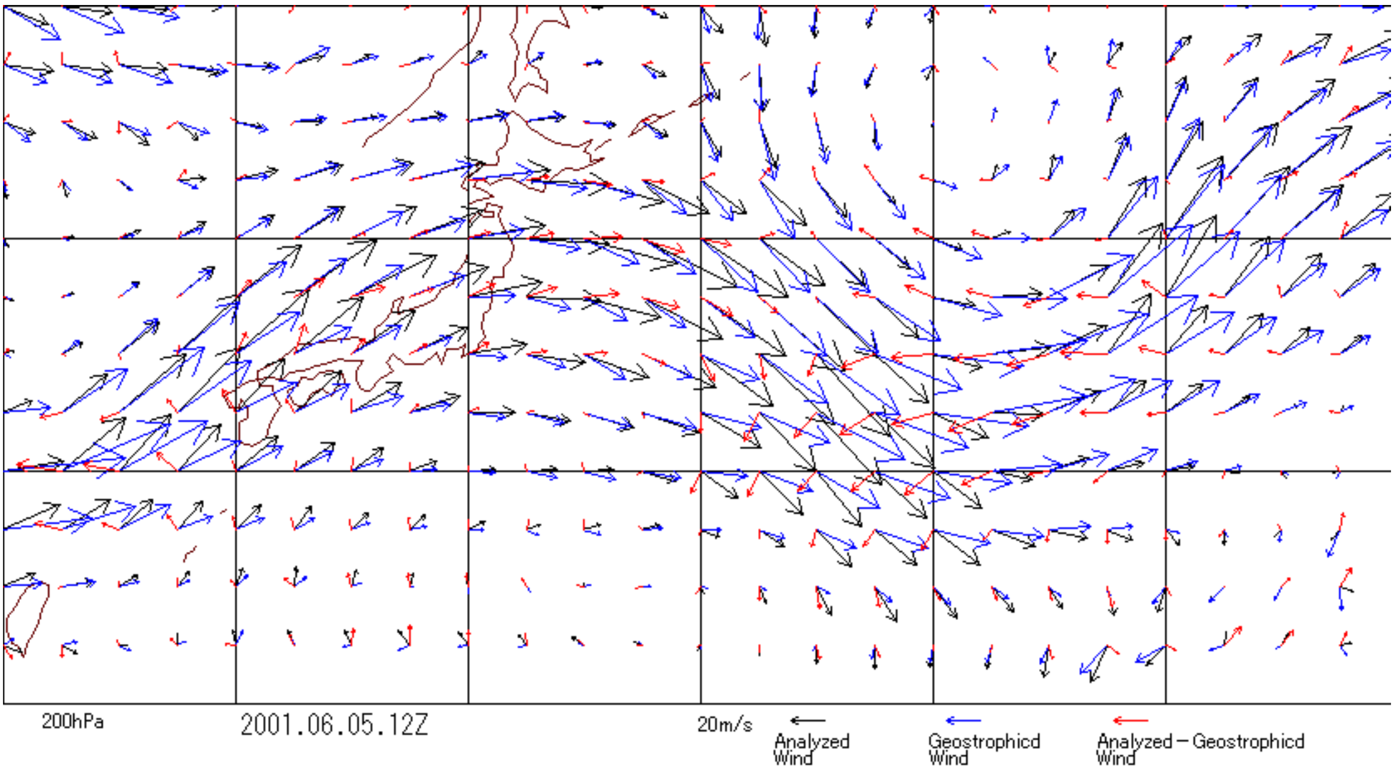


Figure 6. The analyzed winds (black arrows), the geostrophic winds (blue arrows) and the differential vectors(red arrows).

I got the zonal elements and the meridional elements of the analyzed wind velocity from the grid point information of Japan Meteorological Agency. And the zonal elements Ug and the meridional elements Vg of the geostrophic velocity can be obtained from the following equations.

$$Ug = g/f \cdot \partial\varphi/\partial n \qquad = g/f \cdot (\varphi k-1-\varphi k+1)/(2\times a\times \cos(1.25^{\circ}))$$

$$Vg = g/f \cdot \partial\varphi/\partial n \qquad = g/f \cdot (\varphi j-1-\varphi j+1)/(2\times a\times \cos\theta\times \cos(1.25^{\circ}))$$

where n,φ, a and θ denote horizontal distance, geopotential height of isobaric surface, the earth’s radius and latitude, respectively.

In Fig. 6 the magnitudes of the wind velocity and the differential vectors (red arrows) are shown in proportion to the length of arrows. The length of arrows drawn below the chart is equal to the velocity of 20m/s.

If you can find the geostrophic and quasi－geostrophic motions in the real atmosphere, the differential vectors should be very small, but in a lot of regions they are over 20m/s.

