H.F. RESISTANCE AND SELF-CAPACITANCE OF SINGLE-LAYER SOLENOIDS*

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SUMMARY.—This paper contains the results of high-frequency resistance and self-capacitance measurements on about 40 coils, wound with copper wire on grooved Distrene formers. The measuring instrument was a twin-T impedance bridge.

For coils whose turns are widely spaced, the high-frequency resistance measurements are in good agreement with the theoretical values of S. Butterworth.[5] For closely-spaced coils, the measured values are very considerably below those of Butterworth.

A table of values of high-frequency resistance of coils having various values of length/diameter and spacing ratio is derived from these measurements.

It is shown that a good approximation to the high-frequency Q of coils of the type measured is given by the simple expression

$$Q = 0.15\, R\psi\sqrt{f}$$

where R is the mean radius (cm), f the frequency (c/s) and ψ depends on the length/diameter and the spacing ratios. A table of ψ has been calculated.

Measurements of self-capacitance were made with one end of the coils earthed. These measurements show a very considerable divergence from the formula of A. J. Palermo[14] though they are in quite good agreement with other previous experimental work. The self-capacitance of coils of this type is shown to be substantially independent of the spacing of the turns. It is given by an expression of the form

$$C_0 = HD \text{ picofarads}$$

where D (cm) is the mean coil diameter and H depends on the length/diameter.

A table of H is given, based on these measurements.

1. Introduction.

A GREAT deal of theoretical and experimental work has been published concerning the resistances of coils and their variation with frequency. The experimental work, in general, suffers both from its restricted application and from uncertainty as to the absolute error inherent in the method of measurement. The theoretical work, even where it is in reasonable agreement with experiment, tends to produce complicated formulae which lead to very considerable computation. Even now, after over a quarter of a century of work on every type of coil that has been used or proposed, the present writer knows of no reliable data from which Qs of even the simplest coils can be easily and quickly predicted.

The only comprehensive theory extant is that of S. Butterworth[1–5]. He gave formulae which purported to cover single-layer solenoids, wound with round wire, for any frequency, coil dimensions and spacing of turns. The only restriction was that the number of turns had to be large, the case of few turns only being dealt with when the length of the coil was small compared with its diameter, and the turns were not too closely spaced. He suggested modifications to include multi-layer coils and coils wound with stranded wire. All these formulae only dealt with copper losses, dielectric losses being assumed negligible. Dielectric loss must, if necessary, be allowed for separately.

This theoretical work has become so generally accepted that it is quoted as a basis for calculation in standard reference books (see, *e.g.*, ref. 12, pp. 78–80). It will be shown experimentally that Butterworth's theory is only applicable to coils having widely spaced turns (roughly $d/s < 0.5$). More closely spaced coils have a lower high-frequency resistance than that predicted by Butterworth, the discrepancy increasing as the coil length decreases relative to the diameter. For coils having $l/D = 1$, when $d/s = 0.6$ Butterworth's value is too high by 15 per cent, when $d/s = 0.7$ by 25 per cent, when $d/s = 0.8$ by 55 per cent and when $d/s = 0.9$ by 190 per cent.

During the determination of high-frequency resistance and inductance it was necessary to make allowance for the self-capacitance of the coils measured. It was

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thought that self-capacitances calculated from the formula of A. J. Palermo[13] would be sufficiently accurate for this purpose. However, it was found that use of Palermo's formula led to variations, larger than the experimental error, of the apparent measured inductances over quite a small frequency range.

Measurements of self-capacitance of a wide range of single-layer coils were consequently carried out. The results failed to confirm Palermo's claim that the self-capacitance varies steeply with the spacing of turns. They are, instead, in quite good agreement with previous work, which had shown the self-capacitance to be very nearly independent of d/s. For coils having length/diameter = 1, Palermo's formula gives results which are greater than the measured value by 4 : 1 when $d/s = 0.9$, and smaller than the measured value by 1.6 : 1 when d/s = 0.1.

2. List of Symbols

D represents mean diameter of coil (cm)
R represents mean radius of coil (cm)
l represents overall length of coil (cm)
n represents number of turns
d represents diameter of each wire (cm)
s represents distance between centres of adjacent turns (cm)
d/s represents spacing ratio of turns
ρ represents resistivity of wire (ohm-cm)
f represents frequency (c/s)
τ represents power factor of material of coil former
C_0 represents self-capacitance of coil (pF)
L_s represents equivalent series inductance of coil (μH)
R_s represents equivalent series resistance of coil (ohms)
f_0 represents self-resonant frequency of coil (c/s)
ϕ represents ratio of h.f. coil resistance to resistance at same frequency of same length of straight wire.
ψ is a function of l/D and d/s, occurring in the formula for Q,

$$i.e.,\ Q = 0.15\,R\,\psi\sqrt{f}$$

Where $Q = 2\pi f L_s/R_s$.

