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ANTI-GRAVITY WITH PRESENT TECHNOLOGY: IMPLEMENTATION AND THEORETICAL FOUNDATION

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Abstract

This paper proposes a semi-empirical model of the processes leading to the gravitational field based on accepted features of subatomic processes. Through an analogy with methods of cryogenics, a method of decreasing (or increasing) the gravitational force on a vehicle, using presently-known technology, is suggested. Various ways of utilizing this effect in vehicle propulsion are described. A unified field theory is then detailed which provides a more formal foundation for the gravitational field model first introduced. In distinction to the general theory of relativity, it features physical processes which generate the gravitational field.

Introduction

This paper is divided into two parts, I and II, largely independent of one another.

In the first part, a semi-empirical model of the origin of the gravitational field is proposed, based on recognized features of subatomic processes. These features, which concern the annihilation and creation of matter-energy, are shared by all subatomic processes and have been successfully employed in the explanation of the Lamb-Retherford effect, i.e., by the quantum electrodynamics. The model proposed is formulated in semi-classical terms, allowing analogy with the classical theory of gases. By utilizing the analogue of processes occurring in cryogenics, it is possible to formulate a method of reducing the mean energy of the random virtual processes asserted to be responsible for the gravitational field, and therefore to reduce the gravitational force (pat. pend.). The apparatus required is well within the capability of modern technology and an example is presented. Once the gravitational force on a vehicle has been weakened, various methods of vehicle propulsion can be employed, both within and outside the atmosphere of planetary bodies. The model also suggests a method of increasing the gravitational force; it is possible that this effect can be used in a propulsion scheme. Some methods of using the above effects are indicated.

Since the model proposed is different from the one advanced in the general theory of relativity, a more formal justification is desirable than the one described in the first part of the paper. Such a justification is presented in the second part, in which a unified field theory is developed. It is based on the notion of a noisy light signal, the noise being contributed by virtual subatomic processes. It is interpreted in terms of classical dispersion theory. Such a signal is asserted to be closer to reality than the type of light signal introduced in the special theory of relativity, and permits a unified description of matter and radiation. Since the metric of space-time must be altered, owing to the fluctuation phenomena characteristic of virtual processes, new field equations of motion must be introduced. This, in turn, leads to four potentials additional to those of the special theory of relativity. In particular, in the case of an absence of permanent electric charges and currents and in the presence of static mass distributions, one can recover the inverse square law of attraction between two masses, identified with the law of gravitational force. One is then persuaded that the physical mechanism postulated is responsible for the gravitational force. Altering the mean energy associated with these processes can be expected to alter the gravitational field.

Although the above deductions from the theory are of interest, other results can be derived of a more general conceptual value; these are listed below and will be demonstrated in the paper:

- Inertial mass, gravitational mass and the electromagnetic field are exhibited as different states of the same field.
- The theory includes, as special cases, newtonian mechanics, relativistic mechanics of a mass particle, Maxwell's equations for the propagation of the electromagnetic field, the gravitational field and a continuous gradation of these into forces which may be identified with nuclear forces on a sufficiently small scale.
- A unified description of classical particle mechanics and quantum mechanical processes, dependent on the scale of measurement is presented. The probability field of the quantum mechanics is replaced by a matter-energy field, so that some of the conceptual difficulties of the quantum mechanics are eliminated, e.g., the sharp division between radiation and matter.
- The zero-point infinite energy of the quantum mechanics of fields is eliminated and given a new interpretation. The uncertainty principle is reinterpreted.
- The infinite energy of the static electric, magnetic and gravitational field potentials, as separation of test body and source tends to zero, is eliminated.

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o A theory of a fundamental length follows naturally from the formulation.

o Field equations are presented for Bose-Einstein and Fermi-Dirac mass states, as well as associated quantities.

o Since the theory is linear, and the processes described are readily visualized, it becomes easy to formulate experimental tests of theory and to interpret these. Moreover, since the processes discussed are agreed to exist, the theory provides a unified framework into which to fit the many diverse phenomena associated with very small and very large scales of measurement. A consequence of such a formulation is that the experimentalist can expect to be able to predict how each of the fields, which can be generated by present day technology, can affect the other fields represented by this theory. The proposed control of the gravitational field discussed in the first part of the paper is derived by such considerations.

o The derivation of the laws of classical physics and of the quantum mechanics from the proposed theory constitutes an experimental proof of some of the theory. In this sense, it is similar to the molecular and atomic theory of matter before Einstein's paper on the Brownian motion (i.e., many known macroscopic properties of matter could be explained and unified). It is believed that, like the atomic theory, enough new results (aside from the application to the control of gravitation) will be deduced from the theory to make it worthwhile. It is in this expectation that the theory is proposed. However, the clarification of the nature of inertial mass, gravitational mass, and the ether (i.e., its existence) and the provision of a clearly visualized structure to unify the data of modern physics in exchange for a single additional assumption, make the theory worth consideration.

I. A Model for the Generation of the Gravitational Force

1.0 Introduction

The most evident property of the gravitational force is that it appears in the presence of matter (measured by inertial mass) and not otherwise. It is remarkable that whatever the state of matter from cold dust clouds to very hot stars, the same law, Newton's Law of Gravitational Force, remains an accurate representation of the force between two masses, and that this is dependent solely on the masses and separation of the two bodies.

This constancy of behavior argues a common mechanism operating to cause the force, independent of variability of physical state and temperature. Thus we seek common features of all the forms of matter known to exist on an astronomical scale, for which the gravitational force becomes appreciable. One such common feature is the experimentally verified observation that all matter is confined to elementary particles. It is therefore natural to enquire if some property of elementary particles can be responsible for the gravitational force. This we will try to show in the following. In the development of the ideas to follow, we shall not adopt the point of view that a single kind of elementary particle is responsible for the force. For if this were so, it would then be necessary to explain why large extremes of physical states do not result in a variation of the populations of the particles in question, and a possible alteration of the gravitational force as a consequence. A simpler model is chosen below, one which relies on a property to be found also on a classical scale, involving gas kinetic properties which are well understood.

In applying the model it is necessary to show how subatomic particles, whatever their nature and frequency of occurrence, can all give rise to the same kind of force field. The details of the interaction between elementary particles will not be discussed; these should not be necessary if the force is a result of averaging over a feature common to all of them. The model is in the spirit (and was suggested by) the kinematic description of a matter-energy field, developed in Reference 1 and in Part II of this paper. It deduces forces from a known motion or state, rather than relying upon a dynamic description which requires detailed knowledge of the forces of the interaction, from which the motion of the physical system can be deduced. Moreover, once having described a model of the processes which give rise to the force of gravitation, a procedure can be proposed by which the force may be altered, i.e. weakened or strengthened. This is also described below.

We here depart from Einstein's geometrical formulation of the gravitational field, which does not indicate any physical process leading to the origin of the field. Moreover, the model adopted views matter essentially as an aspect of a matter-energy field, a concept developed in a more consistent way in Part II of this paper. The latter concept is familiar to physicists, although a unified description of matter-radiation is not. The remaining concepts employed are taken from modern visualizations of physical processes.

1.1 Kinematic Model for the Gravitational Force

The feature postulated as basically responsible for the gravitational force is the stability of elementary particles during the time they exist. This stability is expressed in the fact that the mass m of the particle is constant, or, to put it somewhat differently, that its Compton wavelength h/mc is constant.

But the Compton wavelength is a measure of the extent of virtual processes associated with a given particle.*

*As pointed out in Reference 1, and more explicitly described in Part II, the Compton wavelength is a measure of the principal extent of an elementary particle matter-energy cloud, essentially a field phenomenon, rather than a particle accompanied by the products of virtual processes. As shown in Reference 1 and in Part II, although h/mc is a small quantity, the average effect of virtual processes leads to an inverse square force law, i.e. these processes extend beyond the Compton wavelength.

In the presence of another particle, and with the above-mentioned non-localizability of both particles, the mass density distribution of the first particle is altered since the fields of the two particles overlap and they share some of their masses with one another. And if this is so, then the mass of the first particle is altered from m to $m+\Delta m$, and its Compton wavelength also altered from h/mc to $h/(m+\Delta m)c$ or to approximately $(h/mc)(1-\Delta m/m)$, a smaller diameter than the initial one. To remain stable, in the sense in which we have defined stability, an alteration in the mass-energy distribution must take place, according to LeChatelier's Principle, which will compress the matter-energy into a smaller volume with the smaller diameter given above and corresponding to the new stability configuration. The alteration is due to the matter-energy contributed by the second particle and results in the mass-energy contributed by this particle being drawn closer to the central location of the first particle. The reaction to this re-distribution is an attractive force exerted by the second particle on the first. Conversely, by symmetric reasoning, the first particle exerts an attractive force on the second particle, again owing to its own stability. The changes must be small in order for these considerations to be valid.

It is asserted that the above mechanism, which depends only on assumptions incorporating the presently accepted view of matter and radiant energy, and does not rely on a detailed description of elementary particle forces, is responsible for the attraction between two masses called the gravitational force. Moreover, it satisfies the requirements outlined in Section 1.0. A more formal discussion of the origin of this force has been given in Reference 1, where the inverse square property has been derived and the attractive nature of the force also derived on more formal grounds. A classical (i.e. non-quantum mechanical) model of the origin of forces of this general nature is provided in Appendix A, but is not necessary for the present exposition; the latter derivation concerns two macroscopic particles suspended in an ideal gas. The ideal gas corresponds to the virtual particle cloud surrounding an elementary particle.

Implementation of an Anti-Gravity Device

2.0 Introduction

The considerations above suggest that a process analogous to those which reduce gas temperature by allowing gas molecules to do work against an external agent, converting internal random disordered motion into an external ordered motion, can be used to reduce the gravitational force between two masses. The "gas" in this case is the cloud of "virtual" particles surrounding every subatomic particle. We discuss the physical mechanism suggested to produce the effect, and estimate its magnitude below.

Summary of the Experiment

For the purpose of reducing the mean energy of the processes which have been characterized as subatomic noise in References 1 and 2, we suggest the use of pulsed dynamic nuclear orientation applied to paramagnetic atoms in a constant magnetic field. The basic mechanism invoked is to allow rapid (i.e., faster than thermal disordering) decay into a disordered state of that part of the oriented magnetic moment of the paramagnetic nucleus due to creation and annihilation processes; this decay is due to interaction with the earth's virtual particle cloud. It is asserted that this procedure will reduce the mean energy of the disordered motion of the earth's virtual particle cloud in the neighborhood of the specimen and lead to a reduction of the earth's gravitational force on the specimen.

For reasons indicated in the following discussion, we suggest use of a specimen composed of a very pure isotope of aluminum (e.g. Al^{27}) with iron inclusions (magnesium plus chromium inclusions are also possible candidates). Acting on the specimen, a pulsed oscillating microwave field orients the nuclei in the iron inclusions by dynamic nuclear orientation. Small quantities of chromium included in the aluminum will increase efficiency of the pulsed field in penetration of the specimen, since its effect will be to increase the skin depth for microwave penetration (the resistivity of chromium is greater than that of aluminum). In addition, chromium is a member of the iron group and thus will aid in the nuclei orientation of the iron inclusions (Reference 3, pp. 63-64).

The nuclear orientation of the iron inclusions will be communicated to the nuclei of the aluminum matrix, diffusing throughout the material. The orientation of the aluminum nuclei has a much longer lifetime with respect to thermal decay than the orientation of the iron nuclei (in the ratio of 9 to 1--Reference 3, p. 64 and p. 83). Hence the matrix acts as a reservoir of nuclei orientation, so that the induced iron nuclei orientation generates a pumping action, amplifying the effect of the forced orientations. Some of the apparatus required is shown in Figure 1.

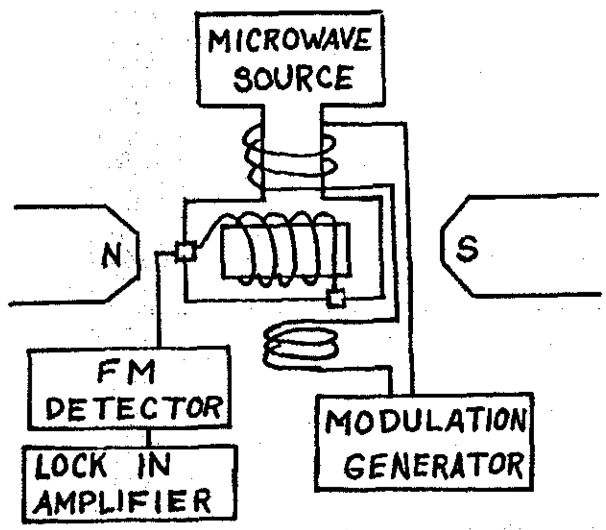

Fig. 1 Some apparatus used for dynamic nuclear orientation.

The interaction of the disordered matter-energy creation-annihilation cloud, generated by *the earth, and the ordered aluminum nuclei (and* inclusions) lead to a disordering of the aluminum nuclei (and inclusions) orientation. This process occurs at the expense of the mean disordered motion of the earth's creation-annihilation cloud and a consequent reduction in the earth's gravitational attraction exerted on the aluminum (and inclusions) specimen.

As a corollary to the above considerations, it is asserted that constantly driving the virtual processes by flipping the paramagnetic nuclei too rapidly to allow the virtual processes to decay appreciably, should lead to an increase of the gravitational force, since then the mean *disordered motion of the earth's* (as well as the specimen's) creation-annihilation field will be increased. This can be accomplished by a periodic reversal of the magnetic field which was held constant above in the usual process of dynamic nuclear orientation.

The remainder of this paper will concern itself with the details and magnitudes of the effects generated by the procedures outlined above.

2.1 Dynamic Nuclear Orientation

The principal source of the information in this section is Reference 3. We describe the physical basis of dynamic nuclear orientation and some of the magnitudes associated with it.

Dynamic nuclear orientation of paramagnetic nuclei consists of first imposing a fixed magnetic field on a given specimen, causing the electrons in a given atom to precess about the magnetic field direction. An oscillating field at right angles to the constant field is then applied, with resonant frequency ν determined by the magnitude of the fixed field $\mathcal{H}$ and the relation

$$\nu = g\,\mu_B \mathcal{H}/\hbar \qquad (2.1)$$

where g is the spectroscopic splitting factor for the electron, μ_B is the magnetic moment of the Bohr magneton, and h is Planck's constant. Typical orders of magnitude are $g \sim 3$, $\mu_B \sim 10^{-20}$ erg/oe.

The oscillating field adds energy at resonant frequencies to the electrons in the atoms, causing transitions to higher energetic spin states, increasing the electron population in these states. Due to their closeness to the nuclei of the atoms, the electron orientation induces nuclei orientation. Appreciable orientation of nuclei orientation in a given sense can be obtained with external fields less than 10^4 oe at temperatures of the order of 0.1°K.

2.2 The Thermal Lifetime of Nuclear Orientation

The rate of relaxation of nuclear orientation in paramagnetic atoms due to thermal processes is very rapid and rises quickly with increase of temperature above very low helium temperatures (e.g. as T^7 or T^9---Reference 3, p. 33). *The above-mentioned immersion of iron* inclusions in an aluminum matrix was proposed to overcome this drawback. The diffusion of orientation into the aluminum matrix takes place in accord with a well-known process (Reference 3, pp. 74-77).

Nuclear thermal relaxation times have been accurately measured, including their dependence on temperature. For Al^{27} the relaxation time at 4°K is about 0.5 sec, and is proportional to 1/T, where T is the absolute temperature of the specimen. If the latter proportionality be extrapolated to room temperature (about 293°K), the relaxation time would be about 0.006 sec; in any case the latter lifetime will be very short.

Thus, if relaxation times are sufficiently large with respect to thermal electron collisions, we can expect that the nuclear orientation (or part of it) will relax owing to other processes. We now estimate the lifetime of the ordered states subject to perturbation by creation and annihilation processes.

2.3 The Lifetime of Virtual States

Quantum electrodynamics has demonstrated that the fractional change in the electron's magnetic moment, owing to creation-annihilation processes, is given by

$$\frac{\mu_e - \mu_{oe}}{\mu_{oe}} = 0.0011596 \qquad (2.2)$$

where μ_e denotes the altered value of the magnetic moment and μ_{oe} its original value $e\hbar/2m$ and $\hbar = h/2\pi$).

Analogously to the estimation of the magnetic moments of proton and neutron by expressions similar to that used for the electron (e.g. $eh/2M_p$ for the proton, where M_p is equal to the mass of the proton), we estimate the effects of virtual processes on the magnetic moment of a nucleon by a relation similar to 2.2

$$\frac{\mu_N - \mu_{oN}}{\mu_{oN}} = 0.0012 \qquad (2.3)$$

Since $\mu_{oN} = 5.1 \times 10^{-24}$ erg/oe, the energy stored in the magnetic interaction with virtual processes is ($g \sim 3$, Reference 3, p. 2---here g is defined somewhat differently than the reference, but the calculation is essentially the same)

$$\Delta E \sim g\,(5.1 \times 10^{-24})\mathcal{H} \qquad (2.4)$$

For a paramagnetic ion, the nucleus experiences a magnetic field, locally, of 10^5 to 10^6 oe, so that (2.4) may reach a value of 1.5×10^{-17} erg per paramagnetic ion nucleus. The frequency associated with the latter energy is then $\Delta E/h$ or about 2.3×10^9 Hz and the complementarity of frequency and time implies a lifetime for the state of about 4.3×10^{-10} sec. That is, the lifetime of the virtual states in question are very much shorter than nuclear orientation thermal lifetimes. A pulsed oscillating field will therefore serve to keep the paramagnetic nuclei oriented insofar as thermal decay is concerned and, properly timed, will allow that part of the magnetic moment due to virtual processes to become disordered. In the relaxation by paramagnetic impurities, the nuclei act as though they are coupled together (Reference 3, p. 73).

2.4 Experiment Parameters and Comments

The above estimates have been in the spirit of Reference 3 (e.g. pp. 2-3 and p. 73), taken more to illustrate the principles proposed than as accurate representations of physical quantities, since the basic parameters do not appear to be accurately known. However these principles have been borrowed from other, and well-known applications in classical physics; the technology of dynamic nuclear orientation is also well-developed. Hence it seems highly desirable and possible to do an experiment of the sort proposed, especially to determine more accurately the parameters estimated. In this connection, the following comments are made:

o The calculation of the energy stored in the ordering of virtual processes was performed for a single nucleon. The estimate was deliberately made much smaller than is likely; this follows from the greater complexity of the proton's structure than that of the electron (e.g. remarks by B.L. Cohen in Reference 4, p. 8). Moreover, the iron nucleus is still more complex than the aluminum nucleus.

o For the proposed experiment, it is likely that the desired effect can be obtained for temperatures much higher than those at which conventional nuclear orientation processes are performed, due to the wide difference in thermal and virtual state lifetimes for inclusions and matrix.

