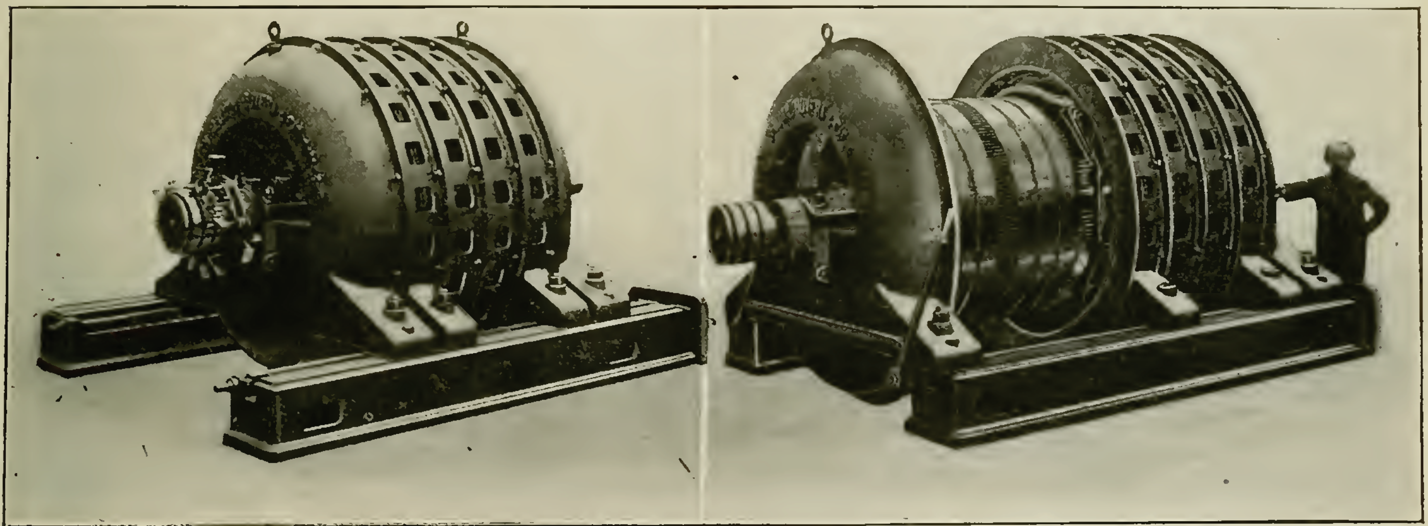
The Largest Induction Motor in the World.

The accompanying photographs represent a two-phase motor for 1000 brake horse-power which has just been turned out from the works of Messrs. Brown, Boveri & Co., of Baden, Switzerland. As this is probably considerably the largest induction motor in the world, some details may be of interest.

The motor is intended to work in the municipal pumping station of the city of Geneva. The existing pumps, of the piston type, are driven direct by turbines, but as no further water power was available for extensions it was decided to put down a centrifugal pump driven by a two-phase motor deriving its power from the electric power house of Chèvres, some miles farther down the river. The pump for the exceptionally high lift of 460 feet is from the works of Sulzer Brothers, of Winterthur, and will be coupled directly to the motor here shown.

The motor is wound with ten poles, so as to run at a speed of 544 revolutions per minute when fully loaded, the frequency of the supply being 46 cycles per second. It is supplied directly with the high-tension current at 2500 volts. The total weight of the motor is 27 tons. It will be seen that the front cover plate is made to slide out on the prolonged bed-plate, so as to facilitate the removal of the rotating part. The circuits of the latter are brought out through the hollow shaft to three contact rings, for the use of a separate starting resistance in the secondary.*



A 1000-HP INDUCTION MOTOR.

The power house at Chèvres is also undergoing complete reconstruction, six new generators of 1200 horse-power each being now in course of construction in Messrs. Brown, Boveri & Co.'s works, as well as a new switchboard for all the generators and circuits, this switchboard being no less than 85 feet in length.

*See also THE ELECTRICAL WORLD January 15, 1898, p. 89.

