CHAPTER XXI.

REACTION MACHINES.

225. In the chapters on Alternating-Current Generators and on Induction Motors, the assumption has been made that the reactance x of the machine is a constant. While this is more or less approximately the case in many alternators, in others, especially in machines of large armature reaction, the reactance x is variable, and is different in the different positions of the armature coils in the magnetic circuit. This variation of the reactance causes phenomena which do not find their explanation by the theoretical calculations made under the assumption of constant reactance.

It is known that synchronous motors of large and variable reactance keep in synchronism, and are able to do a considerable amount of work, and even carry under circumstances full load, if the field-exciting circuit is broken, and thereby the counter E.M.F. E_1 reduced to zero, and sometimes even if the field circuit is reversed and the counter E.M.F. E_1 made negative.

Inversely, under certain conditions of load, the current and the E.M.F. of a generator do not disappear if the generator field is broken, or even reversed to a small negative value, in which latter case the current flows against the E.M.F. E_0 of the generator.

Furthermore, a shuttle armature without any winding will in an alternating magnetic field revolve when once brought up to synchronism, and do considerable work as a motor.

These phenomena are not due to remanent magnetism nor to the magnetizing effect of Foucault currents, because they exist also in machines with laminated fields, and exist if the alternator is brought up to synchronism by external means and the remanent magnetism of the field poles destroyed beforehand by application of an alternating current.

226. These phenomena cannot be explained under the assumption of a constant synchronous reactance; because in this case, at no-field excitation, the E.M.F. or counter E.M.F. of the machine is zero, and the only E.M.F. existing in the alternator is the E.M.F. of self-induction; that is, the E.M.F. induced by the alternating current upon itself. If, however, the synchronous reactance is constant, the counter E.M.F. of self-induction is in quadrature with the current and wattless; that is, can neither produce nor consume energy.

In the synchronous motor running without field excitation, always a large lag of the current behind the impressed E.M.F. exists; and an alternating generator will yield an E.M.F. without field excitation, only when closed by an external circuit of large negative reactance; that is, a circuit in which the current leads the E.M.F., as a condenser, or an over-excited synchronous motor, etc.

Self-excitation of the alternator by armature reaction can be explained by the fact that the counter E.M.F. of self-induction is not wattless or in quadrature with the current, but contains an energy component; that is, that the reactance is of the form $X = h - jx$, where x is the wattless component of reactance and h the energy component of reactance, and h is positive if the reactance consumes power, — in which case the counter E.M.F. of self-induction lags more than 90° behind the current, — while h is negative if the reactance produces power, — in which case the counter E.M.F. of self-induction lags less than 90° behind the current.

227. A case of this nature has been discussed already in the chapter on Hysteresis, from a different point of view.

There the effect of magnetic hysteresis was found to distort the current wave in such a way that the equivalent sine wave, that is, the sine wave of equal effective strength and equal power with the distorted wave, is in advance of the wave of magnetism by what is called the angle of hysteretic advance of phase α. Since the E.M.F. induced by the magnetism, or counter E.M.F. of self-induction, lags $90°$ behind the magnetism, it lags $90 + \alpha$ behind the current; that is, the self-induction in a circuit containing iron is not in quadrature with the current and thereby wattless, but lags more than $90°$ and thereby consumes power, so that the reactance has to be represented by $X = h - jx$, where h is what has been called the "effective hysteretic resistance."

A similar phenomenon takes place in alternators of variable reactance, or what is the same, variable magnetic reluctance.

228. Obviously, if the reactance or reluctance is variable, it will perform a complete cycle during the time the armature coil moves from one field pole to the next field pole, that is, during one-half wave of the main current. That is, in other words, the reluctance and reactance vary with twice the frequency of the alternating main current. Such a case is shown in Figs. 164 and 165. The impressed E.M.F., and thus at negligible resistance, the counter E.M.F., is represented by the sine wave E, thus the magnetism produced thereby is a sine wave Φ, $90°$ ahead of E. The reactance is represented by the sine wave x, varying with the double frequency of E, and shown in Fig. 164 to reach the maximum value during the rise of magnetism, in Fig. 165 during the decrease of magnetism. The current I required to produce the magnetism Φ is found from Φ and x in combination with the cycle of molecular magnetic friction of the material, and the power P is the product IE As seen in Fig. 164, the positive part of P is larger than the

