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A NON-SINGULAR ETHEREAL COSMOLOGY

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Abstract

An attempt is made to construct an ethereal cosmology based on two primary premises; that, with an ether, the universe may be finite and have a finite boundary with a true vacuum or void; and that the expansion in a vacuum of any finite quiescent mass of gas will lead asymptotically to a unique spherically symmetric accelerating outward flow. The ether is taken to be the only known medium which both has Maxwellian statistics and satisfies Planck's energy distribution, with sound-speed and electromagnetic wave speed identical.

It is first necessary to clarify what is meant by 'seeing', 'distance' as distinct from radius of curvature of observed incoming light-waves, curvilinear rays in an unsteady non-uniform flow, and red-shift as distinct from the trivial case of Doppler's principle in a uniform medium at rest.

The asymptotic flow so derived is hypersonic and hyperluminal away from the centre, and the conclusion is reached that the mass of matter in the universe is negligible compared with the mass of ether (i.e. 'dark matter') and that, with such high ethereal velocities over such long periods of time, ether drag ensures that all matter moves with the ethereal flow apart from relatively very small perturbations.

In this asymptotic flow light waves are found to be almost spherical, thus allowing observations to be made with axially symmetric instruments. Hubble's 'law' is derived between recession velocity and distance, but not between red-shift and radius of curvature of incoming light waves. The fact that every observer's rest frame is an accelerating frame of reference provides for a *physical* theory of gravitation which accords precisely with Newton's 'law'. The small perturbations from the ethereal flow of every observer's universal velocity account for the imprecision in measuring G. There seems not yet to be sufficient information to determine the scale of the universe and our position in it precisely but, with a physical meaning for G, only one relation is lacking, so that a full determination can be made for any assumed position of our observations.

With this model, the universe will inevitably go on expanding for ever and, when n times as old as it is now, the speed of light in our location will be $n^{-½}$ times its present value, the background ether temperature n^{-1} times, and Planck's constant, *h*, $n^{½}$ times its present value. There is nothing to suggest that the present expansion phase was not preceded by a convergence, or that matter was not present from the beginning.

> *"That gravity should be innate, inherent and essential to matter, so that one body may act upon another, at a distance, in a vacuum, without the mediation of anything else, by and through which their action may be conveyed one to another, is to me so great an absurdity that I believe no man, who has in philosophical matters a competent facility for thinking, can ever fall into it."*
>
> Isaac Newton

1. Introduction

1.1 The ether concept

It has been shown (Thornhill, 1985b, 1993) that the characteristic wave hypersurfaces and the wave hyperconoid for Maxwell's equations are exactly the same as those for the standard wave equation

$$\nabla^2 \phi = \left(1/\bar{c}^2\right) \partial^2 \phi / \partial t^2 \qquad (1.1.1)$$

in which $\bar{c}$ is a constant wave speed. It is also well-known that Maxwell's equations reduce precisely to the single equation (1.1.1) when there is no current or charge distribution.

The equation (1.1.1) is also found to be (Thornhill, 1993) the equation which governs the propagation of condensational oscillations or sound waves in any general fluid which is in a uniform state at rest. As such, equation (1.1.1), its characteristic wave hypersurfaces and its wave-hyperconoid are not invariant under transformation but unique to one reference-frame. They transform, for any other reference frame moving with a constant relative velocity, into a progressive form which is invariant. This progressive form differs only by a change from the operator $\partial/\partial t$, wherever it occurs, to D/Dt where

$$D/Dt \equiv \partial/\partial t + u_i \partial/\partial x_i \qquad (1.1.2)$$

is Euler's total time-derivative moving with the fluid and $\{-u_i\}$ $(i = 1, 2, 3)$ is the constant velocity of the new reference-frame relative to the old one.

In the same way Maxwell's equations transform under Galilean transformation into a progressive form which is invariant for all other reference-frames and which has the same characteristic equations as the progressive form of the wave equation. More generally, the equations governing the general unsteady flow of a general fluid, and the general equations for the electric and magnetic field-strengths (Thornhill, 1993) are invariant under Galilean transformation and have identical characteristic wave surfaces.

There are no alternative forms of 'physical mathematics' which would permit the same equation, its solution, and its characteristics equations to be treated mathematically or transformed in different ways, according as the equation happened to be regarded at any particular time as applying to different physical phenomena or to be purely a mathematical concept. Thus the equation (1.1.1) must be treated mathematically in the same way whether it is applied to sound waves in a general fluid at rest or to electromagnetic waves, or is regarded purely as a mathematical equation (cf. Thornhill, 1996).

It follows then that, if no error can be found in the comparatively simple mathematics used to derive the characteristics of Maxwell's equations, it is impossible to avoid the conclusion that Maxwell's equations are not general equations invariant under transformation but are unique; and that non-Newtonian relativity and the non-ether concept are therefore mathematically untenable. In such case Maxwell's equations must then refer to electromagnetic waves in a uniform ether at rest; the progressive form of Maxwell's equations must refer to electromagnetic waves in a uniform ether moving at a constant velocity relative to the reference frame; and the general equations for the electromagnetic field must refer to electromagnetic waves in an ether in general unsteady motion (Thornhill, 1993).

A return to the ether concept revives many problems in Newtonian mechanics such as, for instance, the elucidation of stellar aberration, or the refraction of light in a moving medium, by means of fluid dynamics and the theory of characteristics. The most far-reaching of such problems concerns cosmology. It is the purpose here to make a first attempt to construct a non-singular ethereal cosmology which is in accord with present observations and which predicts properties of the universe which may be tested against future observations.

1.2 **The ether**

The first requirement of an ethereal cosmology is an ether. In the course of history there have been many different ethers proposed (see, for example, Whittaker, 1953). One of the last of these, prior to the advent of the non-ether concept, suggested that the ether must behave like an elastic solid, since Maxwell's equations show that electromagnetic waves are transverse. Oscillations in the electromagnetic field-strengths, however, are not condensational oscillations of an ether, and so the suggestion could not be a valid one. Indeed, there are no observations which, taking into account the exceeding low density that the ether around us must have, suggest that the ether should behave other than as a gas (cf. Thornhill, 1985a).

Newtonian mechanics, classical kinetic theory and observation require that an ethereal gas in thermodynamic equilibrium should have Maxwellian statistics and also satisfy Planck's energy distribution for a black-body radiation field. This not merely narrows down the choice for an ether but renders it, at present, unique, since only one gaseous medium is known, at the present time, which satisfies both these requirements. This is an ideal gas (loc. cit.) which has an infinite variety of ether particles whose masses nm $(n = 1 \text{ to } \infty)$ are integral multiples of the mass m of a unit ether particle. All these particles have six degrees of freedom so that the constant first adiabatic index of this ether is $\gamma = 4/3$. The abundance of n particles is given by $\underline{N}_n \propto n^{-4}$. The relation derived between the energy of the particles and the frequency of radiation is not that given by Planck's quantum hypothesis or by Einstein's 'light-quantum' hypothesis but requires that the energy per unit mass, ε, of all the particles, whatever their masses, shall correlate with frequency v according to the relation

$$\varepsilon = hv/m \qquad (1.2.1)$$

where h is what is known as Planck's 'constant'.

Thus radiation of frequency v is not associated, as Planck or Einstein hypothesised, with 'quanta' or 'photons' having a particular energy $E = hv$, but with all ether particles, whatever their masses, which have a particular energy per unit mass ε, and thus have energies $E = nhv$ appropriate to their masses nm.

The black-body state may be taken to correspond to thermodynamic equilibrium and then the ether in our present local $2.7°K$ microwave background black-body radiation is found to have a density of about 0.2×10^{-30} $(kg)/m^3$. The mass m of a unit ether particle is determined as about 0.5×10^{-39} (kg). The product ch of the local wave-speed and the local value of Planck's 'constant' h is found to be a function of the entropy of the ether.

1.3 **Difficulties inherent in ethereal cosmology**

In a simple cosmology based on a uniform ether at rest, or based on the non-ether concept, the wave surfaces emanating from a point source are concentric spheres and the rays are their orthogonal trajectories, the radii of these spheres. In an ethereal cosmology, however, based on a non-uniform, unsteady expanding ethereal flow, the situation is much more complicated. In this case the wave surfaces from a point source are not spherical and the rays are neither their orthogonal trajectories, nor are they straight lines but curvilinear. It is essential, in these circumstances, to re-examine precisely what is meant by such simple terms as 'ray', 'observe', 'distance', etc.

Rays may be defined mathematically. Thus, in two space variables (x_1, x_2) and time t, the characteristic wave surfaces through any point at any time in general envelop a wave conoid; and its curvilinear generators, i.e. the curves along which the wave surfaces through the point touch the conoid, are called bi-characteristic curves (see, for example, Thornhill, 1952). The projections of the sections of this conoid, by planes $t = const$, on to the (x_1, x_2) plane, are the wave fronts from the point source, and the projections of the bi-characteristic curves are the rays.

When axially symmetric telescopes or dishes of any kind are used to make astronomical observations, they do not measure any actual distance at any time between themselves and the source of the wave fronts which reach them. All they can determine is the radius of spherical or near-spherical wave fronts reaching them, and the direction of the normals to these wave fronts. In a simple stationary or non-ether cosmology the observed radius $\Re$ of the spherical wave fronts is usually referred to as 'distance' d, and the time $d/\bar{c}$ as the time from emission at a point source to observation of the wave front, $\bar{c}$ being the local wave speed of light at the point of observation, assumed to be a universal constant.

In ethereal cosmology, in contrast, it is necessary to distinguish between at least six different distances, only two of which could be derived directly from observation of a point source by means of the non-spherical wave fronts reaching an observer, viz.

- (i), (ii) the two principal radii of curvature of the non-spherical wave fronts.
- (iii) the arc length along the curvilinear ray from emitter to observer.
- (iv) the true distance between emitter and observer at the time t_e of emission.
- (v) the true distance between emitter and observer at the time of arrival t_a of the wave front at the observer.
- (vi) the true distance between the emitter at time t_e and the observer at time t_a.

(Note that suffix e will always be used to denote quantities associated with the *e*mitter of a point disturbance; and suffix a will always refer to quantities associated with the *a*rrival of an ensuing wave front at an observer).

It is also possible to observe, by spectrometry for instance, the frequency of arrival from a point source of a recognisable regular series of wave fronts, and thus to measure a spectral shift between the frequencies or wavelengths of emission and reception of a regular train of wave-fronts.

Thus, in a simple cosmology with an assumed universally constant wave speed, the red-shift can be defined uniquely as

$$z = \lambda_a / \lambda_e - 1 = \nu_e / \nu_a - 1 \qquad (1.3.1)$$

In an expanding ethereal cosmology, however, there are two possible definitions of red-shift, namely

$$z_\lambda = \lambda_a/\lambda_e - 1 = (c_a/c_e)\ \nu_e/\nu_a - 1 \qquad (1.3.2)$$

and

$$z_\nu = \nu_e/\nu_a - 1 = (c_e/c_a)\ \lambda_a/\lambda_e - 1 \qquad (1.3.3)$$

These two red-shifts are not, of course, independent since

$$(z_\nu + 1)\ c_a = (z_\lambda + 1)\ c_e \qquad (1.3.4)$$

In practice it is the frequency red-shift z_ν which is important, since it is the emission frequency which is recognised as the universal signature of a particular atom.