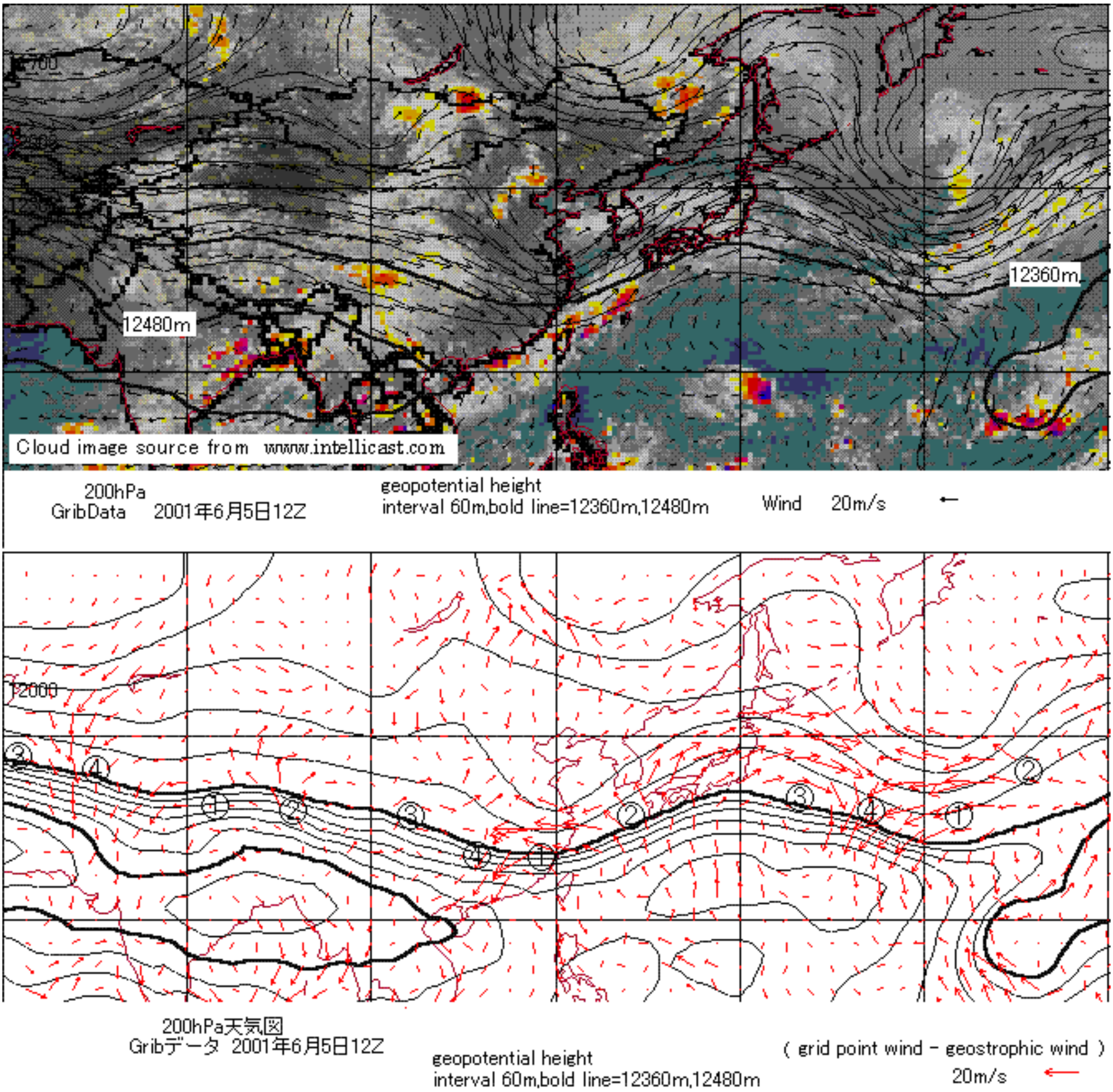


Fig.7 The behavior of Ageostrophic wind

In Fig. 7, the differential vectors V − Vg are illustrated on the chart of the distribution of isobaric heights (Bottom). Figures ①②③ and ④ on the chart correspond to those in Fig.3 and you can see the real atmospheric behavior having a single vibration or rotation.

According to the modern meteorology, the atmosphere on the earth undergoes the geostrophic adjustment. Even if it is initially in the ageostrophic motion, it soon changes into the geostrophic motion because ageostrophic elements are carried away from the region by inertial waves and external gravity waves. If this theory is right, how can ageostrophic elements have such magnitudes as shown in Figs. 6 and 7?

When they estimate the geostrophic adjustment, they assume the rigid wall as the meridional boundary condition in calculating how long it takes ageostrophic elements to be carried away. But in the region between the westerly wind belt and the tropics, meridional elements distinctly cross the height contours, so their assumption is not adequate. There is a clear defect in their estimation of the geostrophic adjustment.

<3. Regions of Deep Convection in the Tropics and the Subtropical High Pressure> In the illustrations on June 5, 2001 (Figs. 6 and 7), you can find ageostrophic winds in the tropics and in the southern part of the westerly wind belt. And you can find them in the real atmosphere almost every day. What causes this ageostrophic motion?

<3.1 An Infant Subtropical High>

The isobaric surfaces are generally flat near the tropics. Air lifted by convective motions there diverges at first into every direction. On account of the conservation of angular momentum, the westerlies (air flowing out poleward) or the easterlies (air flowing out equatorward) get stronger, so especially the meridional flow of air diverges and keeps diverging. The Coriolis force acts on the air which flows out poleward in proportion to its velocity, so the air is gradually deflected westward and then equatorward. As a result, almost occluded circle of air streams is formed and the air is gathered there. If the convective motion is maintained, height of the northeastern part of the convective regions is elevated and a high pressure appears there. This seems to be an infant subtropical high. (refer to Fig. 8)

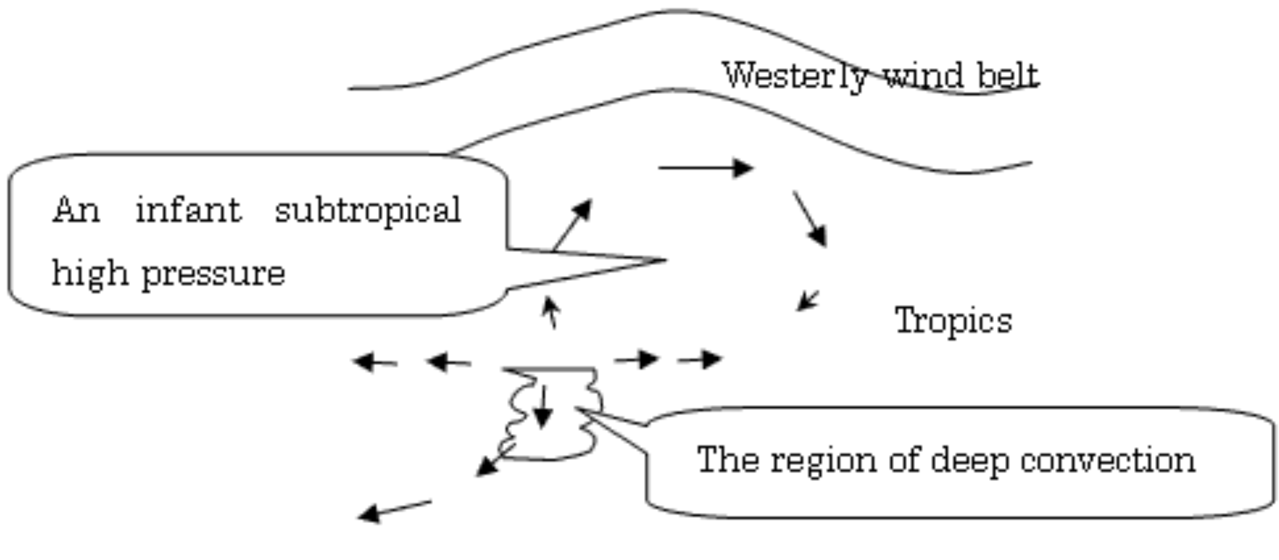


Figure 8. An infant subtropical high pressure.

<3.2 The Subtropical High Pressure and the Subtropical Jet>

The deep convection is maintained in specific regions, such as in the areas around Indonesia from March to May. After a distinct subtropical high appears, convective motion is maintained. To put it in another way, regions of deep convection are established on the southwestern edge of the subtropical high.

Air lifted by cumulus convection developed along the southern part of the subtropical high drives ageostrophic motion and the motion is restricted by the pressure gradient around the subtropical high. Consequently, the stream involving the subtropical jet appears, which is shown in Fig. 9. Horizontal scale of an infant subtropical jet is determined, as told in the theory of the ageostrophic wind, by the horizontal circulation whose period is 12／sinφ{hour}, where φ is latitude.

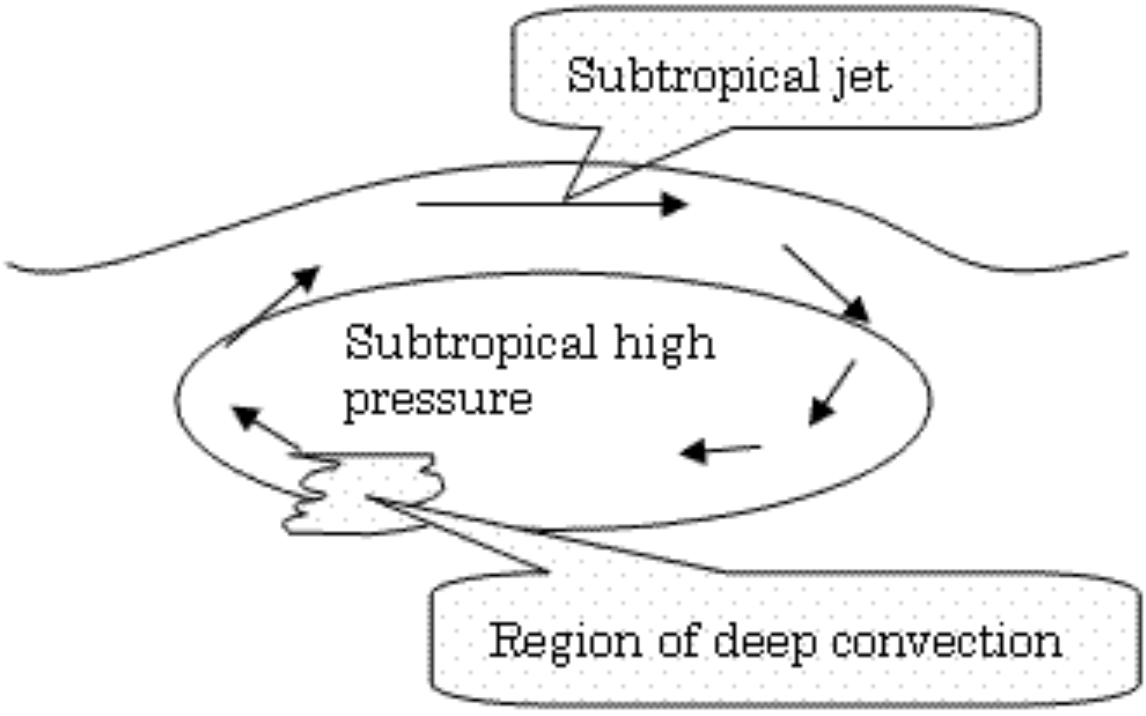


Figure 9. The subtropical high and the subtropical jet.