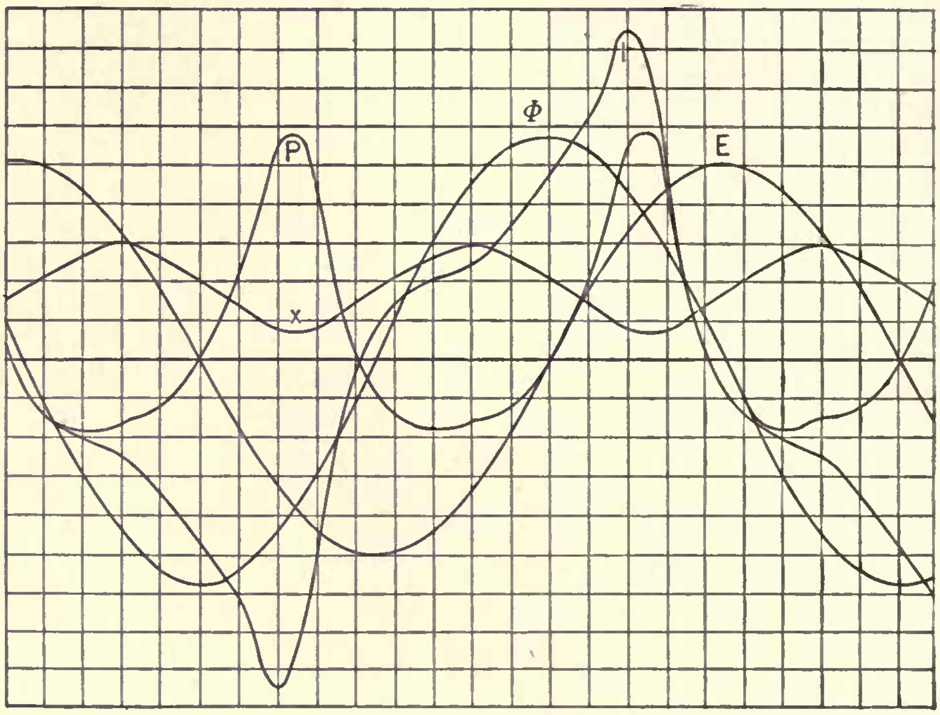

Fig. 164. Variable Reactance, Reaction Machine.

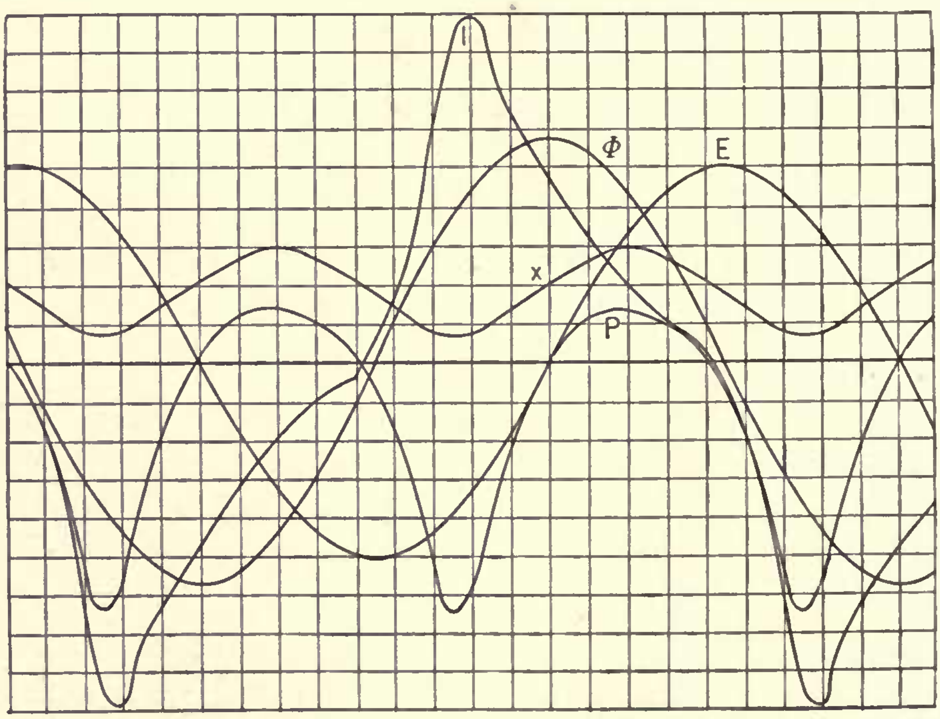

Fig. 165. Variable Reactance, Reaction Machine.