Convention of the Association of Fire and Police Telegraph Superintendents and Municipal Electricians.

This association held its last meeting in Elmira, N. Y., on August 9, 10 and 11. About sixty delegates were in attendance. They were welcomed to the city and entertained pleasantly. Several valuable papers were read and the following officers were elected for the coming year: President, J. W. Aydon, Wilmington, Del.; vice-president, C. T. MacDonald, Ottawa, Can.; treasurer, Adam Boesch, Newark, N. J.; secretary, H. T. Blackwell, Jr., New York; financial secretary, Burt McAllister, Bradford, Pa. The next meeting will be held in Wilmington, Del.

A New Source of Carbon for Calcium Carbide.

In a large lumber yard near Ottawa, Canada, there has been installed a retort for producing carbon from sawdust. It is almost needless to remark that this material is cheap, in fact so cheap that it is a matter of considerable expense in many large lumber yards to dispose of it by burning or otherwise. It is claimed that the charcoal obtained is purer than coke, and is therefore better suited to making calcium carbide.

The Natural Period of a Transmission Line and the Frequency of Lightning Discharges Therefrom.

BY CHARLES PROTEUS STEINMETZ.

The discharge of a condenser through a circuit containing self-induction and resistance is oscillating (provided that the resistance does not exceed a certain critical value depending upon the capacity and the self-induction). That is, the discharge current alternates with constantly decreasing intensity. The frequency of this oscillating discharge depends upon the capacity, C, and the self-induction, L, of the circuit, and to a much lesser extent upon the resistance, so that if the resistance of the circuit is not excessive the frequency of oscillation can with fair, or even close, approximation be derived by neglecting the resistance, by the formula

$$N = \frac{1}{2 \pi \sqrt{C L}}$$

An electric transmission line represents a capacity as well as a self-induction, and thus when charged to a certain potential, for instance, by atmospheric electricity, as by induction from a thunder cloud passing over or near the line, the transmission line discharges by an oscillating current.

Such a transmission line differs, however, from an ordinary condenser, in that with the former the capacity and the self-induction are distributed along the circuit.

The object of the following is to determine the frequency of the oscillating discharge of such a transmission line, which is the frequency of the discharge of atmospheric electricity, or of the oscillation set up when suddenly changing the condition of the circuit, as, for instance, opening the circuit.

For this purpose sufficiently close approximation is derived by neglecting the resistance of the line, which, at the relatively high frequency of oscillating discharges, is small compared with the reactance. This assumption means that the dying out of the discharge current by the resistance of the circuit is neglected, and the current assumed as alternating current of approximately the same frequency and the same intensity as the initial waves of the oscillating discharge current.

Hereby the problem is essentially simplified by the possibility of using the symbolic method of investigation.

Let $c =$ total length of a transmission line;
$r =$ resistance per unit length;
$x =$ reactance per unit length $= 2 \pi N L$.
where $L =$ coefficient of self-induction or inductance per unit length;
$g =$ conductance from line to return (leakage and discharge into the air) per unit length;
$b =$ capacity susceptance per unit length $= 2 \pi N C$
where $C =$ capacity per unit length.
$l =$ the distance from the beginning of the line.*

* "Alternating Current Phenomena," by Charles Proteus Steinmetz, second edition, paragraph 110, pages 161 and 160, equations 14 and 11.

The e. m. f.:

$$E = \frac{1}{g - jb}\left\{(A\varepsilon^{\alpha l} - B\varepsilon^{-\alpha l})\cos\beta l - j(A\varepsilon^{\alpha l} + B\varepsilon^{-\alpha l})\sin\beta l\right\}$$

the current:

$$I = \frac{1}{\alpha - j\beta}\left\{(A\varepsilon^{\alpha l} + B\varepsilon^{-\alpha l})\cos\beta l - j(A\varepsilon^{\alpha l} - B\varepsilon^{-\alpha l})\sin\beta l\right\} \qquad (1)$$

where:

$$\alpha = \sqrt{\frac{1}{2}\left\{\sqrt{(g^2 + b^2)(r^2 + x^2)} + (g\,r - b\,x)\right\}}$$

$$\beta = \sqrt{\frac{1}{2}\left\{\sqrt{(g^2 + b^2)(r^2 + x^2)} - (g\,r - b\,x)\right\}} \qquad (2)$$

ε = basis of natural logarithms, and A and B integration constants.