<3.3 Ageostrophic Wind Entering the Westerly Wind Belt>

Ageostrophic motion which starts in the tropics crosses height contours, goes into the westerly wind belt and exchanges momentum with the stream in the belt. So it forces ageostrophic motion upon the stream in the westerly wind belt.

The stream in the westerly wind belt which starts ageostrophic motion becomes super − geostrophic wind, and then crosses height contours upward by Coriolis force that is over geostrophic balance. So the stream forms the convergence field in the tropics inside of jet axis, as illustrated in Fig. 10.

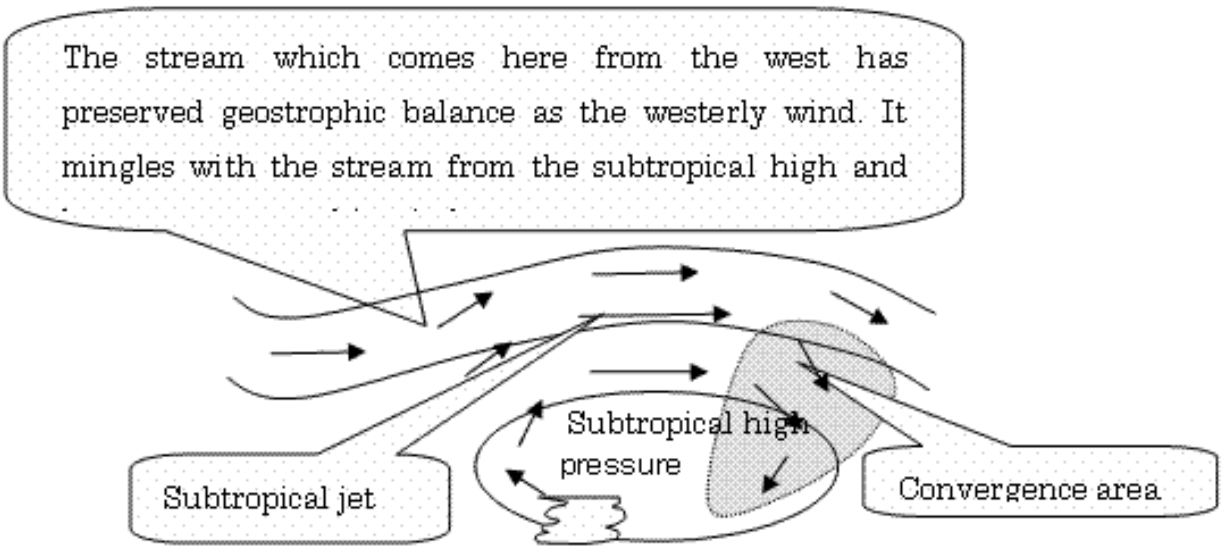


Figure 10. The stream in the westerly wind belt which flows near the subtropical high.

Fig. 10 illustrates the schematic of the stream, whereas Fig. 11 shows an example of the observation on June 5, 2001. Divergence D is obtained from the following equation.

$$D = \frac{1}{A\cos\theta}\frac{\partial u}{\partial \lambda} + \frac{1}{A\cos\theta}\frac{\partial(v\cdot\cos\theta)}{\partial\theta}$$

where A is the earth's radius, u and v denote zonal elements of the velocity and meridional elements of the velocity, respectively, and θ and λ refer to latitude and longitude.

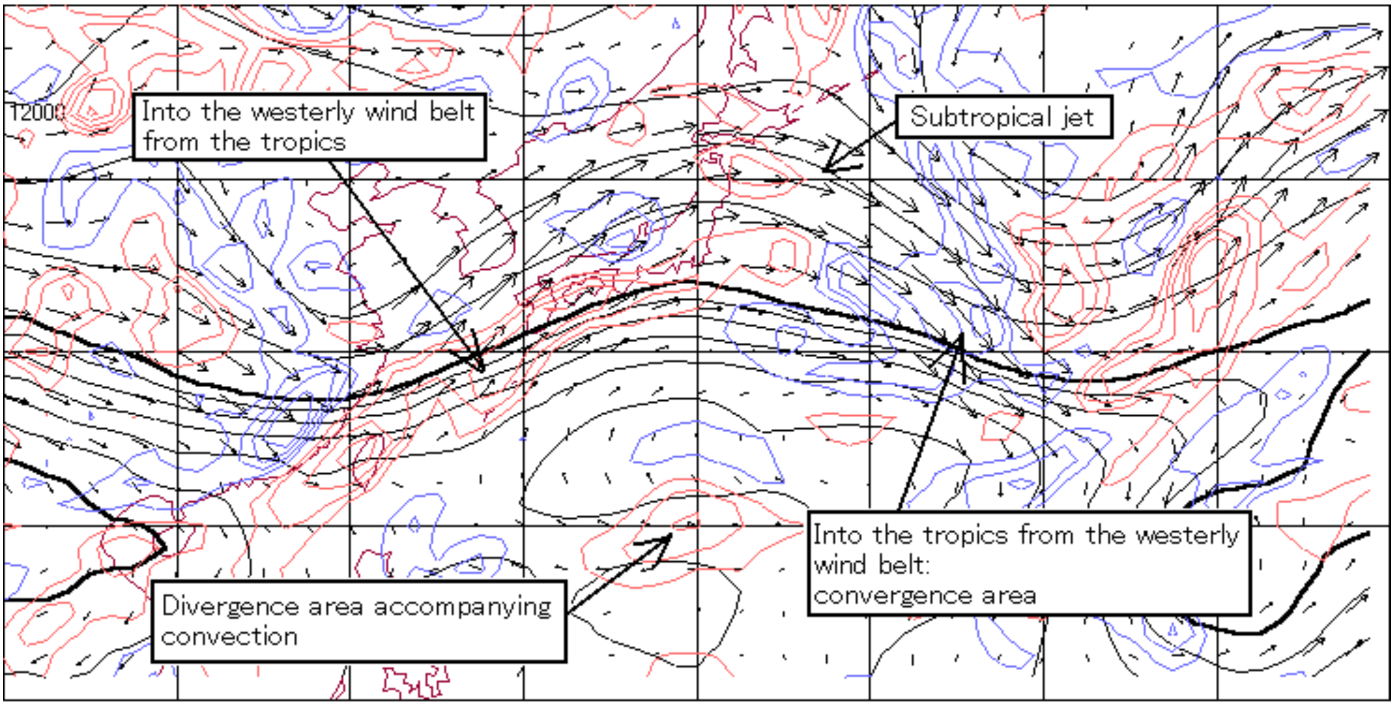


Figure 11 Ageostrophic winds and distributions of divergence (red area) and convergence(blue area) at 200hPa on June 5,2001

Units of divergence (red lines in Fig. 11) and convergence (blue lines) are both 10^{-5} sec^{-1} .

