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Oscillations in an Electrical System
Energized by Means of Periodically Varying Capacities

(Preliminary communication)

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ABSTRACT: The phenomenon of excitation of oscillations by means of periodically changing parameters of a vibrating system has already been known in the physics community for a long time. Back in 1860 Melde [1] had shown that, by periodically changing the tension of a string, it was possible to excite traversal oscillations in it with frequencies half as large as those due to changes of the tension frequencies. In the simplest case, the theoretical path of phenomena related to the occurrence of such "parametric" oscillations leads directly to a differential equation with periodic coefficients. Equations of this sort are met in a lot of other physical problems as well. In regards to these, the mathematical aspect of their details will be herewith explored.

This same series of equations also provides an approach for the analysis of an electrically oscillating system with periodically changing capacity or self-induction. From Lord Raleigh's work [2], oscillations should arise in an oscillating system if the circumstance is specified such that the self-induction varies at a frequency being equal to twice the natural self-resonant frequency of the system. In regards to such principles, once we know the quantity, instructions are available in the current literature for the observational realization of the generator. In the past few years there have appeared two publications [3 and 4] in which the production of oscillations is featured by means of an alternating self-induction. The generation of electrical oscillations by periodically changing self-induction was first realized by us at the Leningrad Electro-Physical Institute beginning in 1931, and has continued onward to where we have reported our efforts in two further publications [5 and 6], which described the results of our experimental work on vibrational topics in the very same year.

When our series of experiments was carried out with the aforementioned generator, it was found that our experimental results differed from those of the experimental installations described in the publications of Guenther-Winter and Watannabe. They focused only on the requirement for the occurrence of oscillations, and did not provide any details concerning the establishment of amplitude (except those referenced to specify the linear differential equation with periodic coefficients). Between this question and that one there is the same basic problem, as well as with the first issue that they had brought up. We therefore feel obligated to inform the scientific community, as quickly as possible, what we have found concerning a parametric generator with periodically changing self-induction, on the one hand, and, on the other hand, to give the approximate theory that deals with the processes of both for the cases of periodically changing self-induction and for capacity - the theory of which, certainly, is only possible if we start with the non-linear equations.

At the present time we would like to note, however, that we anticipate that we will be publishing the results of the self-induction work in the near future, while the present paper concerns our experimental work on the generation of parametric oscillations by means of an alternation of capacity. This case is presented, showing how much we have learned up until now. In this paper we will show that a sparking circuit with a periodically changing capacity should give rise to oscillations. By similar arguments, for simple reasons, one should easily be convinced of the following. There are analogous reasons that these principles should certainly apply in the case of a changing self-induction, as well.

Theory: Let us assume that at some initial moment, t = 0, when a charge q is available on the condenser and the current is equal to zero, we somehow diminish the capacitor C's capacitance a little (ΔC). Thus we accomplish the operation $\frac{\Delta C}{2C^2}q^2$. We leave it then to the condenser to be discharged in a time interval equal to 1/4 of a cycle (T/4) of the system's oscillation frequency, when a fixed amount of energy will transfer into the magnetic field of the inductor from the capacitor plates whose voltage will now be equal to zero, at which point we shall return again to the initial value of capacity. It is possible to do this, and yet not perform any work because the capacitor has no charge on it. Yet, in another 1/4 of a cycle the current in the inductor will be again be equal to zero, and the condenser recharges up to a voltage that will, on average, be larger or smaller, depending on whether there will be larger or smaller energy losses contained in the system at diminishing capacities. In this manner, during ½ of a cycle of the natural oscillations of the system, the changing of the capacities is completed and we can repeat it again. Although initially small (and if there was an initial finite charge on the capacitor), upon further repetitions the process of oscillation in the system will continuously increase if the following sole condition is satisfied

$$\frac{\Delta C}{2C^2}q^2 > \frac{1}{2}Ri^2\frac{\pi}{\omega}$$

or

$$m > \frac{\zeta}{2} \qquad (1)$$

with

$$\zeta = \frac{\pi R}{L\omega}$$

being the mean logarithmic decrement of the system, and the parameter modulation is equal to

$$m = \frac{\Delta C}{2C} = \frac{C_{\max} - C_{\min}}{C_{\max} + C_{\min}} \quad (2)$$

Let us take the size of the change of capacity - and call that the "percentage of modulation" of the parameter. For a sinusoidal (instead of random or highly irregular) change of capacity, the condition (1) becomes