$$z = \pi d\sqrt{\frac{2f}{10^9\rho}} = \frac{1}{\sqrt{2}}\,\frac{\text{wire diameter}}{\text{current penetration depth}}$$

3.—Butterworth's Work on Single-Layer Solenoids

3.1. S. Butterworth's series of papers on solenoidal coils are based on two sets of formulae which he developed for single-layer solenoids. In both cases there is no restriction on the frequency. The first[1] apply to coils having any specified number of turns, the turns being "not too closely spaced," and the coils having lengths small compared with their diameters. The second[3] were evolved as a consequence of some measurements by C. N. Hickman, which were made on coils outside the range of conditions assumed in the formulae of Ref. 1, and consequently failed to agree with the results predicted by these formulae[2,7]. This second group of Butterworth formulae extended the theory to coils having arbitary spacing ratio, and arbitary ratio of length to diameter. The number of turns, however, had now to be assumed to be large. Butterworth's method of deriving his second group of formulae is as follows:

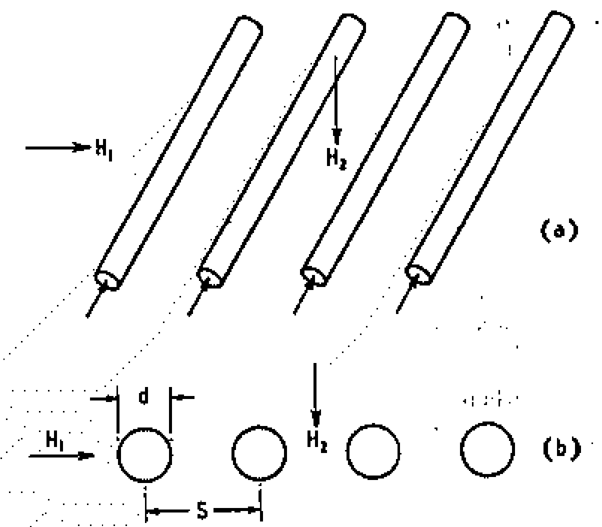

Fig. 1. *Views from above (a) and transverse to the wires (b) of Butterworth's infinite plane system of parallel wires.*

He took as his starting-point a system consisting of an infinite number of parallel wires of equal diameters, equally spaced and lying in the same plane as shown in Fig. 1, which shows a portion of the system seen from above and end-on. He set out to solve three problems, namely, to determine the losses in the system (*a*), when the wires carry equal alternating currents, *i*, flowing in the same direction; (*b*), when they are situated in a uniform alternating magnetic field H_1 parallel to the plane of the wires and perpendicular to their axes; and (*c*), when they are situated in a similar uniform field H_2 perpendicular to the plane of the wires. We shall call H_1 and H_2 the axial and transverse fields respectively.

Each of these problems involved the solution, by a method of successive approximations, of a set of an infinite number of linear equations, each containing an infinite number of variables. Butterworth's solutions are contained in three tables, for a range of d/s from 0.1 to 1.0, in steps of 0.1 (*i.e.*, from widely separated turns to turns touching).

In order to apply these solutions to solenoidal coils, Butterworth worked out the field associated with the coil by adding to the field associated with an infinitely long solenoid a modifying field produced by its ends, considered as a circular disc of poles. This field was resolved into two components, one parallel to the axis of the coil (the axial field) and the other perpendicular to the axis (the transverse field), and the mean-square value of each over the length of the coil deduced.

Now, he pointed out that each short section of wire may, if the wire diameter is small compared with the coil diameter, be treated as part of a plane system, of the kind already considered. He considered that, as a sufficiently close approximation, the axial and transverse fields associated with the coil could be replaced by their mean-square values, these being considered to act uniformly along the length of the equivalent plane system, now taken as being infinitely long. Under such conditions, he showed that the total losses in the system equivalent to the coil could be obtained by summing the separate losses deduced from the solutions of his three problems.

His results for very high frequencies are summarized in a table which is reproduced here as Table I. The quantity tabulated is the ratio of the high-frequency resistance of a coil, assumed to have a large number of turns, to the resistance at the same frequency of a straight wire of the same length and diameter as the wire forming the coil. This straight-wire resistance can be calculated from a well-known formula. The variables of Table I are the ratio length/diameter of the coil, and the spacing ratio d/s.