The magnetic fields and frequencies of the oscillating fields can be selected as those often used in dynamic nuclear orientation. For example, with a fixed magnetic field of 660 oe one can employ a pulsed oscillating field at 3000 MHz. Pulses of the oscillating field are to last for 2 microseconds with a duty cycle of from 2 to 6 milliseconds. The optimum quantity of iron to be embedded in aluminum must be determined by experiment; combination of the two metals is aided by the difference in their melting points. Since iron has a higher melting point than aluminum, iron inclusions can be added to molten aluminum which is then stirred to obtain a uniform distribution in the specimen.

If, for example, the ratio of iron nuclei to aluminum nuclei in one gram of a specimen should be about 1 in 10, then (provided the above estimates are realistic) of the order of a joule of gravitational energy can be removed per duty cycle. At the earth's surface, there are 6.3×10^4 joules per gram mass of gravitational energy, so that an appreciable part of it would be removed in several seconds, on the basis of this simple model.

It would be of interest to perform the proposed experiment so that the model proposed can be tested and improved.

2.5 Vehicle Design

Anti-gravity (i.e., weakening the gravitational force) alone is evidently not enough to propel a vehicle. Its principal use would be to reduce the power needed to propel the vehicle, when near a body of astronomical scale, such as a planet or sun.

Within a planet's atmosphere, there are many possible methods of propelling a vehicle, based on a thrust against the atmosphere of the planet. Since it is likely that the generated field will reduce the inertial mass of the vehicle to some extent, very high velocities and accelerations would be possible.

The method of applying thrust will depend to some extent on the shape of the vehicle. It would be economical to choose a circular cross section since the generator of the anti-gravity field would then be equi-distant from the metal structures making up the vehicle. These are imagined to be sheets of aluminum with iron inclusions, as mentioned above. These sheets will be too thin to supply structural strength to the vehicle and will be attached in panels to the underlying structure. Because of the circular cross section, the propulsive force in the atmosphere will be applied in a symmetrical manner relative to this cross section. For example, if a spherical shape is used, a slotted rim rotating about the center of the craft, parallel to the planet's surface, can be used to lift the vehicle. A central jet engine with exhaust openings about the sides of the vehicle can supply horizontal thrust for maneuverability. Figure 2 illustrates a possible configuration.

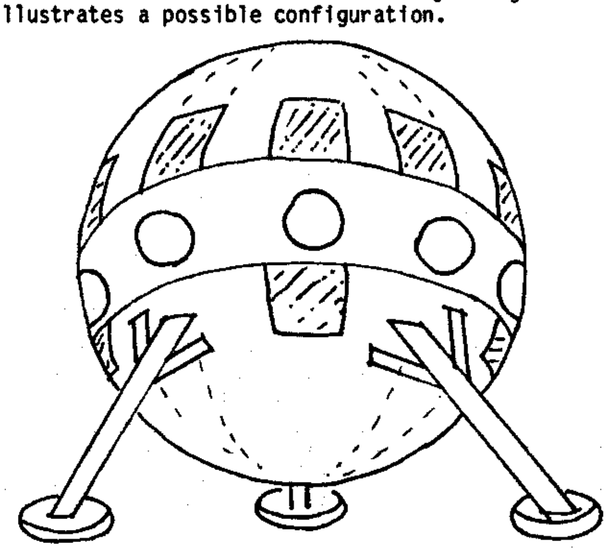

Fig. 2 Conceptual drawing of vehicle using anti-gravity screens.

Because of the harmful effect of microwave radiation on the crew, the microwave generator will be shielded from the crew's quarters; its logical placement would be in the lower part of the sphere.

The design suggested makes this kind of vehicle a good choice for a shuttle vehicle between planet surface and a space ship in orbit about the planet, since it can not only maneuver within the atmosphere of the planet, but also leave its surface to go into a conventionally chosen orbit. For the latter purpose, as well as taking off to explore interplanetary space, the suggested design is useful, since no large take-off acceleration is required. Once in space, small rocket or gas jets offer a convenient means of maneuvering. Since large amounts of propellant do not need to be used in takeoff and landing, it will be possible to accelerate the vehicle for part of an interplanetary trip and to decelerate for the latter part of the trip, thus reducing travel time by a considerable margin.

II. Unified Field Theory

3.0 Introduction

The second part of this paper proposes to provide a formal basis for the gravitation model in Part I. It is not a theory of gravitation alone; it is a unified field theory which presents the electromagnetic field (Maxwell's), the gravitation field and field properties which can be identified with forces on a subatomic level as different aspects of the same field. Both matter and radiation are exhibited as different aspects of the field, i.e. mass particles are a special state of the field.

The question of the value of such a formulation may arise, although the unity of matter and radiation, owing to observed transformations of one into the other, is not in question. In this connection, one is reminded of the advantages which followed from the unification of the electric and magnetic fields, into the electromagnetic field, and the unification performed by the atomic theory, as a justification for the effort. There remains, however, the question of how this unification ought to be carried out and what the properties of such a unification ought to have.

A. Einstein found it necessary to defend his approach to the problem of unifying the fields[5], although he never doubted the desirability of such a unification. As Einstein states, "The characteristics which especially distinguish the general theory of relativity and even more...the unitary field theory, from other physical theories are the degree of formal speculation, the slender empirical basis...and...the fundamental reliance on the uniformity of the secrets of natural law...It is this feature which appears as a weakness to physicists who incline toward realism or positivism, but is especially attractive,...to the speculative mathematical mind."

That Einstein recognizes the weakness of a theory designed primarily for mathematical simplicity, as indicated above, is further emphasized by his remark that "Pure logical thinking can give us no knowledge whatsoever of the world of experience; all knowledge about reality begins with experience and terminates in it."[6] The relevance of the special theory of relativity to experiment, for example, arose out of Einstein's recognition of the empirical fact of the constancy of the speed of light and his assertion of the nature of distant clock synchronization, a feature of the use of light signals (i.e. a field) to establish a coordinate system. His later work was more mathematically oriented.

It was the questioning, by an empiricist, of the geometrization of space-time in the general theory of relativity, which led to experiments claimed to cast doubt on the foundations of the theory.[7]

The above remarks lead one to enquire what a "worthwhile" formulation of a physical theory should be. In this connection, the history of science provides several guidelines for a physical theory's usefulness.

Experience has shown that, to be of general interest to the scientific community, a theory must not only make numerical formulations of correlations between events, but do so in a readily visualized manner. In addition, in the appropriate context, it must reduce to the equivalent of foregoing concepts and theories. It must, also, yield simpler and more direct interpretations of reality than its competitors (e.g. preceding theories). In a final test, a theory must offer attractive opportunities for formulation and interpretation of experiments.

The above considerations have guided the development of the following theory and the course of the discussion below. If the unification of the concepts and fields proposed is correct, then an alternative to the unitary field of A. Einstein has been accomplished. Its resolution of some of the conceptual difficulties of the quantum mechanics and removal of infinities of the Coulomb-like laws is an added useful feature, while the unification of field and particle concepts resolves a difficulty recognized by A. Einstein.[8] If these and other conceptual conveniences were the only features of the theory, it would have only the value of the atomic theory in its initial stages, i.e. offering a unification of already-known observations. However, to the extent that the considerations of Part I are relevant to the formal theory, a basic requirement for a useful theory will have been met.

4.0 General Background

The following theory is based on the addition of a new assumption about the propagation of the real electromagnetic field (in distinction to Maxwell's field), to the assumptions of the special theory of relativity, as well as a closer adherence to practical aspects of field measurements.

There are many reasons to believe that the light signal (i.e. electromagnetic field) behavior used by Einstein should be altered. Since the special theory of relativity was first proposed many new experimental properties of the field have been discovered, one of the most prominent being the conversion of radiation to matter and the converse. These conversions may occur on a scale of the order of h/mc^2 (m = mass of the electron, for example) and also on a relatively long time scale. The former are termed virtual processes and are conceded to have a real existance, as shown by the correctness of the predictions of the quantum electrodynamics.

As an example of how a light signal might be altered by the creation of matter, consider the case of a light source traveling toward the observer at a speed sufficient to raise the frequency of the radiation to the level of gamma rays. The presence of heavy nuclei (to absorb momentum) near the observer will permit pair creation at the expense of the energy of the gamma rays and therefore an alteration in the character of the "light" signal expected. An observer at rest relative to the source would not (according to the special theory of relativity) see such events, suggesting a logical contradiction.*

While the pair creation imagined above occurs on a time scale suitable to atomic physics, virtual processes of creation and annihilation of matter occur on a much shorter time scale. And, although the special theory of relativity predicts an equivalence between inertial mass and radiation, the nature of the equivalence is not featured in the foundations of the theory. However, if the success of the special theory of relativity has been based on a recognition of a property of the electromagnetic field not included in the Maxwell theory of the field (i.e. the constancy of the speed of radiation, independent of the speed of the observer), then it is fitting to enquire what advantage may be derived by adding to the properties of the field the known phenomena of the creation and annihilation of matter.

4.1 Role of the Light Signal in a Unified Field

The model proposed in Reference 1 was motivated by the observation that if there is in reality a unified field, any field model must recognize that experiments such as the Davisson-Germer experiment imply that, on a subatomic scale of phenomena, matter exhibits wave or field properties. In addition, on an atomic scale or smaller it is necessary to allow for the random nature of all physical processes. On the other hand, one must also be able to derive from the model the properties of mass particles on a macroscopic scale, including Newtonian mechanics, as in Reference 1.

*The observer at rest relative to the signal emitter may observe such events on a very small time scale (e.g. virtual pair creation) relative to the scale on which these events are observed in the reference frame in motion relative to the source, owing to time interval dilation. However, this does not alter the assertion that a light signal can have a more complex structure than allowed in the special theory of relativity.

If the metric of space-time is to be established by means of light signals, so that the results of the special theory of relativity may be recovered, then in the context of a unified field theory, the metric is to be established by using only one state of the unified field, the Maxwell electromagnetic field. In accord with the analysis of the Lamb-Retherford line shift and other results of the quantum electrodynamics, an electromagnetic field signal participates in the motion of other states of the unified field and therefore cannot preserve its identity in the presence of matter. And since there must always be a material generator and receiver of a radiation signal, there must always be a finite probability that the light signal will lose its identity for part of its motion between sender and receiver. This probability becomes significant for sufficiently small regions of space and time.

However, if a single state of the unified field is used to establish a metric, the other states of the field will appear as interruptions in the propagation of that single state; this is a notion borrowed from classical dispersion theory. A brief description of the theory is presented in the next section.

4.2 Dispersion Model Applied to the Light Signal

The classical theory of dispersion is an aid to visualizing the route of development chosen, i.e. to preserve the electromagnetic signal in a vacuum characterizing the special theory of relativity, while adding one property to its motion. The latter approach is in accord with the description of the desirable formulation of a theory in application of the scientific method.

We imagine, for simplicity, a plane wave propagated in a vacuum:

$$e^{i(\omega/c)(x-ct)}$$

incident normally on a cubic crystal lattice. Most of the crystal lattice, according to the atomic theory, is composed of a vacuum and yet is capable of altering the phase velocity c of the above wave in a drastic manner. The classical dispersion theory resolves this apparent paradox by imagining the electrical charges of the atoms set into vibration by the plane wave and re-radiating to form (by coherent interference) a new plane wave, again characteristic of propagation in a vacuum. There will be, however, a shift in phase due to the interaction:

$$e^{i(\omega/c)(x-ct+\delta)}$$

Each crystal lattice plane adds another phase shift of the same amount so that when the plane wave emerges from the crystal, the total phase shift is proportional to the thickness of the crystal, call it a. The number of such phase shifts is approximated by a/d, where d is the separation of the lattice planes. Therefore the total phase shift is approximately aδ /d.

Since the thickness of the crystal and the time interval of passage are known, however, it is convenient to interpret the phase shift as a change of phase velocity and to introduce an index of refraction as a measure of this alteration.

In the present theory, we shall introduce phase shifts to indicate processes occurring in the radiation signal propagating in a vacuum, with the difference that the average value of the phase shifts is zero for the electromagnetic field on a macroscopic scale. This condition describes the Maxwell electromagnetic field and will not necessarily be valid for other fields. It is imaginable, for example, that all the phase shifts for the monochromatic waves comprising a wave packet will be equal, as in the classical dispersion theory. In such case, the entire wave packet will be shifted in location from what would be expected on the basis of the electromagnetic theory alone. Corresponding to the possibility of this kind of event and a similar translation in the time scale, we interpret the phase shift as a translation in space and time coordinates, rather than as an alteration in the phase velocity. Thus is preserved the observation of the constant speed of light; moreover, like the dispersion theory, there is the convenience of using a wave motion characteristic of a vacuum.

4.3 Field Theory and Filter

But the introduction of the notion of noisy signals from communication theory is not enough for an adequate correspondence with reality. For, along with the admission of noise as well as signal as fundamental phenomena in establishing a metric of space-time, is the essential concept of filters used by both generator and receiver of a given signal. In a more general sense, every experiment is a filter: a series of operations in which desired effects are to be admitted to observation, if they exist, and undesired effects shut out of observation. But undesired effects, i.e. noise, are never completely eliminated; hence noise, it is proposed, must be taken into account in a fundamental way in establishing a metric of space-time. Moreover, this concept corresponds in a more realistic manner to how measurements of signals are made, as well as, more generally, how physical systems interact through fields.

Relevant to the filter concept, the manner in which the expanded type of motion of the unified field is introduced in Reference 1 characterizes the field in a kinematic manner, as distinguished from a dynamical model, i.e. an assumption is made about the motion of the field rather than about the forces responsible for the fluctuation processes giving rise to the new metric and leading to the additional degrees of freedom. The splitting of the coordinates used into an average and a fluctuation contribution is in consonance with a theorem proved by A. Einstein to hold in an approximate manner.[9] Moreover, this concept corresponds to the manner in which the results of observations of fields, using filters, are analyzed. The latter remark is especially relevant to the logical consistency of the splitting of the coordinates into an average part and a fluctuating part, as successfully carried out for molecular forces in a well-known and analogous analysis of the Brownian motion by Langevin.[10]

5.0 Physical Background

It was first recognized, in connection with the analysis of the spectral energy distribution of the radiation emitted by a blackbody, that the emission and absorption of the radiation occurred in an uncontrollable manner if the scale of energy transfer were sufficiently small.[11] Moreover, the balance of energy in a blackbody cavity was later shown to be closely related to the spontaneous emission of radiation (creation of photons), again an uncontrollable phenomenon.[12] Further, events which occurred outside the control of the experimenter were eventually shown to be inserted into the whole domain of atomic phenomena; this observation became a universal law of nature as the Heisenberg Uncertainty Principle.[13]

More recently, it has been demonstrated that it is necessary to include the spontaneous creation and annihilation of particles in the description of nature. For example, notable success has been achieved with explanation of the Lamb-Retherford line shift in the hydrogen spectrum by means of virtual radiative processes in the presence of a polarized vacuum. The predictions of theory[14] and the experimental data[15] differ by no more than 0.5 MHz,[16] an agreement so accurate that it has been remarked that the polarization of the vacuum, as a consequence of an electron-positron field, is a "well-established phenomenon."[17] Further verification of the reality of the polarization of the vacuum, even if the energy available from the radiant quanta be less than $2m_ec^2$ (m_e = mass of the electron and c=speed of light), follows from the fine structure of the positronium ground state and level shifts of mesic atoms.[14]

Earlier it had been shown that the infrared[11] and the ultra-violet[18] catastrophes in the theory of the scattering of an electron by a central force field, could be removed by accounting for virtual processes. The latter was considered to be in the nature of a formal difficulty, rather than a reflection of reality.

Further, the interaction of nucleons is conveniently imagined, at least in part[19], as due to the creation, followed by the annihilation of particles (mesons). The magnetic moments of neutron and proton[20], as well as the quadrupole moment of the deuteron, are imagined to exist as a consequence of virtual charge creation.

In the above discussion, no sharp distinction was drawn between a virtual and a spontaneous process. In the following, it will be assumed that a virtual process owes its existence to spontaneous creations and annihilations which are not resolved by the scale of measurement employed. Thus, on an atomic scale, no net change in the number of particles or in the net charge will be observed due to virtual processes.

As a consequence of the above-noted phenomena, virtual processes are assumed to have a real existence and to proceed without cessation; they are not considered to be initiated by the action of the observer.

But the existence of additional modes of motion of the real electromagnetic field argues the existence of degrees of freedom additional to those which customarily (in theory) specify the state of the field. In Section 5.1 it will be shown how this consideration effects the formulation of the special theory.

5.1 Light Signals and Spontaneous Processes

By definition, a light signal is an electromagnetic wave packet of specified shape which results from a given series of interactions of material bodies, these interactions terminating at a given time and place. The series of interactions and the time and place of termination (or emission of the packet) will be assumed to be determinable as accurately as required.*

But here a difficulty appears. For so long as space and time intervals of terrestrial magnitude are involved, the series of interactions results in the excitation of many atoms, followed "immediately" by the emission of radiation. But on a smaller scale, if only a few atoms (or one atom) are involved, all the experimenter can do is to excite the atoms and observe whether or not the emission takes place at the given time and place. If the emission of radiation takes place on schedule, it is inferred that the signal has been generated by the experimenter. If the emission of radiation does not take place on schedule, i.e. spontaneously, one may infer that the inherent unpredictibility of atomic phenomena is at fault. But Einstein's analysis of the balance of radiation in a blackbody cavity indicates that this unpredictibility is linked to uncontrollable and spontaneous emission of radiation. One may therefore ascribe the delay to the intervention of spontaneous processes not initiated by the experimenter and it may be expected that no complete description of the generation of a light signal can be obtained without explicit introduction of the effect of spontaneous processes on the signal.

The viewpoint to be developed in the present paper is that the discrepancy between the scheduled and the actual time and place of emission of radiation is evidence of internal motion unresolved by the experimenter, rather than a consequence of an elementary and unresolvable uncontrollability in the excitation of the radiation field.

On an atomic level, this virtual motion can only be inferred; the methods of the quantum mechanics appear adequate to explain all experimental evidence. However, on a smaller scale, the creations and annihilations which give rise to similar discrepancies are resolved and the perturbation methods of the quantum mechanics no longer apply. If, for example, one specifies an electron-positron pair annihilation as the generator of a signal (gamma ray), then there is the possibility that the gamma ray will again create a pair*, etc., the process repeating itself a great number of times. It is then seen that the uniqueness of the time and location of the event of signal generation is itself lost to some degree, the interaction of the material bodies being displaced in space and time.

The latter feature of the model does not introduce any essential difficulties; rather, one can then introduce the concepts of internal motion, virtual processes, etc. relative to the light signal itself after its generation. There is then removed any essential distinction between the light signal and the material system which generated it. These considerations will be expanded in the discussion of the dispersion model.