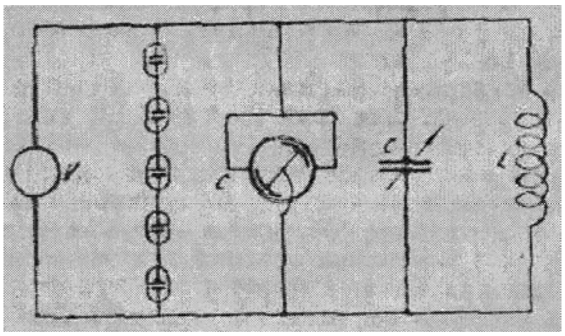

Figure 1: Schematic of Experimental Apparatus.

$$m > \frac{2}{\pi}\delta \quad (3)$$

Notice that, even with the absence of an initial charge on the capacitor, somehow there is always the practical occurrence of residual electrostatic induction (induced charging due to electrical currents, the transfer of atmospherics, etc.) Basically, we always have initial charge by virtue of statistical noise fluctuations in the oscillatory circuit.

Experimental: In the illustration we show an example of an oscillatory system that we can excite into oscillation with no action on it by any electromotive force. In the system there are no current or voltage sources but the mechanically driven capacitance is periodically changing at a frequency twice that of the system's own fundamental frequency.

The effect of exciting electrical oscillations in such a system, which does not contain any current sources, by periodic mechanical change of its capacity, has been accomplished by us in the following installation, which is shown schematically in Figure 1. As is visible from the drawing, the oscillatory system is formed by the capacitor with a periodically changing capacity (C), which has in parallel with it an adjustable oil filled capacitor (c), and an inductor (L), consisting of several sections of the secondary winding of a transformer (but without the iron core). The capacitor with periodically changing capacity consisted of two facing systems: a stator and a rotor (Figure 2). We first constructed the stators, made with 26 square aluminum plates, each with 14 radially symmetric cut-outs; then the rotor, formed from 25 circular discs of 30 cm diameter, also made from aluminum with similar cut-outs. The motive power for the system was provided by a direct current motor giving up to 4000 rev/min. At spin-up, the motor's speed of n revolutions/second periodically changed the capacity of the oscillatory system at a frequency equal to 14n/second.

For determining the onset and intensity of the oscillations, we connected in parallel with the capacitor 5 neon bulbs (each rated for 220 V) and a Hartman & Brown electrostatic voltmeter (V in Figure 1) able to read 1200 V. The neon bulbs also served simultaneously for restricting the build-up of oscillations in case they arose.

The completion of these experiments was obtained through the active participation of V.A. Lazarev who carried out these adjustments with the following results. At the drive motor's operational speed, the system's reactor C (oil filled capacitor) was able to tune the oscillatory system by area changes (by changing oil level), and when this frequency with its readjustments coincides approximately to being one half that of the frequency of the changing capacities, the voltmeter gives a deflection, and neon bulbs light up. A check of the frequency appearing was made by comparison of these fluctuations with audio beats against a standard frequency tuning fork, and showed that this frequency is equal to half of the frequency of the change of the capacity's size, i.e. 7n (where n = the number of rotations of the motor as it was checked on a tachometer). It remains to be said that the readjustment of the oil filled capacitor's area was not changed except when frequency changes are performed and these follow the number of motor rotations.

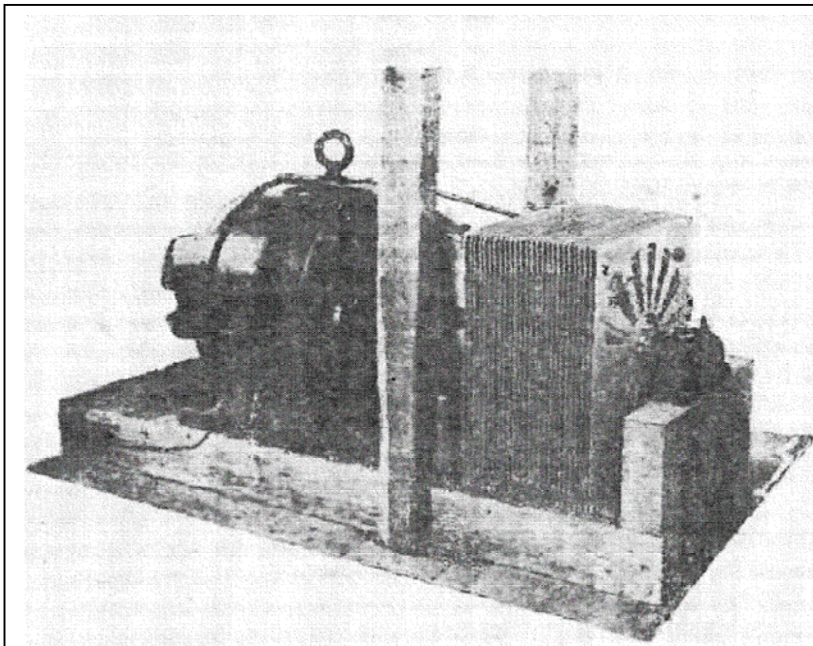

Figure 2: Photograph of the Experimental Apparatus.