The red-shift in an expanding ethereal cosmology must be determined by a full characteristic theory which examines how the interval between the wave fronts from two successive wave peaks or point emissions varies as the light travels from emitter to observer along a curvilinear ray. This is in marked contrast to the trivially simple case, known as the Doppler principle, which is used in simple cosmologies. The Doppler shift was derived originally for wave sources moving relative to a uniform stationary medium, in which case it only applies at distances from a moving wave source which are large compared with the thickness of the viscous boundary layer surrounding it.

1.4 **Observational restrictions**

Some well-known observations, which are easily incorporated into simple cosmologies, place great restrictions on, and provide stringent tests for, ethereal cosmologies. How can it be possible, for instance, with non-spherical wave-fronts in an expanding ethereal flow, for us to 'see' out as far as red-shifts of order 4 to 5 with axially symmetric telescopes or dishes? The simplest explanation would be that, in our locality at the present time, distances out to red-shifts 4 or 5 are very small compared with the present dimensions of the universe, and the age of the present expansion phase so large that, up to such distances, the difference between the two principal radii of curvature of wave-fronts arriving from point sources is negligibly small.

Observations of the red-shifts of distant objects and estimates of their distances led initially to what is now known as Hubble's 'law'. This is an empirical relation suggested by Edwin Hubble around 1929. It was based on the earliest measurements of very small red-shifts and estimates of the relevant 'distances' deduced from the luminosity of Cepheid variables. Hubble conjectured that recession speed was proportional to distance on the basis of an apparent linear relation between observed red-shift z and radius $\Re$ of incoming wave-fronts. In a simple non-ether cosmology with a universally constant wave-speed, $\bar{c}$, $\bar{c}\, z$ is interpreted as recession speed V and radius $\Re$ is interpreted as distance d. The quantity $\bar{c}\, z/\Re$, interpreted as V/d, has dimensions $(\text{time})^{-1}$ and is now known as the Hubble 'constant' H; its reciprocal $t_H = 1/H$ is usually called the Hubble time. It is usually assumed that H is either a universal constant or varies only with time, and is therefore the same for all observers. It is used as a measure of expansion rate on which to base cosmologies in the non-ether concept. Unfortunately, even some 70 years after the conception of Hubble's 'law', when so many more elaborate measurements of red-shifts have been made and 'distances' to objects with larger red-shifts have been estimated by alternative methods, it has not proved possible so far to determine a 'constant' H to within a factor of about two.

It is not possible to use an empirical relation such as Hubble's 'law' as a foundation for an ethereal cosmology, as is done in the construction of some cosmologies. Rather, one of the principal aims of constructing an ethereal cosmology must be to determine the relation between red-shift and radius of incoming wave-fronts and between recession speed and distance. An ethereal cosmology must stand or fall, therefore, according as the relations it yields are in agreement or at odds with present and future observations. An ethereal cosmology must also explain, for instance, why it is not possible at present to determine a precise contemporary value of H to within a factor of about two.

1.5 The Universal ethereal flow

In an ethereal cosmology the characteristic wave-hypersurfaces and wave-hyperconoids through any point at any time must be derived both from the equations of motion of the ether and the general equations for the electric and magnetic field-strengths. Whilst it is true (Thornhill, 1993) that these characteristics can be determined separately, without assuming that the two wave-speeds (thermodynamic and electromagnetic) are equal, reconciliation between the energy distribution in the ether in thermodynamic equilibrium and Planck's energy distribution in a black-body radiation field can only be effected (Thornhill, 1985a) if the two wave-speeds are equal. Moreover, only one family of wave-hypersurfaces and one unique wave-hyperconoid are observed. In what follows, therefore, the thermodynamic and electromagnetic wave-speeds of the ether will be regarded as identical.

For the simple static ethereal cosmology with Maxwell's equations, the differential equation for the wave-hypercone through any point at any time is then derived as

$$(dx_1/dt)^2 + (dx_2/dt)^2 + (dx_3/dt)^2 = \bar{c}^{\,2} \tag{1.5.1}$$

where $\bar{c}$ is the constant thermodynamic and electromagnetic wave-speed. Equation (1.5.1) is a differential equation which can be integrated. This integration is not trivially simple but it can be shown, without significant difficulty, that the general integral of (1.5.1) corresponding to an infinitesimal disturbance at any time t_0 at the point $\{x_{io}\}$ is a right spherical hypercone (or, in three space-dimensions only, a family of concentric spheres), namely

$$(x_1 - x_{10})^2 + (x_2 - x_{20})^2 + (x_3 - x_{30})^2 = \bar{c}^{\,2}(t - t_0)^2 \tag{1.5.2}$$

The equation (1.5.1) and its general integral (1.5.2) are also taken over into the non-ether concept. Thus, in two space-dimensions and time, the solution (1.5.2) reduces to a right circular cone (or, in two space-dimensions, a family of concentric circles), namely

$$(x_1 - x_{10})^2 + (x_2 - x_{20})^2 = \bar{c}^{\,2}(t - t_0)^2$$

and this is often used to 'illustrate' special relativity. The forward part of this cone $(t>t_0)$, the 'region of influence' of characteristic theory, is usually labelled 'the future'; and the backward part $(t<t_0)$, the 'domain of dependence' of characteristic theory, is labelled 'the past'.

In the non-ether concept, however, the differential *equation* (1.5.1) is misinterpreted as the quadratic differential *form* of an imaginary Riemannian geometry in four dimensions, namely x_i and $i\bar{c}\,t$, which is usually referred to 'space-time'. The term 'space-time', in fact, is used indiscriminately both to refer to the real four-dimensional metric $(x_i,\ \bar{c}\,t)$ in which the right spherical hypercone (1.5.2) is located, and to the imaginary four-dimensional Riemannian metric $(x_i,\ i\bar{c}\,t)$ associated with the Lorentz transform and special relativity.

This misinterpretation of the differential equation for the wave hyperconoid, as a quadratic differential form, cannot play any part in the ether concept and so the construction of an ethereal cosmology, with a non-steady non-uniform expanding ether, must necessarily depend entirely on the integration of the general equation for the wave hyperconoid in a general flow, namely

$$(dx_1/dt - u_1)^2 + (dx_2/dt - u_2)^2 + (dx_3/dt - u_3)^2 = c^2 \tag{1.5.3}$$

where $\{u_i\}$ and c are, respectively, the local contemporary values of the ethereal velocity and the wave-speed.

For the ethereal cosmology constructed here, symmetry properties of the flow solution are used to effect considerable reduction and simplification of the general equation (1.5.3) before its general integral is obtained.

Before, however, an equation for the wave-hyperconoid can be obtained for integration it is necessary to specify a model for the universal ethereal flow. There is no reason to suppose *a priori* that an ethereal universe is infinite, as it may often be convenient to do in a non-ether cosmology. The existence of an ether permits there to be a finite boundary to the universe outside which there is a true vacuum or void containing neither ether nor matter. In such case, the outward flow of the expanding ethereal universe must be such that the pressure and density of the ether tend to zero at this boundary. This leads to the idea of the expansion of a finite mass of ether, starting from some initial condition, into a true vacuum or void.

In order to handle and evaluate an ethereal cosmology, it is desirable also to have a complete mathematical solution for the flow, in closed form, which will allow the determination of non-spherical wave-fronts and non-linear rays arising from a point source of disturbance. This raises the question of whether such complete mathematical solutions exist for flows in three-dimensional space and time.

1.6 **Similarity solutions**

Solutions of the type required for the construction of an ethereal cosmology, as now proposed, do exist. They are usually called special, self-similar or similarity solutions, and are, generally, solutions of unsteady flow problems in one space-variable and time, with plane, axial or spherical symmetry, the latter meeting the requirements of a universe in three space-dimensions and time.

The type of similarity solution, which could be applied to an expanding ethereal universe with spherical symmetry, is one in which the variables can be separated so as to give ordinary differential equations in time and in a similarity variable $\eta = r/R$, where r is the radius vector and $R(t)$ the value of r at the outer boundary of the flow. This separation of variables is generally achieved, in Eulerian co-ordinates, by attempting to obtain a solution in which each of the dependent variables is the product of a function of the similarity variable η and a function of time; e.g. for the radial velocity $u = \phi(\eta)\dot{R}$ where $\dot{R}$ is the speed of the outer boundary R. Thus, the separate differential equations obtained in the similarity variable η give the constant 'shape' of the solution and the differential equation in time gives the temporal development of the solution.

Three solutions of this type, which have proved most useful in practical application, may be mentioned briefly. First, the similarity solution obtained in 1941 for a very intense explosion (Taylor, 1950) was used to predict the form of the initial blast produced by the first atomic explosions. This solution uses only the leading terms in the conditions behind a very strong Rankine-Hugoniot shock-wave in air when these are expanded as series in $1/M$, M being the Mach number of the shock with respect to the uniform atmosphere ahead of it. Thus, this solution ceases to be valid as the shock-wave expands and weakens. The blast then ultimately decays more slowly in a way similar to that in a conventional explosion, with a leading shock of moderate strength or very weak.

Second, the similarity solution to the complete problem of internal ballistics (Thornhill, 1966) takes account of the burning phase of the propellant, as distinct from Lagrange's ballistic problem which starts only at the time when the propellant is all burnt. This solution does not cease to be accurate, but is valid for all time, and enables systems of internal ballistics to be scaled and modelled by means of three scale-factors and seven modelling parameters.

Third, and here the most pertinent, is the similarity solution for the expansion into a vacuum of a finite homentropic mass of gas, originally at rest and confined within a spherical boundary. The existence of such a solution was first demonstrated by Keller (1956) who formulated the problem of spherical, cylindrical and plane-symmetric flow of an ideal polytropic gas in Lagrangian co-ordinates. He found a class of special or similarity solutions of his equations by separation of variables and these depend on an arbitrary function related to arbitrary entropy distribution in the gas. In particular, he showed that such solutions, with variable entropy, exist for the expansion of a finite mass of gas into a vacuum, and even predicted that such solutions for the expansion of a sphere of gas may become of interest to astrophysicists.

Such similarity solutions are more easily discovered in Lagrangian co-ordinates but, although more difficult to derive, are much easier to exploit and apply to practical problems in Eulerian co-ordinates. The complete solution for the spherical expansion into a vacuum of a finite homentropic mass of an ideal polytropic gas was given, in Eulerian co-ordinates, by Thornhill (1958). The solution was of interest, at that time, not in connection with astrophysics or cosmology, but in order to estimate the limiting case of blast from a conventional explosion as the altitude increases indefinitely and the atmospheric pressure and density effectively tend to zero. This was necessary because, under sufficiently rarefied atmospheric conditions, the continuum solution for the blast from a conventional explosion, headed by a Rankine-Hugoniot shock-wave, breaks down.

A general account of similarity and dimensional methods is given by Sedov (1959). He discusses, among other things, a particular class of similarity solutions in which the velocity distribution is linear, i.e. $\phi(\eta) = \eta$ and $u = \eta\dot{R}$. Taylor's solution for the very intense explosion is not of this type, but the internal-ballistic solution is. Indeed, the assumption of a linear velocity distribution, in the flow of propellant gases along a gun barrel behind the shot, was first made by Benjamin Robins (1742) although it is now usually known as Lagrange's assumption. This assumption of a linear velocity distribution in the flow along a gun barrel has been used ever since, with great success, in all numerical systems of internal ballistics. It was the success of this assumption which first suggested the possibility of obtaining a similarity solution to the complete problem of internal ballistics more sophisticated than the primitive similarity solutions of Robins and Lagrange to the restricted case of Lagrange's ballistic problem.