negative part; that is, the machine produces electrical energy as generator. In Fig. 165 the negative part of P is larger than the positive; that is, the machine consumes electrical energy and produces mechanical energy as synchronous motor. In Figs. 166 and 167 are given the two hysteretic cycles or looped curves Φ, I under the two conditions. They show that, due to the variation of reactance x, in the first case the hysteretic cycle has been overturned so as to represent not consumption, but production of electrical

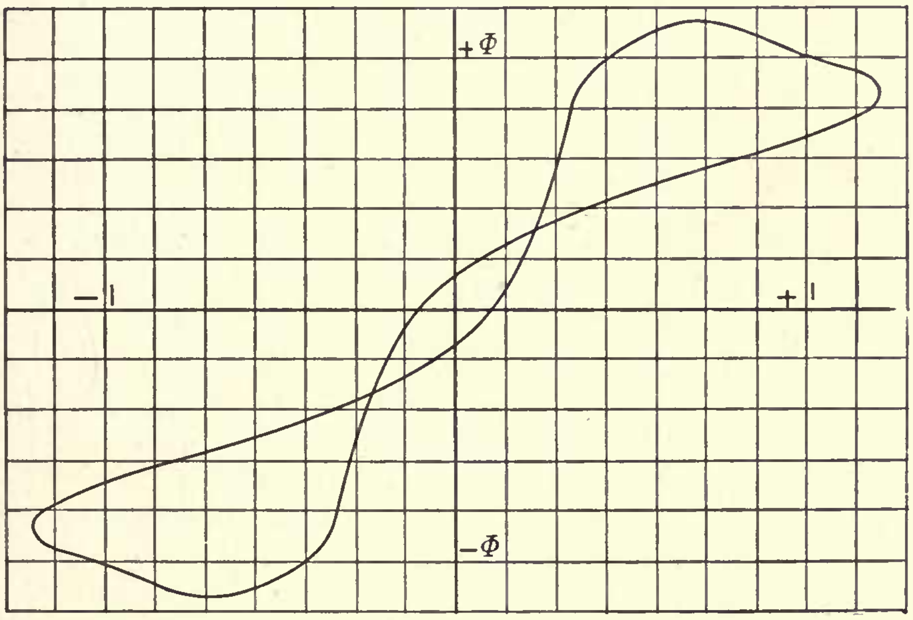

Fig. 166. Hysteretic Loop of Reaction Machine.

energy, while in the second case the hysteretic cycle has been widened, representing not only the electrical energy consumed by molecular magnetic friction, but also the mechanical output.

229. It is evident that the variation of reluctance must be symmetrical with regard to the field poles; that is, that the two extreme values of reluctance, maximum and minimum, will take place at the moment where the armature coil stands in front of the field pole, and at the moment where it stands midway between the field poles.

The effect of this periodic variation of reluctance is a distortion of the wave of E.M.F., or of the wave of current, or of both. Here again, as before, the distorted wave can be replaced by the equivalent sine wave, or sine wave of equal effective intensity and equal power.