The situation described above is not new in the history of natural philosophy; it characterized the growth of the atomic theory. For example, the Brownian motion is caused by processes which could not be resolved by the optical microscopes observing the motion; these processes would then be classed as virtual. Analysis of the inferred processes[21] required postulation of degrees of freedom additional to those descriptive of macroscopic motion. But although the latter, new, degrees of freedom do not differ in their essential character from those descriptive of macroscopic motion, the case of the light signal is unique in that no measurements on other physical entities can be used to

*Although the term "light signal" is usually used in connection with a source of radiation and an observer who (in principle) establishes a coordinate system by observations made with their aid, the point of view adopted here is that the success of such a procedure in establishing a physical theory is evidence that material systems interact with one another through the electromagnetic field on an atomic level of observation and in no other way. It is proposed below that material systems on a subatomic level also interact through fields and in no other way. The metric is a measure of how space-time coordinates are related in this interaction, e.g. whether the interaction is or is not possible.

*The model of creation of pairs intended here differs from that inferred from the formalism of the quantum mechanics. The fact that the formalism to be introduced reduces to that of quantum mechanics, where appropriate, indicates that the model here proposed does not contradict present concepts as they apply to atomic physics.

define the signal in a more elementary manner; it is the determinant of the space-time metric and its behavior cannot be described in terms of itself.

5.2 Dimensionality of Space-time and Spontaneous Processes

At the basis of all physical theory is the recognition that no theoretical construct can be introduced unless it is descriptive of some physical measurement.[22, 23] In particular, the whole imagined structure of space-time is based on the generation and measurement of light signals. This is a recognition that, ultimately, one can reduce physical measurements to interactions between electromagnetic fields and matter, and no further. And, although the success of the special theory has demonstrated the validity of its underlying assumptions on a terrestrial and atomic scale, it is by no means clear that the theory is necessarily valid for a more microscopic scale. The preceding discussion renders it plausible that a light signal transmitted over a distance of, say h/m_ec 10^{-13} cm, or for a time interval of the order of h/m_ec^2 10^{-23} sec (h = Planck's constant), cannot preserve its original structure owing to the creation and annihilation of elementary particles during the course of its motion.

On the other hand, there is no mention of spontaneous creation processes in the model adopted for the establishment of the Minkowski metric. And, although the probability of the event may be very small, there exists the possibility of spontaneous generation of a signal of the type agreed upon in advance. And if this be so, then one must also admit the existence of an uncontrollable means of generating a light signal independently of the experimenter's initiation of the signal. The existence of two such independent means of signal generation requires the introduction of degrees of freedom for the space-time continuum additional to the four of the Einstein-Minkowski geometry*, since a signal cannot be described in terms of itself—it determines the space-time metric.

On a terrestrial scale such events have not been observed, but the probability of observing a given signal, spontaneously generated, will increase as the number of observable spontaneous events increases. The aim here is not the estimation of this probability, but rather the justification for imagining such an event. It is well known, for example, that a similar situation exists in the detection of radar signals when the noise level is comparable to or greater than the signal level.

The spontaneous event imagined is meaningless, from the standpoint of interpreting the present model, unless a probability be associated with the occurrence of the event. It is agreed that this probability is negligible on the terrestrial scale of measurement. Moreover, it is inherent in the manner of their introduction that the new degrees of freedom be associated with statistical distributions descriptive of the processes which give rise to them. Hence the observable quantities to be discussed will be of the nature of expectation values which depend on statistical distributions. A like remark applies to the magnitudes of the new degrees of freedom themselves.

On the other hand, a statistical model is of value if the spontaneous processes introduced above are unresolved and therefore lead to an average effect. If the spontaneous processes are resolved, then there has been obtained a causal description of nature in the sense that one then has dynamical variables which determine the behavior of the individual physical system, not its average behavior alone. This point will be discussed in greater detail in Section 5.3 and the sections following.

5.3 Spontaneous Processes and the Space-time Metric

The axioms of the special theory (of relativity) are now assumed to be in force.[23] Thus, the relation between observations performed by experimenters relative to rectangular Cartesian coordinate systems $S:(x,y,z)$ and $S':(x',y',z')$, in relative motion with constant speed v along a common axis (the x and x' axes), are essentially summarized by the relations between the coordinates x,y,z,t and x',y',z',t' ($\beta = v/c$, $\gamma^{-1} = \sqrt{1-\beta^2}$)

$$\left.\begin{aligned} x' &= \gamma(x-\beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct-\beta x) \end{aligned}\right\} \qquad (5.1)$$

Moreover, it is emphasized that space-time is assumed to be homogeneous, as presumed in the special theory.

For the purpose of extending the special theory to account for the phenomena considered above, other assumptions will be added to those of the special theory:

Postulate I. The character of the space determined by spontaneously generated light signals is identical in structure to and independent of the space-time continuum determined by light signals generated by the experimenter.

A consequence of Postulate I is that point events of a spontaneous origin are related by transformation equations of the same structure as the Einstein-Lorentz equations 5.1.

But, according to Einstein, a coordinate system is derived from an assemblage of operations performed with light sources and clocks at

*It is of interest to compare this remark with A. Einstein's opinion of the validity of field theories which increase the dimensionality of the space-time continuum[24].

rest with respect to an observer.[23] Whether the signals are emitted by design or spontaneously can make no real difference in this definition. Hence, for the sake of consistency, it is required that the Cartesian coordinate systems $S_0:(x_0,y_0,z_0)$ and $S_0':(x_0',y_0',z_0')$, descriptive of spontaneous light signals measured by observers at rest with respect to coordinate systems S and S', are themselves rigidly fixed in the systems S and S'. For simplicity, it is assumed that the axes of the S and S_0 and the S' and S_0' systems are parallel and have the same senses. Hence it follows that

$$\left.\begin{aligned} x_0' &= \gamma(x_0-\beta c t_0) \\ y_0' &= y_0 \\ z_0' &= z_0 \\ ct_0' &= \gamma(ct_0-\beta x_0) \end{aligned}\right\} \qquad (5.2)$$

In the following discussion, any symbol appearing with a subscript or superscript containing the symbol "0" will refer to a coordinate system of the nature of S_0. Sometimes such variables will be referred to briefly as "zero" variables. Subsequently it will be shown that a justification exists for referring to them as "inertial" variables.

In accord with the mode of generation of the coordinate system S_0 (and, of course, S'_0), the coordinates are meaningless without specification of statistical distributions expressive of the processes by which these magnitudes and functions dependent on them arise. Among these distributions is included the causal distribution.[25] More will be deduced below about the properties which such distributions must possess.

Since it is only averages over the zero variables which have physical significance, the origin (and orientation—this has already been agreed upon) of the S_0 system may be chosen arbitrarily. For convenience, the origin of any such system will be assumed to coincide with the origin of the S system unless otherwise stated. Similar considerations are assumed to apply to primed systems S'. An immediate consequence of the letter observation is that the domains of the zero variables are without bound:

$$\left.\begin{aligned} -\infty &< x_0 < +\infty \\ -\infty &< y_0 < +\infty \\ -\infty &< z_0 < +\infty \\ -\infty &< t_0 < +\infty \end{aligned}\right\} \qquad (5.3)$$

It was assumed above that it is possible to separate spontaneous from non-spontaneous events. This assumption was necessary in order that the new degrees of freedom be independent of the customary degrees of freedom. Such an assumption is based on the recognition that, as the refinement of physical measurements advances, one can increasingly expect to be able to separate events due to spontaneous creation and annihilation of particles from those due to the experimenter's initiation. In principle, then, one can imagine this already done. Such refinement, for example, was the case in the development of atomic physics.

However, the separation indicated above is not always possible in experiment. Instead one must frequently reckon with a space-time universe in which both types of events cannot necessarily be separated from one another, and correspondingly, a space-time continuum in which the space-time coordinates of the two kinds do not transform independently of one another. Considerations of symmetry imply that the transformation equations corresponding to (5.1) and (5.2) must be of the form

$$\left.\begin{aligned} x'-x_0' &= \gamma[x-x_0-\beta c(t-t_0)] \\ y'-y_0' &= y-y_0 \\ z'-z_0' &= z-z_0 \\ c(t'-t_0') &= \gamma[c(t-t_0)-\beta(x-x_0)] \end{aligned}\right\} \qquad (5.4)$$

The combined spaces correspond to measurements in which creation processes are virtual and appear as motion internal to the processes observed. As such, transformation (5.4) corresponds to observations on a terrestrial or atomic scale, or approximately so, and the zero variables appear as hidden variables.

One immediately notes a practical difficulty in the assumption as to the transformation equations of the combined spaces. No observation has yet indicated that the Einstein-Minkowski space-time suffers translations in the manner of (5.4). Yet the difficulty is an apparent one; to remove it, one needs only to assume that the average values of the zero variables in a vacuum are all negligible on a macroscopic scale. Indeed, the simplest such assumption is that the average values of these variables are all zero. As will be pointed out in later sections (Section 7.0 and the following) this assumption has a familiar and important meaning for the processes of creation and annihilation of particles and anti-particles, bears directly on the stability of elementary particles, implying a mass quantization, and leads to the gravitational force. Moreover, from a more formal aspect, it eliminates infinities in the energy of the matter-radiation field due to spontaneous fluctuations.

Hence, to Postulate I there is added

<u>Postulate II</u>. The average values of the zero variables, when averaged over spontaneous processes in a vacuum, are all zero.

Symbolically, postulate II is summarized by the relations

$$\left.\begin{aligned} \langle \vec{r}_0 \rangle_0 &= 0 \\ \langle t_0 \rangle_0 &= 0 \end{aligned}\right\} \qquad (5.5)$$

where $\vec{r}_0 = (x_0, y_0, z_0)$, and the symbol $\langle\ \rangle_0$ indicates that an average over spontaneous processes has been calculated. In effect, (5.5) defines the vacuum state.

It will be seen that Postulate II is not *independent of subsequent postulates relative to* the equations of motion of the matter-radiation field.

A consequence of Postulate II is the recovery of the Einstein-Lorentz transformation (5.1) from (5.4), upon averaging over zero variables.

5.4 The Metric

In analogy with the metric introduced by Minkowski,

$$\begin{aligned} s^2 &= x^2 + y^2 + z^2 - c^2t^2 \\ &= r^2 - c^2t^2 \end{aligned} \tag{5.6}$$

(where s denotes the space-time interval between an event at the origin of the four demensional space and the event at the point (x,y,z,ct), while $r^2=x^2+y^2+z^2$), and with consideration of the meaning of equations (5.4), one expects the metric of space-time, including the effects of the spontaneous processes to be

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 - c^2(t-t_0)^2 \tag{5.7}$$

Now it is clear that, although postulate II ensures the reinstatement of the transformation equations of the special theory, such is not possible for the metric (5.2). One obtains, instead,

$$\begin{aligned} x^2 + y^2 + z^2 - c^2t^2 + \langle x_0^2\rangle_0 + \langle y_0^2\rangle_0 \\ + \langle z_0^2\rangle_0 - c^2\langle t_0^2\rangle_0 \end{aligned} \tag{5.8}$$

Thus, as is characteristic of random processes, the behavior of space-time with respect to effects described by quadratic expressions is radically different, on the average, from the effects described by linear expressions. When it is reflected that the equations of motion of a field are dependent on the structure of space-time as determined by a quadratic expression, it is evident that (5.8) implies equations of motion different from those currently employed. More will be pointed out in this connection in the following sections of this paper.

For convenience, it will be assumed that the quadratic form (5.7) be replaced by one equivalent to it under averaging in a vacuum:*

Postulate III. The metric of the space-time continuum generated by both experimenter-initiated light signals and spontaneously initiated light signals is given by

$$\begin{aligned} S^2 &= s^2 + s_0^2 \\ &= r^2 - c^2t^2 + r_0^2 - c^2t_0^2 \end{aligned} \tag{5.9}$$

where $r_0^2 = x_0^2 + y_0^2 + z_0^2$, $s^2 = r^2 - c^2t^2$

and $s_0^2 = r_0^2 - c^2t_0^2$

or by

$$\begin{aligned} dS^2 &= ds^2 + ds_0^2 \\ &= dr^2 - c^2dt^2 + dr_0^2 - c^2dt_0^2 \end{aligned} \tag{5.10}$$

(where $dr^2=dx^2+dy^2+dz^2$, $dr_0^2 = dx_0^2 + dy_0^2 + dz_0^2$, $ds^2 = dr^2 - c^2dt^2$, and $ds_0^2 = dr_0^2 - c^2dt_0^2$. The symbols s, s_0, and S will always refer to the interval between two point events of which one lies at the origin of space-time; the differentials appearing in (5.10) refer to neighboring events.

The metrics (5.9) and (5.10) are invariant under the transformations (5.1) and (5.2). Moreover, the implication of having separated the ordinary from the zero coordinates is that one thereby assumes a causal description of nature to be possible.

For the purpose of analyzing virtual processes into their components, the metrics (5.9) and (5.10) will be assumed valid for the remainder of this work. However, the interpretation of the combined space as generated by a translation of the Einstein-Minkowski continuum will be employed repeatedly for the purpose of bridging the gap between the special theory and the one presently proposed.*

Thus, the equation of the light signal proposed by Einstein

$$\begin{aligned} r^2 - c^2t^2 = 0 \\ \text{or} \quad dr^2 - c^2dt^2 = 0 \end{aligned} \tag{5.11}$$

is now replaced by

$$\begin{aligned} r^2 - c^2t^2 + r_0^2 - c^2t_0^2 = 0 \\ \text{or} \quad dr^2 - c^2dt^2 + dr_0^2 - c^2dt_0^2 = 0 \end{aligned} \tag{5.12}$$

*This assumption reflects the orientation described in Section 3.0. The splitting of the coordinates into mean and fluctuating contributions has been carried out to establish the connection between preceding theories and the present one. In a later section on the relation of this theory to the quantum mechanics, it will be convenient to avoid this separation of variables.

*It is noted that the original discussion of the special theory[23] does not employ the Minkowski metric, although Einstein was well aware of the *invariance of the quadratic form. Einstein gave* a more physically oriented justification for the transformation (5.1). The reasoning employed by Einstein need be altered in no detail by a translation of coordinates, provided that the mean effect of the translation be negligible when averaged over events which occur far more rapidly than those entailed by any measurement he described.

The difference in the expressions (5.11) and (5.12) follows from the fact that, after a translation (5.4), one has averaged over a quadratic form which has reference to events occurring on the surface of a sphere, and this is a more restrictive statement than one concerning light propagation in a line parallel to the direction of relative motion of the coordinate systems S and S'. Moreover, it is remarked that (5.12) has reference to the propagation of a light signal accompanied by particles, while (5.11) contains no such reference. That these particles are "virtual" will be seen below to be summarized by Postulate III.

5.5 Use of the Dispersion Model

Relative to the last remark of the preceding paragraph, we point out the nature of the measurements performed to establish the metrics (5.9) and (5.10).

One agrees at the outset to confine the measurements to light signals recognized to have a specified shape. Until such a signal makes its appearance against the background of the noise consisting of the products of creation and annihilation, no light signal can be presumed to have arrived. That is, the creation and annihilation processes must ultimately result in the signal, if one adheres to the convention agreed upon. Indeed, one may receive the specified signal before the experimenter has generated it. It is noted that $-\infty < t_0 < +\infty$, in connection with the latter remark.

It is proposed that the creation and the annihilation of particles may be treated on the basis of a dispersion model. There is this difference: the average phase shift must be zero if enough random processes are taken into account. Moreover, the shifts must be of such a character as to return the solutions of the equation of propagation of the matter-radiation field to the vacuum form of Section 4.2. One can imagine these random processes to be somewhat of the following structure:

Let $\hbar\omega = 2m_ec^2$ ($\hbar = h/2\pi$) and imagine that at some time t during the propagation of the corresponding wave (Section 4.2), the energy of the wave is transformed into an electron-positron pair which is subsequently annihilated, the whole process occupying a time interval of order h/m_ec^2. When the wave is restored to its original mode of propagation, at time $\bar{t}$, there may well have been a phase shift in space, say of magnitude $(\omega/c)x_0$ (that is, one focuses attention on a wave of frequency ω, as above), while the total time of propagation of the wave in this form is not $\bar{t}$ but $\bar{t} - h/m_ec^2$. Thus the form of the analytical expression for the wave is not as in Section 4.2, but

$$e^{i(\omega/c)(x - x_0 + t - h/m_ec^2)} \tag{5.13}$$

In criticism of the calculation of the preceding paragraph, it is pointed out that the time given as required for the creation and annihilation is a root-mean-square value and not suitable as a phase shift (which, for example, may be negative). Further, application of a scattering model such as proposed, in which the energies in the interaction vary continuously, is not in consonance with the physical basis of the quantum theory, where it has been shown that no energy less than a quantum can be exchanged between a material system and the electromagnetic field. However, it is emphasized that the basis of the proposed theory is classical in character and will yield the same results as the quantum mechanics. These questions will be taken up below.

Moreover, the concept of a radiation field accompanied by material bodies and traveling with the speed c appears to be in contradiction with the basic tenets of the special theory. According to the special theory, no material body can travel with the speed of light, which is a limiting velocity for the motion of matter. The latter conclusion appears to be already implicit in the choice of the electromagnetic field as the agency for signalling. That is, it is assumed that the speed c is the maximum speed available for any signalling system, and this speed is characteristic of the electromagnetic field as conceived by Maxwell. But the discovery that the real electromagnetic field possesses internal degrees of freedom implies that the observed parameter c applies to a more complex entity than heretofore imagined.

Thus, if one wants simultaneously to preserve the concepts of an idealized pure electromagnetic field as the only means of signalling (by agreement), and the combined radiation-particle field propagated with speed c (as observed), then the scattering mechanism is a convenient means by which this can be done. The matter-radiation field is then equated to a pure radiation field with "hesitations"; this hesitation will be identified, in the next section, with the property of matter known as inertia.

The foregoing remarks apply to phase waves. Wave packets necessarily travel at speeds different than c if the matter aspect of the field be dominant. In Section 6.0 and the following it will be shown that one obtains the speed of light c for the radiation aspect of the field, even when propagated as a packet. One likewise obtains a matter-radiation field propagated with the speed of light upon averaging over virtual processes in a vacuum.

An even closer analogy exists between the proposed model and a method of attack utilized by Lorentz in a classic discussion of line broadening by random collisions between emitting molecules of a gas.[26] It is significant that the collisions give rise to an inertial effect (i.e. effectively increased mass) on the electron motion generating the radiation field.[26] A similar result will be evident in a model for the origin of inertial mass below.

5.6 Particle Mechanics and the Metric

As already indicated, the equation

$$dr^2 - c^2dt^2 + dr_0^2 - c^2dt_0^2 = 0 \tag{5.14}$$

has reference to the propagation of an electromagnetic field in a vacuum accompanied by many particles, provided one agrees that the mean values of the random variables $\vec{r}_0$ and t_0 are essentially zero when enough random processes are taken into account. However, although the field is no longer the elctromagnetic field of the special theory, the vanishing of the interval between two events $(x,y,z,t;x_0,y_0,z_0,t_0)$ and $(\bar{x},\bar{y},\bar{z},\bar{t};\bar{x}_0,\bar{y}_0,\bar{z}_0,\bar{t}_0)$ implies here, as it does for the special theory, that a single identifiable feature of the field is specified. Alternatively, one may mean that the two events at the endpoints of the interval can be joined by some identifiable feature of the matter-radiation field, propagated from one event to the other. Equation 5.14 will now be shown to lead to the differential equation of motion of this feature.