<4. The Hadley Circulation>

Where air converges in the upper troposphere, the pressure is higher than that of the surrounding regions from the bottom to the top of the troposphere. Near the surface, the pressure is also higher than the surrounding areas, and if that region locates in the center of a high, the high pressure intensifies. If it is not in the center, the anticyclonic curvature sharpens.

Near the surface, the friction makes winds move across isobars toward low pressure. If the pressure near the center of a high increases, it is clear that divergence intensifies.

Even if the area under the upper convergence field doesn't locate near the center of a high, the pressure there is higher than the neighboring regions, and the anticyclonic curvature of isobars sharpens. In the friction layer, when the velocity doesn't change and the anticyclonic curvature sharpens, divergence intensifies.

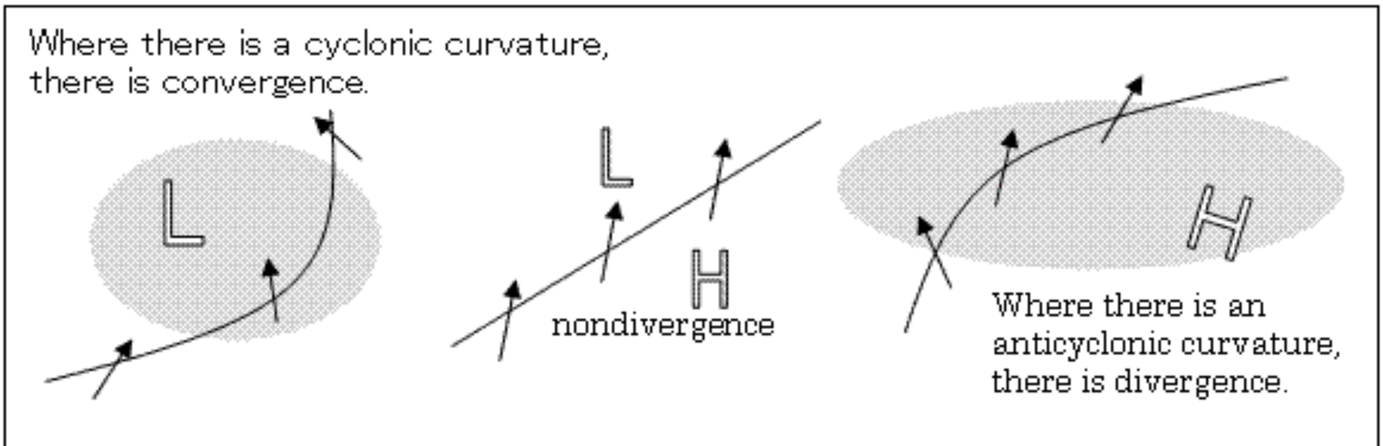


Figure 12. In the friction layer there tend to be divergence under an anticyclonic curvature and convergence under a cyclonic curvature.

curvature and convergence under a cyclonic curvature. When air converges in the upper troposphere, the pressure of the whole atmospheric column increases, so air diverges near the surface.

Near the surface the pressure gets higher than the surrounding regions and an anticyclonic curvature sharpens. Under an anticyclonic curvature the friction makes air flow toward low pressure and diverge. Convergence in the upper troposphere and divergence in the lower friction layer cause subsidence. And air from the subtropical high pressure partly returns to its original area. Figs. 13 and 14 illustrate the above facts.

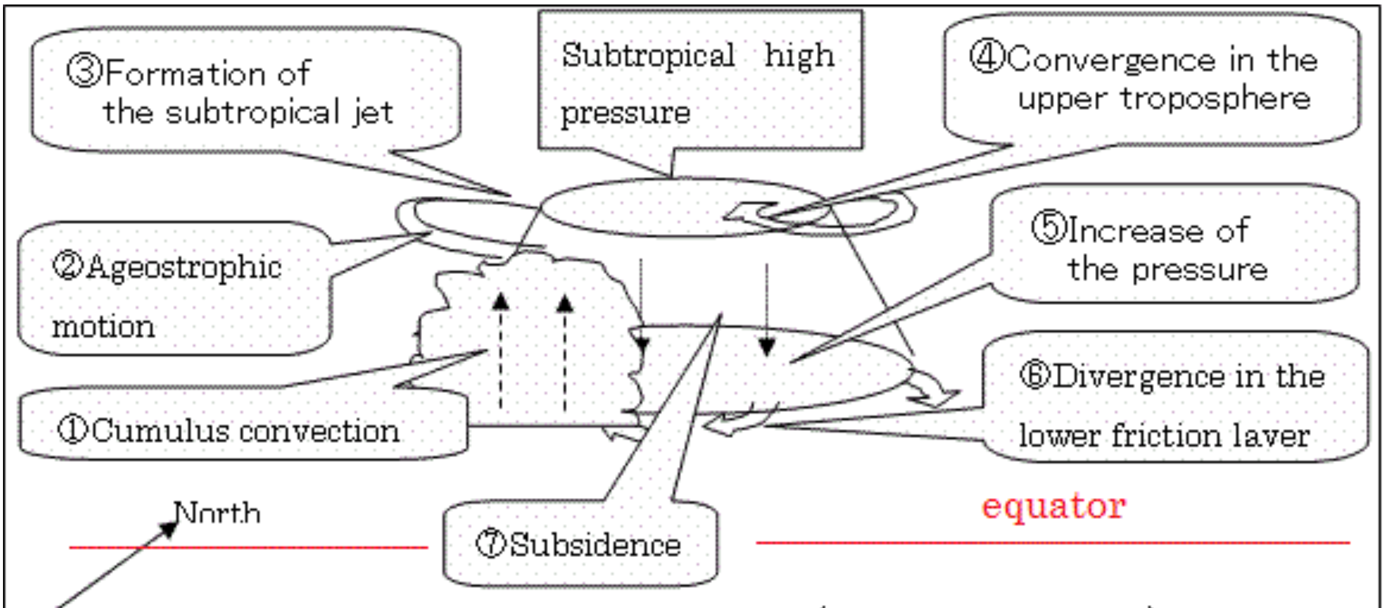


Figure 13. The Hadley circulation model (seen from the south).