Likewise, the similarity solution for the homentropic expansion into a vacuum is also of the type in which $\phi(\eta) = \eta$ and the velocity distribution is linear. It was this property of the solution, which clearly foreshadows Hubble's 'law', that first suggested the possibility of using this solution as the basis on which to construct an ethereal cosmology.

1.7 The ethereal expansion into a vacuum

Some further remarks are appropriate concerning the choice of this solution and the justification for using it as a basis for ethereal cosmology. First, it is the only available solution in closed form that satisfies the requirements for an ethereal cosmology, and it applies to an ideal polytropic gas, the only choice available, at present, for an ether.

There is no *a priori* reason to assume that the entropy distribution in the universal ether is other than uniform. Within the limits of our observations at the present time there is no evidence for any significant variation in the ethereal entropy. It is true, however, that superluminal motion of the tips of some quasar jets has been regularly observed for the last twenty or thirty years and, more recently, superluminal motion through the ether has been detected within the galaxy. This implies entropy changes across the ethereal bow shock waves which must accompany such superluminal motions, but this is not on a scale which indicates any significant alteration to the universal entropy distribution as a whole. Nor are there any observations of reaction or phase-change waves in the ether which would imply any significant changes in the ethereal entropy across the other three types of (non-Rankine-Hugoniot) shock waves which initiate such waves (cf. Thornhill, 1973)

Attention must be drawn, however, to two aspects of the complete similarity solution for a homentropic expansion into a vacuum which merit more detailed consideration.

First, as the outer boundary of the flow is approached, the ethereal pressure, density and wave-speed all tend to zero, whilst the speed of the outer boundary increases monotonically to a constant finite limit. One consequence of this is that the continuum solution must cease to be valid as the outer boundary is approached and the density of the ether tends to zero. Outside the region of validity of the continuum solution, therefore, it is necessary to fit a rarefied gasdynamic solution in order to determine the flow nearer the outer boundary. The question arises then as to whether our present observations are made in a region where the continuum solution could be valid, or outside this region, where they could only be analysed in terms of rarefied gasdynamics. Fortunately, it is found that we are within the region of validity of the continuum solution. With a local contemporary ethereal temperature of about 2.7°K (the micro-wave background black-body temperature), Boltzmann's universal constant *k*, and local contemporary values for the wave-speed *c* and Planck's 'constant' *h*, it is determined (Thornhill, 1985a) that there are, on average at any time, in our vicinity, about 360 ether particles per cm^3 in the ether background in thermodynamic equilibrium, and these, on average, have a mass about 400 times that of the unit ether particle. This result may be compared with the result given by Zeldovich and Novikov (1975), namely *"using as a basis the observational data and a minor theoretical extrapolation, the number density of the primordial (relic) photons has been calculated to be about 400 per* cm^3*, which is* 10^8 *-* 10^{10} *times greater than the average number density of baryons in the universe."*

Another consequence of the conditions at the outer boundary of the complete similarity solution is that the Mach number of the ethereal flow tends to infinity as the outer boundary is approached. There is always, then, a region of flow, in the outer part of the solution, which, on the assumption made here, is both supersonic and superluminal. In the non-ether concept, of course, relative speeds greater in magnitude than the wave-speed are not permissible, but this restriction has no place in Newtonian mechanics. It will become clear that these extremely high flow speeds relative to the centre of the universe, and their persistence in the outer reaches of the flow for most of the duration of the expansion, are essential in order to ensure a motion of matter suspended in the universal ether which is consistent with our observations of the red-shifts of material sources of signals.

Second, the similarity solution demands conditions, in the initial stationary mass of ether, which conform precisely to the constant 'shape' of the solution, i.e. spherical symmetry and a predetermined radial distribution of pressure, density, etc. At first sight this appears to be an impossible restriction to place upon any proposed cosmology and it is precisely for this reason that, so far as practical applications are concerned, such similarity solutions are not generally held in very high regard. In practice, however, this restriction is not so serious.

All complete similarity solutions of this type suffer from this restriction on the initial conditions. Observations, however, of the blast from an atomic explosion (Taylor, 1950) showed that, in spite of not satisfying the initial conditions of the solution, the blast rapidly approached the similarity solution and continued to conform to it until the leading shock became too weak for the similarity solution to remain valid. Likewise, although the initial conditions in a gun barrel during the burning of the propellant and immediately afterwards are remotely different from the requirements of the similarity solution, observation shows that the linear-velocity distribution in the propellant gases is rapidly achieved and measurements then show good agreement with a similarity solution suitably scaled and modelled.

These observations tend then to support the conclusion that similarity solutions of this type are extremely stable and are rapidly or ultimately approached in practice even when the initial conditions are much different from those required by the solution.

For the complete similarity solution for homentropic expansion into a vacuum, however, the situation is not so straightforward. In the early 1960s the advent of space flight aroused considerable interest in the expansion of gas clouds into a vacuum, particularly that of an initially uniform gas cloud (Molmud, 1960; Greifinger and Cole, 1960; Greenspan and Butler, 1962; Mirels and Mullen, 1963; Wedemeyer, 1965; Hubbard, 1966). No analytical solution is available in the case of an initially uniform cylinder or sphere so, among other things, attention was given to numerical solutions in these cases and to empirical approximations to the results they suggested for the asymptotic self-similar form of the flow. The asymptotic forms derived from the numerical solutions were compared with the complete similarity solutions. It was realised (cf. Greenspan and Butler, 1962) that, when the initial cylinder or sphere has finite pressure, etc. at its outer boundary, the ensuing outer boundary of the flow has constant speed $\dot{R} = 2a_2/(\gamma-1)$, where a_2 is the initial sound speed at the outer boundary. In the complete similarity solution the initial cylinder or sphere has zero pressure at its outer boundary; the outer boundary of the flow has zero speed initially and accelerates monotonically to the constant asymptotic speed $\dot{R} = 2a_1/(\gamma-1)\ (\sigma+1)^{\frac{1}{2}}$, where a_1 is the initial sound speed at the centre of the cylinder $(\sigma = 1)$ or sphere $(\sigma = 2)$. This fact, together with attempts to compare the 'asymptotic' forms derived from Greifinger and Cole's numerical solutions with the complete similarity solutions, led to the unanimous conclusion that the expansion of an initially uniform cylinder or sphere of gas into a vacuum does not ultimately approach the form of the complete similarity solution.

If this conclusion were correct, the complete similarity solution could not be regarded as the ultimate form of all such flows, whatever the initial distribution of pressure, etc. It would not then be suitable as a 'universal' solution for the purpose of ethereal cosmology. It would also imply that, in any ethereal cosmology, it would be possible theoretically for any observer, in any location at any time, to make local observations which would enable him to derive information about the initial conditions in the universe at the beginning of the present expansion phase. This would clearly not be possible if all expansions tended asymptotically to the same 'universal' form.

These considerations are of such paramount importance to ethereal cosmology that they must be examined in more detail. In comparing the asymptotic forms derived from numerical solutions with the complete similarity solution, it is important to realise several important requirements.

i) Although the two cases under consideration have different outer boundaries to the flow, it is possible that the mass of gas between the two boundaries tends asymptotically to zero.

ii) Since neither of the flows has any overall momentum the comparison must be made strictly for equal masses, energies and entropy.

iii) For the same distance r from the centres of symmetry of the two flows the values of $\eta_1 = r/R_1$, and $\eta_2 = r/R_2$ are different.

iv) It must not be assumed that the rate of any convergence to the complete similarity solution would be the same for all values of the first adiabatic index γ or that the rate of accumulation of errors, in what is a hyperbolic or marching problem, would be the same in all cases, or for plane, axial or spherical symmetry.

A new comparison of the numerical solutions, so far as they have been taken, with the complete similarity solution, is made here in detail in an appendix, taking into account the above requirements. It is found that the agreement, both in the 'shape' and the time development of the two solutions is remarkably good for small values of $(\gamma - 1)$, but deteriorates as $(\gamma - 1)$ increases or as the number of dimensions increases. There is, however, at present, no evidence to show that, if the computations were taken further, with adequate control of the accumulation of errors, the agreement could not be improved for the higher values of $(\gamma - 1)$ and the number of dimensions. The conclusion may then be reached that all expansions of a gas cloud into a vacuum tend asymptotically to the corresponding complete similarity solution.

It is still possible, therefore, to regard the complete similarity solution as being extremely stable, like the other complete similarity solutions, and to use it as a 'universal' solution for the asymptotic form of an ethereal cosmology. This would imply, as one would expect, that it is impossible to derive information about the initial state of the universe from local observations at large specific times after the beginning of the expansion; and that knowledge of the initial state of the universe can only be obtained by direct observation.

1.8 **Matter**

So far no mention has been made of the material content of the universe which (without prejudice to the possibility that ether is a form of matter) may be discussed as distinct from the ether. Matter must be suspended in the expanding ethereal flow and it is necessary, therefore, to determine how it moves.

The Michelson-Morley experiment is usually interpreted in terms of the non-ether concept and this leads to the Lorentz transform and relativity which, in the present context, are considered to be mathematically untenable. In terms of the ether concept and Newtonian mechanics the results of the Michelson-Morley experiment mean, quite simply, that the ether is moving locally with the apparatus and this implies no more than that the ether like any other gas, has viscosity. When a body, like the Earth, moves relative to a surrounding fluid that has viscosity a viscous boundary layer is formed around its surface across which the relative velocity between the body surface and the fluid tends to zero as the surface is approached. Thus, experiments near the surface of the Earth will give null results or will, at best, over greater ranges which are still small compared with the boundary layer thickness, determine a relative velocity much less than the true velocity of the Earth relative to the mainstream flow outside its boundary layer. It follows that a material body moving relative to the surrounding ether will, therefore, experience viscous ether drag.

So far as our observations are concerned ether drag is exceedingly small. Terrestrial observations are, however, confined to time durations and relative velocities which are very small on a universal scale. The situation is completely different if the ethereal flow has superluminal or hyperluminal velocities relative to the centre of the universe and the time duration extends over the present expansion phase of the universe. Continual ether drag, exerting very large forces when the relative velocity between the ether and matter is very large, and acting over very long periods of time, will accelerate matter suspended in the ether until the relative velocity between matter and ether becomes very small compared with the velocity of the ether relative to the centre of the universe.

Macroscopically, then, matter may be regarded, to good approximation, as ultimately moving with the ether and this will be taken to be so in the construction of an ethereal cosmology.

This necessitates the assumption that the total mass of matter in the universe is very small compared with the mass of the ether, so that the acceleration of matter and the maintenance of its motion in the ether do not significantly affect the overall motion of the universal ether. Observations of the density of matter in our vicinity at the present time, as compared with the density of the surrounding ether, seem, at first, to belie such an assumption. It must be remembered, however, (cf. para 1.4 above) that our observations out to red-shifts of 4 or 5 may only extend over very small distances compared with the size of the universe. They may be, therefore, on the cosmology considered here, typical at most of a narrow spherical annulus between two nearly equal universal radii. Moreover, the present density of the local ethereal flow around us (the micro-wave background radiation field), namely about $0.2 \times 10^{-30} (kg)/m^3$ (cf. para 1.2 above) suggests that we may be located near the outer boundary of the universe. There is no *a priori* reason, therefore, to think that the density of matter around us is typical of the whole universe rather than, say, that most of the matter in the universe is in the outer reaches; and the density of the universal ether, as given by the similarity solution, has its maximum value at the centre, at any time, and declines monotonically to zero at the outer boundary. It is, therefore, quite feasible for the total mass of the universal ether to be much greater than the total mass of matter.