Let it be assumed possible to correlate the proper time interval*

$$\tau = \int d\tau = \int \sqrt{dt^2 - dr^2/c^2} \tag{5.15}$$

with the coordinates of both initiated and spontaneous events, so that the trajectory of an identifiable point of the field is given by

$$\vec{r} = \vec{r}(\tau), t = t(\tau), \vec{r}_0 = \vec{r}_0(\tau), t_0 = t_0(\tau) \tag{5.16}$$

and let there be defined the momenta

$$\left.\begin{aligned} \vec{p} &= m_0 d\vec{r}/d\tau \\ E/c &= m_0 c\, dt/d\tau \\ \vec{p}_0 &= m_0 d\vec{r}_0/d\tau \\ E_0/c &= m_0 c\, dt_0/d\tau \end{aligned}\right\} \tag{5.17}$$

where m_0 is identified with the inertial mass associated with a specified feature of the matter-radiation field. Then the average values of p_0 and E_0 in a vacuum are zero when averaged over virtual processes:

$$\langle \vec{p}_0 \rangle_0 = 0, \langle E_0 \rangle_0 = 0 \tag{5.18}$$

Thus, the mean energy and the mean momentum associated with spontaneous fluctuations in a vacuum are zero.

Equation (5.14) now implies that

$$p^2 + p_0^2 = (E/c)^2 + (E_0/c)^2 \tag{5.19}$$

Relation (5.19) may be obtained from a similar formula for the electromagnetic field

$$p = |\vec{p}| = E/c \tag{5.20}$$

by a translation,

$$|\vec{p} - \vec{p}_0|^2 = [(E - E_0)/c]^2 \tag{5.21}$$

followed by averaging over virtual processes, with due account of the vacuum condition (5.5).

In particular, let the specified feature of the field be a point at rest relative to the observer, and with which is associated inertial mass m_0 (hence a "point mass"—it will be shown below that specifying a particle in this manner does not prevent it from having a finite, measurable, diameter). Then

$$\left.\begin{aligned} dS &= 0 \\ d\vec{r} &= 0 \text{ (hence } \vec{p} = 0) \end{aligned}\right\} \tag{5.22}$$

whence it follows that

$$E = m_0 c^2 \tag{5.23}$$

and

$$(E/c)^2 - p_0^2 - (E_0/c)^2 = (m_0 c)^2 \tag{5.24}$$

Relations (5.23) and (5.24) are independent of any average over zero variables and may be expected to hold, essentially, for low velocities. Hence one recovers Einstein's relation between energy and momentum whenever creation processes do not play a dominant role, and with it the Newtonian particle mechanics for low velocities. It will be shown in what sense Einstein's relation for $p \neq 0$ holds in the present formalism. Thus, the total energy of a particle at rest, as measured by processes initiated by the observer, is equal to m_0c^2 and owes its existence to a residual motion of the point mass field, due to spontaneous creation and annihilation processes.

There has now been obtained, and given physical content, the well known relation of the special theory between mass and radiant energy, originally derived from more formal considerations.[23] It is seen that the new degrees of freedom are closely related to the inertial mass of the particles they describe. For this reason, they will often be referred to as <u>inertial</u> variables, and the associated energy and momentum as the <u>inertial energy</u> and <u>inertial momentum</u>; the corresponding field will be termed an inertial field.

With reference to the pure electromagnetic field, the condition (5.20) coupled with (5.19), implies that the conditions

$$p = |\vec{p}| = E/c$$

and

$$p_0 = |\vec{p}_0| = E_0/c \tag{5.25}$$

are equivalent and serve to specify the pure electromagnetic field. Relations (5.25) have the interesting implication that a pure electromagnetic field generated by an observer cannot exist as such unless the spontaneous variations in the field are due to creation and annihilation of electromagnetic radiation alone. Moreover, neither type of motion can exist without the other, if one admits the validity of the metric.

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*A more consistent definition of $d\tau$ would be $\sqrt{-dS^2/c^2}$. The choice made is a first approximation and chosen for more convenient comparison with the special theory of relativity.

6.0 Notation

To summarize, there has been derived a metric suitable to describe the propagation of a matter-radiation field. The metric lends itself to the definition of a "point mass" and leads to the introduction of momenta

$$\vec{p} = m_0(d\vec{r}/d\tau) \qquad (6.1)$$
$$\vec{p}_0 = m_0(d\vec{r}_0/d\tau)$$

and energies

$$E = m_0 c^2 (dt/d\tau) \qquad (6.3)$$

$$E_0 = m_0 c^2 (dt_0/d\tau) \qquad (6.4)$$

where $d\tau$ is a proper time interval measured by means of light signals initiated by the observer. The momenta and energies are related by the quadratic expression

$$p^2 + p_0^2 = (E/c)^2 + (E_0/c)^2 \qquad (6.5)$$

obtained by imposing the condition that an identifiable point feature of the field has been singled out

$$P_S = m_0(dS/d\tau) = 0 \qquad (6.6)$$

where

$$dS^2 = ds^2 + ds_0^2 = dr^2 - c^2dt^2 + dr_0^2 - c^2 dt_0^2 \qquad (6.7)$$

In the following discussion, contravariant quantities will be written with a superscript, while convariantly transforming quantities will be written with a subscript. Greek Indices will vary from one to eight, while Latin indices will vary from one to four. The appearance of 0 as either a superscript or a subscript refers to the coordinates generated by spontaneous light signals. It is agreed that

$$x = x^1, y = x^2, z = x^3, ict = x^4, x_0 = x^5, y_0 = x^6 = x_0^2, \ldots$$

and since the covariant metric tensor $g_{\mu\nu} = \delta_{\mu\nu} =$ the Kronecker delta, it follows that contravariant and covariant representations coincide. Hence one may write $x_1 = x^1 \ldots$ and $x^5 = x_5 = x_0^1 = x_1^0$, etc. Eight-vectors will be denoted by $(x^\mu) = (x^1, x^2, x^3, x^4, \ldots, x^8)$, four-vectors by $(x^k) = (x^1, x^2, x^3, x^4)$, and three-vectors by $\vec{r} = (x^1, x^2, x^3)$. Abbreviations similar to

$$\frac{\partial}{\partial x^\mu} = \partial_\mu, \frac{\partial}{\partial x_\mu} = \partial^\mu, \frac{\partial}{\partial x_k^0} = \partial_k^0, \frac{\partial}{\partial x_k^0} = \partial_0^k$$

will be employed while $\{\vec{r},t\}$ and $\{\vec{r}_0,t_0\}$, will denote the sets of all ordered number quadruples $(\vec{r},t)$ and $(\vec{r}_0,t_0)$, respectively. The eight dimensional space will be denoted by $\{\vec{r},t;\vec{r}_0 t_0\}$. Alternatively, $\{\vec{r},t\}$ and $\{\vec{r}_0,t_0\}$ will be referred to as observer (or experimenter) space and inertial space, respectively. The summation convention will be adopted, e.g.

$$dS^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

and the Laplacian operator Δ may be written

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad \Delta_0 = \frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2} + \frac{\partial^2}{\partial z_0^2} \qquad (6.8)$$

for three dimensions,

$$\partial_k \partial^k = \Box = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \qquad (6.9)$$

for four dimensions (the d'Alembertian), and

$$\partial_\mu \partial^\mu = \Box + \Box_0 \qquad (6.10)$$

for eight dimensions.

In the present paper, it will be shown how much of the classical concept of the point mass can be preserved. The equations of motion of the field will be postulated and some of their consequences will be discussed. Among these are to be mentioned a quantization of mass and a model for nuclear forces.

6.1 Model for the Matter-Radiation Potentials

On a scale of observation near the terrestrial, as pointed out above, one may consider virtual processes to be described by a translation of the Einstein-Minkowski space-time. The latter translation has been linked to the inertial properties of matter. Thus, one may expect to gain insight into the nature of the matter-radiation potentials by carrying out a translation of the arguments of the Maxwell theory.

For simplicity, let there be considered a static field in a vacuum, and the Coulomb potential $V(\vec{r})$ due to some charge distribution. Then, according to the proposed prescription, $V(\vec{r})$ is to be replaced by $V(\vec{r}-\vec{r}_0)$, where $\langle\vec{r}_0\rangle_0 = 0$.

Expanding $V(\vec{r}-\vec{r}_0)$ in a power series in $\vec{r}_0$,

$$V(\vec{r}-\vec{r}_0) = V(\vec{r}) - \vec{r}_0\cdot\nabla V(\vec{r}) + \frac{(\vec{r}_0\cdot\nabla)^2}{2!}V(\vec{r}) \qquad (6.11)$$

and averaging over virtual processes, one finds

$$V(\vec{r}-\vec{r}_0) = V(\vec{r}) + \frac{\langle x_0^2\rangle}{2}\frac{\partial^2}{\partial x^2}V + \frac{\langle y_0^2\rangle}{2}\frac{\partial^2}{\partial y^2}V + \frac{\langle z_0^2\rangle}{2}\frac{\partial^2}{\partial z^2}V + \cdots \qquad (6.12)$$

And, assuming the average values of the zero variables to be equal (a symmetrical distribution), one finds

$$\langle V(\vec{r}-\vec{r}_0)\rangle_0 = V(\vec{r}) + \frac{1}{2}\langle r_0^2\rangle_0 \Delta V(\vec{r}) + \cdots \qquad (6.13)$$

The preceding discussion has been introduced to provide a physical model as closely related as possible to recognized models implying the existence of a correction to the Coulomb field. To what extent one can employ such a model in calculations has been explored[27], and will not be discussed here. Only one numerical result will be borrowed from the work cited, and that is the estimate of r_0^2 as being of the order of magnitude $(e^2/\hbar c)(\hbar/m_e c)^2$ for an electron interacting with a fluctuating electric field. The estimate is convenient, but not necessary, in the considerations to follow; it illustrates the order of magnitude of the correction to classical theory entailed by the translation.

The approach outlined above leads essentially to perturbation corrections, which are unlikely to result in a unified field theory. The next section will introduce potentials for the complete unified field.

6.2 *The Field Potentials*

Noting that the scalar product

$$\vec{p}\cdot\vec{r}-\left(\frac{E}{c}\right)ct+\vec{p}_0\cdot\vec{r}_0-\left(\frac{E_0}{c}\right)ct \tag{6.14}$$

is invariant under the group of rotations

$$\left.\begin{aligned} x' &= \gamma(x-\beta ct)\\ y' &= y\\ z' &= z\\ ct' &= \gamma(ct-\beta x)\end{aligned}\right\} \tag{6.15}$$

$$\begin{aligned} x_0' &= \gamma(x_0-\beta ct_0)\\ y_0' &= y_0\\ z_0' &= z_0\\ ct_0' &= \gamma(ct_0-\beta x_0)\end{aligned} \tag{6.16}$$

(an equivalent to (6.14), in a vacuum, is $(\vec{p}-\vec{p}_0)\cdot(\vec{r}-\vec{r}_0)-(E-E_0)(t-t_0)$, it follows that a suitable phase wave for description of the field is

$$e^{i/\hbar\,(\vec{p}\cdot\vec{r}-Et+\vec{p}_0\cdot\vec{r}_0-E_0t_0)} \tag{6.17}$$

In (6.17), $\hbar$ is Planck's constant divided by 2π and serves for the present as a dimensional constant. No quantum effects are thereby introduced.

The identification between (6.17) and expressions like (5.13) can be made more explicit by

Postulate IV. (*de Broglie*). *The frequencies* ν and ν_0 and wavelengths λ and λ_0 of the matter-radiation waves (11.4) are related to the energies E and E_0 and momenta $\vec{p}$ and $\vec{p}_0$ by the proportionalities

$$\begin{aligned} E &= h\nu = \hbar\omega\\ E_0 &= h\nu_0 = \hbar\omega_0\\ \vec{p} &= \hbar\vec{k}\ (|\vec{k}|=k=2\pi/\lambda)\\ \vec{p}_0 &= \hbar\vec{k}_0 (|\vec{k}_0|=k_0=2\pi/\lambda_0)\end{aligned} \tag{6.18}$$

The symbol k in (6.18) will be referred to as a wave number, although it differs by a factor of 2π from the usual definition. No quantum effects are necessarily implied by Postulate IV.

To affirm the existence of the potentials, there is introduced

Postulate V. There exist potentials $\vec{a}(\vec{r},t;\vec{r}_0,t_0)$ $V(\vec{r},t;\vec{r}_0,t_0)$, $\vec{a}_0(\vec{r},t;\vec{r}_0,t_0)$, $V_0(\vec{r},t;\vec{r}_0,t_0)$ and which describe the radiation-matter field and respectively transform as $(\vec{r},ct;\vec{r}_0,ct_0)$. The eight potential will be denoted by (a_μ); at times one denotes a_5, a_6 a_7 and a_8 by a_1^0, a_2^0, a_3^0 and a_4^0, respectively. It is noted that $(a_\ell - a_\ell^0)$ transforms as $(\vec{r},t)$.

Postulate V is not independent of the succeeding Postulate VI relative to the equations of motion; in any case, it is clear that one should obtain, for example,
where V(r,t) denotes a solution of Maxwell's equation.

Although some ambiguity is introduced by this notation, considerable economy and suggestiveness are attained. If no arguments are indicated for a given function, it will be understood that these are r,t,r_0 and t_0.

In formulating the equations of motion of the field, a tensor notation would be convenient; *for the present it is more instructive to defer* such a treatment. Eventually, it will appear that the field to be described as a reasonable generalization of the electromagnetic field may be termed a "Bose-Einstein" field.

Since the exponentials (6.17) can be superposed linearly to obtain an extensive class of functions (owing to the completeness of the set of complex exponentials), it is convenient to formulate the equations of motion of the field such that each exponential is a solution of the equations, by reason of (6.5). To this end, the prescription employed in the quantum mechanics is used, in which replacements

$$\left.\begin{aligned} \vec{p} &\to \tfrac{\hbar}{i}\nabla\\ E/c &\to (i\hbar/c)\,\partial/\partial t\\ \vec{p}_0 &\to \tfrac{\hbar}{i}\nabla_0\\ E_0/c &\to (i\hbar/c)\,\partial/\partial t_0\end{aligned}\right\} \tag{6.19}$$

are carried out in a given function, for the purpose *of transforming the expression into an operator*. It will be shown that the substitution (6.19) leads to recovery of the classical equations of motion, either of the particle mechanics or of the electromagnetic field. It follows as well that the Hamilton-Jacobi equation of motion is likewise recovered in the appropriate context; that the Hamilton-Jacobi equation represents a geometrical optical model and provides a bridge *between classical mechanics and the Schrodinger* equation is well known.

Thus, it is proposed that

Postulate VI. The equations of motion of the generalized electromagnetic field in free space (or Bose-Einstein field—briefly, the BE field) are

$$\partial_\nu\partial^\nu a_\mu = \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta_0 - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}\right)a_\mu = 0 \tag{6.20}$$

6.3 The Relation of the Matter-Radiation Field to Classical Fields and Elementary Particle Force Fields

In the present section, some consequences of the equations of motion (6.20) will be presented. These will be directed toward establishing relations between the formalism of the proposed theory and preceding field theories, along with suitable interpretation.

Invoking the electromagnetic field conditions (5.25), the form of the elementary solutions (6.17), and the agreement (6.19), the equations of motion for the electromagnetic field are

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) Q_\mu = 0 \qquad (6.21)$$

$$\left(\Delta_0 - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}\right) Q_\mu = 0 \qquad (6.22)$$

Equation (6.21) refers to the field observed through the medium of a filter, while (6.22) refers to the field generated spontaneously; both of the equations may be derived from (6.20) by the phase restriction (5.25). The dependence of Q_μ on r_0^0 and t_0 in (6.21) and of Q_μ on $\vec{r}$ and t in 6.22 can be removed by averages over these variables.

Hence, in (6.21) and (6.22), one recovers Maxwell's wave equations for the two fields, each propagated with speed c. These equations emphasize the symmetry of the theory and the essential identity of structure of the two fields, differing as they do only in the mode of their generation.

On the other hand, it has been pointed out that an unweighted average of Q_μ over virtual processes should lead to the potentials of the electromagnetic field in observer space. The average effect of the r_0, t_0 motion should result in the electromagnetic field described by (6.21). Such is indeed the case.

Averaging both sides of (6.20) over $\vec{r}_0$ and t_0, invoking Green's theorem over a sphere of unboundedly large diameter $2s_0$ in $\{\vec{r}_0, t_0\}$ space, and presuming the derivatives of Q_μ tend to zero as s_0 increases without bound, one obtains

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\langle Q_\mu \rangle_0 = 0 \qquad (6.23)$$

Equations (6.23) are Maxwell's equations for an electromagnetic field propagated in a vacuum, at least for $\mu = 1,2,3,4$. The meaning of $\langle Q_\mu \rangle_0$ for $\mu = 5,\ldots,8$ evidently rests on an interpretation of the forces implied by the $\vec{r}_0, t_0$ motion. The latter point will be undertaken below.

Thus, one obtains the electromagnetic field, propagated with its characteristic speed, either as a phase condition or as a resultant of averaging over virtual processes. The latter view-point is discussed in Section 6.4.

If, in (6.23) $\langle Q_\mu \rangle_0$ be independent of the time t, one obtains

$$\Delta \langle Q_\mu \rangle_0 = 0 \qquad (6.24)$$

Laplace's equation for a source-free field. On the other hand, the Green's function for an infinite region in $\{\vec{r}\}$ space,

$$1/r \qquad (6.25)$$

($r \neq 0$), is a solution of (6.24) and represents the field of a point source at the origin. The Green's function (6.25) is likewise a solution for Poisson's equation for a field due to a source density which varies as a delta function, $\delta(r)$.

With the latter remarks in mind, the Green's functions for infinite regions in $\{\vec{r}, t; \vec{r}_0, t_0\}$ space and some subspaces of lesser dimensionality will be studied.

The Green's function for $\{\vec{r}, t; \vec{r}_0, t_0\}$ space is[28]

$$\frac{1}{2\pi^4}\,(s^2 + s_0^2)^{-3} \qquad (6.26)$$

$|s| + |s_0| \neq 0$. With due regard for the method of deriving (6.23), one notes that the unweighted average of (6.26) in s_0 is proportional to

$$\int_0^\infty ds_0\, s_0^3 (s^2 + s_0^2)^{-3} \qquad (6.27)$$

and (6.27) is, in turn, proportional to the Green's function for $\{\vec{r}, t\}$ space[27],

$$\frac{1}{4\pi^2}\, s^{-2} = \frac{1}{4\pi^2}(r^2 - c^2t^2)^{-1} \qquad (6.28)$$

($s \neq 0, r \geq 0, t \geq 0$). The statistical weight function employed in (6.27) is essentially the causal distribution[25], equal to unity in the integral.