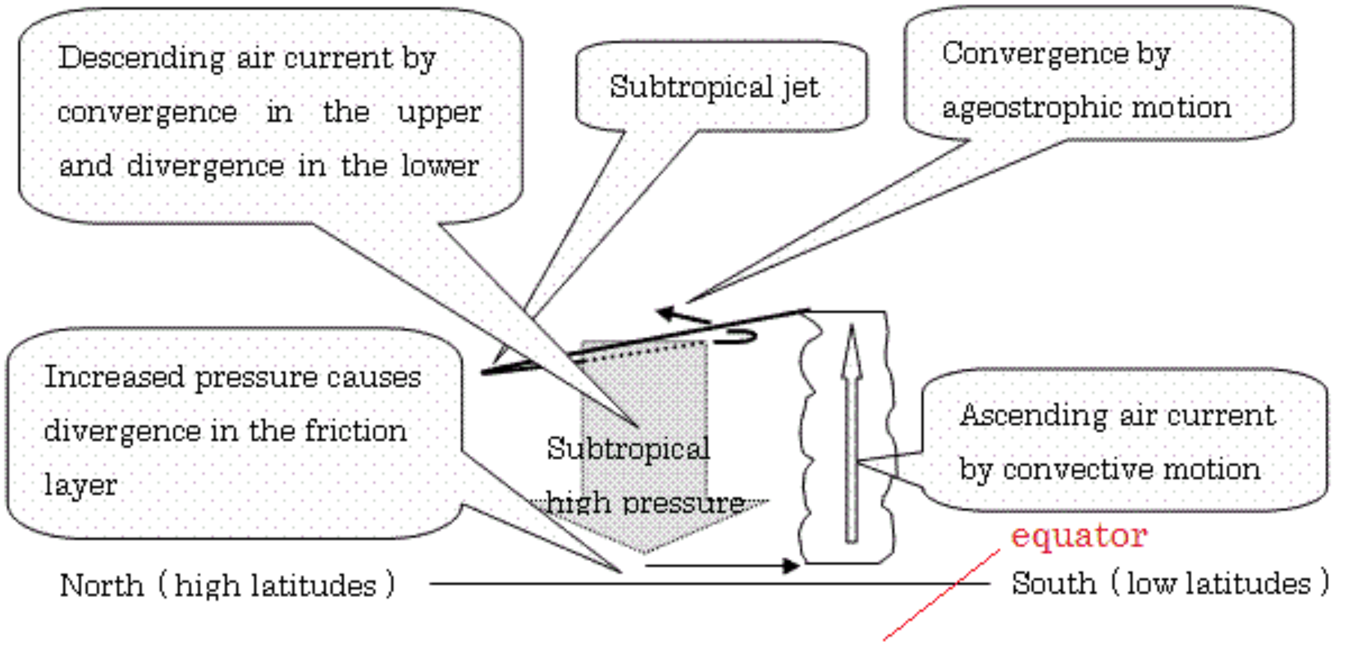


Figure 14 The Hadley circulation in the Northern Hemisphere (seen from the west).

In the present Hadley circulation model, most people believe that the high pressure is maintained by subsiding air in the descending branch of the circulation at high latitude. But the descending air current doesn’t have a direct relationship to the pressure, nor does it maintain the high pressure. The mechanism of maintaining the subtropical high pressure has not been made clear.

In my new circulation model, I can explain this mechanism clearly. That is, when the difference between the amount of convergence in the upper troposphere and that of divergence in the lower troposphere is positive, the subtropical high pressure intensifies, and when it is negative, the high pressure weakens.

<Conclusions>

Until today it was believed that in the Hadley circulation, air lifted at low latitude moved toward high latitude and descended there. So the Hadley circulation was inconsistent with the subtropical jet.

But in the real atmosphere, as explained, ageostrophic component is too strong to ignore. Thinking that there is ageostrophic air current, I have found that air which moves toward high latitude, lowering the slope of isobaric surface, accelerates at first by the pressure gradient force and after becoming super－geostrophic wind called “the subtropical jet”, by using its energy, air runs up the mountain of the subtropical high. In the general circulation, it was said, air must descend after it diverged. But instead of descending, air compresses the whole mountain by running up the mountain. This mechanism can produce the circulation.

About the mechanism maintaining the height field of the subtropical high there was often a misunderstanding that the intensity of the meridional circulation, namely, the intensity of the ascending or descending air current was related with the power of the subtropical high. But I should estimate the ebb and flow of the subtropical high by estimating the convergence and divergence through the whole atmospheric column. About this mechanism of maintaining the subtropical high, I believe I could construct the satisfactory theory.

During the northern summer most of the air current which diverges in the upper troposphere at low latitude flows out into the Southern Hemisphere. In spite of that, the power of the subtropical jet in the Northern Hemisphere is still strong. This fact can be explained by estimating the difference between the amount of convergence in the upper troposphere and that of divergence in the lower troposphere. In the Southern Hemisphere most of the convergence region in the upper troposphere locates at low latitude where the Coriolis force is weak, so it is easy for air to diverge in the lower troposphere. This means the difference between the amount of convergence in the upper troposphere and that of divergence in the lower troposphere is small, so in spite of the strong influence of the circulation, you cannot find the distinct subtropical high. These are my conclusions.

＜References＞

i. Nitta, T. The General Circulation of the Atmosphere (Tokyo－do Pub., Tokyo, 1980)
ii. Kurihara, N. Introduction to Atmospheric Dynamics (Iwanami, Tokyo, 1979)
iii. Aihara, M. Basic Equations and Theory of Energy Meteorology Research Paper No. 134, 24 ~ 32 pp. (1978)

Leave a Comment

APRIL 5, 2013

Helmholtz Decomposition is wrong (image version 1/2)

Helmholtz Decomposition is wrong (https://taka19440606.wordpress.com/2013/04/04/helmholtz-decomposition-is-wrong-image-version-12/).

Leave a Comment

APRIL 4, 2013

Helmholtz Decomposition is wrong (image virsion 2/2)

get back (https://taka19440606.wordpress.com/2013/04/04/helmholtz-decomposition-is-wrong-image-version-12/)

I can show you that there are many flows which do not include **W**.

You have got following expression from the vector triple product identity.

$\mathbf{V}=\nabla\chi+\nabla\times\mathbf{A}$

It is given on terms and conditions as required by $\chi=\nabla\cdot\mathbf{W}$, $\mathbf{A}=\nabla\times\mathbf{W}$

That is, χ and **A** are deriven from the common function **W**.

If χ is decided from **W**, then **A** should be decided uniquely at the same instance.

And, the flow given by $\nabla\times\mathbf{A}$ is supposed to exist independently from other flows.

It is called solenoidal flow. That means a flow like in the tube. According to Helmholtz Decomposition there exists such a flow.

I do not think that such a solenoidal flow exists in the real world .

(https://taka19440606.files.wordpress.com/2013/04/hd_7.png)

If there were such solenoidal flows, I would be able to show the collapses of Helmholtz decomposition.

Assuming that Helmholtz decomposition theorem is correct. you can consider two flows as following.

$\mathbf{F}_1 = -\nabla^2 \mathbf{W}_1 = \nabla \cdot \chi_1 + \nabla \times \mathbf{A}_1$

$\mathbf{F}_2 = -\nabla^2 \mathbf{W}_2 = \nabla \cdot \chi_2 + \nabla \times \mathbf{A}_2$

χ_1 and $\mathbf{A}_1$ are functions which are derived from $\mathbf{W}_1$. and, χ_2, and $\mathbf{A}_2$ are derived from $\mathbf{W}_2$.

Here, because an arbitrary flow (vector function) must be possible, you can consider the flow $\mathbf{F}_3$ which includes divergent component of $(\nabla \cdot \chi_1)$ and rotational component of $(\nabla \times \mathbf{A}_2)$.

(https://taka19440606.files.wordpress.com/2013/04/hd_8.png)

$\mathbf{F}_3 = \nabla \cdot \chi_1 + \nabla \times \mathbf{A}_2$

Here, $\chi_1 = \nabla \cdot \mathbf{W}_1$, $\mathbf{A}_2 = \nabla \times \mathbf{W}_2$

I must say that again $\mathbf{F}_3$ should have $(\nabla \cdot \chi_1)$ as divergent component, and have $(\nabla \times \mathbf{A}_2)$ as rorational component.

But according to Helmholtz decomposition, $\mathbf{F}_3$ can be decomposed into two flows only by the vector triple product identity.

Then,

$\mathbf{F}_3 = -\nabla^2 \mathbf{W}_3 = \nabla \cdot \chi_3 + \nabla \times \mathbf{A}_3$

Therfor, $\mathbf{F}_3$ has $(\nabla \cdot \chi_3)$ as divergent component, and has $(\nabla \times \mathbf{A}_3)$ as rorational component.