" Very-high frequency " has to be defined in terms of the diameter and electrical constants of the wire. It is a frequency higher than that at which the current penetration depth is some arbitrary fraction, say, 1/10th, of the wire diameter. It is convenient to express the high-frequency resistance of a round wire in terms of a quantity z, which is defined by the relation

$$z = \pi d\sqrt{\frac{2f}{10^9\rho}} = 0.107\, d\, \sqrt{f} \text{ for copper.}$$

Now, the current penetration depth in copper at frequency f

$$= \frac{1}{2\pi}\sqrt{\frac{10^9\rho}{f}}$$

Hence, it follows immediately that the criterion, given above, that the frequency should be " very high " may be written in the form

$$z > 10/\sqrt{2}$$

$$\text{i.e., } z > 7 \text{ approximately.}$$

3.2. Certain curious features are apparent in this Butterworth table. Particularly surprising are the high values that appear when the turns are closely spaced. Suppose for example, that we take a coil having a length/diameter = 0.4 and spacing ratio = 0.8—that is to say, with turns quite close. If we bring the turns a little closer to give a spacing ratio of 0.9, the length/diameter ratio being thereby only slightly changed, we are to expect the h.f. resistance to increase in the ratio of about 2.7 to 1. This would be surprising. For a spacing ratio of 1.0 the value infinity appears in the table; *i.e.*, when the turns are brought very close the h.f. resistance becomes infinitely large.

TABLE I

$\frac{d}{s}$	Coil Length/Coil Diameter											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	3.41
0.9	18.2	17.5	16.1	14.6	13.2	11.9	8.02	5.27	4.39	3.96	3.78	3.11
0.8	6.49	6.32	5.96	5.57	5.23	4.89	3.91	3.20	3.04	2.97	2.92	2.82
0.7	3.59	3.53	3.43	3.29	3.17	3.07	2.74	2.61	2.51	2.51	2.50	2.52
0.6	2.36	2.35	2.32	2.29	2.26	2.23	2.16	2.15	2.14	2.16	2.16	2.22
0.5	1.73	1.74	1.75	1.75	1.75	1.76	1.77	1.85	1.85	1.86	1.86	1.93
0.4	1.38	1.39	1.41	1.42	1.44	1.45	1.49	1.56	1.57	1.59	1.60	1.65
0.3	1.16	1.19	1.21	1.22	1.22	1.24	1.28	1.34	1.34	1.35	1.36	1.39
0.2	1.07	1.08	1.08	1.10	1.10	1.10	1.13	1.16	1.16	1.17	1.17	1.19
0.1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05

Butterworth's theoretical values of the ratio of the h.f. coil resistance to the resistance at the same frequency of the same length of straight wire.

However, although this infinite value of the h.f. resistance holds for coils whose length varies from zero to a value indefinitely large, when the length actually becomes infinite there is a discontinuity, the h.f. resistance assuming a value 3.41 times the straight-wire resistance.

It will be useful to consider where, in Butterworth's calculation, these unexpectedly high values of h.f. resistance arise.

We have said that Butterworth evaluates separately three sets of losses, which he subsequently combines. These are (1) the losses due to the current in the wire under consideration and the currents in adjacent wires, (2) the losses due to the axial field (H_1 in Fig. 1) and (3) the losses due to the transverse field (H_2). Losses (1) and (2) show no surprising behaviour when the wires are closely spaced; the abrupt rise in h.f. resistance for close spacing is all due to loss (3). Butterworth tabulates a quantity g, varying with d/s, which when multiplied by $(d/s)^2$ and a factor depending on the length/diameter ratio of the coil, gives the contribution of the transverse field losses to the total losses. This is reproduced as Table II.

Table II gives values, worked out by Butterworth, of the ratio of the losses in one wire of his infinite parallel-wire system, acted on by a transverse field, to those in an isolated wire acted on by the same field. The table shows the variation of this quantity with the spacing ratio. If we work through the theory again, taking account only of the two adjacent wires (besides the wire in question), we find for a spacing ratio of 1.0 that g comes out as 3.40 instead of ∞. Values of g for wider spacings will now lie between 3.40 and 1. Consequently, it is the more remote wires that, for close spacings, make the largest contribution to Butterworth's values of g given in Table II. In the case of a coil, the transverse field will be concentrated near the ends of the coil, that is to say, it will be effective over the last few turns. Consequently, the contribution of the effect of the transverse field to the total coil losses would be expected to be of the order of the value of g that we have just worked out, rather than of that given by Butterworth.

So far, we have only considered the results given by Butterworth for "very high frequency." We shall see later (Section 14) that it is also necessary to proceed with caution when trying to make use of his low-frequency results.

TABLE II

d/s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
g	1.017	1.069	1.166	1.326	1.585	2.03	2.87	4.83	12.5	∞

That part of the resistance due to the transverse field rises steeply for spacing ratios over 0.7, becoming infinite for d/s = 1.0. In this latter case, the parallel-wire system becomes a continuous infinite metal screen at right angles to the field.