It is necessary also to consider the origin of the matter in the universe. Contemporary theories, based on relativity, rely on very extreme conditions at an initial mathematical singularity to explain the beginning of time or the present expansion phase and to deduce that these conditions led to the evolution of matter. In the cosmology constructed here, however, attempts to determine completely the scale of the universe suggest that the conditions at the beginning of the present expansion phase were not nearly so extreme and leave open the possibility that matter was present from the beginning.

The effect of expansion into a vacuum is ultimately to convert all the energy into kinetic energy and, as a result of ethereal viscosity, the end result is that matter and ether have the same kinetic energy per unit mass. Moreover, if matter is formed out of ether during the expansion process, then matter will have the same kinetic energy per unit mass as the ether from the time of its creation. Whether, therefore, matter is present *ab initio*, or is formed during the expansion process, it is again necessary to assume that the total mass of matter in the universe is always very small compared with the mass of ether so that the overall motion of the ether, as given by the similarity solution, is not significantly altered by the presence or creation of matter.

It is, indeed, possible to look for observational confirmation of the preponderance of ether mass. For, if this is so, it should be possible to detect the presence of such a large mass of ether by the gravitational force it exerts. The ether cannot, of course, be 'seen' for 'seeing' is only the observational process of detecting the arrival of ethereal waves; the ether must, then, be utterly dark. In fact, the gravitational forces due to such an invisible mass, estimated to be very much larger than the mass of observed matter, are observed. This mass is aptly referred to as 'dark matter'.

If matter moves, for practical purposes, with only slight perturbations from the ethereal flow, a significant simplification is introduced into ethereal cosmology. For then the red- shift observed by a material observer in the light from a material source will be practically the same as the red-shift due to, and derivable from, the ethereal flow as given by the similarity solution. An obvious difficulty arises, then, for small distances and red-shifts when the corresponding recession velocity given by the similarity solution is not large compared with the perturbation velocities of the source and the receiver. This accords with the failure to determine the Hubble 'constant', to within a factor of two, from observations of red-shifts sufficiently small for the 'distance' of the source to be reliably estimated.

All observers moving with the ethereal flow (Thornhill, 1993) have the same local wave-hyperconoid, given by the differential equation

$$(dx_1/dt)^2 + (dx_2/dt)^2 + (dx_3/dt)^2 = c^2 \qquad (1.8.1)$$

where c is the (variable) local wave speed. This simple local wave hyperconoid appears, therefore, to be invariant under Galilean transformation to all such observers, and does not require, for its invariance, any alternative transformation like the Lorentz transform introduced by the interpretation of the Michelson-Morley experiment in the non-ether concept.

1.9 **Gravitation**

Another principal objective of an ethereal cosmology must be to provide a *physical* theory of gravitation. Indeed, without such a physical theory, knowledge of the local contemporary value of Newton's 'constant' of gravitation cannot be used in the determination of the scale of the universe. The acceleration due to gravity is real and apparently universal. Newtonian mechanics requires, therefore, that the source of this acceleration be explained physically. It cannot be dismissed as a 'force of Nature' or as some mystical curvature property of an imaginary four-dimensional Riemannian metric.

In the cosmology constructed here the ether is accelerating away from the centre of the universe up to a finite asymptotic radial velocity. It is clear, therefore, that a frame of reference whose origin moves with the ethereal flow, i.e. the rest-frame of a material observer travelling with the ether, is an accelerating frame of reference. In Newtonian mechanics, Galilean transformation decrees that all masses in such a rest-frame experience an acceleration (i.e. a force per unit mass) in the reverse direction, namely *towards* the centre of the universe. This force per unit mass towards the centre of the universe experienced by all masses (matter and ether) in the rest-frames of all materials observers is seen to provide completely for the phenomenon of gravitation.

The acceleration given by the similarity solution for any material observer can be expressed in the form of Newton's empirical 'law' of gravitation, namely a force per unit mass GM/r^2, where M is the total mass contained with the sphere of radius r about the centre of the universe, which passes through the observer. G is found to be independent of time, but varies with the relative position, r/R, of the observer in the universe. Thus, for any given observer, travelling with the ethereal flow, G is constant for all time. (This accords with the amusing anagram of 'constant of gravitation', namely '*O*, so in fact *G* not *t*-variant'!)

The universal gravitational potential is determined and found to be the amount by which the kinetic energy per unit mass falls short of its asymptotic value. Thus, for all observers throughout the universe, the universal gravitational potential tends asymptotically to zero.

1.10 Comparison with observations

The success of an ethereal cosmology depends entirely on the extent to which its properties conform to, and its predictions are verified by, observation. The similarity solution enables all the relevant information pertaining to an emitter and receiver, not too far apart, to be derived as series expansions in powers of a small quantity which is, essentially, $(t_a - t_e)/t_a$ in some form or other. [Suffix *a* refers to the *a*rrival at an observer, at time t_a, of a point source signal *e*mitted by a source at time t_e. So far as our own observations are concerned, then, t_a is the present epoch and suffix *a* refers to our location in the universe.]

The common leading term in the series expansions for the two principal radii of curvature of an incoming wave front is non-directional, but subsequent coefficients contain directional elements whose influence declines as t_a increases. The first two terms in these expansions are identical and, as t_a tends to infinity, the third terms also become identical. This accords with the fact that observations can be made, out to red-shifts of order 4 or 5, with axi-symmetric instruments, indicating that, at least up to such distances, incoming wave fronts are practically spherical with radius $\Re$.

Hubble's 'law' is derived exactly insofar as the true source distance *d* and the recession speed *V*, at time t_a are such that d/V is a function of t_a only which tends to t_a as t_a increases.

On the other hand the observation usually referred to as source 'distance' is the radius of curvature $\Re$ of the practically spherical wave fronts arriving at the observer, so that, what is normally called the red-shift-distance relation is the relation between z_v and $\Re$. The derived relation between z_v and $\Re$ does not accord with Hubble's 'law', but the series expansion for $\Re / c_a z_v t_a$ tends to unity as $\Re$, z_v, $(t_a - t_e)$ all tend to zero; i.e. $\Re / c_a z_v$ tends to t_a as z_v tends to zero.

Thus, even though the relation between z_v and $\Re$ is not linear at any time, as Hubble envisaged, its slope for small red-shifts would provide a means of determining t_a as the Hubble time, were it not for the fact that the perturbational motion of source and observer make it difficult to determine the Hubble time with any precision.

1.11 The scale of the universe

The final objective of an ethereal cosmology must be to determine the scale of the universe, our position in it, and the time which has elapsed since the beginning of the present expansion phase.

The observational and theoretical information available at present do not permit any such determination to be made precisely. It appears that five relations are required for such a precise determination but, at present, only four are available even when a physical meaning is attributed to the gravitational 'constant' G which permits its local value to be used.

It is possible, with only four relations, however, to evaluate the complete systems corresponding to different values of our relative position, r/R, in the universe. This is done on the basis that the present age of the universe is sufficiently large for simplifications to be made such as have already been indicated to occur when t_a is sufficiently large. Uncertainty in the determination of the Hubble time, however, necessitates that the calculations be made for three values of t_H, an upper limit, a median value and a lower limit.

The present radius of the universe is found to be of order 10^{28}m; the total mass of the universe of order 10^{57}(kg); the total number of ether particles of order 10^{96}; the initial ethereal temperature of order 5 x 10^4°K; and the initial speed of light of order 10^{10}m/s.

It is possible also, with the complete solution, to determine what will happen in the future. For example, when the universe is n times as old as at present, the speed of light in any particular location will be $n^{-½}$ times its present value and the ethereal temperature will be n^{-1} times its present value. Thus, in our location, when the universe is twice its present age, the speed of light will have fallen to about 2.1 x 10^8m/s and the ethereal temperature to about 1.35°K.

This first attempt to construct a complete ethereal cosmology thus falls short of any precise and unique determination of the present expansion phase of the universe and our position in it. It does, however, conform to all our present observations, explain various observational difficulties and make predictions for the future. It also purports to solve one of the three great questions unanswered in present day theoretical physics, namely the physical *raison d'être* of gravitation. It does not, however, contribute towards the elucidation of the physical nature of electricity and magnetism. In the ether used here, ether particles have six degrees of freedom (Thornhill, 1985a) only three of which are required for the three components of velocity. It seems likely that the physical nature of electricity and magnetism derives from the remaining three degrees of freedom, probably rotation or vibration.

It will be interesting to see from where, and how soon, a fifth relation will come which will permit a precise evaluation of the cosmology constructed here; and even more interesting, whether ultimately a sixth and further relations give consistency.

Perhaps the most surprising aspect of the cosmology constructed here is that it is so antithetic to the theoretical physics and cosmology which have come to be accepted during the twentieth century. Here there is no denial of an ether, no assumption that the speed of light is a universal constant, no upper limit to relative speed, no big bang or other mathematical singularity, no imaginary Riemannian geometry, no specious explanation of gravitation - and no strings attached. Whereas, in the cosmologies accepted at the present time, the mass of the universe is entirely the mass of matter permeated by a 'non-ether' of massless photons, in the cosmology here the mass of matter in the universe is negligible compared with the mass of ether.

2. **The similarity solution**

The similarity solution is concerned with the expansion in vacuo of a homentropic sphere of an ideal polytropic gas with a constant first adiabatic index γ. Initially, at time $t = o$, the gas is at rest, the radius of the sphere is L and the pressure, temperature and wave-speed at its centre are, respectively, p_1, T_1 and c_1. The radial distribution of pressure, density, etc., in the initial gas sphere is the particular constant radial distribution required by the similarity solution. At subsequent times the radius of the sphere is $R(t)$ and, at the outer boundary, where $r = R(t)$, pressure, density and wave-speed all tend to zero mathematically, for the purpose of deriving the solution, even though physically the continuum solution must necessarily cease to be valid before such conditions are reached.

The solution is rendered non-dimensional in terms of a fundamental mass, length, time and temperature as defined by p_1, c_1, T_1 and L. Thus $\underline{P} = P/[P]$, where P stands for any physical quantity and $[P]$ denotes that combination of the fundamentals that has the same dimensions as P; e.g. $\underline{r} = r/L$, $\underline{p} = p/p_1$, $\underline{u} = u/c_1$, $\underline{t} = tc_1/L$ but density $\underline{\rho} = \rho c_1^2/p_1 = \gamma\rho/\rho_1$.