Neglecting the line resistance: $r = O$, and the conductance (leakage, etc.): $g = O$, gives:

$$\alpha = O$$
$$\beta = \sqrt{b\,x} \qquad (3)$$

These values substituted in (1) give:

$$E = \frac{j}{b}\left\{(A - B)\cos\sqrt{b\,x}\,l - j(A + B)\sin\sqrt{b\,x}\,l\right\}$$

$$I = \frac{j}{\sqrt{b\,x}}\left\{(A + B)\cos\sqrt{b\,x}\,l - j(A - B)\sin\sqrt{b\,x}\,l\right\} \qquad (4)$$

If the discharge takes place at the point: $l = O$, that is, the distance is counted from the discharge point to the end of the line: $l = c$, it is:

At: $l = O$: $E = O$
At: $l = c$: $I = O$

Substituting these values in (4) gives:
For: $l = O$:

$$A - B = O \qquad A = B$$

These values substituted in (4):

$$E = \frac{2A}{b}\sin\sqrt{b\,x}\,l$$

$$I = \frac{2jA}{\sqrt{b\,x}}\cos\sqrt{b\,x}\,l \qquad (5)$$

and at $l = O$:

$$I_0 = \frac{2jA}{\sqrt{b\,x}} \qquad (6)$$

At $l = c$: $I = O$, thus, substituted in (5):

$$\cos\sqrt{b\,x}\,c = O \qquad (7)$$

hence:

$$\sqrt{b\,x}\,c = \frac{(2k + 1)\pi}{2};\ k = O, 1, 2, \ldots \qquad (8)$$

that is, $\sqrt{b\,x}\,c$ is an odd multiple of $\frac{\pi}{2}$. And at $l = c$:

$$E_c = \pm\frac{2A}{b} \qquad (9)$$

Substituting in (5)

$$b = 2\pi N C$$
$$x = 2\pi N L$$

gives:

$$2\pi N\sqrt{C\,L}\,c = \frac{(2k + 1)\pi}{2}$$

$$N = \frac{2k + 1}{4c\sqrt{C\,L}} \text{ the frequency of the oscillating discharge, where}$$
$$k = O, 1, 2, \ldots \qquad (10)$$

That is, the oscillating discharge of a transmission line of distributed capacity does not occur at one definite frequency (as that of a condenser), but the line can discharge at any one of an infinite number of frequencies, which are the odd multiples of the fundamental discharge frequency:

$$N_1 = \frac{1}{4c\sqrt{C\,L}} \qquad (11)$$

Since

$C_0 = c\,C$ = total capacity of transmission line,
$L_0 = c\,L$ = total self inductance of transmission line, (12)

it is:

$$N = \frac{2k + 1}{4\sqrt{C_0 L_0}} \text{ the frequency of oscillation,} \qquad (13)$$

or natural period of the line, and:

$$N_1 = \frac{1}{4\sqrt{C_0 L_0}} \text{ the fundamental,} \qquad (14)$$

or lowest natural period of the line.

Substituting (10) gives:

$$b = 2\pi N C = \frac{(2k + 1)\pi}{2c}\sqrt{\frac{C_0}{L_0}} \qquad (15)$$

$$\sqrt{b\,x} = \frac{(2k + 1)\pi}{2c} \qquad (16)$$

thus, substituted in (5):

$$E = \frac{4c}{(2k + 1)\pi}\sqrt{\frac{L_0}{C_0}}\,A\sin\frac{(2k + 1)\pi}{2}\,\frac{l}{c}$$

$$I = \frac{4jc}{(2k + 1)\pi}\,A\cos\frac{(2k + 1)\pi}{2}\,\frac{l}{c} \qquad (17)$$

The oscillating discharge of a line can thus follow any of the forms given by substituting $k = 0, 1, 2, 3 \ldots$ into equation (17).