We can show, qualitatively, that Butterworth's high h.f. resistance values, for coils having closely-spaced turns, may be expected to be absent in practice. Butterworth has substituted for the actual transverse field of a coil a uniform field whose square is the mean-square value—obtained by a method of approximate integration—of the transverse field. This uniform field is supposed to be imposed on a system consisting of an infinite number of parallel wires, and we have to consider the effect on any one wire of this field as modified by the eddy-currents set up in that wire and in all other wires of the system. Now, we shall show that the high losses predicted by Butterworth in each wire of this system are due chiefly to the presence of the more remote wires.

4. Previous Experimental Work on the A.C. Resistance of Single-layer Solenoids

Butterworth, in his second paper[3], compares his theoretical values of the a.c. resistances of single-layer solenoids with two sets of experimental results, those of C. N. Hickman[7] and G. W. O. Howe.[6]

C. N. Hickman used coils wound with very thick wire (0.518 cm diameter), each coil being 96 cm in length, the ratio of length/diameter varying from 3 to 17. He used a d/s value of 0.86. He employed a bridge method of measurement, at frequencies of 1, 2 and 3 kc/s. The coils were wound on "well-seasoned wooden cylinders." Comparison of Butterworth's predicted values of resistance and Hickman's measured values shows that the former are from 2 to 13 per cent higher than the experimental.

G. W. O. Howe, using a thermal method, measured coil resistances at radio frequencies. He gave results for two coils, each having a length/diameter ratio of about 10, wound with wire of diameters 0.163 cm and 0.254 cm respectively. The respective values of d/s were 0.49 and 0.90. The results for the widely-spaced coil are close to those predicted by Butterworth. Those for the closely-spaced coil are lower than the theoretical values by 5½ per cent at the low-frequency end of Howe's frequency range, and 20 per cent at the high-frequency end. Howe made his measurements on long coils because, at that time, no satisfactory formulae had been suggested for short coils. He pointed out that short coils, such as occur almost invariably in practice, would be expected to give results considerably different from those predicted by the long-coil theory, and he suggested a tentative modification of this theory to make it applicable to short coils.

Not a great deal of additional experimental work has been published, since the publication of Butterworth's papers. The most important is that of Dr. Willis Jackson.[8] He pointed out that Butterworth's formulae had not hitherto been satisfactorily verified. The two principal difficulties that he mentioned were that "precision uses of bridge networks are not available at radio frequencies," and that the losses in a tuning capacitor, which can at radio frequencies be comparable with those of the coil being measured, could not usually be measured separately. He avoided these difficulties by using an ingenious method suggested by E. B. Moullin, involving measurements on a number of coils having the same dimensions but wound with wire of different metals. The method is elaborate, and it would not be practicable to use it for measuring more than a small number of coils. It constitutes a check on an existing theory, rather than a method of making absolute measurements, and in addition Jackson found that it did not give results sufficiently accurate to allow more than tentative conclusions to be drawn. He worked at about 1 Mc/s, his coils having a spacing ratio of 0.63.

Sets of Q-meter readings, corresponding to coils of various dimensions, have been published from time to time. Typical of these are the values quoted in an article by Art H. Meyerson.[9] Results are given from 25 to 60 Mc/s in the form of a table, and some very irregular curves are based on the measurements. These results seem characteristic of the Q-meter rather than of the coils.

Of the methods of measurement so far considered, only the thermal method of Howe seems likely to give reliable results. The objection to Howe's method is the practical one of the length of time required for each measurement.

5. Description of Coils used in the Present Series of Measurements

The intention of the present series of measurements was to provide an empirical substitute for Butterworth's theoretical h.f. resistance table (Table I). We shall see that the conditions that need to be fulfilled before Butterworth's table becomes applicable—i.e., that z should be high and the number of turns large (see below)—tend to become incompatible over part of the range of the table, more especially in the bottom left-hand region. That is to say, practically useful coils cannot be wound such that they work at a large z value, have a large number of turns, have a wide turn spacing and have a small length/diameter. To cover this region, some compromise had to be made, and the results are of theoretical rather than practical interest. Butterworth, constructing his table exclusively from theoretical formulae, did not encounter this difficulty. However, most high-frequency single-layer coils actually used do fall within the region covered by the body of the table, which is the practically important region.