The instantaneous value of magnetism produced by the

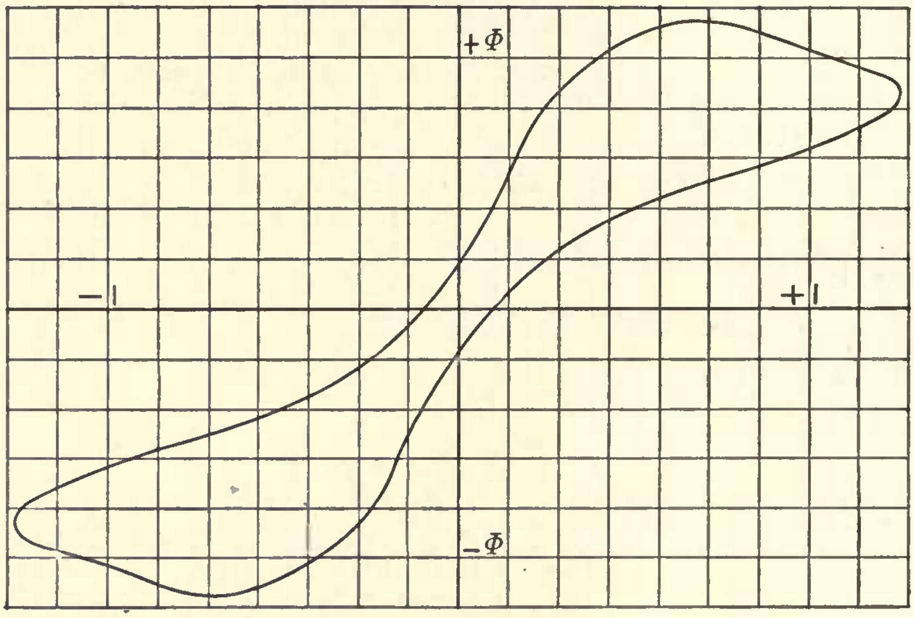

Fig. 167. Hysteretic Loop of Reaction Machine.

armature current — which magnetism induces in the armature conductor the E.M.F. of self-induction — is proportional to the instantaneous value of the current, divided by the instantaneous value of the reluctance. Since the extreme values of the reluctance coincide with the symmetrical positions of the armature with regard to the field poles, — that is, with zero and maximum value of the induced E.M.F., E_0, of the machine, — it follows that, if the current is in phase or in quadrature with the E.M.F. E_0, the reluctance wave is symmetrical to the current wave, and the wave of magnetism therefore symmetrical to the current wave also. Hence the equivalent sine wave of magnetism is of equal phase with the current wave; that is, the E.M.F. of self-induction lags 90° behind the current, or is wattless.

Thus at no-phase displacement, and at 90° phase displacement, a reaction machine can neither produce electrical power nor mechanical power.

230. If, however, the current wave differs in phase from the wave of E.M.F. by less than 90°, but more than zero degrees, it is unsymmetrical with regard to the reluctance wave, and the reluctance will be higher for rising current than for decreasing current, or it will be higher for decreasing than for rising current, according to the phase relation of current with regard to induced E.M.F., E_0.

In the first case, if the reluctance is higher for rising, lower for decreasing, current, the magnetism, which is proportional to current divided by reluctance, is higher for decreasing than for rising current; that is, its equivalent sine wave lags behind the sine wave of current, and the E.M.F. or self-induction will lag more than 90° behind the current; that is, it will consume electrical power, and thereby deliver mechanical power, and do work as synchronous motor.

In the second case, if the reluctance is lower for rising, and higher for decreasing, current, the magnetism is higher for rising than for decreasing current, or the equivalent sine wave of magnetism leads the sine wave of the current, and the counter E.M.F. at self-induction lags less than 90° behind the current; that is, yields electric power as generator, and thereby consumes mechanical power.

In the first case the reactance will be represented by $X = h - jx$, similar as in the case of hysteresis; while in the second case the reactance will be represented by $X = -h - jx$.

231. The influence of the periodical variation of reactance will obviously depend upon the nature of the variation, that is, upon the shape of the reactance curve. Since, however, no matter what shape the wave has, it can always be dissolved in a series of sine waves of double frequency, and its higher harmonics, in first approximation the assumption can be made that the reactance or the reluctance vary with double frequency of the main current; that is, are represented in the form,

$$x = a + b \cos 2\beta.$$

Let the inductance, or the coefficient of self-induction, be represented by —

$$L = l + \phi \cos 2\beta$$
$$= l(1 + \gamma \cos 2\beta)$$

where γ = amplitude of variation of inductance.

Let

$\hat{\omega}$ = angle of lag of zero value of current behind maximum value of inductance L.