Reduced from symbolic representation to absolute values, by multiplying E with $\cos 2\pi N t$ and I with $\sin 2\pi N t$ and omitting j, and substituting N from equation (13), it is:

$$E = \frac{4c}{(2k + 1)\pi}\sqrt{\frac{L_0}{C_0}}\,A\sin\frac{(2k + 1)\pi}{2}\,\frac{l}{c}\cos\frac{(2k + 1)\pi}{2}\,\frac{t}{\sqrt{C_0 L_0}}$$

$$I = \frac{4c}{(2k + 1)\pi}\,A\cos\frac{(2k + 1)\pi}{2}\,\frac{l}{c}\sin\frac{(2k + 1)\pi}{2}\,\frac{t}{\sqrt{C_0 L_0}} \qquad (18)$$

where A = integration constant, depending upon the initial distribution of voltage, before the discharge, and t = time after discharge.

The fundamental discharge wave is thus, for: $k = O$:

$$E_1 = \frac{4c}{\pi}\sqrt{\frac{L_0}{C_0}}\,A\sin\frac{\pi l}{2c}\cos\frac{\pi t}{2\sqrt{C_0 L_0}}$$

$$I_1 = \frac{4c}{\pi}\,A\cos\frac{\pi l}{2c}\sin\frac{\pi t}{2\sqrt{C_0 L_0}} \qquad (19)$$

With this wave the current is a maximum at the beginning of the line: $l = O$, and gradually decreases to zero at the end of the line: $l = c$.

The voltage is zero at the beginning of the line, and rises to a maximum at the end of the line.

Thus the relative intensities of current and potential along the line are as represented by Fig. 1, where the current is shown in dotted lines, the potential in drawn lines.

The next higher discharge frequency, for: $k = l$, gives:

$$E_3 = \frac{4c}{3\pi}\sqrt{\frac{L_0}{C_0}}\,A\sin\frac{3\pi l}{2c}\cos\frac{3\pi t}{\sqrt{C_0 L_0}}$$

$$I_3 = \frac{4c}{3\pi}\,A\cos\frac{3\pi l}{2c}\sin\frac{3\pi t}{2\sqrt{C_0 L_0}} \qquad (20)$$

Here the current is again a maximum at the beginning of the line: $l = O$, and gradually decreases, but reaches zero at one-third of the line: $l = \frac{c}{3}$, then increases again, in opposite direction, reaches a second but opposite maximum at two-thirds of the line: $l = \frac{2c}{3}$, and decreases to zero at the end of the line. There is thus a nodal point of current at one third of the line.

The e. m. f. is zero at the beginning of the line: $l = O$, rises to a maximum at one third of the line: $l = \frac{c}{3}$, decreases to zero at two thirds of the line: $l = \frac{2c}{3}$, and rises again to a second but opposite maximum at the end of the line: $l = c$. The e. m. f. thus has a nodal point at two thirds of the line.

The discharge waves: $k = 1$, are shown in Fig. 2, those with $k = 2$, with two nodal points, in Fig. 3.

Thus k is the number of nodal points or zero points of current and of e. m. f. existing in the line.

In case of a lightning discharge the capacity C_o is the capacity of the line against ground, and thus has no direct relation to the capacity of the line conductor against its return. The same applies to the inductance L_o.