If, for convenience, the function of (6.28) be continued analytically to domain $-\infty < t < +\infty$* and Fourier analyzed in t, one obtains a transform which varies essentially as

$$\frac{e^{i(\omega/c)r}}{r} \qquad (6.29)$$

, which is a solution of

$$\left[\Delta + (\omega/c)^2\right]\langle Q^\mu_\omega \rangle_0 \qquad (6.30)$$

*There can be no objection to this continuation on physical grounds, since the choice of origin for the t variation is arbitrary. Greater symmetry in the structure of $\{\vec{r}, t\}$ and $\{\vec{r}_0, t_0\}$ spaces is thus obtained.

where a^{μ}_{ω} is the transform of a^{μ}. Expression (6.29) is a Green's function for (6.30) and represents the field due to a point source at the origin with oscillating strength. It is noted that exp $(i\omega/c)$ r is proportional to the characteristic function[25] of the causal distribution, and that an average over the frequency with this as weight function leads to the Green's function (6.28). Moreover, setting $\omega = 0$ to obtain a static field, one obtains (6.24) from (6.30) and, correspondingly, (6.25) from (6.29). In addition, averaging $1/s^2$ over the time with a causal distribution, one obtains $1/r$, as before.

The results of the latter paragraph serve to demonstrate gain the equivalence of a phase condition and an average, as well as the symmetry of behavior of functions defined in observer space and in inertial space.

The foregoing remarks serve to define a heirarchy of Green's functions defined on regions of progressively smaller dimensionality. Each function appears as a point source and can be interpreted as a resultant of and due to processes over which an unweighted average has been calculated. The sources discussed have had reference to generation of an electromagnetic field; one can proceed somewhat differently to extend the meaning of the singular solution (6.26).

Thus, for a particle of mass m_0 at rest, equation (5.24) and relations (6.20) imply

$$\left(\Delta_0 - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}\right) g_\mu = -\varkappa^2 g_\mu \qquad (6.31)$$

($\varkappa = m_0 c/h$), with elementary solution, in polar coordinates*

$$e^{i\varkappa s_0 \cos\alpha} \qquad (12.32)$$

*The point $(x_\ell) = (\vec{r}, t)$ is represented in polar coordinates by $(s, \alpha, \beta, \varphi)$, where[28]

$$\left.\begin{aligned} x &= s \sin\alpha \sin\beta \cos\varphi \\ y &= s \sin\alpha \sin\beta \sin\varphi \\ z &= s \sin\alpha \cos\beta \\ ict &= s \cos\alpha \end{aligned}\right\}$$

If, for example, $s = i\sigma$ with $0 < \sigma < +\infty$, then $0 \leq \alpha \leq \pi$, $0 \leq \beta \leq \pi$, and $0 \leq \varphi < 2\pi$; if s be real, similar considerations follow. The Laplacian equation (6.9) then becomes[28]

$$\Big[\frac{\partial^2}{\partial s^2} + \frac{3}{s}\frac{\partial}{\partial s} + \frac{2}{s^2}\cot\alpha\frac{\partial}{\partial\alpha} + \frac{1}{s^2}\frac{\partial^2}{\partial\alpha^2} + \frac{1}{s^2\sin^2\alpha\sin^2\beta}\frac{\partial}{\partial\beta}\left(\sin\beta\frac{\partial}{\partial\beta}\right) + \frac{1}{s^2\sin^2\alpha\sin^2\beta}\frac{\partial^2}{\partial\varphi^2}\Big] u(s,\alpha,\beta,\varphi) = 0$$

If the four vector $(k_\ell) = (k_1, k_2, k_3, k_4)$ be directed along the ict axis, then $(k_\ell)(x^\ell) = ks\cos\alpha$, where $k^2 = k_\ell k^\ell$, $s^2 = r^2 - c^2t^2$, and α is the angle between (x_ℓ) and the ict axis.

Hence, for a particle nearly at rest, (6.20) implies

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right) g_\mu = 0 \qquad (6.33)$$

(g_μ may now be replaced by $\langle g_\mu \rangle_0$, in (12.33), and equations (12.33) are observed to correspond to the Proca wave equations.[29] Thus, (6.20) represents the motion of a family of mass states of intrinsic integral spin and may be termed a Bose-Einstein (BE) field.[29]

Repeating the argument whch led to (6.23), one finds that the Green's function of (6.33) for an infinite region is proportional to ($J_n(\varkappa s_0)$ is a Bessel function of order n)

$$\int_0^\infty ds_0 \frac{s_0^2 J_1(\varkappa s_0)}{(s^2 + s_0^2)^3} \qquad (6.34)$$

For an unweighted average over t, and considerations similar to those leading to (6.23), one must obtain the Green's function (Yukawa potential[29])

$$e^{-\varkappa r}/r \qquad (6.35)$$

($r \neq 0$). The potential (6.35) is a solution of

$$(\Delta - \varkappa^2) g_\mu = 0 \qquad (6.36)$$

obtained by averaging (6.33) over t, and the foregoing conclusion, relative to the average of (6.34), follows from the uniqueness of this solution.

Fourier analysis of the Green's function for (6.33) in the time and subsequently setting $\omega = 0$ must lead again to the Yukawa potential.

The conclusion proposed as a consequence of the above considerations is that the initial Green's function (6.26) represents a potential suitable to describe a field arising from superposition of all mass states, $0 < \varkappa < \infty$, without weighting any state in preference to another. Each state, if static, has spatial range of the order of $1/\varkappa$.

But with reference to (12.32), a plane wave advancing along the ict_0 axis has an approximate representation $e^{i\varkappa s_0}$. A superposition of such exponentials suffices to represent any reasonable type of packet advancing in the ict_0 direction. And of such packets, one may assert

$$\sqrt{\langle s_0^2 \rangle_0 \langle \varkappa_0^2 \rangle_0} \gtrsim \hbar \qquad (6.37)$$

so that ($s_0^2 > 0$)

$$\langle S^2 \rangle_0 = s^2 + \langle s_0^2 \rangle_0 \gtrsim s^2 + \frac{\hbar^2}{\langle \varkappa^2 \rangle_0} \qquad (6.38)$$

or, for a single mass state, $\varkappa = m_0 c/\hbar$

$$\langle S^2 \rangle \gtrsim s^2 + (\hbar/m_0 c^2)^2 \quad . \tag{6.39}$$

Thus, in (6.39) there has been obtained a theory of a fundamental length[30] in a natural manner from consideration of creation processes in which one mass state has been resolved. Moreover, for such a situation it is apparent that the range of fluctuation effects is of the order of a Compton wavelength for that state, and is, on the average, negligible beyond this limit. More will be pointed out in this connection below.

More generally, it has been shown that with an identifiable feature of the matter-radiation field there is necessarily associated an uncontrollable blurring due to spontaneous processes. This fuzziness prevents an arbitrary degree of accuracy in observing the postulated location, say $(\vec{r},t)$, of the feature. The space-time extension of the blurring is of the order of $\sqrt{\langle S_0^2 \rangle}$ and this is approximately $h/m_0 c$ for a single mass state. Hence, for the experimental refinement in question, a point mass cannot be defined except as a convenient construct. On the other hand, one is then freed from the necessity of introducing the Dirac delta function to describe a real mass or charge distribution.**

At the same time, there is obtained an operational definition of an elementary particle of non-zero diameter.

Considerations similar to those above apply whenever several mass states are unresolved, each point of space-time (observer) having associated with it a "blur sphere", to borrow a term having an analogous meaning from optics. The suggested term refers (as does its analogue) to the smallest measurable entity corresponding to a point source.

The potential (6.26) is now suggested to represent a field in a region of the diameter of a Compton wavelength enclosing the source. Thus, outside the sphere of confusion for a point source at $\vec{r}'$, the average over virtual processes leads to a Coulomb potential. Now, as a test body is brought closer to the point $\vec{r}'$ so that $S \sim h/m_0 c$, the spontaneously generated processes become resolved, an unweighted average is no longer suitable (see Section 7.1), and one obtains a potential which varies approximately as

$$\frac{(\hbar/m_0 c)^5}{[r^2 + \zeta(\hbar/m_0 c)^2]^3} \tag{6.40}$$

where $\zeta \sim 1$, and other factors have been deleted.

*The estimate given in (6.37) may be improved by averaging over α, but cannot alter the essential meaning of the relation. Thus, it is noted that

$$\frac{1}{\pi}\int_0^\pi d\alpha\, e^{i\varkappa s_0 \cos\alpha} = J_0(\varkappa s_0)$$

which has asymptotic behavior

$$e^{i\varkappa s_0}/\sqrt{\varkappa s_0}$$

Then, if $\varkappa s_0 \gg 1$, (6.39) holds also for the average over α.

**A brief statement of the difficulties introduced by this necessity, and a proposed method of avoiding them, is given by Ken-iti Gotô.[31]

If the distribution is such as to favor a single mass state, one obtains the Yukawa potential.

It is notable that, as a consequence of the existence of virtual processes, the potential has no singularity if $r \neq 0$. Moreover, this is in the nature of things: if s_0 be small, then the number of $\varkappa$ states must be large (6.37), and if mass states appear with equal statistical weights, then the average effect is simply a Coulomb potential. But if r be small, then the Coulomb potential is no longer suitable and a relation of the nature of (6.40) applies. Further, if only a few $\varkappa$ states exist, clustered about some central value, then the spread in s_0 values must be large, and the potential (6.26) cannot lead to singularities.

Thus, it has been shown in this section that

a) virtual processes lead to and are responsible for electromagnetic field phenomena;

b) the existence of a fundamental length is directly traceable to virtual processes;

c) postulation of a point mass in $\{\vec{r},t\}$ space can be reconciled with the existence of elementary particles of finite diameter, as a measurable phenomenon, and without contradiction;

d) the singular Coulomb and Yukawa potentials can be replaced by non-singular potentials, equivalent to these upon suitable averaging, and valid in the range of nuclear forces.

Before passing on to the next section, we point out a physical aspect of the existence of an inertial field, bearing upon the concept of action at distance.

A Coulomb potential 1/r seems to indicate an instantaneous transport of force through empty space from source to receptor: hence action at a distance. Yet with the introduction of virtual processes, the concept of empty space becomes untenable and action at a distance unnecessary.

For a static field in observer space is represented by a solution of

$$(\Delta + \Box_0)\,\mathcal{Q}_\mu = 0 \tag{6.41}$$

and an elementary solution to (6.41) is

$$\tag{6.42}$$

with $k^2 + \varkappa^2 = 0$, so that the elementary solution assumes the form

$$e^{i\varkappa \hat{n}\cdot\vec{r}}\; e^{-\varkappa s_0 \cos\alpha} \tag{6.43}$$

and any solution of (6.41) can be written as a superposition of functions (6.43).

But the functions (6.43) imply a relation of observer space to inertial space. Averaging over r_0 and t_0 does not destroy the significance of the dependence $k = \pm i\varkappa$: a static field is a superposition of states $e^{i\varkappa\vec{R}\cdot\vec{r}}$ in observer space, and states $e^{i\varkappa s\cos\alpha}$ in inertial space; hence of mass states specified by $\varkappa$.

Thus one can imagine a Coulomb static field, or a mass static field (e.g. a particle at rest), to be a resultant of a time varying inertial field responsible for propagation of the static field. Just as the time dependent electromagnetic field is conceived to be a special, limiting form of matter, propagated with maximum speed c in $\{\vec{r},t\}$ space, the static radiation-matter field (electric, magnetic, or matter at rest) is likewise a special form of matter propagated with zero speed in $\{\vec{r},t\}$ space. Both limiting cases are consequences of the interference of various inertial states with one another.

Hence, the concept of empty space surrounding a source of a static field can be given up, and replaced by the concept of a superposition of states which transport the field from source to test body. "Action at a distance" then becomes an unnecessary concept.

6.4 Brownian Motion and Creation Processes

Thus far the inertial mass m_0 has been assumed to vary continuously and appears as a constant of proportionality between a residual energy and the square of the speed of light (or better, $\omega/c = E/\hbar c = \varkappa$, a proportionality between wave number and frequency). Although the continuous variation is reasonable on a terrestrial scale, *the relation (12.17) indicates* that a restriction on the scale of observation of s_0 (and hence of S) will lead to restrictions on corresponding values of $\varkappa$, and conversely. Thus, if $\varkappa \sim m_0c/\hbar$, then $s_0 \sim h/m_0c$. Indeed, experiment demonstrates that such must be the case whenever the scale of measurement is of the order of the Compton wavelength of a given mass state.

This section proposes to show how the model featured in the preceding sections can result in constraints on the inertial field, giving rise to discrete mass states. For this purpose, however, it is convenient to develop still further the analogy to Brownian motion.

It will be recalled that one model of Brownian motion[21] concerns the motion of a particle large in comparison with the molecules which cause its motion. The particle is assumed subject to the same forces as a molecule of the fluid surrounding it; these forces are

a) *a rapidly varying and random force*,
and b) a viscous damping force.

The random force is attributed to random collisions tending to create a flow and mixing of matter thoughout the fluid, while the viscous force tends to oppose large accelerations of the molecules. Thus, forces a) and b) are opposed to one another in a dynamic equilibrium, on the average.[21] Hence the net displacement of the Brownian particle is zero.

A further model[21] eliminates the viscous force previously assumed to hold, on the average, and considers only random collisions of molecules with the Brownian particle. It is significant that one obtains identical results for both the proposed models, <u>on the average.</u>

Yet it is recognized by both models that only one mechanism is in operation: molecular collisions. The viscous force, a macroscopic effect, is elevated to the standing of a force acting in the same interval of time as the much more rapid random collisions. There is no contradiction if one enquires after effects resulting from averages taken over periods of time in which viscous force may be assumed to have some real existence.

Thus, the Brownian motion, as observed by optical means, can be viewed as a superposition of two kinds of motion:

a) a very rapid motion not observable by optical instruments, due to a rapid and random force, and

b) a slower motion observable by optical means, and giving rise to a viscous force.

The first motion is internal to the second, and separated from it as a matter of convenience.

A study by Lorentz[26] demonstrates how the viscous force may arise from random interruptions of an unforced motion, to a first approximation. Indeed, it follows from the existence of a fixed mean value for a given coordinate, that there *must exist a force opposing any departure of the* physical state described by the coordinate, from that mean value. The latter consideration can be formalized in a theorem having particular relevance to the present theory (Appendix C). From the point of view of the present paragraph, the Brownian motion appears as a superposition of two kinds of motion, each an interruption of the other. *Each motion occurs at the expense of the* other.

The model adopted for description of the creation processes is essentially that of the latter paragraph. Thus, the observer-initiated signal is propagated in a vacuum and interrupted by spontaneous processes and conversely. It is emphasized that the spontaneous processes may occur at a very high rate and then constitute a motion internal to the observer-initiated processes.

The latter feature corresponds to similar features of the Brownian motion, and the formalism further illustrates the point. Thus, the *propagation of an identifiable feature of the* field is described by

$$s^2 = -s_0^2 \qquad (6.44)$$

or $$ds^2 = -ds_0^2 \tag{6.45}$$

or $$p_s^2 = -p^2{}_{s_0} \tag{6.46}$$

($p_s = m_0 ds/d\tau$, $p_{s_0} = m_0 ds_0/d\tau$). Hence, increase of a squared space-time interval s^2 occurs at the expense of the interval s_0^2. Alternatively, if s^2 (or ds^2) be space-like, then s_0^2 (or ds_0^2) must be time-like. Hence the squared magnitudes of p_s and p_{s_0} are always opposite in sign; these relations are independent of averaging.

With reference to the discussion of Appendix C, it is asserted that the vacuum condition imposes a constraint on the creation processes, opposing any large variation in density of the created particles. The forces called into play may be deduced with the aid of the theorem and a knowledge of the appropriate statistical distribution in r_0 and t_0. Rayleigh's method[32] provides a means of deducing the statistical distribution required, if one interprets the inertial variables as phase shifts induced in an electromagnetic wave. The derivation of the distribution is given in Appendix A and indicates that the inertial mass m_0 serves as a root-mean-square measure of the quantity of motion stored in the $\{\vec{r}_0, t_0\}$ space. Moreover, one may then view the $\vec{r}_0, t_0$ motion as <u>responsible</u> for the appearance of an inertial mass. <u>It will be shown below</u> (Section 7.5) that the inertial motion likewise gives rise to gravitational mass.

Since the forces due to inertial motion oppose any departure of the field potentials from the vacuum condition $\langle\vec{r}_0\rangle_0 = 0$ and $\langle t_0\rangle_0 = 0$, there is, in effect, an attractive force field with seat at the origin of $\langle\vec{r}_0, t_0\rangle$ space. In the next section it will be shown how this effective force field leads to a quantization of mass.

The latter paragraph emphasizes the kinematic nature of the vacuum assumption (5.5). One is, by its aid, led to an equivalent force field which is asserted to be responsible for the stability of the matter state of the field. The manner in which the condition is introduced forbids looking for more fundamental causes in the framework of the present theory; it is asserted to be an elementary property of nature. Moreover, as will be seen, the condition is equivalent to the requirement that particle and anti-particle be created together (Section 7.1), emphasizing again the fundamental character of the condition.

6.5 The Quantization of Mass

In Section 6.4 it was shown that the vacuum condition leads to an effective attractive force field. The equivalence of kinematic-statistical and dynamic descriptions was pointed out. The present section applies these considerations in obtaining a quantization of mass states from both points of view.

A. Kinematic Viewpoint

For brevity, let W denote a potential of the spontaneous creation field, pertaining to a mass at rest and localized in a region about the origin of inertial space of radius $r_0 = a$. It will be assumed that W depends on r_0 alone; the meaning of the restricted dependence will be discussed in part B of this section.

Then (6.31)

$$(\Delta_0 + \varkappa^2) W = 0, \; r_0 < a \tag{6.47}$$

$$\Delta_0 W = 0, \; r_0 > a \tag{6.48}$$

which may be rewritten in the form ($\ell = 0, 1, 2, \ldots$)

$$\left[\frac{d^2}{dr_0^2} + \frac{2}{r_0}\frac{d}{dr_0} + \frac{\ell(\ell+1)}{r_0^2} + \varkappa^2\right] R(r_0) = 0, \; r_0 < a \tag{6.49}$$

$$\left[\frac{d^2}{dr_0^2} + \frac{2}{r_0}\frac{d}{dr_0} + \frac{\ell(\ell+1)}{r_0^2}\right] R(r_0) = 0, \; r_0 > a \tag{6.50}$$

or

$$\left[\frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} + \frac{\ell(\ell+1)}{\rho^2} + 1\right] R = 0, \; \rho < \varkappa a \tag{6.51}$$

$$\left[\frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} + \frac{\ell(\ell+1)}{\rho^2}\right] R = 0, \; \rho > \varkappa a \tag{6.52}$$

It follows that

$$R = j_\ell(\rho), \; \rho \leq \varkappa a \tag{6.53}$$

$$R = \rho^{n(\ell)}, \; \rho > \varkappa a \tag{6.54}$$

with $j_\ell(\rho) = (\pi/2\rho)^{1/2} J_{\ell+1/2}(\rho)$ and $J_{\ell+1/2}$ is a <u>Bessel function</u> of order $\ell + 1/2$, and $n(\ell) = -\frac{1}{2} - \sqrt{\frac{1}{4} + \ell(\ell+1)}$.