(https://taka19440606.files.wordpress.com/2013/04/hd_9.png)

But, because $\mathbf{F}_1 \neq \mathbf{F}_3$, and $\mathbf{F}_2 \neq \mathbf{F}_3$

$\mathbf{W}_3 \neq \mathbf{W}_1$, $\mathbf{W}_3 \neq \mathbf{W}_2$

So,

$\mathbf{F}_3 = -\nabla^2 \mathbf{W}_3 \neq -\nabla(\nabla \cdot \mathrm{W}_1) + \nabla \times (\nabla \times \mathbf{W}_2)$

Here, you must say that $\mathbf{F}_3$ which has combined with divergent component of $(\nabla \cdot \chi_1)$ and rotational component of $(\nabla \times \mathbf{A}_2)$ can not decomposed into divergent component of $(\nabla \cdot \chi_1)$ and rotational component of $(\nabla \times \mathbf{A}_2)$.

Or we may have to say that there is no $\mathbf{W}$ functions in $\mathbf{F}_3$.

I can show you many flows like this kind of flow.

(https://taka19440606.files.wordpress.com/2013/04/hd_10.png)

---

postscript

Electromagnetics is out of my hands.

But, I may say that the vector triple product identity is applicable just in electromagnetics.

Because, there exists χ as electric field, and $\mathbf{A}$ as magnetic field in real world. There is not such phisical potential in hydrodynamics.

In Electromagnetics, there is dynamics in the electric field and in the magnetic field. But there is not dynamics in velocity potential and in vector potential.

If you use ageostrophic winds as substitute for the winds from $\nabla\chi$, there is dynamics, and you can calculate the divergence of the winds.

(https://taka19440606.files.wordpress.com/2013/04/hd_11.png)

If you agree with me, please mail me.

(e－mail : taka19440606@yahoo.co.jp)

Don't be kind to teach me "the true things", even if you find what I have mistook. Because, many kind, gentle doctors had already teached me a lot. They have been teaching me the same thing again and again that there exist χ and A in any flow.

(https://taka19440606.files.wordpress.com/2013/04/hd_12.png)

1 Comment

APRIL 2, 2013

Helmholtz Decomposition is wrong

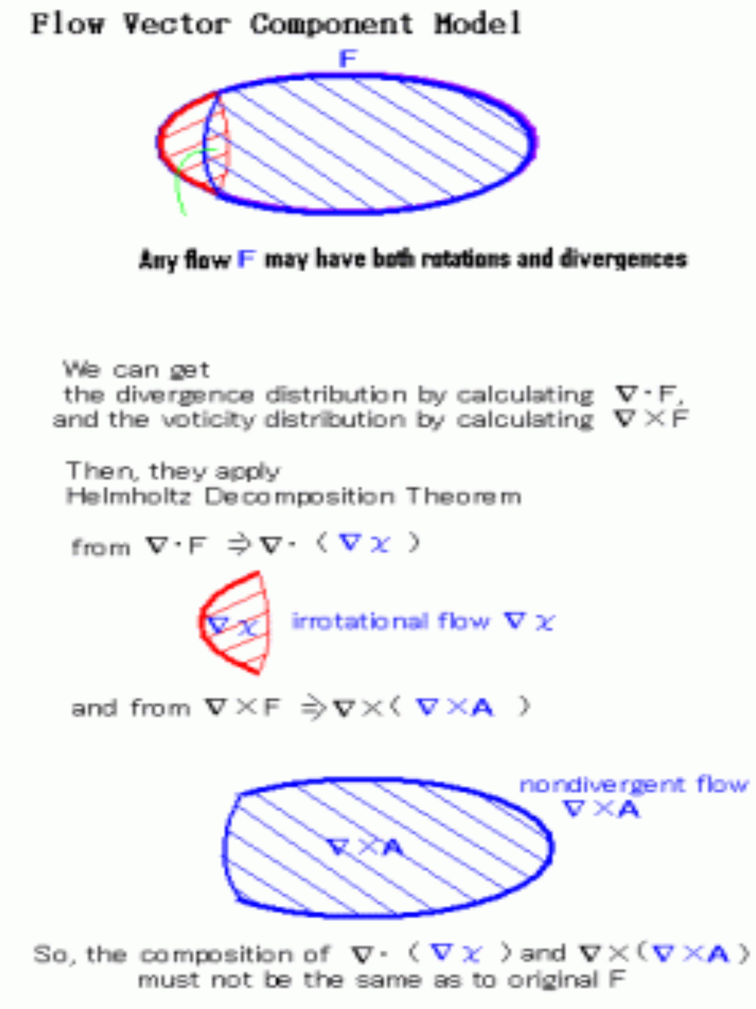
There is a theorem called Helmholtz Decomposition that is believed among meteorologists and hydrodynamicists.

That theorem says that any flow can be decomposed into a curl-free flow and a divergence-free flow.

That is,**F**=$\nabla\chi$ +$\nabla\times\mathbf{A}$

Here, **F** shows any flow, ∇ shows differential operator, χ shows velocity potential, **A** shows vector potential.

So, the first term of right hand is considered as a curl-free flow, and the second term is considered as a divergence-free flow. But,I have found some problems on this theorem. This theorem has been established on many mistakes.



(https://taka19440606.files.wordpress.com/2013/04/fig1_component.gif)

mistake 1

We can calculate the distribution of ($\nabla\cdot\mathbf{F}$) and ($\nabla\times\mathbf{F}$) from any flow **F**.

It is not needed to divide this flow to calculate those distribution.

If we have not proved that Helmholtz Decomposition is right yet, we should assume that any flow has curl component, divergence component and the component which play both role of curl and divergence as shown in right illustration. You can not brush aside the last component, because that is to use preliminarily Helmhltz Decomposition theorem.

They have mistook these distributions of $\nabla\cdot\mathbf{F}$ and $\nabla\times\mathbf{F}$ to be decomposed. And then, they think to be able to calculate **uniquely** $\nabla\chi$ and $\nabla\times\mathbf{A}$.

Here, I want to define **G**=$\nabla\chi$ +$\nabla\times\mathbf{A}$.

As you can see in upper illustration, aparently $G \neq F$. **(https://taka19440606.files.wordpress.com/2013/05/g_is_not_f.gif)**

For example, someone has explained about Helmholtz Decomposition in **Wikipedia (http://en.wikipedia.org/wiki/Helmholtz_decomposition)** as follows,

Statement of the theorem

Let $\mathbf{F}$ be a vector field on a bounded domain V in $\mathbb{R}^3$, which is twice continuously differentiable. Then $\mathbf{F}$ can be decomposed into a curl-free component and a divergence-free component:[1]

$$\mathbf{F} = -\nabla\varphi + \nabla\times\mathbf{A},$$

where

$$\varphi(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\cdot\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V' - \frac{1}{4\pi}\int_S \frac{\mathbf{F}(\mathbf{r}')\cdot\mathrm{d}\mathbf{S}'}{|\mathbf{r}-\mathbf{r}'|},$$

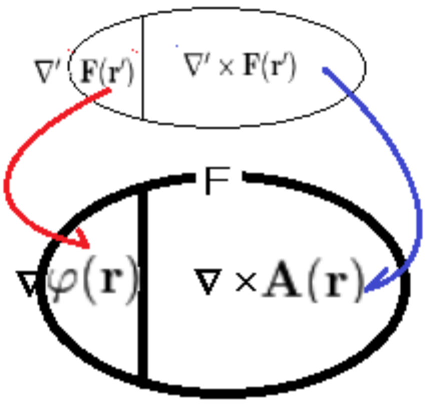
$$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\times\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V' + \frac{1}{4\pi}\int_S \frac{\mathbf{F}(\mathbf{r}')\times\mathrm{d}\mathbf{S}'}{|\mathbf{r}-\mathbf{r}'|}.$$