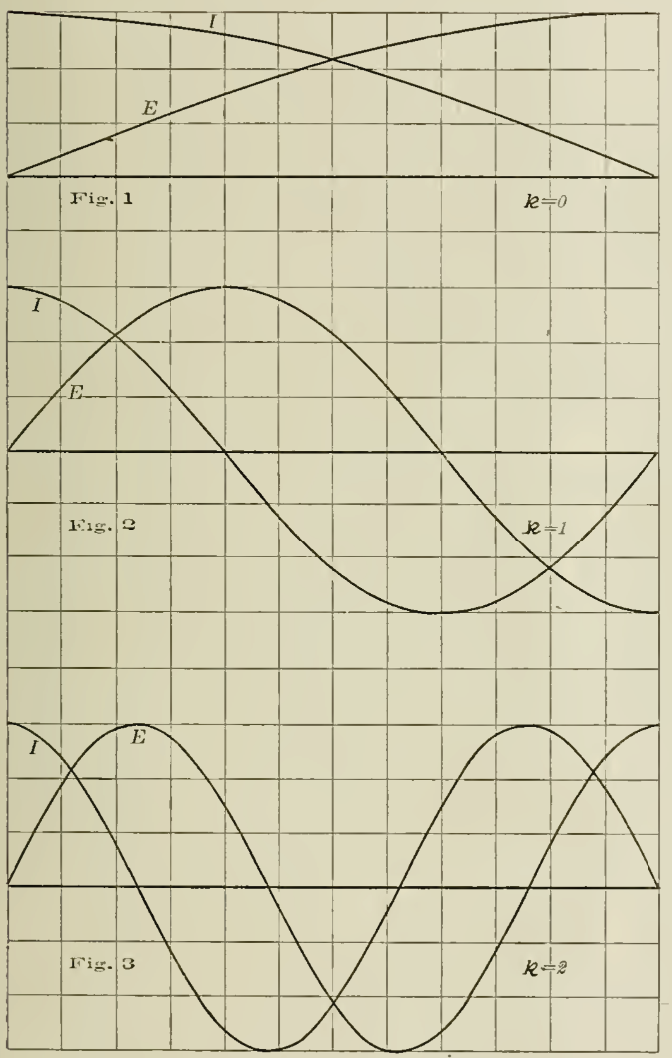
If $d =$ diameter of line conductor,
$D =$ distance of conductor above ground,
and $c =$ length of conductor,

the capacity is:

$$C_o = \frac{1.11 \times 10^{-6}\, c}{2\, lg \frac{4D}{d}}\ mf$$

the self-inductance:

$$L_o = 2 \times 10^{-6}\, c\, lg \frac{4D}{d}\ mh \qquad (21)$$



The fundamental frequency of oscillation is thus, by substituting (21) in (14):

$$N_1 = \frac{1}{4\sqrt{C_o L_o}} = \frac{7.5 \times 10^9}{c} \qquad (22)$$

That is, the frequency of oscillation of a line discharging to ground is independent of the size of line wire and its distance from the ground, and merely depends upon the length c of the line, being inverse proportional thereto.

We thus get the numerical values:

Length of line	10	20	30	40	50	60	80	100 miles
=	1.6	3.2	4.8	6.4	8	9.6	12.8	16×10^6 cm.
hence frequency: $N_1 =$	4680	2340	1560	1170	937 5	780	585	475 cycles.

As seen, these frequencies are comparatively low, and especially with very long lines almost approach alternator frequencies.

The higher harmonics of the oscillation are the odd multiples of these frequencies.

Obviously all these waves of different frequencies represented in equation (18) can occur simultaneously in the oscillating discharge of a transmission line, and in general the oscillating discharge of a transmission line as thus of the former:

(by substituting: $a_k = \frac{A_k}{2k+1}$)

$$E = \frac{4c}{\pi}\sqrt{\frac{L_o}{C_o}}\left\{a_1 \sin\frac{\pi l}{2c}\cos\frac{\pi t}{2\sqrt{C_o L_o}} + a_3 \sin\frac{3\pi l}{2c}\cos\frac{3\pi t}{2\sqrt{C_o L_o}} + \ldots\right\}$$

$$I = -\frac{4c}{\pi}\left\{a_1 \cos\frac{\pi l}{2c}\sin\frac{\pi t}{2\sqrt{C_o L_o}} + a_3 \cos\frac{3\pi l}{2c}\sin\frac{3\pi t}{2\sqrt{C_o L_o}} + \ldots\right\} \qquad (23)$$

where $a_1\ a_3\ a_5, \ldots$ are constants depending upon the initial distribution of potential in the transmission line, at the moment of discharge, or at $t = O$, and calculated therefrom.