Continuity at $r_0 = a$ requires that

$$\varkappa a j_1(\varkappa a) = j_0(\varkappa a), \; \ell = 0 \tag{6.55}$$

$$(2\ell+1)\frac{n_\ell}{\varkappa a} = \ell \frac{j_{\ell-1}(\varkappa a)}{j_\ell(\varkappa a)} - (\ell+1)\left[\frac{j_{\ell+1}(\varkappa a)}{j_\ell(\varkappa a)}\right]_{\ell \neq 0} \tag{6.56}$$

The condition (6.55) is equivalent to $\cos \varkappa a = 0$ or to $\varkappa a = \pi/2, 3\pi/2, \ldots$, for $\ell = 0$, and to

$$3n(1) - 1 = 2\varkappa a - 6/na + (6 - \varkappa a)\cot \varkappa a$$

for $\ell = 1$.

For given a (say $a = 10^{-13}$ cm) there is implied a discrete series of levels for m_0 corresponding to various angular momentum numbers ℓ. The dependence of a on m_0 has been assumed to be negligible in relations (6.47) to (6.54).

Since it is not the purpose here to attempt a detailed correspondence of theory and experiment, no calculation of the values of will be given. Moreover, there exists a good possibility that any detailed discussion must lead to selection rules which will modify the above considerations (e.g. as must occur for the dipole moment distributions of spontaneous charges and currents).

In connection with the significance of ℓ, one may speculate on its relation to the intrinsic spins attributed to the elementary particles, the anomalous magnetic moments of proton and neutron, and the strangeness number.[33]

B. Dynamic Viewpoint

It has been shown that interruption of the observer-generated electromagnetic field gives rise to the observed inertial character of matter. But the converse is perforce true: the process may be viewed as interruption of the spontaneously generated field, again giving rise to the inertial property. Moreover, the latter interruption is observer-generated, and if a single mass state thereby is induced, then there is implied a constraint on the inertial field, generated by the interaction with the observer. The constraint must be such that one mass state is produced as a result of the interaction; this may be described as a process leading to selection of the states $(\vec{k}_0, \omega_0)$ such that $\varkappa^2 = k_0^2 - (\omega_0/c)^2 = (m_0 c/\hbar)^2$ = constant. Other mass states can only appear as virtual states.

Thus, one is led, in the Maxwell equation

$$\left[\nabla_0 - (E_0/\hbar c)^2\right] W(\vec{r}_0) = 0 \qquad (6.57)$$

describing propagation of an electromagnetic field generated spontaneously and of frequency E_0/h, to substitute $\nabla_0 - i/\hbar\, \vec{a}$ and $E_0 - (i/\hbar)V$ for ∇_0 and E_0, respectively. Here $\vec{a}$ and V transform as a four vector and describe the observer's intervention in the spontaneous processes to select the mass state m_0.

It is now assumed that both $\vec{a}$ and E_0 are negligible and $V = -m_0c^2$, if $r_0 \leq a$ and V=0 if $r > a$; hence it is enquired of the derived equation as to the condition for the appearance of a field of zero energy corresponding to a binding energy m_0c^2 in a square well of range $r_0 = a$. One then obtains

$$(\Delta_0 + \varkappa^2)W = 0,\ r_0 \leq a \qquad (6.58)$$

$$\Delta_0 W = 0,\ r_0 > a \qquad (6.59)$$

as before.

The discussion in Part B has run counter to the spirit of the theory preceding, but is equivalent to it. The viewpoint is adopted by way of illuminating the discussion in Part A and in Section 6.4.

7.0 Introduction to the Formalism

It has been shown above how the basis of the special theory of relativity may be extended to include the matter field as an integral part of the theory. Of necessity, the source-free electromagnetic field served as an initial basis of discussion, and its generalization, the Bose-Einstein field, reflected the initial bias.

In the present sequence of Sections 7.0....., the formalism begun in Section 6.0, etc. will be extended to the BE field with sources, and to interaction with other fields. The generalized Fermi-Dirac (FD) field will be introduced. The neutrino field will be placed in a position relative to the FD field analogous to that occupied by the electromagnetic field among the BE fields.

The formalism is shown to be equivalent to classical quantum mechanics and the vacuum condition is related to Dirac's hole theory.

The theory is shown to encompass gravitational effects and relates part of the dipole magnetic fields of celestial bodies to their angular moments.

7.1 Tensor Formulation: Source-Free Field

It is convenient to formulate the equations of motion and quantities derived from the eight-potentials as tensor relations. In the present section, this will be done for the free field, and in the following sections the formalism will be extended to the forced field. The physical significance of the tensors will be touched upon.

In analogy to the tensor formulation of the special tneory, one defines a gauge invariant generalized field intensity tensor

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \qquad (7.1)$$

The form 7.1 is justified when one observes that $\langle f_{\ell m} \rangle_0$ is a tensor identical with the corresponding tensor of the special theory. If μ and ν are greater than four, one obtains a tensor with components averaging to zero

$$\langle f_{\ell+4,m+4} \rangle_0 = 0 \qquad (7.2)$$

Moreover, since the field generated spontaneously differs from that measured only in source, but not in structure, the following notation is adopted, with due regard for the remarks following (6.7) and Postulate V: $(f_{\mu\nu}) \equiv$

$$\begin{pmatrix} 0 & \mathcal{H}_3 & -\mathcal{H}_2 & -i\mathcal{E}_1 & f_{15} & f_{16} & f_{17} & f_{18} \\ -\mathcal{H}_3 & 0 & \mathcal{H}_1 & -i\mathcal{E}_2 & f_{25} & f_{26} & f_{27} & f_{28} \\ \mathcal{H}_2 & -\mathcal{H}_1 & 0 & -i\mathcal{E}_3 & f_{35} & f_{36} & f_{37} & f_{38} \\ i\mathcal{E}_1 & i\mathcal{E}_2 & i\mathcal{E}_3 & 0 & f_{45} & f_{46} & f_{47} & f_{48} \\ -f_{15} & -f_{25} & -f_{35} & -f_{45} & 0 & \mathcal{H}_3^0 & -\mathcal{H}_2^0 & -i\mathcal{E}_1^0 \\ -f_{16} & -f_{26} & f_{36} & -f_{46} & -\mathcal{H}_3^0 & 0 & \mathcal{H}_1^0 & -i\mathcal{E}_2^0 \\ -f_{17} & -f_{27} & -f_{37} & -f_{47} & \mathcal{H}_2^0 & -\mathcal{H}_1^0 & 0 & -i\mathcal{E}_3^0 \\ -f_{18} & -f_{28} & -f_{38} & -f_{48} & i\mathcal{E}_1^0 & i\mathcal{E}_2^0 & i\mathcal{E}_3^0 & 0 \end{pmatrix} \qquad (7.3)$$

One notes that $\langle \vec{\mathcal{E}}(\vec{r},t;\vec{r}_0,t_0) \rangle_0$ and $\langle \vec{\mathcal{H}}(\vec{r},t;\vec{r}_0,t_0) \rangle_0$ coincide with the electric and magnetic field intensities of the Maxwell theory. Moreover, $\langle \vec{\mathcal{E}}^0(\vec{r},t;\vec{r}_0,t_0) \rangle_0$ and $\langle \vec{\mathcal{H}}^0(\vec{r},t;\vec{r}_0,t_0) \rangle_0$ the corresponding field intensities due to spontaneous processes, are zero. By previous convention, then, one writes

$$\left.\begin{aligned} \langle \vec{\mathcal{H}}^0(\vec{r},t;\vec{r}_0,t_0) \rangle_0 &= \vec{\mathcal{H}}^0(\vec{r},t) - \vec{\mathcal{H}}_0(\vec{r},t) = 0 \\ \langle \vec{\mathcal{E}}^0(\vec{r},t;\vec{r}_0,t_0) \rangle_0 &= \vec{\mathcal{E}}^0(\vec{r},t) - \vec{\mathcal{E}}_0(\vec{r},t) = 0 \end{aligned}\right\} \qquad (7.4)$$

which is seen to correspond to the vacuum condition (5.5), but is not equivalent to it.

The cross terms $f_{\ell,m+4}$ correspond to coupling of the average and inertial fields, and the averages of these terms are, in general, not zero. The coupling is a reflection of the assumption that radiation-matter transitions may occur in free space.

One now further restricts the variation of a_μ by the guage condition

Postulate VII

$$\partial_\mu a^\mu = 0 \tag{7.5}$$

which is seen to pass over into the corresponding relation of the special theory when averaged

$$\partial_\ell \langle a^\ell \rangle_0 = 0 \tag{7.6}$$

It follows at once that

$$\partial_\mu f^{\mu\nu} = 0 \tag{7.7}$$

and the latter relations become the Maxwell equations when averaged.

There is now defined the energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi}\left[f_{\mu\alpha} f^{\alpha}_{\ \nu} + \frac{1}{4}\delta_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta}\right] \tag{7.8}$$

by analogy with the special theory. The choice of the numerical factor $1/4\pi$ will be justified in section 7.4, as well as the fitness of the definition. One notes that

$$\partial^\mu T_{\mu\nu} = 0 \tag{7.9}$$

as a consequence of (7.7).

It is evident that $\langle T_{\mu\nu} \rangle_0$ is not equal to the corresponding tensor of the special theory, except as a first approximation in terms of virtual processes. This has been characteristic of the foregoing theory, the behavior of quadratic expressions being markedly dissimilar, on the average, from that of linear expressions.

Similarly, by analogy, one defines the eight-current density in observer space by

$$(j_\nu) = (\vec{j}, i\rho, j_{\ell+4}) \begin{cases} \vec{j} = \frac{i}{c} T_{4k}, \ k = 1,2,3 \\ \rho = T_{44}, \ j_8 = i T_{48} \\ j_{\ell+4} = \frac{i}{c} T_{4,\ell+4}, \ \ell = 1,2,3 \end{cases} \tag{7.10}$$

and in inertial space by

$$(j^0_\nu) = (j^0_k, \vec{j}^0, i\rho^0) \begin{cases} j^0_k = \frac{i}{c} T_{8k}, \ k = 1,2,3 \\ \vec{j}^0 = \frac{i}{c} T_{8\ell}, \ \ell = 5,6,7 \\ i\rho^0 = i T_{88}, \ j^0_4 = i T_{84} \end{cases} \tag{7.11}$$

Hence, corresponding to the energy density of the special theory, one has

$$\rho = T_{44} = \frac{\mathcal{E}^2 + \mathcal{H}^2}{8\pi} - \frac{\mathcal{E}_0^2 - \mathcal{H}_0^2}{8\pi} + \frac{1}{2} f_{\ell,k+4} f^{\ell,k+4} - f_{4,\ell+4} f^{4,\ell+4} \tag{7.12}$$

$$\rho_0 = T_{88} = \frac{\mathcal{E}_0^2 + \mathcal{H}_0^2}{8\pi} - \frac{\mathcal{E}^2 - \mathcal{H}^2}{8\pi} + \frac{1}{2} f_{\ell,k+4} f^{\ell,k+4} - f_{8\ell} f^{8\ell} \tag{7.13}$$

while the current densities are

$$\vec{j} = \frac{\vec{\mathcal{E}} \times \vec{\mathcal{H}}}{4\pi c} + \frac{i}{4\pi c}\left(f_{4,\ell+4} f^{\ell+4,k}\right), \ k = 1,2,3 \tag{7.14}$$

$$\vec{j}_0 = \frac{\vec{\mathcal{E}}_0 \times \vec{\mathcal{H}}_0}{4\pi c} + \frac{i}{4\pi c}\left(f_{8\ell} f^{\ell k}\right), \ k = 1,2,3 \tag{7.15}$$

and, for example,

$$j_5 = \frac{i}{c} f_{4\alpha} f^{\alpha 5} \tag{7.16}$$

It is evident that the averages of current densities and densities are not equal to their correspondents of the special theory. On the other hand, the averages of these quantities over $\vec{r}_0$ and t_0 satisfy a conservation law in observer space

$$\nabla \cdot \langle \vec{j} \rangle_0 + \frac{1}{c}\frac{\partial}{\partial t}\langle \rho \rangle_0 = 0 \tag{7.17}$$

so that the integral of the averaged density, over r, is a constant in the time t (note that $\langle j_{\ell+4} \rangle_0 = 0$, $\ell = 1,2,3$)

$$\frac{\partial}{\partial t}\int \langle \rho \rangle_0 \, d\vec{r} = 0. \tag{7.18}$$

Similarly, bypassing the meaning of the operation for the moment, if one averages (7.9) over $\{\vec{r}, t\}$ space, with suitable restriction of the indices, one finds (note that $\langle j_{k+4} \rangle = 0$, $k = 1,2,3$)

$$\nabla_0 \cdot \langle \vec{j}_0 \rangle + \frac{1}{c}\frac{\partial}{\partial t_0}\langle \rho_0 \rangle = 0 \tag{7.19}$$

where $\langle \ \rangle$ refers to the indicated average. Then

$$\int \langle \rho_0 \rangle \, d\vec{r}_0 \tag{7.20}$$

is a constant in the time coordinate t_0.

Thus, although the current and density are not equivalent to their correspondents of the special theory, their averages satisfy a conservation law identical with that of the special theory. The significance of the integrals (7.18) and (7.20), as well as the meaning of the average over r, t space, will be undertaken in Section 7.2.

Although the field is unforced, there exist matter states in the field. These do not play the role of sources, but are an inevitable consequence of creation processes. Moreover, if no electric charges or currents exist in average space, so that

$$\left.\begin{aligned}\vec{\mathcal{E}}(\vec{r},t) &= 0\\ \vec{\mathcal{H}}(\vec{r},t) &= 0\end{aligned}\right\} \qquad (7.21)$$

there is no guarantee that $\langle\rho\rangle_0$ or $\langle\rho_0\rangle$, for example, will likewise vanish. The latter is clearly due to the quadratic character of the currents and densities. The cross terms do not represent interactions between fields in the usual formal meaning of the term, but signify the essential unity of matter and radiant energy. The generalized Coulomb potential (6.26) indicates that there exist non-zero currents and densities in the field, even if $\vec{\mathcal{H}}(\vec{r}, t) = 0$; moreover, this can only occur because the potential depends on s_0.

7.2 *The Formalism and Quantum Mechanics*

It is the purpose of this section to establish an equivalence between the present formalism and that of classical, non-relativistic quantum mechanics. Further insight into the nature of the currents and densities defined above, as well as some implications of the vacuum condition, will then follow.

To obtain the indicated approximation, one assumes

i) a_μ varies essentially as $e^{im_0c^2t/\hbar}$ in the time t (there may remain a comparatively weak dependence on t);

ii) a gauge such that $\langle a_4\rangle_0 = 0$, and hence $\partial^\ell\langle a_\ell\rangle_0 = 0(\ell=1,2,3)$ or, alternatively $\nabla\cdot\vec{a}(\vec{r}, t)=0$

iii) the direction of $\vec{a}(\vec{r},t)$ varies slowly in comparison with the rate of variation of $|\vec{a}(r,t)| = a$.

With restrictions i) to iii) it will be shown that can be identified, essentially, as the probability function of the classical quantum mechanics. The identification can be made complete by altering the formalism somewhat to include complex field potentials.

Imposing the non-relativistic condition, $E/m_0c^2 \ll 1$, one obtains $\left(\langle a_\ell\rangle_0 = \sum_n a_n e^{-iE_nt/\hbar}\right)$

$$-\frac{\hbar^2}{2m_0}\Delta\langle a_\ell\rangle_0 = i\hbar\frac{\partial}{\partial t}\langle a_\ell\rangle_0 \qquad (7.22)$$

$\ell = 1,2,3$, and hence, with the aid of ii) and iii)

$$-\frac{\hbar^2}{2m_0}\Delta a = i\hbar\frac{\partial a}{\partial t} \qquad (7.23)$$

Moreover, to a first approximation, the density is given by

$$\langle\rho\rangle_0 = \frac{\mathcal{E}^2+\mathcal{H}^2}{8\pi} = \frac{(m_0c)^2}{8\pi}a^2 \qquad (7.24)$$

while the current density is

$$\langle\vec{j}\rangle_0 = \frac{c}{4\pi}\vec{\mathcal{E}}\times\vec{\mathcal{H}} = \frac{m_0c^2}{4\pi}a\nabla a \qquad (7.25)$$

The conservation of density and current is assured by (7.17) to the same approximation. Pauli [34] has shown that a current and density defined by (7.24) and (7.25), with a law of motion (7.22) cannot lead to a conservation law (7.17) if a be real. It is therefore of interest to repeat the above discussion allowing the potentials to be complex, re-defining observable quantities to ensure their reality.

Thus, let a_μ be a complex function satisfying equations (6.33) and let a_μ^* denote its complex conjugate. As before, define

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \qquad (7.26)$$

$$a = \sqrt{\langle a_1\rangle_0^2+\langle a_2\rangle_0^2+\langle a_3\rangle_0^2} \qquad (7.27)$$

and note that $f_{\mu\nu}^* = \partial_\mu a_\nu^* - \partial_\nu a_\mu^*$ with $(1/2)\,(f_{\mu\nu} + f_{\mu\nu}^*)$ representing a measured field strength.

Similarly, define

$$T_{\mu\nu} = \frac{1}{4\pi}\left(f_{\mu\alpha}^* f^{\alpha}{}_{\nu} + f_{\nu\alpha}^* f^{\alpha}{}_{\mu} + \frac{1}{2}\delta_{\mu\nu}f_{\alpha\beta}f^{\alpha\beta*}\right) \qquad (7.28)$$

so that

$$\rho = -T_{44} = \frac{\vec{\mathcal{E}}\cdot\vec{\mathcal{E}}^* + \vec{\mathcal{H}}\cdot\vec{\mathcal{H}}^*}{2\pi} \qquad (7.29)$$

and

$$j_k = -T_{4k} = \frac{ic}{4\pi}\left[(\vec{\mathcal{E}}\times\vec{\mathcal{H}}^*)_k - (\vec{\mathcal{E}}^*\times\vec{\mathcal{H}})_k\right] + \cdots \qquad (7.30)$$

with

$$\partial^\ell\langle j_\ell\rangle_0 = 0 \qquad (7.31)$$

It follows that, with approximations essentially those of i) to iii) of this section

$$-\frac{\hbar^2}{2m_0}\Delta a = i\hbar\frac{\partial a}{\partial t} \qquad (7.32)$$

and

$$\vec{j} \propto i\,(a\nabla a^* - a^*\nabla a),\ \rho \propto aa^* \qquad (7.33)$$

Hence, for the approximations (7.33), the conservation of current and density holds exactly and the identification of a with the probability *function* ψ *of the classical quantum mechanics* is complete.