Since, in the absence of neon bulbs, the oscillatory system investigated by us is essentially linear in its behavior, and is described by linear differential equations with periodical coefficients, which, as is known, does not give the inner unstable areas stationary oscillations. Therefore, when the neon bulbs are switched off, it follows that we observed that fluctuations will arise in the system and continuously accrue until the electrical insulation breaks down. Experimentally, we have already shown that, with the amount of neon bulbs on hand, electrical activities in the reactor reached 600-700 V and were steady, where in the absence of the bulbs, the voltage continued to accrue until it reached up to 2000—3000 V, yet did not slip (stop working) even with sparking between the capacitor plates. It is interesting to note that, in agreement with the theory, by slowing down the oscillations in the oscillatory system (so they are no longer at half the frequency of the change of size of the capacitor) the frequency of sparking in the insulation breakdown process gradually decreased according to a reduction of the capacitor fluctuation speed as it approaches the borders of the accented area of the graph in Figure 3. (A = stable areas corresponding to linear differential equations with periodic coefficients.)

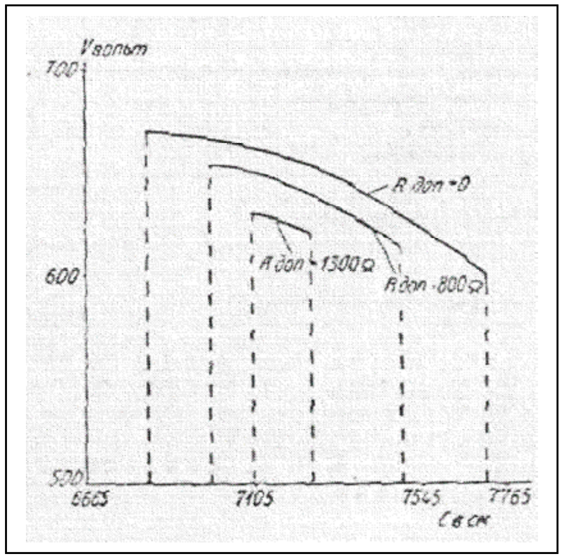

Figure 3: Performance Curves for Apparatus.

In order for a more complete judgment to be made about that conclusion we cite the following data:

The self-inductance of oscillatory system was equal to ….. …………………..L = 16.6 Henrys
Maximal size of the rotating reactor ….……………………………………… C_{max} = 6020 cm (6682 pF)
Minimal size of the rotating reactor….. ……………………………………… C_{min} = 3480 cm (3862 pF)
Magnitude of the additional size (of the oil filled capacitor) at a resonance … c = 2500 cm (2775 pF)

Modulation depth parameter:

$$m = \frac{C_{max} - C_{min}}{2c + C_{max} + C_{min}} = 0.175$$

The number of rotations the motor made was:

$$n = \frac{3740}{60} = 62.33 \quad \textbf{revolutions/second.}$$

The frequency of the capacitor's changing was measured, based on audible beats, by comparing it with a tuning fork frequency: the resulting parametrically excited fluctuations equaled 435 oscillations/second. This coincides (within the limits of accuracy of measurements) with the frequency calculated by the formula: N = 7n = 436.3 (the capacitor's mechanically changing frequency was twice this, or 870 oscillations per second).

The nature of the dependence between the amplitude of the fluctuations of an electrical current in the reactor and the damping of the system is presented on Figure 3. Here it is shown that increasing the damping of the system coincides with diminished regions of parametrical excitation (and stable oscillations). For comparison, Table 1 shows the size of parametrical excitation for various values of system decrement ξ, by experiment on the one hand, and, on the other hand, calculated from the theoretical formula for sinusoidally changing parameters

$$\frac{C_2 - C_1}{C} = \sqrt{m^2 - \frac{4\xi^2}{\pi^2}} \quad (4)$$

for the same value ξ.

Table 1

Supplementary Circuit Resistance R in Ohms (Ω)	Circuit Decrement ξ	Relative Magnitude of Excitation	
		Calculated from Formula (4)	Experimentally Measured
0	0.181	0.136	0.120
800	0.233	0.093	0.066
1300	0.269	0.039	0.022
1400	0.275	0.000	0.000

From the table, concurrence among these values is apparently quite satisfactorily if it is understood that the experiments were mainly qualitative in nature.

Leningrad

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10 July 1933

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