It is specified in Butterworth's table that the number of turns shall be large, though there is no indication of the effect on the h.f. resistance of employing a finite number of turns. We can form a very rough estimate of the order of magnitude of this effect as follows. If we assume that the part of the losses in each wire due to proximity effect is to be attributed largely to the two immediately adjoining wires, one on each side, the effect of using a finite number of wires will be, principally, to diminish the proximity effect in two turns, those at each end of the coil. As a first approximation, the proximity effect in each of these end turns will be diminished by half, since there is only one wire adjacent to each. Thus the total proximity effect losses will be diminished by $2 \times \frac{1}{2} \times \frac{100}{n}$ %; i.e., by $\frac{100}{n}$ %.

Since, in practice, the proximity effect on each turn of more remote turns is not negligible, this expression will be too low. Since, however, proximity losses form only part of the total losses, this expression should give the order of magnitude of the effect of using a finite number of turns. It is not in violent disagreement with Butterworth's figures for very short coils, of not too close spacing, having a finite number of turns.

From this consideration, it was decided that the minimum number of turns that would be used was 30. In practice, the number of turns ranged from 30 for the smallest length/diameter coils to 50 for the largest.

The condition that the number of turns shall be large may, over part of Butterworth's table, be difficult to reconcile with the condition that the frequency shall be high, or, more correctly, that z should be large. From the expression for z, i.e.,

$$z = 0.107\, d\sqrt{f}$$

it is evident that both the wire diameter and the frequency must be as large as possible. As regards the frequency, since as we shall see later, we shall want to make use of a large tuning capacitance, in order to work at a high enough frequency our coil inductance must be as low as possible. That is to say, we must use the smallest permissible coil diameter and the smallest number of turns. The minimum number of turns has already been decided. As for the coil diameter, since the number of turns has a lower limit, and the spacing of turns and the ratio of coil length to coil diameter are determined by the position of the coil in Table I, we can only decrease the coil diameter by decreasing the diameter of wire. But it is evident from the form of the expression for z, that, for satisfying the "high-frequency" criterion, a large d is more important than a large f.

Specifically, since for a given number of turns and a given spacing ratio, the length of coil, l, is proportional to the diameter of wire, d, then the coil diameter D, for a given l/D ratio, will also be proportional to d. But f is proportional to $1/\sqrt{L}$, i.e., to $1/\sqrt{D}$, and hence to $1/\sqrt{d}$. So $\sqrt{f}$ is proportional to $1/(d)^{1/4}$, and hence z is proportional to $(d)^{3/4}$.

It is apparent that the upper limit to d is determined by the maximum permissible coil diameter. In the present work, the largest formers used had a diameter of 2½in. This involved the use of wires of 18 and 20 S.W.G. for most of the coils. Furthermore, since for a given maximum coil diameter, the upper limit of z becomes increasingly severely diminished with decreasing value of length/diameter, no coils were constructed having a length/diameter less than 0.4.

The coils were wound on Distrene rod, diameters ranging from 2½in to ⅝in. In one case, to attain a sufficiently high length/diameter ratio with a spacing ratio of 0.9, a coil of 20 S.W.G. double-silk-covered wire was wound on a ¼-in diameter former. The special virtues of Distrene rod for the present purpose are its low power factor (about 0.0003) and, from the mechanical point of view, the ease with which it can be grooved and the ends of the coil anchored.

The effect of dielectric losses on the total losses may be estimated if we assume that the coil turns are completely embedded in the former. Since in reality, each turn is roughly half surrounded by air and half by former material, we may expect that this assumption will cause us to over-estimate the actual dielectric losses. With this limitation, the percentage increase in the copper-loss series resistance due to dielectric losses is given by

$$\frac{\tau\omega^3 L^2 C_0}{R} \times 100\ \%$$

where τ is the power factor of the former material,
ω is the angular frequency,
L is the coil inductance,
C_0 is the coil self-capacitance,
R is the equivalent series resistance due to copper losses.

This may be written as

$$\tau Q (f/f_0)^2 \times 100,$$

where f is the working frequency and f_0 the self-resonant frequency,
or as

$$\tau Q (C_0/C) \times 100$$

where C is the tuning capacitance at the working frequency. Q, for the coils measured, was never higher than about 250, and C_0 was usually about 3 or 4 pF. Minimum C was about 850 pF. Thus, the maximum value of the percentage dielectric losses that we may expect is

$$\frac{0.0003 \times 250 \times 4 \times 100}{850}$$
$$= 0.035\ \%\ \text{approx.}$$

The ends of the wires (mostly 18 and 20 gauge copper wires) were soldered to leads of 12 or 14 gauge copper wire, which had been previously tinned and sunk about ¼in into the former (using the heat of a soldering iron pressed against the lead just above the former). There is a tendency for the leads to twist in their holes, if they are not bent carefully.