As instance is calculated the discharge equation of a line charged to a uniform potential e over its entire length, and then discharging at $l = O$.

The harmonics shall be determined up to the 11th—that is, a_1, a_3, a_5, a_7, a_9, a_{11}.

Those six unknown quantities require six equations, which are given by assuming $E = e$ for $l = \frac{c}{6}, \frac{2c}{6}, \frac{3c}{6}, \frac{4c}{6}, \frac{5c}{6}, \frac{6c}{6}$.

At $t = O$, $E = e$, equation (22) assumes the form

$$e = \frac{4c}{\pi}\sqrt{\frac{L_o}{C_o}}\left\{a_1 \sin\frac{\pi l}{2c} + a_3 \sin\frac{3\pi l}{2c} + \ldots + a_{11} \sin\frac{11\pi l}{2c}\right\} \qquad (24)$$

Substituting herein for l the values: $\frac{c}{6}, \frac{2c}{6}, \ldots \frac{6c}{6}$

gives six equations for the determination of $a_1, a_3 \ldots\ldots a_{11}$. These equations solved give:

$$E = e\,(1.26 \sin\omega\cos\varphi + .40 \sin 3\omega\cos 3\varphi + .22 \sin 5\omega\cos 5\varphi + .12 \sin 7\omega\cos 7\varphi + .07 \sin 9\omega\cos 9\varphi + .02 \sin 11\omega\cos 11\varphi)$$

$$I = e\sqrt{\frac{C_o}{L_o}}\,(1.26 \cos\omega\sin\varphi + .40 \cos 3\omega\sin 3\varphi + .22 \cos 5\omega\sin 5\varphi + .12 \cos 7\omega\sin 7\varphi + .07 \cos 9\omega\sin 9\varphi + .02 \cos 11\omega\sin 11\varphi) \qquad (25)$$

where:

$$\omega = \frac{\pi l}{2c}, \qquad \varphi = \frac{\pi t}{2\sqrt{C_o L_o}} \qquad (26)$$

Instance:

Length of line: $c = 25$ miles $= 4 \times 10^6$ cm.

Size of wire: No. 000 B. & S. G., thus: $d = 1$ cm.

Height above ground: $D = 18$ feet $= 550$ cm.

Let $e = 25{,}000$ volts $=$ potential of line in the moment of discharge.

It is then:

$$E = 31.500 \sin\omega\cos\varphi + 10{,}000 \sin 3\omega\cos 3\varphi + 5500 \sin 5\omega\cos 5\varphi + 3000 \sin 7\omega\cos 7\varphi + 1750 \sin 9\omega\cos 9\varphi + 500 \sin 11\omega\cos 11\varphi.$$

$$I = 61.7 \cos\omega\sin\varphi + 19.6 \cos 3\omega\sin 3\varphi + 10.8 \cos 5\omega\sin 5\varphi + 5.9 \cos 7\omega\sin 7\varphi + 3.4 \cos 9\omega\sin 9\varphi + 1.0 \cos 11\omega\sin 11\varphi.$$

$$\omega = .39\, l\, 10^{-6}$$

$$\varphi = 1.18\, t\, 10^{+4}$$

A simple harmonic oscillation as a line discharge would require a sinoidal distribution of potential on the transmission line at the instant of discharge, which is not probable, so that probably all lightning discharges of transmission lines or oscillations produced by sudden changes of circuit conditions are complex waves of many harmonics, which in their relative magnitude depend upon the initial charge and its distribution—that is, in the case of the lightning discharge, upon the atmospheric electrostatic field of force.

The fundamental frequency of the oscillating discharge of a transmission line is relatively low, and of not much higher magnitude than frequencies in commercial use in alternating current circuits.