The general solution is then obtained (Thornhill, 1958), in terms of time $\underline{t}$ and the similarity variable $\eta = \underline{r}/\underline{R}$, as

$$\underline{p} = (1-\eta^2)^{\gamma/(\gamma-1)}\, \underline{R}^{-3\gamma}\,;\ \underline{\rho} = \gamma(1-\eta^2)^{1/(\gamma-1)}\, \underline{R}^{-3},$$

$$\underline{c}^2 = \underline{T} = (1-\eta^2)\, \underline{R}^{-3(\gamma-1)}$$

together with the radial velocity distribution, $\underline{u} = \eta\dot{\underline{R}}$, where $\dot{R}$ satisfies

$$\dot{\underline{R}} = [(2/\surd 3)/(\gamma\text{-}1)]\,[1 - \underline{R}^{-3(\gamma-1)}]^{½}$$

For the ether $\gamma = 4/3$ (Thornhill 1985a) and, with

$$\eta = \sin\alpha,\ \underline{R} = \cosh^2\xi,\ \underline{r} = \sin\alpha\cosh^2\xi, \tag{2.1}$$

the solution for the ethereal flow reduces to

$$\underline{p} = (\cos\alpha/\cosh\xi)^8\ ;\ \underline{\rho} = 4(\cos\alpha/\cosh\xi)^6/3\ ;\ \underline{c} = \cos\alpha/\cosh\xi \tag{2.2}$$

$$\dot{\underline{R}} = 2\surd 3\tanh\xi \text{ or } d\underline{t} = \cosh^2\xi d\xi/\surd 3 \tag{2.3}$$

$$\underline{u} = 2\surd 3\sin\alpha\tanh\xi \tag{2.4}$$

Thus, as $\underline{t} \to \infty$, $\underline{R} \to \infty$, and $\xi \to \infty$, $\dot{\underline{R}} \to 2\surd 3$; and equation (2.3) may be integrated to give, satisfying $\xi = 0$ when $\underline{t} = 0$,

$$\underline{t} = (\surd 3/6)\,(\sinh\xi\cosh\xi + \xi) \tag{2.5}$$

The flow is supersonic (and therefore superluminal) if

$$2\surd 3\tan\alpha > \operatorname{cosech}\xi;\ \text{or}\ \eta^2 > 1/(12\underline{R}-11) \tag{2.6}$$

In the $(\underline{r}, \underline{t})$-plane the world-lines or universe-lines are given by

$$\eta = \underline{r}/\underline{R} = \underline{r}/\cosh^2\xi = \text{constant}, \tag{2.7}$$

whilst the characteristics in the $(\underline{r}, \underline{t})$-plane, namely $d\underline{r}/d\underline{t} = \underline{u} \pm \underline{c}$ become

$$d(\sin\alpha\cosh^2\xi)/dt = 2\surd 3\sin\alpha\tanh\xi \pm \cos\alpha/\cosh\xi$$

These integrate at once to give

$$\surd 3\alpha \pm gd\xi = \text{constant} \tag{2.8}$$

where the Gudermannian is defined by

$$gd\,x = \int_o^x \quad sech\,xdx = tan^{-1}(sinhx) = 2tan^{-1}e^x - \pi/2 \qquad (2.9)$$

3. The wave hyperconoid

For general flow in three space variables and time, the general equation for the wave hyperconoid is

$$(dx_1/dt - u_1)^2 + (dx_2/dt - u_2)^2 + (dx_3/dt - u_3)^2 = c^2 \qquad (3.1)$$

where $\{u_i\}$ and c are, respectively, the ethereal velocity and the wave-speed. Here, the symmetry properties of the similarity solution enable considerable simplifications to be made in the general differential equation (3.1). First, since the flow in the similarity solution is spherically symmetric, it follows that the wave-fronts from a point disturbance at any time are all axially symmetric about the diameter on which the point source of the disturbance lies. It is sufficient, therefore, to consider only the flow in a diametral plane and its wave-conoids in two space variables and time. Second, the flow is radial in a diametral plane so that further simplification is possible by using polar co-ordinates. Thus, the differential equation for the wave conoid through any point on a diametral plane is

$$(d\underline{r}/d\underline{t} - \underline{u})^2 + \underline{r}^2\,(d\theta/d\underline{t})^2 = \underline{c}^2 \qquad (3.2)$$

If the values of $\underline{u}$ given by equation (2.4), $\underline{c}$ given by equation (2.2) and $\underline{r} = sin\,\alpha\,cosh^2\,\xi$, are substituted in equation (3.2) and a new time-variable τ is introduced, defined by

$$\tau = (1/\surd 3)\,gd\,\xi\,;\; d\tau = (1/\surd 3)\,d\xi/cosh\,\xi \qquad (3.3)$$

then the differential equation (3.2) for the wave-conoid reduces to

$$(d\alpha)^2 + tan^2\alpha\,(d\theta)^2 = (d\tau)^2 \qquad (3.4)$$

Remarkably, in this form, equation (3.4) may be integrated by standard methods.

Let $\tau(\alpha, \theta)$, $(\tau_\alpha = p\ ;\ \tau_\theta = q)$ be a particular integral of the equation (3.4) i.e. the equation of a wave-surface. If $(\tau_1, \alpha_1, \theta_1)$ is a neighbouring point on this wave surface, then

$$\tau_1 - \tau = (\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1 \tag{3.5}$$

so that, by equation (3.4)

$$(\alpha_1 - \alpha)^2 + \tan^2\alpha_1(\theta_1 - \theta)^2 = (\tau_1 - \tau)^2 = [(\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1]^2 \tag{3.6}$$

The envelope of (3.6) for different values of α, θ, i.e. a wave-conoid, must also be a wave-surface. Hence, by partial differentiation with respect to α, θ,

$$(\alpha_1 - \alpha) = p_1\,[(\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1]$$

and

$$\tan^2\alpha_1(\theta_1 - \theta) = q_1[(\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1] \tag{3.7}$$

Thus

$$\frac{(\alpha_1 - \alpha)}{p_1} = \frac{(\theta_1 - \theta)\tan\alpha_1}{q_1 \cot\alpha_1} = \frac{[(\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1]}{1} \qquad \text{by equation (3.7),}$$

$$= \frac{[(\alpha_1 - \alpha)p_1 + (\theta_1 - \theta)q_1]}{(p_1^2 + q_1^2\cot^2\alpha_1)^{1/2}} \qquad \text{by equation (3.6),}$$

so that

$$p^2 + q^2\cot^2\alpha = 1 \tag{3.8}$$

The relation (3.8) is of the form $\phi(\alpha,p) = \psi(\theta,q)$ which is a standard form; e.g. it is the standard III of Forsyth (1933). The procedure is as follows. Put q equal to an arbitrary constant a, so that

$$\tau = a\theta + f_1(\alpha). \tag{3.9}$$

Then, by equation (3.8)

$$p = \delta\tau/\delta\alpha = (1 - a^2\cot^2\alpha)^{1/2}$$

This may be integrated to give

$$\tau = -a\cos^{-1}(a\cot\alpha) + (a^2+1)^{½}\cos^{-1}[(a^2+1)^{½}\cos\alpha] + f_2(\theta) \qquad (3.10)$$

The equations (3.9) and (3.10) can now be combined to give, if $\alpha = \alpha_o$, $\tau = \tau_o$, when $\theta = o$,

$$\tau - \tau_o = a\theta - a\cos^{-1}(a\cot\alpha) + a\cos^{-1}(a\cot\alpha_o) + (a^2+1)^{½}\cos^{-1}[(a^2+1)^{½}\cos\alpha] - (a^2+1)^{½}\cos^{-1}[(a^2+1)^{½}\cos\alpha_o] \qquad (3.11)$$

The general integral, i.e. the wave conoid, is now the envelope of (3.11) for different values of a and so must also satisfy the following relation obtained by differentiating (3.11) with respect to a.

$$0 = \theta - \cos^{-1}(a\cot\alpha) + \cos^{-1}(a\cot\alpha_o) + a\cos^{-1}[(a^2+1)^{½}\cos\alpha]/(a^2+1)^{½} - a\cos^{-1}[(a^2+1)^{½}\cos\alpha_o]/(a^2+1)^{½} \qquad (3.12)$$

By combining the two relations (3.11) and (3.12), the solution for the wave conoid is finally given, in terms of the parameter a, by

$$\tau - \tau_o = \cos^{-1}[(a^2+1)^{½}\cos\alpha]/a^2+1)^{½} - \cos^{-1}[(a^2+1)^{½}\cos\alpha_o]/a^2+1)^{½} \qquad (3.13)$$

and

$$\theta + a(\tau - \tau_o) = \cos^{-1}(a\cot\alpha) - \cos^{-1}(a\cot\alpha_o) \qquad (3.14)$$

4. The rays

The solution, obtained in the previous section, for the wave conoid of the flow in a diametral plane, is not in a form suitable for practical application. It is necessary, therefore, to start again with a different approach which can be ultimately reconciled with the equations (3.13) and (3.14).

Figure 1 shows the flow in the diametral plane defined by the polar co-ordinates $\eta = \sin\alpha$ and θ. The point P, $(\sin\alpha, \theta)$, is on the wave-front DPE, at time τ, emanating from a point disturbance at P_e, $(\sin\alpha_e, o)$, at time τ_e. P_eP is a curvilinear ray which, at time τ_e, is inclined at an angle β to the radius vector $\theta = 0$; and, at time τ, is inclined at an angle ψ to the radius vector OP. On any wave-front τ has a constant value. The wave-conoid, projected on to the diametral plane $(\sin\alpha, \theta)$ gives, then, two families of curves, namely the wave-fronts like DPE along which τ is constant but β varies; and the curvilinear rays, like P_eP, along which β is constant but τ varies.

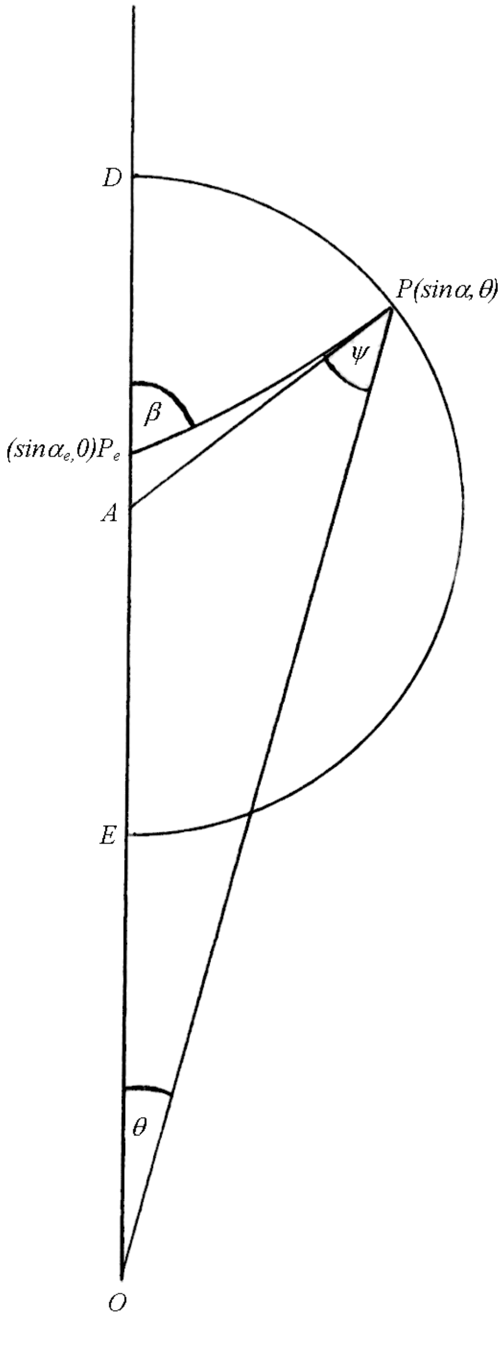


Figure 1

In the $(\sin\alpha, \theta)$-plane, the flow is stationary, and therefore the rays must be the orthogonal trajectories of the wave-fronts. It is, then, possible to write down at once the two equations

$$\tan\alpha\ \theta_\beta/\alpha_\beta = -\cot\psi;\ \tan\alpha\ \theta_\tau/\alpha_\tau = \tan\psi \tag{4.1}$$

