GPS Carrier-Phase Ambiguity Resolution

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Topic Outline

- Basics of GPS
- Measurement problems and their mitigation
- Differential Techniques
  - Code
  - Carrier-Phase
- Ambiguity Resolution
  - Geometry independent—in measurement space
  - Geometry dependent
    - In position space
    - In ambiguity space
  - Third frequency
- References

Basics of GPS

The satellites

The signals

The receiver measurements

The GPS Satellites

- Currently 26 satellites in ~12 hour circular orbits at 55 degree inclination to the equator (~25,500 Km. Radius)
- Currently broadcast on two L-band frequencies with a pseudorandom bi-phase modulation.
- Multiple atomic clocks for control of system timing.

GPS Signals

L1	1575.42 MHz = 154 * 10.23 = 77 * 20.46; C/A & P
L2	1227.60 MHz = 120 * 10.23 = 60 * 20.46; P (C/A)
(Lc	1176.45 MHz = 115 * 10.23; P)
C/A Code	1.023 MHz chip rate; 1000 Hz repetition rate.
P Code	10.23 MHz chip rate; 1 per week repetition rate.
Anti-Spoof (Y Code)	~500 KHz chip rate; 1 per week repetition rate.
Binary Message	50 Hz

Selective Availability has been discontinued (orbit and clock inaccuracy)

Satellites are identified by their specific pseudorandom C/A and P codes

BI-PHASE MODULATED SIGNAL

GPS SPECTRUM (NO MODULATION)

SPREAD SPECTRUM POWER DENSITY

Code (Pseudorange) Measurements

- Measurement of transit time multiplied by “c”
- Correlation of code to maximize power
- Code modulation is pseudorandom 180 phase shifts
- High sensitivity to measurement epoch accuracy (154 GHz)
- Solution requires measurements from at least four satellites to solve (X, Y, Z, and Rx Clock)

Pseudorange Measurements

P-Code Autocorrelation Function

Measurement Comparison

Psuedoranging Geometry

CODE POSITION SOLUTION TECHNIQUE

Carrier-Phase Measurements

- Recovery of suppressed carrier via:
  - Code removal, squaring, cross-correlation
- Integral of (Sat clock rate minus Rx clock rate)
- Unknown (ambiguous) whole-cycle constant of integration
- Very precise measurement—less than 200th of one cycle (1mm)
- Low sensitivity to measurement epoch accuracy (5KHz + clock drift rate)
- Direct solution uses range change (hyperbolic) measurements

Measurement Problems & Mitigation

- Multipath (reflections)
- Ionospheric Refraction
- Tropospheric Refraction
- Satellite orbit and clock
- Antenna phase center variation
- Receiver clock divergence between L1 and L2
- Receiver noise

Multipath

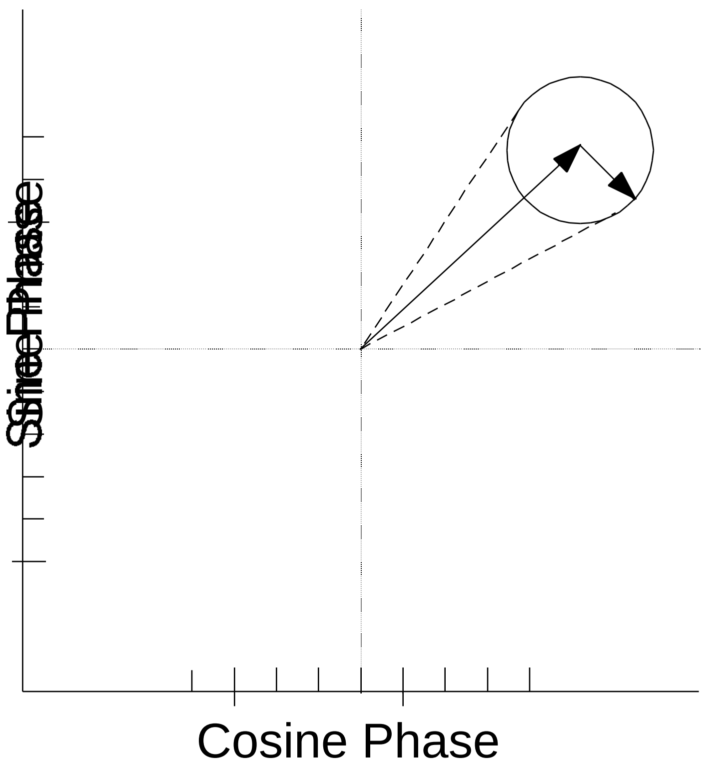
- Reflected signal mixes with direct signal
- Affects code much worse than carrier-phase
- Affects C/A-code about 3 times P-Code
- Effect can be reduced by:
  - Antenna design, e.g. choke ring antennas
  - Narrow-correlator peak detector (code only)
  - Double delta correlator design
  - Signal to noise pattern detect
  - Smoothing the code measurements with the carrier-phase measurements