If V is $\mathbb{R}^3$ itself (unbounded), and $\mathbf{F}$ vanishes sufficiently fast at infinity, then the second component of both scalar and vector potential are zero. That is,[2]

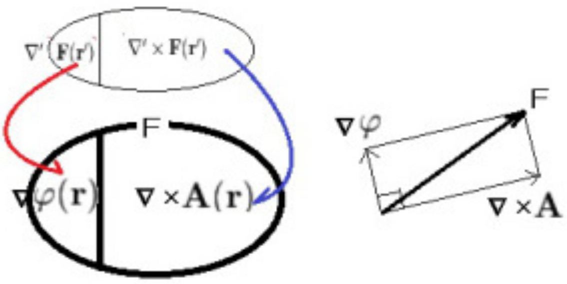
$$\varphi(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\cdot\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V',$$

$$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\times\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V'.$$

He preliminarily beleave Helmholtz Decomposition is right as follow



If you can prove that $\nabla\times$ is perpendicular to $\nabla\times\mathbf{A}$, the original flue has not such component which play both role of curl and divergence . Therefor Helmholtz Decomposition is right.



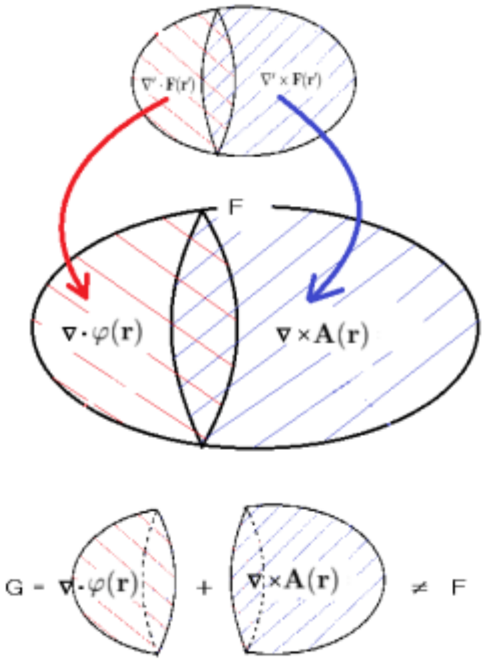
(https://taka19440606.files.wordpress.com/2013/04/preliminarilydecompose2.jpg)

But, we sometime see actual "divergence wind" is not perpendicular to "the wind from steam function". So, we should consider any flow has the component which play both roles of curl and divergence.

Even if there is a component which paly both roles, he can calculate the distributions of the fields of $\nabla\cdot\mathbf{F}$ and $\nabla\times\mathbf{F}$.

And, if he thinks those fields are separated, he may calculate $\nabla\chi$ and $\nabla\times\mathbf{A}$

He (or She) has confused G with F as shown in next illustration.



mistake 2

There are two of basically mathematical **theorems (http://blogs.yahoo.co.jp/taka19440606/34994516.html)** on any flow(vector function).

One of them says that if a flow has culr(voticity) component, there is no velocity potential χ, and if there are velocity potential χ in a flow, the flow has not curl. That is, not to have curl component is the necessary and sufficient condition for existing of velocity potential χ.

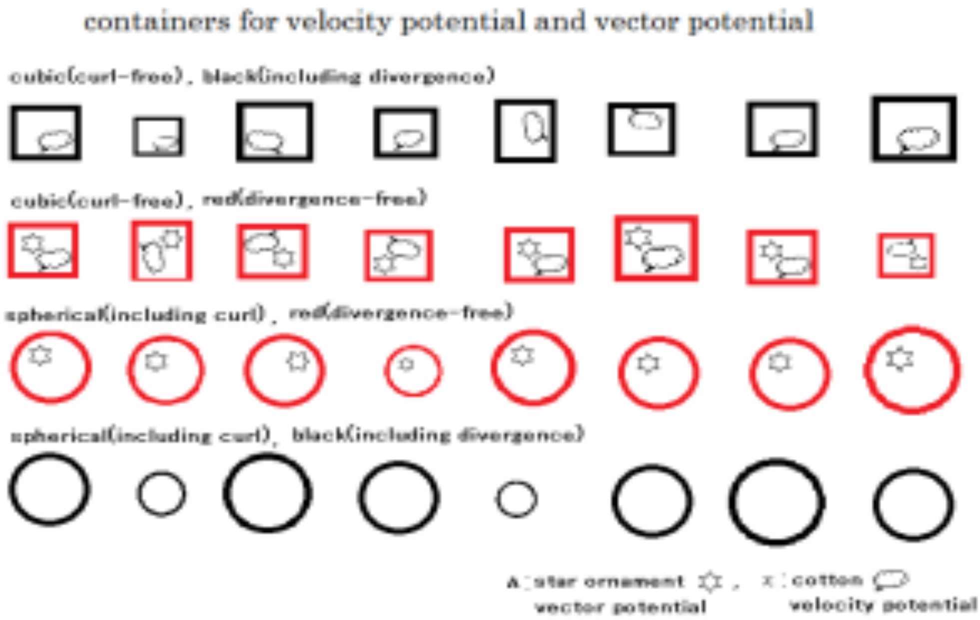
And other says that if a flow has divergent component in the flow, there is no vector potential **A**, and if there are vector potentials **A** in the flow, the flow has no divergence. That is, not to have divergence in the flow is the necessary and sufficient condition for exiting of vector potential **A**.

So, I can definitively say that there is no χ and **A** in the flow which has curl and divergence.

But, almost of authorities of Meteorology and Hydrodynamics don't think so. They think that any flow can be divided into two kinds of flow. They are a curl-free flow and a divergence-free flow.

I can show you some good example.

I would like to liken all kinds of flows to container boxes.



(https://taka19440606.files.wordpress.com/2013/04/helmholtz_container.png)

Then, there are just two kinds of shape of container, one of them is spherical shape, and other is cubic. A cubic shape container means a curl-free flow, and a spherical shape container means a flow including curl.

Then, I check all of containers(flow), and if it's a cubic container(curl-free), I put cotton( ) into it. And if it's a spherical container(including curl), I confirm that there is no cotton( ) in it.

And then, all container is painted with only two colors, red or black. Red one means a divergence-free flow, and black one means a flow including divergence.

Then, I check all of containers(flow), and if it's a red container(divergence-free), I put star ornaments(**A**) into it. And if it's a black container(including divergence), I confirm that there is no star ornament(**A**) in it.

Aren't you sure that there is no cotton nor star ornament in any spherical black container.

I can't believe that those clever persons think that there are cotton and star ornament in a spherical black container.

Why do they think so? I think they confuse G with F in < mistake 1 >

The spherical black container is **F**, not **G**. There is not nor **A** in **F**.

mistake 3

If you want to prove that any vector **F** can be devided into a irrotational vector **V**e and a solenoidal vector **V**r, you need to find some identity which can be described by like **F**=**V**e + **V**r.

As far as I know, the only equation is the vector triple product identity.

For example, "The vector triple product identity" is posted in the next homepage.