But, from the equation (3.4) for the wave conoid, it follows, since β is constant along a ray, that

$$\alpha_\tau^2 + \tan^2\alpha\ \theta_\tau^2 = 1$$

so that

$$\alpha_\tau = \cos\psi;\ \tan\alpha\ \theta_\tau = \sin\psi \tag{4.2}$$

In the equations (3.13) (3.14) of the previous section 3, the parameter a is independent of τ and so must be a function of β only. Differentiating equation (3.13) with respect to τ gives

$$a^2 = \tan^2\alpha\sin^2\psi \tag{4.3}$$

whilst differentiating equation (3.14) with respect to τ gives a quartic equation in a, namely

$$(\sin\psi\cot\alpha + a)^2\ (1 - a^2\cot^2\alpha) = a^2\ \mathrm{cosec}^4\alpha\cos^2\psi \tag{4.4}$$

It is easily verified that $a = \tan\alpha\sin\psi$ is a root of equation (4.4) and so both equations (3.13) and (3.14) are satisfied by $a = \tan\alpha\sin\psi$. Since also a is constant along a ray, it follows that

$$a = \tan\alpha\sin\psi = \tan\alpha_e\sin\beta \tag{4.5}$$

and so

$$\psi_\tau = -\sin\psi/\sin\alpha\cos\alpha \tag{4.6}$$

Further, if this value of a is substituted in the equations (3.13) and (3.14) it yields the results

$$\mu - \mu_e = (1 + \sin^2\beta\tan^2\alpha_e)^{1/2}\ (\tau - \tau_e) \tag{4.7}$$

where $\sin\mu = \sin\alpha\cos\psi;\ \sin\mu_e = \sin\alpha_e\cos\beta,$

and $\beta - \theta - \psi = \sin\beta \tan\alpha_e (\tau - \tau_e)$ (4.8)

The equations (4.5) to (4.8) are the key relations required to determine the relation between a signal sent at time τ_e by an emitter at P_e and the corresponding wave-front which arrives, at time t_a at an observer P_a.

5. The interpretation of observations

Before setting out to determine the various quantities connecting an observer with an emitter it is first necessary to move from the $(\sin\alpha, \theta)$ frame of reference of Figure 1, in which the flow is stationary, to the physical plane, with polar co-ordinates $(\sin\alpha \cosh^2\xi, \theta)$, in which the outer boundary of the solution is $\underline{R} = \cosh^2\xi$, as shown in Figure 2.

Thus, in Figure 2, P_{ee} is the position of the emitter at time $\underline{t}_e$, P_{ae} is the position of the observer at time $\underline{t}_e$; and the wave-front $E_e\, P_{ae}\, D_e$, the ray $P_{ee}\, P_{ae}$ and the normal $A_e\, P_{ae}$ simply reproduce Figure 1 with all the radii vectores about O multiplied by $\underline{R}_e = \cosh^2\xi_e$. Likewise, P_{ea} is the position of the emitter at time $\underline{t}_a$, P_{aa} is the position of the observer at time $\underline{t}_a$; and the wave-front $E_a\, P_{aa}\, D_a$, the ray $P_{ea}\, P_{aa}$ and the normal $A_a\, P_{aa}$ reproduce Figure 1 with all the radii vectores about O now multiplied by $\underline{R}_a = \cosh^2\xi_a$.

According to the similarity solution a point disturbance emitted at time $\underline{t}_e$, when the emitter is at P_{ee} and the observer at P_{ae}, will arrive at the observer at time $\underline{t}_a$ when the emitter is at P_{ea} and the observer at P_{aa}. The observer at P_{aa}, lying on the true physical ray, $P_{ee}\, P_{aa}$, will observe the disturbance by means of the full three-dimensional axi-symmetric wave-front whose diametral section is $E_a\, P_{aa}\, D_a$ and will be able to observe its two principal radii of curvature and the direction $P_{aa}\, A_a$ of its normal.

Again, before setting out to determine the various relations between emitter and observer, it is necessary to consider the type of expansion in power series, for 'small' distances and time intervals, which is appropriate to the observer's measurements. Clearly, power series are required in which the coefficients pertain to the observer's conditions at P_{aa} at time $\underline{t}_a$, and these are achieved by means of reverse Taylor series.

As an example of this, the conversion of series in powers of $(\tau_a - \tau_e)$ to series in powers of $(\xi_a - \xi_e)$ or $(\underline{t}_a - \underline{t}_e)$ may be considered. Thus the reverse Taylor series.

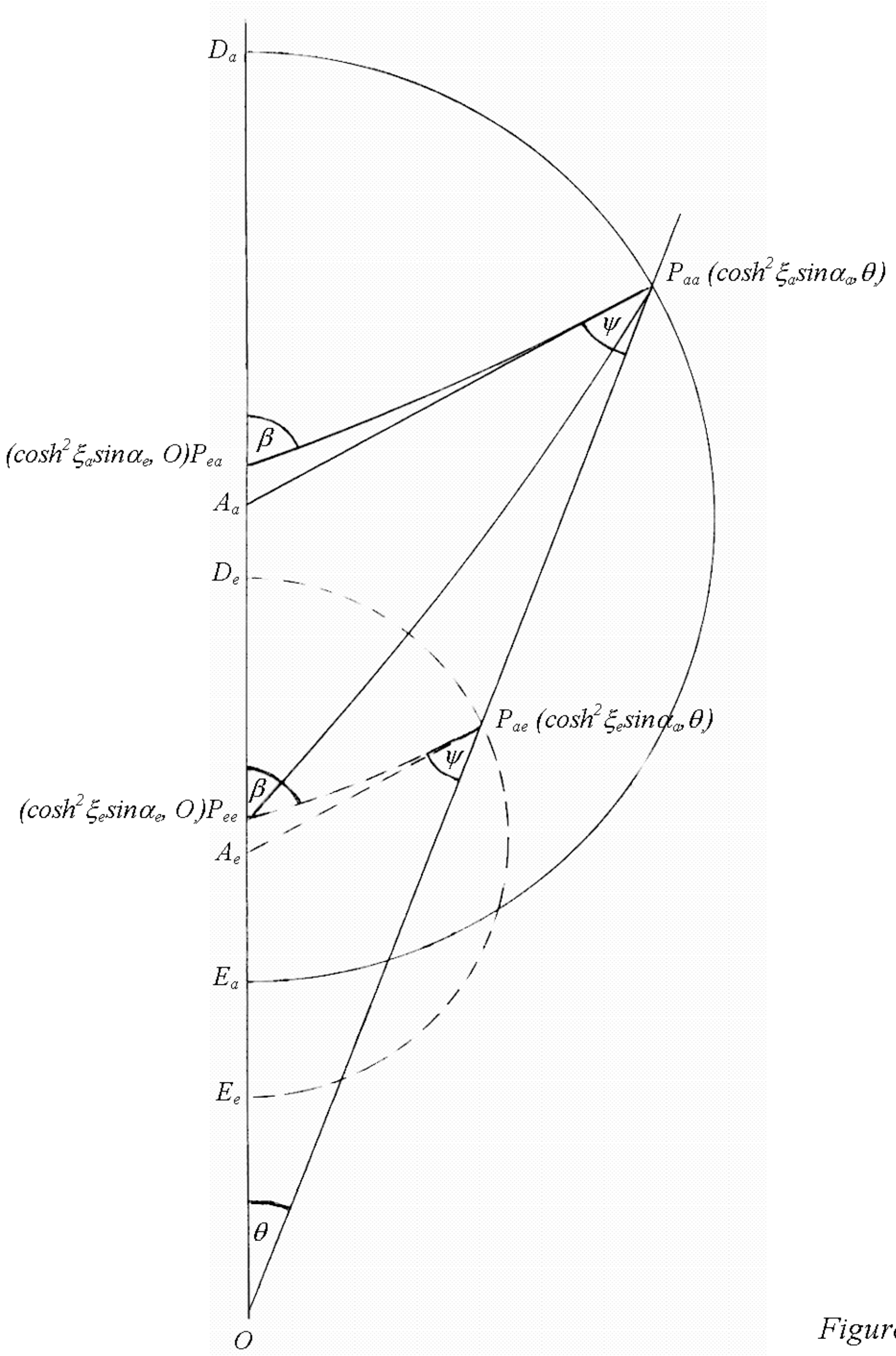

D_a
P_{aa} $(cosh^2 \xi_a sin\alpha_a, \theta)$
ψ
β
$(cosh^2 \xi_a sin\alpha_e, O)P_{ea}$
A_a
D_e
P_{ae} $(cosh^2 \xi_e sin\alpha_a, \theta)$
ψ
β
$(cosh^2 \xi_e sin\alpha_e, O)P_{ee}$
A_e
E_a
E_e
θ
O

Figure 2

$$\tau_e = \tau_a + (\tau_\xi)_a (\xi_e - \xi_a) + (\tau_{\xi\xi})_a (\xi_e - \xi_a)^2/2 + \ldots\ldots\ldots,$$

together with $d\tau = (1/\surd3)d\xi/\cosh\xi$, [equation (3.3)], gives

$$\tau_\xi = 1/(\surd3 \cosh \xi); \ \tau_{\xi\xi} = -(1/\surd3) \sinh\xi/\cosh^2\xi;$$

$$\tau_{\xi\xi\xi} = (1/\surd3) (\sinh^2\xi\text{-}1)/\cosh^3\xi$$

and hence

$$(\tau_a - \tau_e) = (1/\surd3) (\xi_a - \xi_e)/\cosh \xi_a + (\surd3/6) (\xi_a - \xi_e)^2 \sinh \xi_a/\cosh^2 \xi_a$$

$$+ (\surd3/18) (\xi_a - \xi_e)^3 (\sinh^2\xi_a - 1)/\cosh^3\xi_a + \ldots\ldots\ldots \quad (5.1)$$

Similarly, using $d\underline{t} = (1/\surd3) \cosh^2\xi \, d\xi$,

$$(\xi_a - \xi_e) = \surd3(\underline{t}_a - \underline{t}_e)/\cosh^2 \xi_a + 3\sinh \xi_a (\underline{t}_a - \underline{t}_e)^2/\cosh^5 \xi_a$$

$$+ \surd3(4\sinh^2\xi_a - 1) (\underline{t}_a - \underline{t}_e)^3/\cosh^8 \xi_a + \ldots\ldots\ldots \quad (5.2)$$

and

$$(\tau_a - \tau_e) = (\underline{t}_a - \underline{t}_e)/\cosh^3\xi_a + (3\surd3/2) \sinh \xi_a(\underline{t}_a - \underline{t}_e)^2/\cosh^6\xi_a$$

$$+ (3/2) (\underline{t}_a - \underline{t}_e)^3 (5\sinh^2\xi_a - 1)/\cosh^9\xi_a + \ldots\ldots\ldots \quad (5.3)$$

The series (5.1) - (5.3) enable series expansions to be obtained which can be interpreted physically. Further simplification of some of these series is achieved, when the solution has progressed to cover a volume very large compared with the initial volume at $t=0$, by noting that, as $\xi \to \infty$, $\sinh \xi \to e^\xi/2$, $\cosh \xi \to e^\xi/2$ and so, by equation (2.4),

$$\underline{t} = (\surd3/6) (\sinh\xi \cosh \xi + \xi) \to (\surd3/24)e^{2\xi} \quad (5.4)$$

for large ξ..