It is then, assuming the current as sine wave, or replacing it by the equivalent sine wave of effective intensity I,
Current,

$$i = I\sqrt{2} \sin(\beta - \hat{\omega}).$$

The magnetism produced by this current is,

$$\Phi = \frac{Li}{n},$$

where n = number of turns.
Hence, substituted,

$$\Phi = \frac{lI\sqrt{2}}{n} \sin(\beta - \hat{\omega})(1 + \gamma \cos 2\beta),$$

or, expanded,

$$\Phi = \frac{lI\sqrt{2}}{n} \left\{ \left(1 - \frac{\gamma}{2}\right) \cos\hat{\omega} \sin\beta - \left(1 + \frac{\gamma}{2}\right) \sin\hat{\omega} \cos\beta \right\}$$

when neglecting the term of triple frequency, as wattless.

Thus the E.M.F. induced by this magnetism is,

$$e = -n \frac{d\Phi}{dt}$$

$$= -2\pi N n \frac{d\Phi}{d\beta}$$

hence, expanded —

$$e = -2\pi N l I \sqrt{2} \left\{ \left(1 - \frac{\gamma}{2}\right) \cos \hat{\omega} \cos \beta + \left(1 + \frac{\gamma}{2}\right) \sin \hat{\omega} \sin \beta \right\}$$

and the effective value of E.M.F.,

$$E = 2\pi N l I \sqrt{\left(1 - \frac{\gamma}{2}\right)^2 \cos^2 \hat{\omega} + \left(1 + \frac{\gamma}{2}\right)^2 \sin^2 \hat{\omega}}$$

$$= 2\pi N l I \sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}.$$

Hence, the apparent power, or the voltamperes —

$$P_0 = IE = 2\pi N l I^2 \sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}$$

$$= \frac{E^2}{2\pi N l \sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}}.$$

The instantaneous value of power is

$$p = e i$$

$$= -4\pi N l I^2 \sin(\beta - \omega) \left\{ \left(1 - \frac{\gamma}{2}\right) \cos \omega \cos \beta + \left(1 + \frac{\gamma}{2}\right) \sin \omega \sin \beta \right\}.$$

and, expanded —

$$p = -2\pi N l I^2 \left\{ \left(1 + \frac{\gamma}{2}\right) \sin 2\omega \sin^2 \beta - \left(1 - \frac{\gamma}{2}\right) \sin 2\omega \cos^2 \beta + \sin 2\beta \left(\cos 2\omega - \frac{\gamma}{2}\right) \right\}$$

Integrated, the effective value of power is

$$P = -\pi N l I^2 \gamma \sin 2\hat{\omega};$$

hence, negative, that is, the machine consumes electrical, and produces mechanical, power, as synchronous motor, if $\hat{\omega} > 0$; that is, with lagging current; positive, that is, the machine produces electrical, and consumes mechanical, power, as generator, if $\hat{\omega} > 0$; that is, with leading current.

The power factor is

$$f \doteq \frac{P}{P_0} = \frac{\gamma \sin 2\hat{\omega}}{2\sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}};$$

hence, a maximum, if,

$$\frac{df}{d\hat{\omega}} = 0;$$

or, expanded,

$$\cos 2\hat{\omega} = \frac{1}{\gamma} + \frac{\gamma}{4} \pm \frac{1}{4}\sqrt{8 + \gamma^2}.$$

The power, P, is a maximum at given current, I, if

$$\sin 2\hat{\omega} = 1;$$

that is,

$$\hat{\omega} = 45°$$

at given E.M.F., E, the power is

$$P = -\frac{E^2 \gamma \sin 2\hat{\omega}}{4\pi N l\left(1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}\right)};$$

hence, a maximum at

$$\frac{\delta P}{\delta \omega} = 0;$$

or, expanded,

$$\cos 2\hat{\omega} = \frac{\pm \gamma}{1 + \frac{\gamma^2}{4}}.$$

232. We have thus, at impressed E.M.F., E, and negligible resistance, if we denote the mean value of reactance,

$$x = 2\pi N l.$$

Current

$$I = \frac{E}{x\sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}}.$$

Voltamperes,

$$P_0 = \frac{E^2}{x\sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}}.$$

Power,

$$P = -\frac{E^2 \gamma \sin 2\hat{\omega}}{2x\left(1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}\right)}.$$

Power factor,

$$f = \cos(E, I) = \frac{\gamma \sin 2\hat{\omega}}{2\sqrt{1 + \frac{\gamma^2}{4} - \gamma \cos 2\hat{\omega}}}.$$

Maximum power at

$$\cos 2\hat{\omega} = \frac{\gamma}{1 + \frac{\gamma^2}{4}}.$$

Maximum power factor at

$$\cos 2\hat{\omega} = \frac{1}{\gamma} + \frac{\gamma}{4} \pm \frac{1}{4}\sqrt{8 + \gamma^2}$$

$\hat{\omega} > 0$: synchronous motor, with lagging current,
$\hat{\omega} < 0$: generator, with leading current.