For spacing ratios up to 0.8, the formers were grooved and the coils were wound with bare wire. For a spacing ratio of about 0.9, double-silk-covered wire was wound on ungrooved formers, the turns being as close as possible. This procedure gave spacing ratios ranging from 0.885 to 0.92. One coil (length/diameter = 1.30) was wound with single-silk covered 20 gauge wire (d.s.c. wire, from which the outer layer was stripped) on an ungrooved former, a spacing ratio of about 0.95 being obtained.

There is an appreciable tolerance on wire sold by gauge number, and in any case, the wires are stretched before winding to remove kinks. Consequently, the wire diameter for each coil was taken as the mean of three or four roughly equally spaced measurements, by a centimetre micrometer, along the length of the wire, these measurements being made during the winding.

Each coil was dried for about twelve hours in a desiccator, before measurement.

6. Use of the Twin-T Impedance Measuring Bridge

The twin-T Impedance-Measuring Circuit manufactured by the General Radio Company, Cambridge, Mass., is one of the most recent developments in the technique of measuring high-frequency impedances. Detailed accounts of the theory and its practical application are given in references 10 and 11. We shall very briefly describe its use for the measurement of high-frequency coil impedances.

The twin-T operates over a frequency range from 460 kc/s to 30 Mc/s. It is a null instrument, a balance being obtained by the adjustment of two capacitors. One of these capacitors is calibrated in micro-mhos, and, after multiplying by a factor involving the square of the frequency, gives the effective parallel conductance of the measured impedance. The other, calibrated in picofarads, gives directly the effective parallel capacitance.

The capacitance dial has a range of from 100 to 1,100 pF, and it is calibrated at intervals of 0.2 pF. The scale of the conductance dial is not linear, the calibration intervals ranging from 2½ to 10 per cent of the measured conductance. This dial can usually be read to 1 per cent or better.

In the present series of measurements, the oscillator consisted of a Marconi Signal Generator, covering a frequency range from 85 kc/s to 25 Mc/s. A number of receivers were used from time to time, the necessary qualifications being that the receiver should be well shielded, that it should have a sensitivity of the order of 1 to 10 microvolts, and that it should be provided with a local oscillator to beat with the incoming signal, so as to produce an audible note.

7. Method of Measurement

Each coil was measured over a small band of frequencies at the low-frequency end of the available working range; i.e., at frequencies at which it tuned with capacitances of 800–1,000 pF. There were two reasons for working at the lowest available frequencies: (1) because the conductance-dial reading falls off rapidly as the frequency increases (being inversely proportional to $f^{3.5}$, for the present type of coil) and the error in the reading becomes correspondingly larger, and (2) because the effect of the resistance of the twin-T tuning capacitor is minimum at the lowest frequencies.

Each set of measurements was entered up on a standard sheet. We have reproduced, as a typical example, that relating to coil No. 31. Nine measurements were carried out on each coil, the first at a frequency at which the coil tuned with about 1,000 pF, and the remainder at frequencies increasing progressively by the smallest dial intervals of the signal generator. It was thought preferable to perform each measurement at a different, rather than at the same, frequency to avoid, if possible, systematic errors. The values of $Q/\sqrt{f}$ thus obtained, after various corrections had been made (see over), usually showed a spread of from 3 to 5 per cent.

Various necessary formulae were reproduced, for convenience, on each sheet.

8. Corrections Applied to the Twin-T Measurements

Six corrections had to be applied to the original measurements. Four of them will be described in this section, and the remaining two will be given a section each.

TABLE III

Coil No. 31. 29 . 8 . 45. Mean temperature, 20.5° C.

Freq. Mc/s	C_{p1} (pF)	C_{p2} (pF)	Conductance dial reading	$C_{p2}-C_{p1}$ + 3.4 (pF)	Corrected conductance dial reading	$-\delta G$	True conductance (μmhos)	Q	$\frac{Q}{\sqrt{f}}$	L (μH)
0.78	100	1,080.3	34.9	983.7	21.2	0.1	21.1	228	0.258	42.32
0.79	100	1,056.2	32.9	959.6	20.5	0.1	20.4	233	0.262	42.29
0.80	100	1,031.8	32.0	935.2	20.5	0.1	20.4	230	0.257	42.32
0.81	100	1,007.9	30.8	911.3	20.2	0.1	20.1	231	0.257	42.36
0.82	100	984.8	29.1	888.2	19.6	0.1	19.5	235	0.260	42.41
0.83	100	962.6	28.0	866.0	19.3	0.1	19.2	235	0.258	42.46
0.84	100	943.5	27.0	846.9	19.1	0.1	19.0	235	0.256	42.39
0.85	100	923.4	26.0	826.8	18.8	0.1	18.7	236	0.256	42.40
0.86	100	904.0	25.2	807.4	18.6	0.1	18.5	236	0.255	42.42
								Mean	0.2577	42.37