6. The two principal radii of curvature

One of the two principal radii of curvature $(\underline{\Re}_1)$ of the axially-symmetric wave-surface, of which $E_a \, P_{aa} \, D_a$ in Figure 2 is a diametral section, is, by Meunier's theorem, the intercept $A_a \, P_{aa}$ made, on the normal, by the axis of symmetry $\theta = 0$. Thus, by simple trigonometry

$$\underline{\Re}_1 = \cosh^2\xi_a \sin\alpha_a \sin\theta / \sin(\theta + \psi) \qquad (6.1)$$

By the method outlined, using the key relations (4.5) - (4.8), it is found that

$$\underline{\Re}_1 = \frac{\cosh\xi_a \cos\alpha_a (\xi_a - \xi_e)}{\sqrt{3}}\left\{1 + (\xi_a - \xi_e)\left[\frac{\sinh\xi_a}{2\cosh\xi_a} + \frac{\cos\psi\tan\alpha_a}{\sqrt{3}\cosh\xi_a}\right]\right.$$

$$+ (\xi_a - \xi_e)^2\left[\frac{(\sinh^2\xi_a - 1)}{6\cosh^2\xi_a} + \frac{\sinh\xi_a\cos\psi\tan\alpha_a}{2\sqrt{3}\cosh^2\xi_a} + \frac{\sinh\xi_a}{2\sqrt{3}\cosh^2\xi_a} + \frac{(2\cos^2\psi\tan^2\alpha_a - 2 - \tan^2\alpha_a)}{9\cosh^2\xi_a}\right]$$

$$\left. + \ldots \right\} \qquad (6.2)$$

It can now be seen that the expansion for $\underline{\Re}_1$ has coefficients with a directional element specified by the angle ψ. This dependence on direction, however, decreases as the solution progresses. Thus, as $\xi_a \to \infty$,

$$\underline{\Re}_1 \to (\sqrt{3}/6)e^{\xi a}\cos\alpha_a(\xi_a - \xi_e)\left[1 + (\xi_a - \xi_e/2 + (\xi_a - \xi_e)^2/6 + \ldots\right] \qquad (6.3)$$

and, by equation (5.2)

$$\xi_a - \xi_e \to 4\sqrt{3}e^{-2\xi_a}(\underline{t}_a - \underline{t}_e) + 48e^{-4\xi_a}(\underline{t}_a - \underline{t}_e)^2 + 256\sqrt{3}e^{-6\xi_a}(\underline{t}_a - \underline{t}_e)^3 + \ldots \qquad (6.4)$$

Also, by equation (5.4), as $\xi_a \to \infty$, $\underline{t}_a \to \sqrt{3}e^{2\xi a}/24$, so that, finally, as $\underline{t}_a \to \infty$,

$$\xi_a - \xi_e \to \frac{1}{2}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right) + \frac{1}{4}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right)^2 + \frac{1}{6}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right)^3 + \ldots \qquad (6.5)$$

and

$$\underline{\Re}_1 \to \frac{e^{\xi a}\cos\alpha_a}{4\sqrt{3}}\left[\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right) + \frac{3}{4}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right)^2 + \frac{5}{8}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a}\right)^3 + \ldots\right] \qquad (6.6)$$

If s denotes arc length along the wave-front EPD in Figure 1, then the second principal radius of curvature of the wave surface, at the point P_{aa} on the diametrical section $E_aP_{aa}D_a$ of Figure 2, is given by

$$\underline{\Re}_2 = \cosh^2\xi_a\, s_\beta/(\theta + \psi)_\beta \qquad (6.7)$$

whilst, by equation (4.1)

$$-\cos\alpha_a \alpha_\beta / \sin\psi = \sin\alpha_a \theta_\beta / \cos\psi = s_\beta \tag{6.8}$$

and so

$$\underline{\Re}_2 = \cosh^2\xi_a \sin\alpha_a \, \theta_\beta \sec\psi/(\theta + \psi)_\beta \tag{6.9}$$

Using the key relations (4.5) - (4.8), it is found that

$$\underline{\Re}_2 = \frac{\cosh\xi_a \cos\alpha_a(\xi_a - \xi_e)}{\sqrt{3}} \left\{ 1 + (\xi_a - \xi_e)\left[\frac{\sinh\xi_a}{2\cosh\xi_a} + \frac{\cos\psi \tan\alpha_a}{\sqrt{3}\cosh\xi_a} \right] \right.$$

$$\left. + \frac{(\xi_a - \xi_e)^2}{\cosh^2\xi_a}\left[\frac{\left(\sinh^2\xi_a - 1\right)}{6} + \frac{\sinh\xi_a \cos\psi \tan\alpha_a}{\sqrt{3}} + \frac{\left(1 + 3\sin^2\alpha_a \cos^2\psi - 3\sin^2\psi\right)}{9\cos^2\alpha_a} \right] + \ldots \right\} \tag{6.10}$$

Again, the coefficients are directional, depending on the angle ψ, but it may be seen at once that the first two terms of the expansions for $\underline{\Re}_1$ and $\underline{\Re}_2$ are identical. As $\xi_a \rightarrow \infty$ $\underline{\Re}_2$ becomes identical with $\underline{\Re}_1$ at least as far as the first three terms of the expansion. To this order, therefore, the wave-front becomes a sphere, with

$$\underline{\Re}_1 = \underline{\Re}_2 = \underline{\Re} = \frac{e^{\xi_a} \cos\alpha_a}{4\sqrt{3}} \left[\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a} \right) + \frac{3}{4}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a} \right)^2 + \frac{5}{8}\left(\frac{\underline{t}_a - \underline{t}_e}{\underline{t}_a} \right)^3 + \ldots \right] \tag{6.11}$$

The leading term in this expansion for $\underline{\Re}$ is, of course, simply $\underline{c}_a(\underline{t}_a - \underline{t}_e)$ the result always obtained in simple cosmologies.

7. Recession speed and distance

The distance between the emitter and the observer at time $\underline{t}_a$ is $P_{ea} P_{aa}$ in Figure 2. Thus

$$\underline{d}_a = \cosh^2\xi_a(\sin^2\alpha_e + \sin^2\alpha_a - 2\sin\alpha_e \sin\alpha_a \cos\theta)^{1/2} \tag{7.1}$$

Again, in Figure 2, the emitter at P_{ea} at time $\underline{t}_a$, has radial velocity $2\sqrt{3}\, \sin\alpha_e \tanh\xi_a$; the observer, at P_{aa} at time $\underline{t}_a$, has radial velocity $2\sqrt{3}\, \sin\alpha_a \tan h\xi_a$. The recession speed at time $\underline{t}_a$ is thus

$$\underline{V}_a = 2\sqrt{3}\, tanh\xi_a (sin^2\alpha_e + sin^2\alpha_a - 2sin\alpha_e\, sin_a \cos\theta)^{½} \qquad (7.2)$$

Hence

$$\underline{d}_a/\underline{V}_a = (\sqrt{3}/6)\, cosh^3\xi_a / sinh\xi_a \qquad (7.3)$$

i.e. $\underline{d}_a/\underline{V}_a$ is a function of $\underline{t}_a$ only. This is the 'law' conjectured by Edwin Hubble on the basis of an apparent linear relation between the observable quantities red shift z_ν and radius of curvature $\Re$. In the cosmology considered here it is an exact relation in the form of equation (7.3). It will be shown that it is not an exact relation between z_ν and $\Re$.

As $\underline{t}_a$ increases, $\underline{d}_a/\underline{V}_a$ tends to $\underline{t}_a$ and so

$$d/V \to t_a \qquad (7.4)$$

When the expression (7.1) for $\underline{d}_a$ is evaluated in series form it gives the result

$$\underline{d}_a = \frac{cos\alpha_a(\underline{t}_a - \underline{t}_e)}{cosh\xi_a}\left\{1 + \frac{(\underline{t}_a - \underline{t}_e)}{2cosh^3\xi_a}\left[3\sqrt{3}\, sinh\xi_a + cos\,\psi\, tan\alpha_a\right] + \ldots\right\} \qquad (7.5)$$

Again, the leading term is $\underline{c}_a(\underline{t}_a - \underline{t}_e)$ as in simple cosmologies, and the directional element introduced by the angle ψ decreases as $\underline{t}_a$ increases. When $\underline{t}_a$ is large

$$\underline{d}_a \to \underline{c}_a(\underline{t}_a - \underline{t}_e)\left[1 + \frac{3}{4}\frac{(\underline{t}_a - \underline{t}_e)}{\underline{t}_a} + \ldots\right] \qquad (7.6)$$

8. The red-shift

In Figure 3, P_{ee} $(cosh^2\xi_e\, sin\, \alpha_e, 0)$ is the position of an emitter at time t_e whose radial velocity is $2\sqrt{3}\, sin\, \alpha_e\, tanh\xi_e$ and who is transmitting a train of waves of frequency ν_e. One wave-peak leaves P_{ee} at time t_e. P_{aa} $(cosh^2\xi_a\, sin\, \alpha_a, \theta)$ is the position of an observer at time t_a whose radial velocity is $2\sqrt{3}\, sin\, \alpha_a\, tanh\, \xi_a$ and who is receiving the train of waves at a frequency ν_a. The wave-peak which left P_{ee} at time t_e arrives at P_{aa} at time t_a along a curvilinear ray $P_{ee}P_{aa}$.

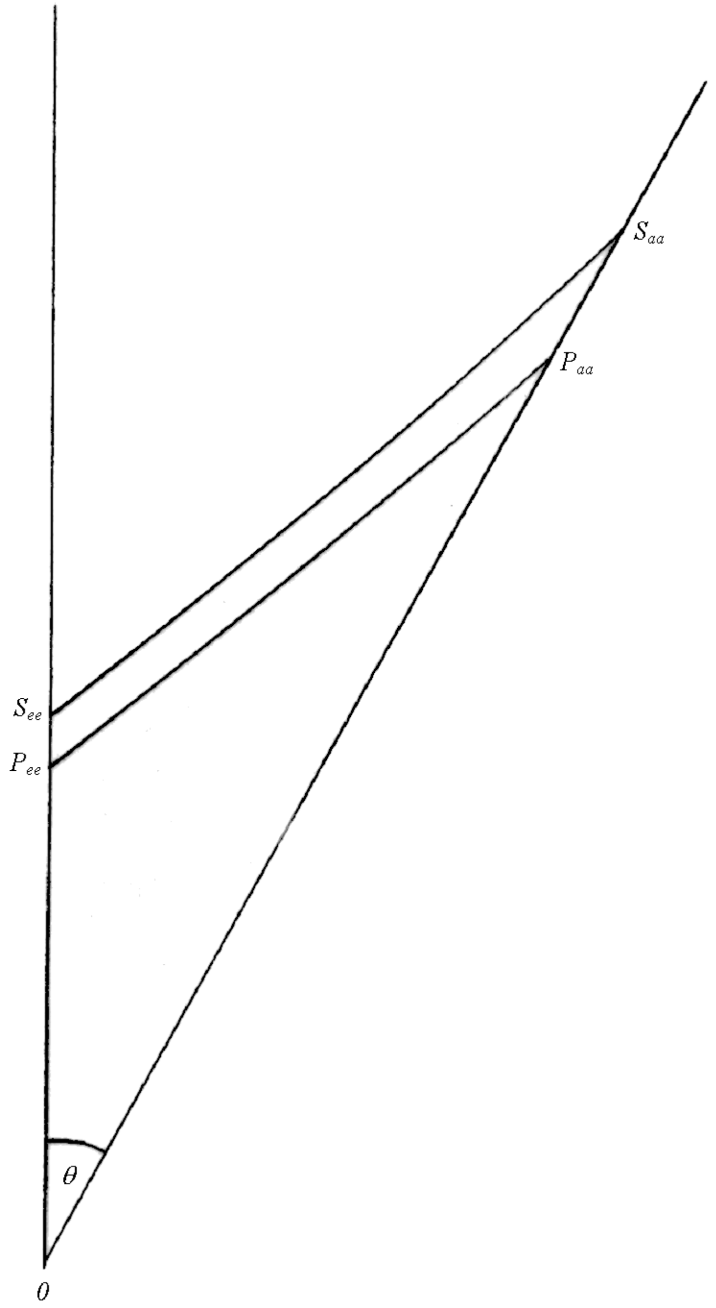


Figure 3

At time $(t_e + 1/\nu_e)$ the next wave peak will leave the emitter who will then be at S_{ee} where $P_{ee}\, S_{ee} = 2\sqrt{3}\, sin\alpha_e\, tanh\xi_e/\nu_e$. This wave-peak will reach the observer at time $(t_a + 1/\nu_a)$ when he is at S_{aa} where $P_{aa}\, S_{aa} = 2\sqrt{3} sin\alpha_a\, tanh\xi_a/\nu_a$.