C/A Code and Carrier Relative Noise

Code Multipath Effects

Code Multipath Effects

Carrier-Phase Multipath Effects

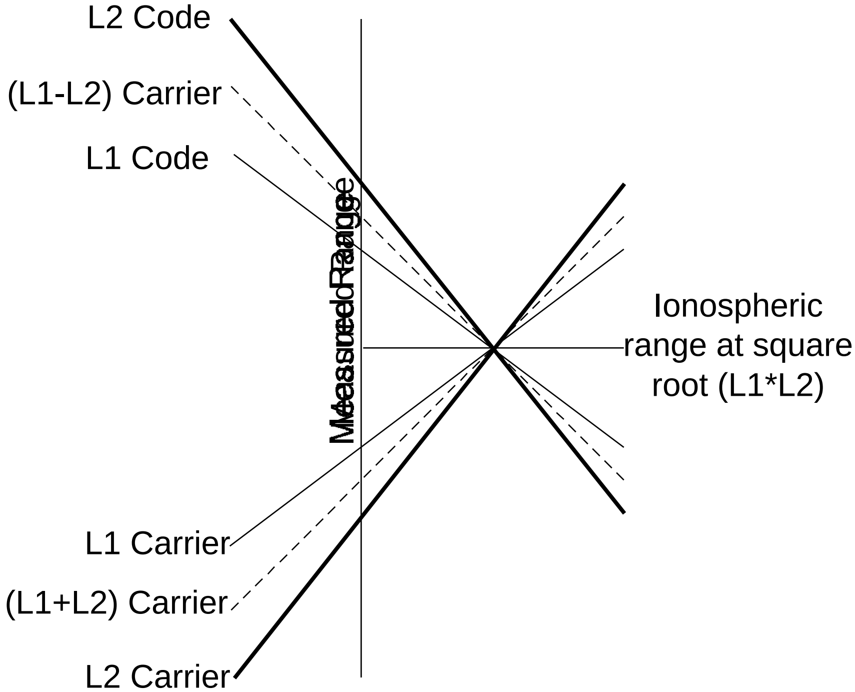


Ionospheric Refraction

- Effect is inversely proportional to the frequency squared
  - Causes a time delay (longer range) in code measurements
  - Causes a phase advance (shorter range) in carrier-phase measurements
- Modeling with transmitted coefficients can reduce the effect by 50% to 75%
- WAAS uses multiple reference stations to create a more accurate model
- Dual-frequency receivers can be used to eliminate the effect—with an associtaed amplification of the noise

Overhead Ionospheric Intensity Map

Ionospheric Effect on Measured Range



Dual Frequency Extended Code Smoothing to Minimize Multipath

- Refraction correction amplifies the multipath
- Code smoothing limited by ionospheric divergence

So:

- Form linear combination of carrier-phase measurement which matches the ionospheric effect of the corresponding code measurement
- This allows unlimited smoothing

Combinations which Match Ionosphere

- *P1* matches $\dfrac{(f_1^2 + f_2^2)}{(f_1^2 - f_2^2)}\varphi_1\lambda_1 - \dfrac{2f_2^2}{(f_1^2 - f_2^2)}\varphi_2\lambda_2$

  (5.0914556) *(4.0914556)*

- *P2* matches $\dfrac{2f_1^2}{(f_1^2 - f_2^2)}\varphi_1\lambda_1 - \dfrac{(f_1^2 + f_2^2)}{(f_1^2 - f_2^2)}\varphi_2\lambda_2$

  (4.0914556) *(3.0914556)*

Frequency Weighted Average Code Matches Difference of the Carrier Phase

- Average code
  - Reduced Multipath (apx. 1/6th)

$$P_a = \frac{P_1}{\lambda_1} + \frac{P_2}{\lambda_2} \quad \leftrightarrow \quad \frac{f_1 - f_2}{f_1 + f_2}$$

$$P_a = \left(\frac{f_1 P_1 + f_2 P_2}{f_1 + f_2} \right) \lambda_d$$

MATCHES:

- Difference of carrier
  - Longer wavelength (Wider lane)

$$\varphi_d \lambda_d = (\varphi_1 - \varphi_2) \lambda_d$$

Frequency Weighted Difference of the Code Matches Average of the Carrier Phase

- Difference of code
  - Amplified Multipath (apx. 3 times)

$$P_d = \frac{P_1}{\lambda_1} - \frac{P_2}{\lambda_2} \leftrightarrow \frac{f_1 + f_2}{f_1 - f_2}$$

$$P_d = \left[\frac{f_1 P_1 - f_2 P_2}{2(f_1 - f_2)}\right]\lambda_a$$

MATCHES:

- Average of Carrier
  - Minor wavelength change (Apx. unchanged lane)

$$\varphi_a \lambda_a = \left[\frac{\varphi_1 + \varphi_2}{2}\right]\lambda_a$$

Refraction Corrected Code Matches Refraction Corrected Carrier Phase

- Refraction Corrected Code

$$P_{rc} = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2$$

  - Amplifies Multipath (apx. 3 times)

(2.5457) (1.5457)

MATCHES:

$$\varphi_{rc}\lambda_s = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1\lambda_1 - \frac{f_2^2}{f_1^2 - f_2^2}\varphi_2\lambda_2$$

- Refraction Corrected Carrier Phase

$$\varphi_{rc}\lambda_s = ¾\,\frac{f_1}{f_1 - f_2}\varphi_1 - \frac{f_2}{f_1 - f_2}\varphi_2 \sqrt{/}\lambda_s$$

(4.5294) (3.5294)

  - Short Wavelenth
  - No effect from L1/L2 clock divergence

Tropospheric Refraction

- Primary effect is due to slower speed of light in the lower atmosphere
- Magnitude of the effect is a function of the temperature, pressure and humidity
- Often separate models for dry (temp. & pres.) and wet (humidity) effects
- Dry accounts for typically 95% of the effect and can be modeled with good accuracy
- Wet is highly variable and is a function of path

Tropospheric Mitigation

- A number of models are available to correct for the dry component
  - Separate model of overhead effect and
  - Tipping angle effect
  - e.g. Hopfield, Saastamoinen, Berman, Chao, (WAAS)
- Direct measure of effect using water vapor radiometer—very expensive
- Solve for effect as a slowly varying stockastic random walk—MIT, JPL, Scripps

Satellite Orbit & Satellite Clock Errors

- Recent removal of Selective Availability makes these errors much less severe
- Most effective method for removal is to use differential GPS—i.e. measure the errors via a reference site or sites
  - Differential code (DGPS)
  - Differential carrier phase (Kinematic GPS)

Differential & Kinematic GPS

- Amplifies independent noise
  - Multipath effects
  - Receiver noise
  - Antenna phase center variations
- Minimizes or cancels correlated noise
  - Orbit and clock errors
  - Ionospheric refraction effects
  - Tropospheric refraction effects (short distance) (watch out for height differences)

Other DGPS Comments

- Most users generate corrections rather than transmitting reference site code measurements—less sensitive to latency
- With removal of Selective Availability no correction rate terms are needed
- Partial cancellation of tropospheric refraction allows the use of lower elevation satellites
- Reference station can improve integrity by assessing the health of satellites
- NavCom obtains 25 centimeter (one sigma) navigation across the U.S. with dual frequency refraction corrected extended smoothing—using 6 reference stations

Kinematic (Carrier Phase) Differential GPS—Initial Comments

- Most users send raw data from reference site(s) rather than corrections—more sensitive to latency and can require much higher data rate communication link—requires knowledge of the location of the reference site
- If reference phase data has an integer number of whole-cycles added to bring into approximate agreement with code measurements and corrections generated, any integer error can be lumped with user whole-cycle ambiguity. (Corrections equivalent to differencing across sites)
- Major problem is user ambiguity resolution
- Accuracy near one centimeter horizontal over approximately 10 kilometer baseline separation