No. of turns = 40
Wire gauge = 20 s.w.g.
D = 5.19—0.09 cm
= 5.1 cm
R = 2.55 cm
length = 7.01 cm
$\frac{\text{Length}}{\text{diameter}} = 1.375$
d = 0.0910 cm
$s = \frac{6.92}{39}$ cm

$$\frac{d}{s} = \frac{0.0910 \times 39}{6.92} = 0.513$$

$$Q = \frac{6.283 fC}{G}$$

$$L = \frac{0.02533}{f^2 C}$$

$$R_s = \frac{6.283L}{Q/\sqrt{f}}\sqrt{f}$$

$$\phi = \frac{[(\text{resist})/10^{-6}\ \sqrt{f}] \times d}{0.5218\ Rn}$$

$R\,exp.$ = 1,033 × $10^{-6}\sqrt{f}$ ohms
Resistance of leads
= 7 × $10^{-6}\sqrt{f}$ ohms
Resistance of coil
= 1,026 × $10^{-6}\sqrt{f}$ ohms
ϕ = 1.745
= 1.743 at 20° C
= 1.71 at 20° C $f \rightarrow \infty$
Resistance of leads
= 0.1661 × $10^{-6}\frac{l}{d}\sqrt{f}$
where l = ½ (total length of leads).

8.1. The conductance-dial reading has to be multiplied by a factor of the form $\left(\frac{f}{f_0}\right)^2$, where f is the working frequency and f_0 is either 1, 3, 10 or 30 Mc/s according to the frequency range which is used. The results of this correction are in column 6 of the measurements sheet.

8.2 The corrected conductance value thus obtained has to be further corrected on account of the resistance, R_c, of the metal structure of the main capacitor. According to Sinclair's paper,[11] R_c, in a twin-T whose residual parameters were measured by him, had the value 0.026 ohm at 30 Mc/s. As will be shown later, even a large departure from this value in the machine actually used, will not seriously affect the present measurements. Assuming this resistance to be proportional to the square root of the frequency, the error it introduces into the conductance reading can be corrected by adding algebraically a factor δG, which has the form

$$\delta G = -\{0.184(f)^{2.5} (C_{p2}^2 - C_{p1}^2) \times 10^{-6}\ \mu\text{mhos.}$$

f represents the frequency (Mc/s), and C_{p1} and C_{p2} are respectively the initial and final settings of the tuning capacitor (pF).

This is a very cumbersome expression to use, if one has to work out a large number of values of δG. The labour of evaluation can be considerably decreased by using a graphical procedure. The first part of the expression, $0.184(f)^{2.5}$, is plotted on double-logarithmic graph paper (Fig. 2), for a range of f from 0.1 to 10. The second part, $(C_{p2}^2 - C_{p1}^2) \times 10^{-6}$, is plotted as the third line of a nomograph (Fig. 3) the two base lines being C_{p1} and C_{p2}, each ranging from 100 to 1,150 pF. δG is now obtained as the product of the two quantities read off from Figs. 2 and 3.

As an example: for coil No. 31, at a frequency of 0.78 Mc/s C_{p1} was 100 pF, and C_{p2} 1,080.3 pF.

Now, $0.184(f)^{2.5} = 0.184(0.78)^{2.5}$
$= 0.1$ from Fig. 2
and $(C_{p2}^2 - C_{p1}^2) \times 10^{-6} = 1.16$ from Fig. 3
so $\delta G = -0.1 \times 1.16$
$= -0.1\ \mu$mho approx.

While the value of R_c, for the particular twin-T used, was probably not identical with that of the twin-T measured by Sinclair, it was not thought necessary to repeat Sinclair's measurements. There were two reasons for this: (1) the correction for R_c, at the frequencies used, is a small fraction of the measured conductance (usually of the order of 1 or 2 per cent), and the correction would not be appreciably affected by any likely deviation of R_c from Sinclair's value, and (2) the value assumed for R_c, at frequencies of the order of 1 Mc/s is, in any case, an approximation, since R_c has to be measured at a frequency round about 30 Mc/s and the doubtful assumption is made that R_c will vary precisely as the square root of the frequency.