As a possible criticism of the preceding remarks, it is noted that the present discussion deals with the Bose-Einstein field while the classical quantum mechanics was principally concerned with the motion of a Fermi-Dirac particle (the electron). On the other hand, the approximation invoked in both cases renders the motion independent of spin; hence the relevance of the analysis.

The analysis above, coupled with (7.18), permits one to interpret $\langle \rho \rangle_0$ as a probability density, after suitable normalization. Moreover, the symmetry is behavior of observer and inertial spaces, as evidenced in (7.20), permits identification of $\langle \rho_0 \rangle$ with a probability density in inertial space.

But the latter identification implies a restriction on the variation of ρ_0 in a vacuum, for one must now write

$$\int d\vec{r}_0 dt_0 \langle \rho_0 \rangle \vec{r}_0 = 0, \quad \int d\vec{r}_0 dt \langle \rho_0 \rangle t_0 = 0 \qquad (7.34)$$

as an expression of the vacuum condition. And (7.34) is secured in a simple manner if one requires

$$\rho_0(\vec{r}, t; \vec{r}_0, t_0) = \rho_0(\vec{r}, t; -\vec{r}_0, -t_0) \qquad (7.35)$$

or the equivalent statement

$$(\rho_0)_{\vec{k}_0, \omega_0} = (\rho_0)_{-\vec{k}_0, -\omega_0} \qquad (7.36)$$

where $(\rho_0)_{\vec{k}_0, \omega_0}$ is the Fourier transform of ρ_0 in $\vec{r}_0$ and t_0.

Condition (7.36) is thus an alternative statement of the vacuum condition, and as such expresses the fact that equal statistical weights are assigned to creation and annihilation of both particle and anti-particle. It is clear that (k_0, ω_0) and $(-k_0, -\omega_0)$ refer to the same mass state $\varkappa = \sqrt{k_0^2 - (\omega_0/c)^2}$. It is therefore suggested that (7.36) represents the condition that an electron and a positron be created together near a heavy nucleus through the agency of a gamma ray, and hence agreement with reality in this case. It is true that the creation does not occur in a vacuum, but it is the closest approximation to such a condition observed in this connection and to this extent verifies (7.36).

Moreover, condition (7.36) and its meaning will be proposed as replacement of Dirac's hole model of electron-positron creation, and is worthy of note here in reference to BE fields. Thus, Dirac's assertion that negative energy states are conceived to be occupied by electrons becomes transformed to the assertion that negative inertial four-momentum' states are anti-particles (e.g. positrons); the exclusion principle prevents transitions into occupied states. The assumption that the negative energy states (e.g. positrons) produce no electromagnetic or gravitational effects is now altered to assert that (along with other virtual states) the negative four-momentum states are responsible, in the mean, for classical electromagnetic and gravitational effects.

Although the BE field is not subject to the exclusion principle, similar interpretations apply to it.

The symmetry in the $\{\vec{r}, t\}$ and $\{\vec{r}_0, t_0\}$ spaces, and the existence of a probability density for $\{\vec{r}, t\}$ space now justifies the averages over $\vec{r}$ and t mentioned earlier (Sections 6.3 and 7.1). Except perhaps for the vacuum condition, the correspondence in behavior of the two spaces is complete to this extent. But this condition is more a reflection of a limited type of experiment than a true picture of the field structures in the two spaces.

7.3 The Unified Field and Quantum Mechanics

The significance of the foregoing theory for the quantum mechanics can be illustrated by some physical considerations.

There are two features of physical observation at the basis of the quantum mechanics. They are:

a) the de Broglie hypothesis recognizing the wave character of matter; and

b) the indivisibility and uncontrollability of transfer of a quantum of energy between physical systems.

A theory seeking to supplant the quantum mechanics must account for these features.

The foregoing discussion has been based on a wave model of the matter state and embodies the de Broglie hypothesis; it does not, however, exclude the particle aspect of matter.

For definiteness, let us consider the case where observations do not resolve virtual processes; this is the atomic level of observation. This is expressed in the present theory by the assumption that the coordinates measured are $x-x_0$, $y-y_0$, $z-z_0$, and $t-t_0$, denoted by X, Y, Z and T, respectively. The corresponding metric is

$$dS^2 = dX^2 + dY^2 + dZ^2 - c^2 dT^2 \qquad (7.37)$$

with energy E and momentum P defined in the usual manner.* In addition one may define the action variable for a particle in a periodic orbit, by

$$J = \oint P\,dQ \qquad (7.38)$$

where $Q=q-q_0$ and $P=p-p_0$ are generalized coordinate and momentum. Averaging over the virtual processes, one finds

$$J' = \oint p\,dq + \langle \oint p_0 dq_0 \rangle \qquad (7.39)$$

*This metric could have been chosen from the outset with the understanding that the coordinates (e.g. Section 6) are random variables. The approximation implied by the separation would not then be in effect, but comparison with preceding theories would not be as direct.

For sufficiently small values of the first integral (i.e. on an atomic level) the second becomes of comparable value. Since $\sqrt{\langle p_0^2\rangle_0\langle q_0^2\rangle} > h$ the *minimum* value of the second integral is estimated to be h. Similarly, differences in the values of J have a minimum value of h.

But, with reference to a light wave of frequency ν , a variation in the energy $E_0 = h\nu$ of the wave (see (5.25)) is given by

$$\Delta E_0 \sim \frac{\partial E_0}{\partial J'} \Delta J' \sim h\nu \qquad (7.40)$$

No smaller energy transfer than $h\nu$ can be expected, under the conditions assumed.

It is felt that many of the conceptual difficulties of the quantum mechanics arise from the division between particle and field. The uncertainty principle, for example, concerns the indivisible transfer of energy between a radiation field and particle. Because of the need to use radiation to observe the motion of a particle, that motion is perturbed. The degree of the perturbation is measured by the complementary relation between the momentum and location of the particle. Thus the observer enters into the determination of the reality of physical observations. On the other hand, in the context of the present theory, there is necessarily a complementary relation between frequency and time interval necessary to define the field, or, alternately, between the bandwidth of the filter required to observe the desired feature within the time interval allowed. A similar remark applies to the momentum and location of a given feature of a particle field. In the latter case, the reality of the physical world is not considered to be inaccessible to the observer, due to his interference with it, but its perception is limited by the type of instrumentation used.

In addition, the present theory replaces the probability field of the quantum mechanics described by a function by the potentials of the field equations, chosen to be invariant under transformations leaving the above metric invariant. Thus, instead of an auxiliary Ψ field which guides the motion of an assembly of particles, or describes the probability of the behavior of a single particle, one has a potential which can be directly related to the density of the matter/radiation field. No change in the formalism of the quantum mechanics appears to be necessary; there is simply a re-interpretation of methods used.

For example, the mysterious notion of a particle "leaking through" a potential barrier, (as in radioactive decay) since the Ψ function of quantum mechanics is of a wave-like nature and carries the particle along with it, is replaced by an alternate concept. In the alternate model, the fluctuations in the energy of the particle constrained within the barrier are sometimes sufficient to carry the particle over the barrier, whence it escapes from the potential well, giving up its excess energy to the well as it leaves. The latter model, it is felt, is more easily visualized than the former, and more in consonance with the model advanced in preceding sections. Another model is seen to arise from identifying the particle with the unified field for which leakage through a barrier by a somewhat similar mechanism can be visualized, again a more direct description than possible with the auxiliary Ψ field.

In closing, it is noted that the need to appeal to spontaneous processes to provide a more classical picture of quantum effects is well known, e.g. the rôle of spontaneous emission of radiation in the derivation of the Planck black body radiation distribution. And, although these spontaneous processes lead to an infinite zero point energy for the radiation field, they are necessary to trigger any emission of radiation by a system in a stationary state. The present theory provides a mechanism for these effects, otherwise unresolved as spontaneous processes.

7.4 The BE Field with Sources; Interactions Between Fields.

The formalism developed in the preceding sections has had reference to the unforced BE field. As pointed out above, the restriction does not exclude matter currents from the field, these currents being an integral part of the field. Yet one does not obtain the classical electromagnetic field equations with sources from the present theory, in the appropriate context, and hence the formalism must be extended.

Postulate VIII. In the presence of sources, specified by the eight-current density (7.10), the equations of motion of the generalized radiation-matter BE field potentials are

$$\partial_\mu \partial^\mu a_\nu = 4\pi j_\nu \qquad (7.41)$$

It is emphasized that Postulate VII remains valid.

Averaging (7.41) over virtual processes, one obtains (note $\langle j_{k+4}\rangle_0 = 0$)

$$\partial_\ell \partial^\ell \langle a_k\rangle_0 = 4\pi \langle j_k\rangle_0 \qquad (7.42)$$

in agreement with the classical theory (k,ℓ =1,2,...4). Moreover, Postulates VII and VIII imply that

$$\partial^\mu f_{\mu\nu} = 4\pi j_\nu \qquad (7.43)$$

and hence that

$$\partial^\ell \langle f_{\ell k}\rangle_0 = 4\pi \langle j_k\rangle_0 \qquad (7.44)$$

Equations (7.44) are the classical equations of motion for the field intensities if ℓ, k =1,2,3,4.

Defining the generalized force density $\mathcal{F}_\mu$ by

$$\mathcal{F}_\mu = f_{\mu\nu} j^\nu \qquad (7.45)$$

equations (7.43) imply

$$\mathcal{F}_\nu = \partial^\mu T_{\mu\nu} \tag{7.46}$$

where $T_{\mu\nu} = T_{\nu\mu}$ is the energy-momentum tensor defined in relation (7.8); the use of the factor $1/4\pi$ is then justified.

A forcing action may likewise be obtained by interaction of two fields. Formal expression of this situation is obtained by replacing ∂_μ in the above theory by

$$D_\mu = \partial_\mu - \frac{i}{\hbar}B_\mu \tag{7.47}$$

where (B_μ) is the eight-potential descriptive of the interacting field.

It would lead this preliminary study too far from the present purpose to detail further properties of the BE field; introduction of the generalized FD field will next be considered.

7.5 The Fermi-Dirac Field

With respect to the meaning of equations (6.20) and (6.25), it is seen that these are essentially equivalent to the Proca field equations and hence describe mass states of integral spin. The results of Section 6.5 indicate that a single set of equations of motion for both BE and FD fields may be possible. On the other hand, it is easier to employ existing analytic techniques, as well as being of interest to examine the meaning of these methods in the light of the present viewpoint. For the foregoing reasons, the Dirac equation will be extended to obtain the equations of motion of a generalized Fermi-Dirac matter-radiation field, characterized by half-integral intrinsic spin.

Thus, with the aid of the matrices

$$S_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{7.48}$$

one defines the eight row and column Hermitian matrices

$$\sigma_1 = \begin{pmatrix} S_1 & 0 & 0 & 0 \\ 0 & S_1 & 0 & 0 \\ 0 & 0 & S_1 & 0 \\ 0 & 0 & 0 & S_1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} S_2 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 \\ 0 & 0 & S_2 & 0 \\ 0 & 0 & 0 & S_2 \end{pmatrix} \tag{7.49}$$

$$\sigma_3 = \begin{pmatrix} S_3 & 0 & 0 & 0 \\ 0 & S_3 & 0 & 0 \\ 0 & 0 & S_3 & 0 \\ 0 & 0 & 0 & S_3 \end{pmatrix}$$

$$\rho_1 = \begin{pmatrix} 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0 & 0 & 0 & -iI \\ 0 & 0 & iI & 0 \\ 0 & -iI & 0 & 0 \\ iI & 0 & 0 & 0 \end{pmatrix} \tag{7.50}$$

$$\rho_3 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -I \end{pmatrix}$$

$$\beta_1 = \begin{pmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 0 & 0 & iI & 0 \\ 0 & 0 & 0 & iI \\ -iI & 0 & 0 & 0 \\ 0 & -iI & 0 & 0 \end{pmatrix} \tag{7.51}$$

$$\beta_3 = \begin{pmatrix} -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

The components of each of the triples $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, $\vec{\rho} = (\rho_1, \rho_2, \rho_3)$, and $\vec{\beta} = (\beta_1, \beta_2, \beta_3)$ commute with the components of any other triple, and anticommute among themselves. All the matrices satisfy the relations (α_k, with k=1,2,3, denotes kth component of a given triple of matrices)

$$\alpha_k^2 = I \tag{7.52}$$

$$\alpha_k\alpha_\ell + \alpha_\ell\alpha_k = 2\delta_{k\ell} I, \tag{7.53}$$

$\ell = 1,2,3$

$$\alpha_k\alpha_\ell = i\alpha_m, \quad m = 1,2,3 \tag{7.54}$$

(where k, ℓ , m are permuted cyclically), and

$$(\vec{\alpha}\cdot\vec{A})(\vec{\alpha}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\vec{\alpha}\cdot(\vec{A}\times\vec{B}) \tag{7.55}$$

(where $\vec{A}$ and $\vec{B}$ are vectors which commute with $\vec{\alpha}$). The traces of all the matrices are equal to zero.

With the aid of the matrices (7.49) to (7.51) one may factor the fundamental relation between momenta and energies (6.5) into the square of the equation

$$\beta_1\vec{\sigma}\cdot\vec{p} + \beta_2\vec{\rho}\cdot\vec{p}_0 = \beta_1\frac{E}{c} + \beta_2\frac{E_0}{c} \tag{7.56}$$

Now, letting

$$\psi = (\psi_\nu) = (\psi_1, \ldots, \psi_8) \quad (7.57)$$

denote a row matrix whose elements are the potentials of the FD field, there is introduced

Postulate IX. The equations of motion of the free Fermi-Dirac (FD) matter-radiation field are assumed to be

$$\left[\beta_1\left(\vec{\sigma}\cdot\nabla + \frac{1}{c}\frac{\partial}{\partial t}\right) + \beta_2\left(\vec{\rho}\cdot\nabla_0 + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\right]\psi = 0 \quad (7.58)$$

It is clear that (7.58) has been obtained by means of the substitution (6.19) in equation (7.56). Moreover, for closer identification with the Dirac equation to which (7.58) will be shown to reduce in the mean, it will be assumed from the outset that the potentials ψ_ν may be complex numbers.

Upon averaging (7.58) over $\vec{r}_0$ and t_0, and assuming ψ_ν to vanish at infinity in $\{\vec{r}_0, t_0\}$ space, one finds

$$\beta_1\left(\vec{\sigma}\cdot\nabla + \frac{1}{c}\frac{\partial}{\partial t}\right)\langle\psi\rangle_0 = 0 \quad (7.59)$$

which is to be compared with the equation of motion for the free electromagnetic field. Indeed, upon substituting for $\langle\psi\rangle_0$ the expenential $\exp\frac{i}{\hbar}(\vec{p}\cdot\vec{r} - Et)$, one finds

$$\frac{i}{\hbar}\beta_1\left(\vec{\sigma}\cdot\vec{p} - E/c\right) = 0$$

so that $\vec{\sigma}\cdot\vec{p} = E/c$, or $p^2 = (E/c)^2$, and the latter is nothing else than the phase condition for a free electromagnetic field, when applied to the appropriate equations of motion. It will be shown that (7.58) refers to the equations of motion of a field of half-integral spin, and since (7.59) applies to a field of zero rest mass, it (and the corresponding phase condition) is suggested to be the equation of motion of the neutrino field. Thus the neutrino field is the analogue of the electromagnetic field, both as a phase condition and as the result of averaging over virtual processes.

If a single mass state of non-vanishing rest mass m_0 be specified, so that

$$\psi = \psi(\vec{r},t)e^{i(\vec{k}_0\cdot\vec{r}_0 - \omega_0 t_0)} \quad (7.60)$$

with $\varkappa = \sqrt{k_0^2 - (\omega_0/c)^2} = m_0 c/\hbar$, then

$$\left(\vec{\rho}\cdot\nabla_0 + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\psi = i\psi(\vec{r},t)\left(\vec{\rho}\cdot\vec{k}_0 - \frac{\omega_0}{c}\right)e^{i(\vec{k}_0\cdot\vec{r}_0 - \omega_0 t_0)} \quad (7.61)$$

and since

$$\vec{\rho}\cdot\vec{k}_0 - \frac{\omega_0}{c} = \varkappa\left[1 + \text{terms less than unity}\right],$$

it follows that (7.58) becomes, to the indicated approximation,

$$\left[\vec{\alpha}\cdot\nabla + \frac{1}{c}\frac{\partial}{\partial t} + i\beta\varkappa\right]\psi(\vec{r},t) = 0 \quad (7.62)$$

with $\vec{\alpha} = \vec{\sigma}$ and $\beta = \beta_1\beta_2$. Equation (7.62) is recognized as the Dirac equation. And with (7.62) one recovers the Dirac formalism as well as the classical quantum mechanics in the appropriate non-relativistic limit.

The relativistic invariance of both (7.58) and (7.62) may be proved in a manner too well described to be repeated here.

Defining the eight-current density by

$$\left.\begin{aligned} \vec{j} &= c\psi^*\vec{\sigma}\psi, & \vec{j}_0 &= c\psi^*\beta_1\beta_2\vec{\rho}\psi \\ \rho &= \psi^*\psi, & \rho_0 &= \psi^*\beta_1\beta_2\psi \end{aligned}\right\} \quad (7.63)$$

and with the aid of the equations of motion (7.58), it is easily shown that the current is conserved:

$$\partial^\mu j_\mu = \nabla\cdot\vec{j} + \frac{1}{c}\frac{\partial\rho}{\partial t} + \nabla_0\cdot\vec{j}_0 + \frac{1}{c}\frac{\partial\rho_0}{\partial t_0} = 0 \quad (7.64)$$

and that

$$\nabla\cdot\langle\vec{j}\rangle_0 + \frac{1}{c}\frac{\partial\langle\rho\rangle_0}{\partial t} = 0 \quad (7.65)$$

so that upon integration

$$\frac{\partial}{\partial t}\int\langle\rho\rangle_0\, d\vec{r} = 0 \quad (7.66)$$

and hence $\langle\rho\rangle_0$ may be interpreted as a probability density.

One further defines the Hamiltonian operator

$$H = \frac{\hbar c}{i}\left[\beta_1\vec{\sigma}\cdot\nabla + \beta_2\left(\vec{\rho}\cdot\nabla_0 + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\right] \quad (7.67)$$

noting that this commutes with the generalization of the angular momentum tensor

$$M_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu \quad (7.68)$$

and with the operators*

$$\begin{aligned} &\vec{r}\times\vec{p} + \tfrac{1}{2}\hbar\vec{\sigma} \\ &\vec{r}_0\times\vec{p}_0 + \tfrac{1}{2}\hbar\vec{\rho} \end{aligned} \quad (7.69)$$

and hence are constants of the motion.