Ambiguity Resolution

- Geometry Independent—insensitive to tropospheric refraction—more degrees of freedom—simple verification
  - In measurement space
    - Uses smoothed code for wide lane ambiguity resolution, then wide-lane resolved value to step to narrow lane.
- Geometry dependent—sensitive to tropospheric refraction—fewer degrees of freedom—tougher verification
  - In position space—(Counselman) ambiguity function
  - In ambiguity space—searches for minimum residuals as a function of ambiguity combinations

Ambiguity Resolution in Measurement Space (1)

- Uses extended smoothed differentially corrected code combination which matches the ionospheric effects of the corrected carrier phase differences

$$N_d = N_1 - N_2 = \frac{f_1 P_1 + f_2 P_2}{(f_1 + f_2)\lambda_d} - (\varphi_1 - \varphi_2)$$

Stepping from Wide Lane to Narrow

Ambiguity Resolution in Measurement Space (2)

- Use resolved wide-lane measurement to step to narrow lane—depends upon ionospheric error being small—thus, apx. 10 kilometer baseline

$$N_s = N_1 + N_2 = \frac{(\varphi_1 - \varphi_2 + N_d)\lambda_d}{2\lambda_s}$$

$$N_1 = \frac{(\varphi_1 - \varphi_2 + N_d)\lambda_d}{\lambda_s} \leftrightarrow \frac{f_1}{f_1 + f_2} \qquad (0.562)$$

$$N_2 = \frac{(\varphi_1 - \varphi_2 + N_d)\lambda_d}{\lambda_s} \leftrightarrow \frac{f_2}{f_1 + f_2} \qquad (0.438)$$

Ambiguity Resolution in Measurement Space (3)

- Verification is that solution residuals are small
- Can flag wide or narrow lane resolution if questionable (large fractional error) and do partial ambiguity search on only those satellites

Ambiguity Resolution in Position Space

- Ambiguity Function technique originally developed by Counselman
- Developed further and streamlined by Mader
- Is simple but computationally intensive
- Not widely used

Ambiguity Resolution in Ambiguity Space (1)

- Many variations of techniques to minimize the computational intensity and to minimize the number of epochs of data required
  - Least Squares Search (Hatch)
  - Fast Ambiguity Search Filter (Chen & Lachapelle)
  - Fast Ambiguity Resolution Approach (Frei & Beutler)
  - Modified Cholesky search (Euler et al.)
  - Lambda Decomposition (Teunissen)

Ambiguity Resolution in Ambiguity Space (2)

- Advantages of each:
  - LSS – simple and minimizes search combinations
  - FASF & FARA – takes advantage of correlation between ambiguity values to minimize number of epochs required
  - CS – very efficient computationally
  - Lambda – uses combination of measurements to minimize the correlation between ambiguity values

Ambiguity Resolution in Ambiguity Space (3)

- Verification is generally a ratio test between smallest residuals to second smallest residuals
- Dual frequency (wide lane) ambiguity search is much much quicker than single frequency ambiguity search (~4.5 cubed fewer possible ambiguity values in same position search volume) (~4.5 to (n-1) power fewer possible ambiguity values with n satellites in same ambiguity search space)
- Persistence of false solutions is reduced in time

Ambiguity Resolution in Ambiguity Space (4)

- Illustrations
  - Two dimensional illustration of phase measurements from multiple satellites
  - Of residuals as function of lane error
  - Of persistence of residuals with time
  - Of accuracy

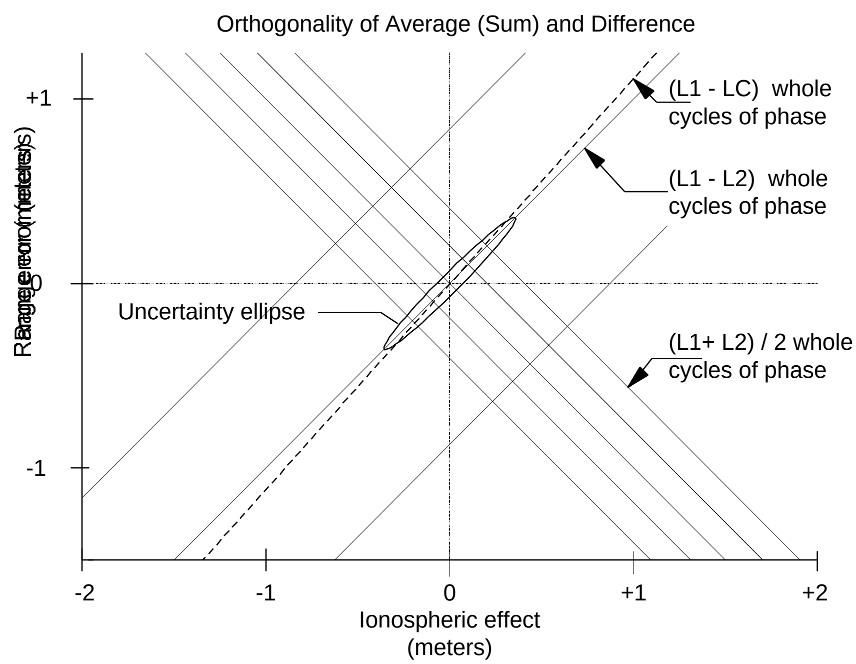
Two Dimensional – Multiple Satellites Illustration