8.3 The correction for losses in the leads is most conveniently applied by subtracting it from the equivalent series resistance. The expression for the h.f. resistance of a straight

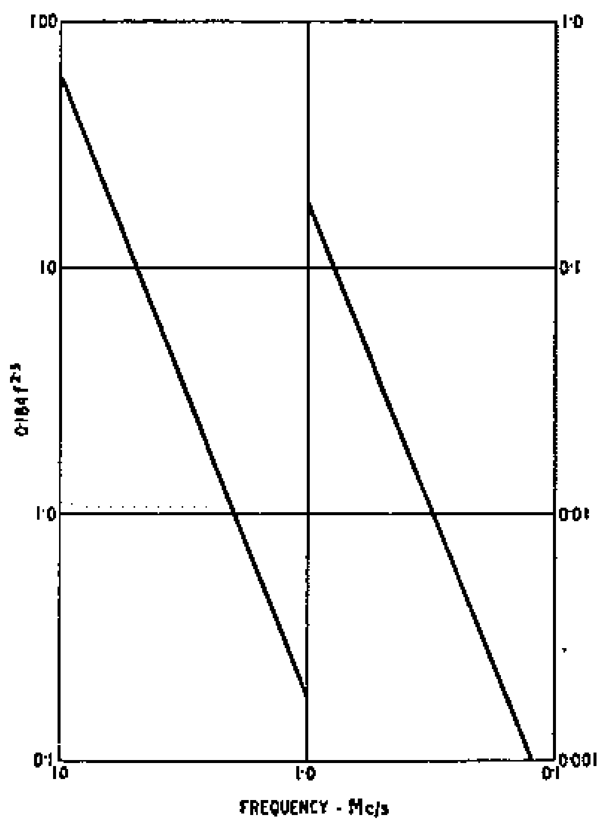

Fig. 2. *Correction for R_c (1).*

isolated copper wire was considered a good enough approximation to this loss. The correction was usually less than 1 per cent of the total series losses.

8.4 Temperature variation of the dimensions of the coils will not produce an appreciable effect on the series resistance over the temperature range (from about 16° to 26° C) encountered. However, temperature variation of resistivity has to be taken into account. Taking the temperature coefficient of resistivity of copper as 0.0039 at 20° C, we have to multiply the apparent value of ϕ (the ratio of the coil resistance to the resistance of the same length of straight wire at

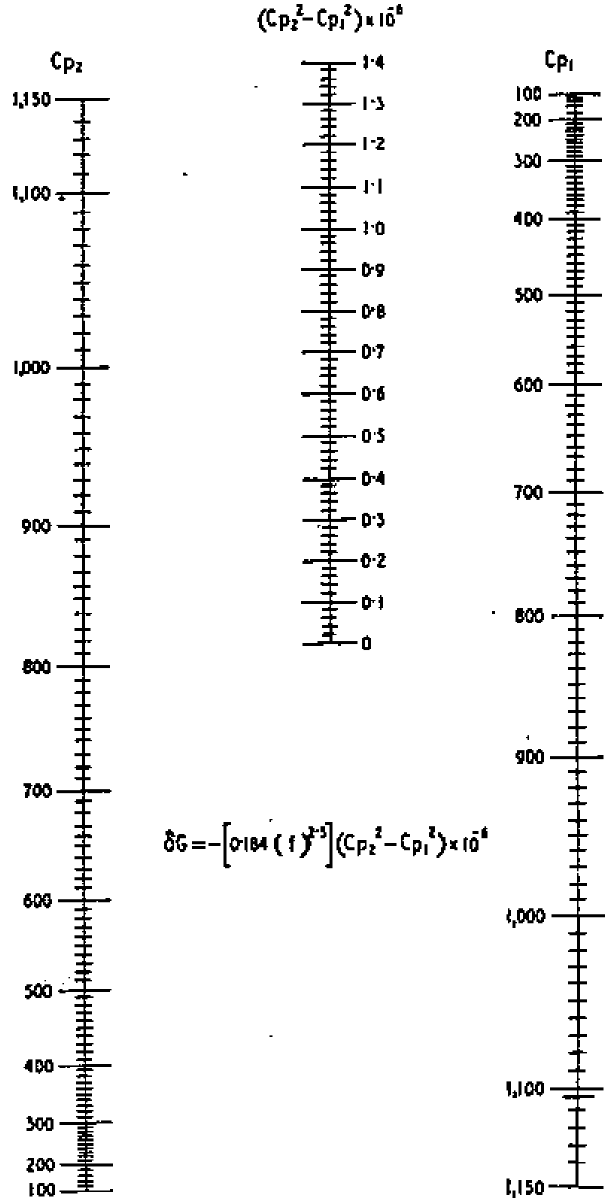

Fig. 3. *Correction for R_c (2).*

the same frequency) by one of the following factors.

$$\frac{1}{\sqrt{1 + 0.0039(t - 20)}} \text{ if } t > 20^\circ \text{ C}$$

or $\sqrt{1 + 0.0039(20 - t)}$ if $t < 20^\circ$ C

where t°C is the temperature of measurement.

8.5 The remaining two corrections are (5), for coil self-capacitance and (6), a frequency correction to the final value of ϕ. We shall consider each of these in a separate section.

(*To be concluded*)

For references see end of article in March issue.