7.6 The Coupled FD and BE Fields

The coupling of the FD and BE fields may be expressed by the formalism in a variety of ways. For example, one may substitute the eight-current density defined in Section 7.5 in equations (7.41) as a forcing term, and the potentials ψ_ν for B_ν in (7.47). Further, by analogy with the introduction of the current sources in (7.41), one may imagine a like generalization of the equations of motion for the free FD field (7.58).

*It is again pointed out that the quadratic expressions of the special theory are generalized by extending the domain of the summation indices from four to eight. Those variables defined in terms of r_0, t_0 alone are given an interpretation analogous to that in observer space for corresponding expressions. In this connection, one notes the effect of a translation of observer space on the angular momentum operator:
$\vec{r}\times\vec{p} \longrightarrow (\vec{r}-\vec{r}_0)\times(\vec{p}-\vec{p}_0) = \vec{r}\times\vec{p} - \vec{r}\times\vec{p}_0 - \vec{r}_0\times\vec{p} + \vec{r}_0\times\vec{p}_0$

On the other hand, the purpose here is a discussion closely analogous to that employed by Dirac in deriving the spin properties of the electron and demonstrate that the mass states of the field under discussion do indeed possess half integral spin.

Hence, in analogy with the customary procedure, the substitutions (suitable for a point charge; previously interaction potentials for charge and current distributions have been featured)

$$\vec{p} \to \vec{p} - \frac{i\varepsilon}{\hbar c}\vec{a},\ \vec{p}_o \to \vec{p}_o - \frac{ig}{\hbar c}\vec{a}_o$$
$$\frac{1}{c}\frac{\partial}{\partial t} \to \frac{1}{c}\frac{\partial}{\partial t} - \frac{i\varepsilon\hbar}{c}V,\ \frac{1}{c}\frac{\partial}{\partial t_o} \to \frac{1}{c}\frac{\partial}{\partial t_o} - \frac{i\hbar g}{c}V \qquad (7.70)$$

are carried out in equations (7.58). One then obtains

$$\left\{\beta_1\left[\vec{\sigma}\cdot\left(\nabla - \frac{i\varepsilon}{\hbar}\vec{a}\right) + \frac{1}{c}\left(\frac{\partial}{\partial t} + \frac{i\varepsilon}{\hbar}V\right)\right]\right.$$
$$\left. + \beta_2\left[\vec{\rho}\cdot\left(\nabla_o - \frac{ig}{\hbar}\vec{a}_o\right) + \frac{1}{c}\left(\frac{\partial}{\partial t_o} + \frac{ig}{\hbar}V_o\right)\right]\right\}\psi = 0 \qquad (7.71)$$

and for the plane wave $\psi = e^{i/\hbar(\vec{p}\cdot\vec{r} - Et)}\, e^{i/\hbar(\vec{p}_o\cdot\vec{r}_o - E_o t_o)}$

equations (7.71) become

$$\beta_1\left[\vec{\sigma}\cdot\left(\vec{p} - \frac{\varepsilon}{c}\vec{a}\right) + \frac{1}{c}\left(-E + \varepsilon V\right)\right]$$
$$+ \beta_2\left[\vec{\rho}\cdot\left(\vec{p}_o - \frac{g}{c}\vec{a}_o\right) + \frac{1}{c}\left(-E_o + gV_o\right)\right] = 0 \qquad (7.72)$$

Upon squaring both sides of equations (7.72), and employing (7.55) in the forms

$$\left[\vec{\sigma}\cdot\left(\vec{p} - \frac{\varepsilon}{c}\vec{a}\right)\right]^2 = \left(\vec{p} - \frac{\varepsilon}{c}\vec{a}\right)^2 - \frac{\hbar\varepsilon}{c}\vec{\sigma}\cdot\vec{\mathcal{H}}$$
$$\left[\vec{\rho}\cdot\left(\vec{p}_o - \frac{g}{c}\vec{a}_o\right)\right]^2 = \left(\vec{p}_o - \frac{g}{c}\vec{a}_o\right)^2 - \frac{\hbar g}{c}\vec{\rho}\cdot\vec{\mathcal{H}}_o \qquad (7.73)$$

it is readily seen that

$$\left(\vec{p} - \frac{\varepsilon}{c}\vec{a}\right)^2 - \left(\frac{E}{c}\right)^2 + p_o^2 - \left(\frac{E_o}{c}\right)^2$$
$$- \frac{\varepsilon\hbar}{c}\vec{\sigma}\cdot\vec{\mathcal{H}} - \frac{g\hbar}{c}\vec{\rho}\cdot\vec{\mathcal{H}}_o + \cdots = 0 \qquad (7.74)$$

But, $p_o^2 - \left(E_o/c\right)^2 = (m_o c)^2$ and setting $E = E_1 + m_o c^2$ ($E_1/m_o c^2 << 1$), one finds

$$E_1 = \frac{1}{2m_o}\left(\vec{p} - \frac{\varepsilon}{c}\vec{a}\right)^2 - \frac{\hbar}{2m_o c}\left(\varepsilon\vec{\sigma}\cdot\vec{\mathcal{H}} + g\vec{\rho}\cdot\vec{\mathcal{H}}_o\right) + \cdots \qquad (7.75)$$

Equations (7.75) is recognized as the nonrelativistic expression for the energy of a particle with magnetic moment $\varepsilon\hbar/2m_o c$ interacting with the magnetic field $\mathcal{H}$, and magnetic moment $g\hbar/2m_o c$, interacting with the magnetic field $\mathcal{H}_o$. Only the first two terms survive upon averaging over virtual processes. One must conclude, however, that the particle described by (7.75) has half integral spin in observer space, with a similar behavior in inertial space. The significance of the charge g will be taken up in the next section.

7.7 Magnetic Moments of Rotating Rigid Bodies and the Unified Field

The discussion in Section 7.6 found it necessary to introduce a quantity g with the dimensions of electric charge. The charge g cannot be equal to ε since charge creation and annihilation may proceed in the neighborhood of a particle observed to have no net electric charge (e.g. the neutron), so that although $\varepsilon = 0$, it does not follow that q=0. In the latter connection, it has been implicitly assumed that the scale of observation does not permit separating out the products of charge creation; hence g must be considered to be a kind of average charge. For the present, it will suffice that one can expect the effective charge g to generate a magnetic field in the neighborhood of rotating bodies. This conclusion follows from considerations similar to those encountered in discussions of the Einstein-deHaas-Barnett experiments[35].

Thus it is asserted that, since the effect of g is small in comparison with that of a permanent charge ε, a magnetic field should be perceptible on a macroscopic scale in the neighborhood of a large, sufficiently electrically neutral, body in rotation. Moreover, the ratio of the magnetic moment to the angular momentum of such a body should be

$$g_o\left(g/m_o c\right) \qquad (7.76)$$

Celestial bodies serve as the most well-known examples of large masses in in rotation which possess magnetic fields, and of these, the best known is the Earth. The magnetic field of the Earth cannot be explained on the basis of a small and permanent electric charge, and it is of the nature of a dipole field with axis roughly coincident with the axis of rotation of the Earth.

As long ago as 1947, Blackett[36] proposed a ratio analogous to 7.76 (G = gravitation constant)

$$\zeta\, \frac{G^{1/2} m_o}{m_o c} \qquad (7.77)$$

(where ζ is of the order of unity), to hold for all rotating celestial bodies, i.e.

$$g \sim G^{1/2} m_o$$

Although the agreement with observation was strikingly accurate for many cases, later observations disagreed with the predictions of 7.77. However, the theory presented above makes it possible to evaluate g_o for cases in which magnetohydrodynamic effects do not predominate (for simplicity). Observation of rotating rigid bodies at very high rotational speeds is an example of such an experiment, if the experiment can be carried out.

7.8 Summary

In the considerations above, it has been shown that a unified field theory with a comparatively simple formalism can be derived by splitting the space-time continuum into two spaces. These spaces correspond to two independent methods of signal generation, one a mean effect and the other arising spontaneously, characterized as uncontrollable noise.

With a few reasonable assumptions about the structure of space-time one may obtain insight into (among other subjects):

1) a causal description of nature, including the quantum mechanics, as an integral part of the theory;

2) a more satisfactory conception of "action at a distance";

3) an insight into the nature of elementary particles, as it relates to the matter/energy field and a mechanism for the quantization of mass;

4) the genesis of elementary and gravitational forces and an implementation of an anti-gravity device.

The extensive agreement of the theory proposed above, and those theories already found to be in agreement with experiment, constitutes a partial proof of the validity of the theory. In addition, when the proposed theory is employed as a moderate extension of accepted theories, one may expect, by extrapolation, e.g., increased insight into the subtraction of infinities of the quantum mechanics, as well as more accurate predictions and analyses of phenomena not presently included in physical theory.

Appendix A
Kinetic Theory Model of the Gravitation Field

The following semi-quantitative discussion is a kinetic theory model of the generation of an attractive force in the gravitation field by a mechanism analogous to that proposed for the unified field. It is not a detailed description, but being formulated in terms more familiar than the theory of Reference 1, it may be easier to see how the attractive force arises in the unified field. It also serves to emphasize what features are significant in the mechanism of the force's generation.

We first consider a single particle of mass M immersed in an ideal gas and in thermal equilibrium with the gas at absolute temperature T. The particle is subject to no other forces than those due to collisions of the molecules of the gas, which are assumed to have mass m and all to move with speed v. We assume that m M. The molecules interact strongly with the large particle and very weakly with one another. We will show below that the speed of the molecules of the gas, as well as the density of the gas, must, contrary to the initial assumption, vary throughout the gas volume.

The speed of each molecule is given by

$$v = (3kT/m)^{1/2} \tag{A1}$$

and of the large, massive, particle by

$$V = (3kT/M)^{1/2} \tag{A2}$$

so that

$$\left(\frac{v}{V}\right) = \left(\frac{M}{m}\right)^{1/2} \tag{A3}$$

In a single collision between a molecule and the large mass, the ratio of the changes in speeds, $\Delta v/\Delta V$ is in the ratio M/m. Thus on the average, the massive particle is essentially at rest and gains very little in speed in any single collision; multiple collisions lead to the condition summarized in equation A3, again essentially a rest condition. Moreover, multiple collisions with molecules is essentially symmetric as far as the large particle is concerned, so that the large particle tends to remain in a single location. The situation is very different for each colliding molecule since it is brought nearly to rest with every collision with the large particle. Since the pressure of the gas p at any point is given by

$$p = \frac{1}{3}\rho v^2 \tag{A4}$$

where ρ refers to the average mass density of the molecules (i.e. ρ = nm, where n is equal to the number of molecules per unit volume), there is implied a reduction in pressure in the neighborhood of the large particle and consequent flow of molecules toward the particle. Assuming thermal equilibrium to continue to hold, the pressure near the massive particle can only increase if the particle density increases. Once equilibrium in mass flow has been established, we can expect that the increase in density over the original density is effective over, approximately, several mean free path lengths of the molecules. The mean free path L varies inversely as the density of the gas ρ.

Let us now consider a configuration of two massive particles immersed in an ideal gas, under the same conditions originally postulated for the single particle above. We will, in addition, assume that the particle density distribution corresponding to a state of thermal equilibrium exists separately for each massive particle, without perturbation by the distribution associated with the other particle. This assumption, as a first approximation, in the spirit of the discussion above for a single massive particle, corresponds to the principle of linear superposition for the unified field. Like the assumption made in the preceding discussion for the single particle, it leads to a further mass flow.

For each particle senses a departure from the equilibrium condition of gas density which would exist if it were the only large particle in the gas. To re-establish this condition, there must be a mass flow from each element of volume toward each of the large particles until a symmetric density distribution with smaller mean free path (due to the increased density) and a smaller region in which the density varies is established. The reaction upon the large particles, of compelling this mass flow, is a force urging the particles toward one another, taking into account the symmetry of the processes acting.

Appendix B

By means of Rayleigh's method[32], it is possible to gain further insight into the role played by random phase shifts in generating that property of matter-radiation known as inertia.

Let N denote the total number of combinations of n independent unit sinusoidal motions with resultant $(x_0, y_0, z_0, ict_0) = (s_0, \alpha_0, \beta_0, \varphi_0)$ (see footnote of Section 12) and let

$$N F(n, x_0, y_0, z_0, ict_0)\, dx_0\, dy_0\, dz_0\, d(ict_0)$$

denote the number of combinations lying in the interval x_0, x_0+dx_0, $y_0, y_0+dy_0, z_0, z_0+dz_0, ict_0+d(ict_0)$. It is assumed that the phases of the vibrations are distributed randomly and that both N and n are large.

Let $P(x_0, y_0, z_0, ict_0)$ be the coordinates of a point after each of the N combinations has received one additional vibration, so that n increases to n+1. Then before the addition, the coordinates of P must have been

$$\begin{aligned} x_0' &= x_0 - s_0 \sin\alpha_0 \sin\beta_0 \cos\varphi_0 \\ y_0' &= y_0 - s_0 \sin\alpha_0 \sin\beta_0 \sin\varphi_0 \\ z_0' &= z_0 - s_0 \sin\alpha_0 \cos\beta_0 \\ ict_0' &= ict_0 - s_0 \cos\alpha_0 \end{aligned} \tag{B1}$$

and the number in the volume element $dx_0dy_0dz_0d(ict_0)$ after the addition, and averaging over phases, must be given by the left side of

$$N\, dx_0\, dy_0\, dz_0\, d(ict_0) \langle F(n, x_0', y_0', z_0', ict_0')\rangle_\varphi = N F(n+1, x_0, y_0, z_0, ict_0)\, dx_0\, dy_0\, dz_0\, d(ict_0) \tag{B2}$$

But, upon expanding $F(n, x_0, y_0, z_0, ict_0)$ in a power series about the point (x_0, y_0, z_0, ict_0) and carrying out the indicated average, one finds that only terms quadratic in the variables remain. Further, expanding $F(n+1, x_0, y_0, z_0, ict_0)$ in a power series in the neighborhood of n, and equating quadratic terms figuring in (B2), one finds

$$\left(\Delta_0 - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}\right) f = \frac{1}{\sigma^2}\frac{\partial f}{\partial n} \tag{B3}$$

In equation (B3), it has been assumed that

$$\langle(x_0'-x_0)^2\rangle_\varphi = \langle(y_0'-y_0)^2\rangle_\varphi \cdots = \langle[ic(t_0'-t_0)]^2\rangle_\varphi = \sigma^2 \tag{B4}$$

and $F(n, r_0, ict_0) = f(n, r_0, ct_0)$.

An elementary solution of (B3) is $e^{i(\vec{k}_0\cdot\vec{r} - \omega_0 t_0)}e^{-\sigma^2\varkappa^2 n}$ where $\varkappa^2 = k_0^2 - \omega_0^2/c^2$, so that the effective diameter $\mathcal{R}$ of the domain of $f(n, r_0, ct_0)$ in n is of the order of magnitude $1/\sigma^2\varkappa^2$. That is, the factor $e^{-\sigma^2\varkappa^2 n}$ governs the amplitude of the elementary solution, with

$$\mathcal{R} \sim 1/\sigma^2\varkappa^2 \tag{B5}$$

But σ measures the effective range of the inertial motion; hence, it follows that $\varkappa$ (or the inertial mass m_0) is inversely proportional to (approximately)

i) the square root of the maximum effective value of n, and

ii) the diameter of the domain of the inertial motion.

Appendix C

Although the following considerations admit of a more general formulation, the analysis will be directed toward equations of the type featured in this work:

$$\Box V(\vec{r}, t) = W(\vec{r}, t) \tag{C1}$$

The application to the equations sometimes introduced in the analysis of Brownian motion

$$\frac{d^2\vec{r}_0}{dt^2} = -\beta\frac{d\vec{r}_0}{dt} + \vec{A}(t) \tag{C2}$$

(i.e. Langevin's equation, where $\vec{r}_0$ is the position vector of the Brownian particle, β is a constant, and A(t) is a fluctuating vector function), will be evident.

Let it be supposed that some boundary condition has been assigned to $V(\vec{r}, t)$ for the point $\vec{r} = \vec{r}_0$, $t = t_0$. Then one may indicate the dependence of $V(\vec{r}, t)$ on this choice by setting

$$V(\vec{r}, t) = V(\vec{r}, t; \vec{r}_0, t_0) \tag{C3}$$

Any ambiguity introduced by the notation introduced in (C3), will be avoided by always indicating the arguments of the function under discussion.

One then imagines the motion of the potential field $V(\vec{r}, t)$ constantly interrupted, each time beginning a new with different values of $\vec{r}_0$ and t_0, such that

$$\begin{aligned} \langle\vec{r}_0\rangle_\varphi &= 0 \\ \langle t_0\rangle_\varphi &= 0 \end{aligned} \tag{C4}$$

But

$$V(\vec{r}, t; \vec{r}_0, t_0) = V(\vec{r}, t; 0, 0) + \vec{r}_0\cdot\nabla_0 V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} + t_0\frac{\partial}{\partial t_0}V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} + \cdots \tag{C5}$$

whence it follows that

$$\Box V(\vec{r},t;0,0) = W(\vec{r},t) - \vec{r}_0 \cdot \nabla_0 \Box V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} - t_0 \frac{\partial}{\partial t_0} \Box V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} - \dots \tag{C6}$$

where the terms in $V(\vec{r},t;\vec{r}_0,t_0)$ on the right side of (C6) are exhibited as forcing terms on the same scale of observation as $W(r,t)$. Moreover,

$$\Box V(\vec{r},t;0,0) = W(\vec{r},t) - \left\langle \frac{(\vec{r}_0 \cdot \nabla_0)^2}{2!} \Box V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} \right\rangle_0 - \left\langle \frac{t_0^2}{2!} \frac{\partial^2}{\partial t_0^2} \Box V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} \right\rangle_0 - \dots \tag{C7}$$

and

$$V(\vec{r},t;0,0) = \Box^{-1} W(\vec{r},t) - \vec{r}_0 \cdot \nabla_0 V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} - t_0 \frac{\partial}{\partial t_0} V\Big|_{\substack{\vec{r}_0=0\\ t_0=0}} \tag{C8}$$

where the symbol $\Box^{-1}$ is defined by the relation

$$\Box^{-1} \Box V = V \tag{C9}$$

Thus, the behavior of $V(\vec{r},t;\vec{r}_0 t_0)$ in the neighborhood of the common average value of $\vec{r}_0$ and t_0 (i.e. the value most frequently taken on by these arguments) is a result of forcing terms additional to $W(r,t)$. On the average, only terms of even order in $\vec{r}_0$ and t_0 contribute to the variation of $V(\vec{r},t;0,0)$; in the absence of an average, the first two such forcing terms are linear in these variables. And if the latter terms dominate the behavior of the series in $\vec{r}_0$ and t_0 in (C8), then they must act to oppose any variation of $V(\vec{r},t;\vec{r}_0,t_0)$ from the value $V(\vec{r},t;0,0)$.

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