As an instance is shown in Fig. 168, with angle $\hat{\omega}$ as abscissæ, the values of current, power, and power factor, for the constants, —

$$E = 110$$
$$x = 3$$
$$\gamma = .8$$

hence,

$$I = \frac{41}{\sqrt{1.45 - \cos 2\hat{\omega}}}$$

$$P = \frac{-2017 \sin 2\hat{\omega}}{1.45 - \cos 2\hat{\omega}}$$

$$f = \cos(E, I) = \frac{.447 \sin 2\hat{\omega}}{\sqrt{1.45 - \cos 2\hat{\omega}}}$$

As seen from Fig. 152, the power factor f of such a machine is very low — does not exceed 40 per cent in this instance.

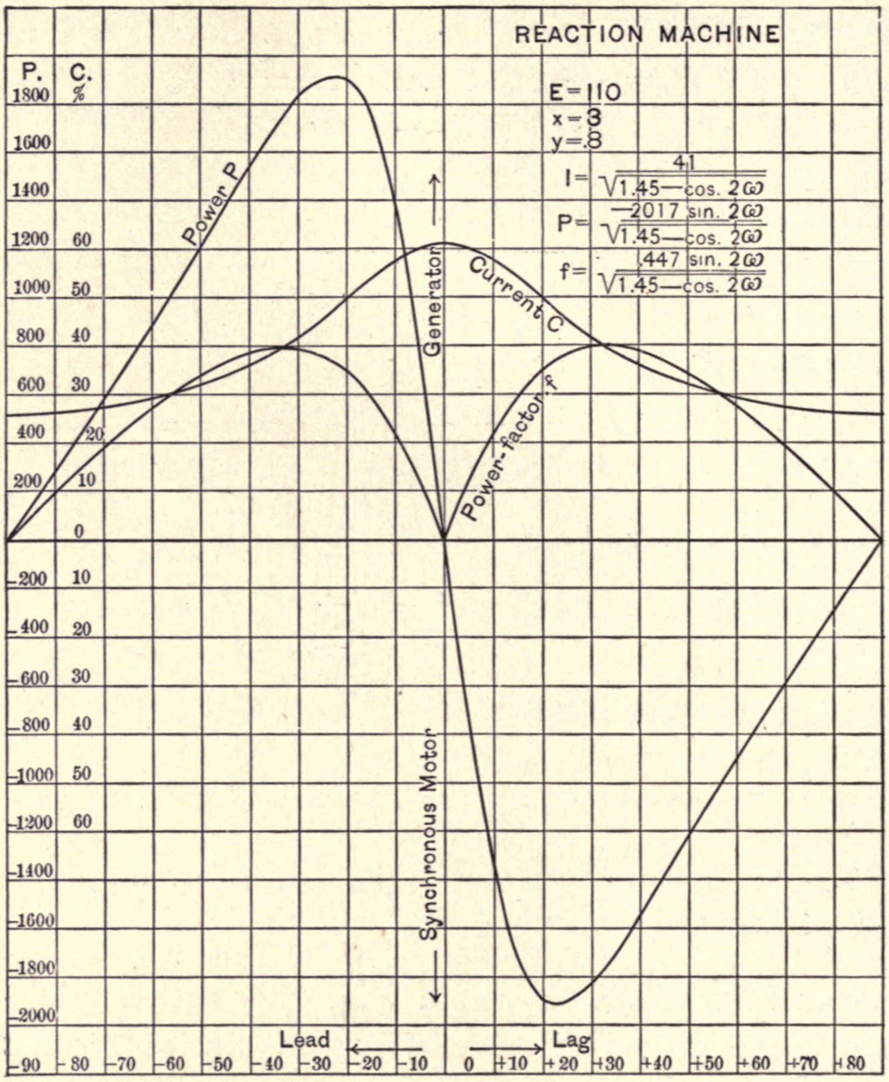

Fig. 168. Reaction Machine.