(reference:**http://en.wikipedia.org/wiki/Triple_product#Proof (http://en.wikipedia.org/wiki/Triple_product#Proof)**)

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\, \mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\, \mathbf{w}$$ **(https://taka19440606.files.wordpress.com/2013/04**

/vector_triple_product.png)

If you replace **U** and **V** with ∇, you may get $\nabla\times(\nabla\times\mathbf{W})=\nabla(\nabla\cdot\mathbf{W})-\nabla^2\mathbf{W}$ **(https://taka19440606.files.wordpress.com/2013/04/nabla_triple.jpg)**

So,you can get

$\mathbf{F}=-\nabla^2\mathbf{W}=-\nabla(\nabla\cdot\mathbf{W})+\nabla\times\nabla\times\mathbf{W}=-\nabla\chi+\nabla\times\mathbf{A}$ **(https://taka19440606.files.wordpress.com/2013/04/f_equal.png)**

In above equation, the first term of the right hand shows a curl-free flow, and the second one shows a divergence-free flow.

So, you might say that Helmholtz decomposition is perfectly proved.

But, the third mistake is that they take $-\nabla^2\mathrm{W}$ as any flow.

Any flow **F** certainly exist. But **W** is not guaranteed to exist. You need to make sure that there exists **W** for any flow **F**.

And if **W** exists, at the same time, $\nabla\chi$ and $\nabla\times\mathbf{A}$ are decided **uniquely** with **W**, and they need to appear together.

I can show you that there are many flows which do not include **W**.

You have got following expression from the vector triple product identity.

F=-$\nabla\chi$ +$\nabla\times\mathbf{A}$

It is given on terms and conditions as required by $\chi\equiv\nabla\cdot\mathbf{W},\ \mathbf{A}\equiv\nabla\times\mathbf{W}$ **(https://taka19440606.files.wordpress.com/2013/04/chi_and_a.png)**

That is, χ and **A** are deriven from the common function **W**.

If χ is decided from **W**, then **A** should be decided uniquely at the same instance.

And, the flow given by $\nabla\times\mathbf{A}$ is supposed to exist independently from other flows.

It is called solenoidal flow. That means a flow like in the tube. According to Helmholtz Decomposition there exists such a flow.

I do not think that such a solenoidal flow exists in the real world.

If there were such solenoidal flows, I would be able to show the collapses of Helmholtz decomposition.

Assuming that Helmholtz decomposition theorem is correct. you can consider two flows as following.

$\mathbf{F}_1=-\nabla^2\mathbf{W}_1=\nabla\cdot\chi_1+\nabla\times\mathbf{A}_1$ **(https://taka19440606.files.wordpress.com/2013/04/f1_equal.png)**
$\mathbf{F}_2=-\nabla^2\mathbf{W}_2=\nabla\cdot\chi_2+\nabla\times\mathbf{A}_2$ **(https://taka19440606.files.wordpress.com/2013/04/f2_equal.png)**

χ_1 and $\mathbf{A}_1$ are functions which are derived from $\mathbf{W}_1$. and, χ_2, and $\mathbf{A}_2$ are derived from $\mathbf{W}_2$.

Here, because an arbitrary flow (vector function) must be possible, you can consider the flow $\mathbf{F}3$ which includes divergent component of ($-\nabla\chi$ 1) and rotational component of ($\nabla\times\mathbf{A}$ 2).

$\mathbf{F}_3 = -\nabla^2\mathbf{W}_3 = -\nabla\chi_1 + \nabla\times\mathbf{A}_2$ **(https://taka19440606.files.wordpress.com/2013/04/f3_equal_comb.png)**

Here, $\chi_1 = \nabla\cdot\mathbf{W}_1, \quad \mathbf{A}_2 = \nabla\times\mathbf{W}_2$ **(https://taka19440606.files.wordpress.com/2013/04/origin_of_comb.png)**

I must say that again $\mathbf{F}3$ should have ($\nabla\chi$ 1) as divergent component, and have ($\nabla\times\mathbf{A}$ 2) as rorational component.

But according to Helmholtz decomposition, $\mathbf{F}3$ can be decomposed into two flows only by the vector triple product identity.

Then,

$\mathbf{F}_3 = -\nabla^2\mathbf{W}_3 = -\nabla\chi_3 + \nabla\times\mathbf{A}_3$

Therfor, $\mathbf{F}3$ has ($\nabla\chi$ 3) as divergent component, and has ($\nabla\times\mathbf{A}$ 3) as rorational component.

But, because $\mathbf{F}_1 \neq \mathbf{F}_3$, and $\mathbf{F}_2 \neq \mathbf{F}_3$ **(https://taka19440606.files.wordpress.com/2013/04/f3_notequal_f1_f2.png)** $\mathbf{W}_3 \neq \mathbf{W}_1, \mathbf{W}_3 \neq \mathbf{W}_2$ **(https://taka19440606.files.wordpress.com/2013/04/w3_notequal_w1_w2.png)**

So,

$\mathbf{F}_3 = -\nabla^2\mathbf{W}_3 \neq -\nabla(\nabla\cdot\mathbf{W}_1) + \nabla\times(\nabla\times\mathbf{W}_2)$ **(https://taka19440606.files.wordpress.com/2013/04/f3not_equal.gif)**

Here, you must say that $\mathbf{F}3$ which has combined with divergent component of ($\nabla\chi$ 1) and rotational component of ($\nabla\times\mathbf{A}$ 2) can not decomposed into divergent component of ($\nabla\chi$ 1) and rotational component of ($\nabla\times\mathbf{A}$ 2).

Or we may have to say that there is no **W** functions in $\mathbf{F}3$.

In real world, “The vector triple product identity” just means that there exit many electromagnetic waves such as X-rays and radio waves. They have their own electric fields(corresponding to velocity potential χ) and their own magnetic fields（corresponding to vector potential **A**）.

That does not mean there exist any electromagnetic wave which has an electric field radiated from “CNN” antenna and a magnetic field radiated from “ABC” antenna.

As I showed above, any vector function in Helmholtz Decomposition can not be able to have arbitrary potential velocity χ **(https://taka19440606.files.wordpress.com/2013/04/chi.png)** and arbitrary vector potential **A**. Their partners are definitive with same **W**.

That is, “the function **F** in Helmholtz Decomposition” is not “any function”, but the very special function with their own χ **(https://taka19440606.files.wordpress.com/2013/04/cross.png)** and their own **A**.

Simply, “The vector triple product” does not give proof of Helmholtz Decomposition.

Helmholtz Decomposition theorem is believed even in electromagnetics, but they must not need this theorem. They should be enough to have Maxwell’s equations.

They does not needed to divide any vector function.

I think many articles on Helmholtz Decomposition are written by authority of electromagnetics. They believe the two components are perpendicular to each other, and theoretically they should be so.

This theorem is for fluid dynamics. The authorities of Meteorologist roughly aplly this theorem to the real winds in some plane, and make “the potential velocity” and “the stream function”. If the two components are perpendicular to each other, isolines of these two function should be parallel to each other. But as we can see in **NASA home page (http://www.cpc.ncep.noaa.gov/products/hurricane/)**, they are not parallel.

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postscript

Electromagnetics is out of my hands.

But, I may say that the vector triple product identity is applicable just in electromagnetics.

Because, there exists χ as electric field, and **A** as magnetic field in real world. There is not such phisical potential in hydrodynamics.

In Electromagnetics, there is dynamics in the electric field and in the magnetic field. But there is not dynamics in velocity potential and in vector potential.

If you use ageostrophic winds as substitute for the winds from $\nabla\chi$, there is dynamics, and you can calculate the divergence of the winds.

If you agree with me, please mail me.

(e－mail : taka19440606@yahoo.co.jp)

4 Comments

Taka's memo