Residuals as Function of Lane Error Illustration

Persistence of Small Residuals

Accuracy Illustration – Horizontal

The Problem with the New Third Frequency – Lc



Three Frequency Wide Laning

- Excellent (generally one epoch) resolution of ambiguities over short distances (5.86 meter L1-Lc wavelength)
- Improved ability to extend ambiguity resolution to regional networks
- Still very difficult to resolve ambiguities over large baseline distances
- Long distance needs a frequency at 900 MHz or below or at 1800MHz or above

Kinematic Applications

- Farming – yield monitoring—auto steering
- Mining
- Dredging
- Surveying
- Photogrammetry
- Attitude determination
- Snow Removal
- Automated vehicles

Overview References

- Chen & Lachapelle (1994) “A Comparison of the FASF and Least-Squares Search Algorithms for Ambiguity Resolution On-the-Fly,” KIS Symposium, Banff, Canada, Aug.
- Hatch & Euler (1994) “Comparison of several AROF Kinematic Techniques,” *Proceedings of ION GPS-94,* Salt Lake City, Sept., p 363
- Han & Rizos (1997) “Comparing GPS Ambiguity Resolution Techniques,” *GPS World,* **8,** 10, Oct. p54

Measurement Space References

- Hatch (1982) "The Synergism of Code and Carrier Measurements," *Proceedings of the Third International Symposium on Satellite Doppler Positioning,* Las Cruces, NM, Feb.
- Melbourne (1985) "The Case for Ranging in GPS-Based Geodetic Systems," *Proceedings of the First International Symposium on Precise Positioning with GPS,"* Rockville, MD, April
- Hatch (1986) "Dynamic Differential GPS at the Centimeter Level," *Proceedings of the Fourth International Geodetic Symposium on Satellite Positioning*, Las Cruces, NM, April

Position Space References

- Counselman & Gourevitch (1981) “Miniature Interferometer Terminals for Earth Surveying: Ambiguity and Multipath with Global Positioning System,” *IEEE Transactions on Geoscience and Remote Sensing*, **GE-19,** 4, Oct.
- Remondi (1991) “Pseudo-kinematic GPS Results Using the Ambiguity Function Method,” *Navigation*, **38**, 1, Spring
- Mader (1992) “Kinematic GPS Initialization Using the Ambiguity Function,” *Proceedings of the Sixth International Geodetic Symposium on Satellite Positioning*, Columbus, Ohio, March

Ambiguity Space References

- Frei & Beutler (1989) "Some Considerations Concerning an Adaptive, Optimized Technique to Resolve the Initial Phase Ambiguities," *Proceedings of the Fifth International Geodetic Symposium on Satellite Positioning*, Las Cruces, NM, March
- Hwang (1990) "Kinematic GPS: Resolving Integer Ambiguities On-the Fly," *Proceedings IEEE PLANS-90*, Las Vegas, NV, March
- Hatch (1990) "Instantaneous Ambiguity Resolution," *KIS-90 Symposium*, Banff, Canada, Aug.

Ambiguity Space References (Continued)

- Abidin (1991) “New Strategy for On-the-Fly Ambiguity Resolution,” *Proceedings ION GPS-91*, Albuquerque, NM, Sept.
- Landau & Euler (1992) “On-the-Fly Ambiguity Resolution for Precise Differential Positioning,” *Proceedings ION GPS-92*, Albuquerque, NM, Sept.
- Teunissen (1994) “A New Method for Fast Carrier Phase Ambiguity Estimation,” *Proceedings of IEEE PLANS*, Las Vegas, NV, April