If the ethereal flow accords with the similarity solution, the triangles $P_{ee}\, O\, P_{aa}$, $S_{ee}\, O\, S_{aa}$ must be similar so that

$$\frac{OS_{ee}}{OS_{aa}} = \frac{OP_{ee}}{OP_{aa}} = \frac{S_{ee}P_{ee}}{S_{aa}P_{aa}}$$

and hence

$$\frac{\nu_e}{\nu_a} = \frac{cosh^3\,\xi_a\; sinh\,\xi_e}{cosh^3\,\xi_e\; sinh\,\xi_a}$$

The frequency red-shift z_ν is then

$$z_\nu = \frac{\nu_e}{\nu_a} - 1 = \frac{cosh^3\,\xi_a\; sinh\,\xi_e}{cosh^3\,\xi_e\; sinh\,\xi_a} - 1 \qquad (8.1)$$

For large t_a and t_e then

$$z_\nu \rightarrow t_a/t_e - 1 \qquad (8.2)$$

or

$$z_\nu \rightarrow e^{2(\xi_a - \xi_e)} - 1 = 2(\xi_a - \xi_e)\left[1 + (\xi_a - \xi_e) + 2(\xi_a - \xi_e)^2/3 + \ldots\right] \qquad (8.3)$$

The wave-length red shift z_λ is given by

$$z_\lambda = \frac{cosh^2\,\xi_a\; sinh\,\xi_e\; cos\,\alpha_a}{cosh^2\,\xi_e\; sinh\,\xi_a\; cos\,\alpha_e} - 1 \qquad (8.4)$$

so that, for large t_a, t_e

$$z_\lambda \rightarrow (t_a/t_e)^{½}\; cos\alpha_a/cos\alpha_e - 1 \qquad (8.5)$$

or

$$z_\lambda = c_a t_a / c_e t_e - 1$$

9. The relation between red-shift and radius of curvature

From the expansions for the principal radii of curvature of observed waves, (6.2) and (6.10), and the expansion for the observed red-shift z_v (8.3), it is clear that Hubble's 'law' does not hold for the relation between red-shift and radius of curvature. Using the leading terms in these expansions, however, it is found that, when $\underline{t}_a$ is large,

$$\mathop{Lt}_{z_v \to 0} \quad \underline{\Re}/\underline{c}_a z_v = \sinh\xi_a \cosh^3\xi_a / \sqrt{3}(2\cosh^2\xi_a - 3) \tag{9.1}$$

This is a function of $\underline{t}_a$ only, but it is not the same function of $\underline{t}_a$ as that derived for $\underline{d}_a/\underline{V}_a$, (equation 7.4). As $\underline{t}_a$ increases, however, both these functions tend to $\underline{t}_a$.

More generally, when $\underline{t}_a$ is sufficiently large for the directional elements in the coefficients of the expansions for $\underline{\Re}_1$ and $\underline{\Re}_2$ to be neglected, then from equation (6.3)

$$\underline{\Re}_1 \simeq \underline{\Re}_2 \simeq \underline{\Re} \simeq (\sqrt{3}/6)e^{\xi a} \cos\alpha_a(\xi_a - \xi_e)[1 + (\xi_a - \xi_e)/2 + (\xi_a - \xi_e)^2/6 + ...] \tag{9.2}$$

and from equation (8.3)

$$(\xi_a - \xi_e) \simeq \tfrac{1}{2} \ln(1+z_v) \simeq \tfrac{1}{2}z_v(1-z_v/2+ z_v^2/3 - ...) \tag{9.3}$$

so that

$$\Re/c_a\, z_v \simeq t_a\, (1 - z_v/4 + z_v^2/8\, ...) \tag{9.4}$$

or, using equation (8.2)

$$\frac{\Re}{c_a z_v} \square\, t_a \left[1 - \frac{1}{4}\left(\frac{t_a - t_e}{t_a}\right) - \frac{1}{8}\left(\frac{t_a - t_e}{t_a}\right)^2 + ... \right] \tag{9.5}$$

10. Gravitation

Up to the present time no viable *physical* theory of gravitation has yet emerged. By using the similarity solution without any gravitational forces, however, as a basis for a cosmology, it has been implicitly assumed here throughout that there are no gravitational forces in a Newtonian frame of reference at rest or in uniform motion. Such an assumption implies, in turn, that gravitation is a consequence of the acceleration of an observer's rest-frame.

The need for a *physical* theory of gravitation has long been recognised and many such theories have been proposed, going back to the vortical theories of Descartes (17th century) and others, and the ultra-mundane corpuscles of John Louis LeSage (18th century), (cf. Whittaker 1953).

Newton's empirical 'law' of gravitation is very often referred to as a theory of gravitation regardless of the fact that he always vigorously denied any knowledge of the physical mechanism which causes gravitation. Whilst, therefore, it is possible to speak of the mathematical theory of Newtonian gravitation, there is no Newtonian physical theory of gravitation.

Likewise, Einstein's theory of general relativity is often referred to as a theory of gravitation. In fact it is no more than a geometric allegory describing gravitation in terms of the curvature properties of a general imaginary four dimensional Riemannian metric. This is a Riemannian metric obtained by generalising the simple imaginary metric of special relativity imposed as a consequence of Minkowski's unfortunate misinterpretation of the differential equation.

$$(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = \bar{c}^{\,2} \qquad (10.1)$$

This equation (10.1) is no more than the common differential equation (Thornhill 1993, 1996) for the wave hypersurfaces both of sound waves in any general fluid at rest and of electromagnetic waves as given by Maxwell's equations.

In the cosmology constructed here all material observers move, apart from slight perturbations, with the universal ethereal flow which is accelerating outwards from the centre of the universe up to a finite asymptotic radial velocity. Galilean transformation therefore, to any observer's rest-frame requires that all masses experience, in this rest frame, an acceleration towards the centre of the universe. This acceleration, or force per unit mass, is given by

$$-\ddot{\underline{r}} = -6\sin\alpha/\cosh^4\xi \qquad (10.2)$$

Now the mass contained within the sphere on which lies the observer at radius $\underline{r} = \sin\alpha\,\cosh^2\xi$ is given by

$$\underline{m}(\alpha) = (\xi const.)\int_0^{\sin\alpha\cosh^2\xi} 4\pi \underline{r}^2 \rho d\underline{r}$$

which, since $d\underline{r} = \cos\alpha \cosh^2\xi\, d\alpha + 2 \sin\xi \cosh\alpha \sinh\xi d\xi$, leads to

$$\underline{m}(\alpha) = \int_0^{\alpha} (16\pi/3) \sin^2\alpha \cos^7\alpha d\alpha \quad (10.3)$$

and thus to

$$\underline{m}(\alpha) = (16\pi/9) \sin^3\alpha(1 - 9\sin^2\alpha/5 + 9 \sin^4\alpha/7 - \sin^6\alpha/3) \quad (10.4)$$

The force per unit mass towards the centre may then be written as

$$- \ddot{\underline{r}} = - \underline{G}(\alpha)\, \underline{m}(\alpha)/\underline{r}^2 \quad (10.5)$$

where the gravitational 'constant' $\underline{G}(\alpha)$ is given by

$$\underline{G}(\alpha) = (27/8\pi)(1 - 9 \sin^2\alpha/5 + 9 \sin^4\alpha/7 - \sin^6\alpha/3)^{-1} \quad (10.6)$$

so that $\underline{G}(\alpha)\, \underline{m}(\alpha) = 6\sin^3\alpha$ (10.7)

Thus $\underline{G}(\alpha)$ is independent of time and, for any observer travelling with the universal ethereal flow at a fixed α, remains constant for all time.

If $\underline{V}_g$ denotes the universal gravitational potential of an observer at $\underline{r} = \sin\alpha\cosh^2\xi$, then

$$(d\underline{V}_g/d\underline{r})_{\alpha const} = - \ddot{\underline{r}}$$

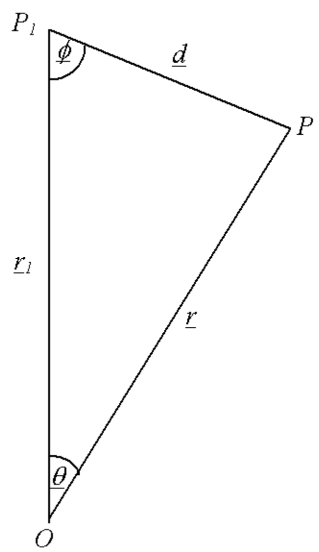
and so $\underline{V}_g = \int_{(\alpha const)} -(12 \sin^2\alpha \sinh\xi / \cosh^3\xi) d\xi$

Since $\underline{V}_g = O$ when $\alpha = o$ this leads to

$$\underline{V}_g = 6\sin^2\alpha/\cosh^2\xi = 6\sin^2\alpha - 6\sin^2\alpha\tanh^2\xi \quad (10.8)$$

or $\underline{V}_g = \underline{G}(\alpha)\, \underline{m}(\alpha)/\underline{r}$ (10.9)

Now $6\sin^2\alpha\tanh^2\xi$ is the kinetic energy per unit mass of the flow at $\underline{r} = \sin\alpha \cosh^2\xi$ and $6\sin^2\alpha$ is its asymptotic value as $\xi \to \infty$. Thus the universal gravitational potential on each flow surface $\alpha = const.$ is the amount by which the kinetic energy per unit mass of the flow falls short of its asymptotic value. Let P_1 be a point in the universe with polar co-ordinates $(\underline{r}_1, o)$ and P any other point at $(\underline{r}, \theta)$, such that $P_1P = \underline{d}$. Then since, in general, $\ddot{\underline{r}}/\underline{r} = 6/cosh^6\xi$, a function of t only, it follows that the triangle P_1OP may be used at any time as a triangle of accelerations. Thus, in the rest frame of P_1, the force per unit mass $-\ddot{\underline{r}}_1$, felt by a mass at P_1, is the vector sum of $\ddot{\underline{d}}$ *towards* P and $-\ddot{\underline{r}}$ along PO, equal and opposite to P's acceleration away from the centre O. Likewise, in the rest frame of P, the force per unit mass, $-\ddot{\underline{r}}$, felt by a mass at P, is the vector sum of $\ddot{\underline{d}}$ *towards* P_1 and $-\ddot{\underline{r}}$ along P_1O, equal and opposite to P's acceleration away from the centre O. Thus, any mass in the universe *in its own unique rest-frame* feels an attraction towards every other mass, and the resultant of all these attractions is a force per unit mass $-\ddot{\underline{r}}$ towards the centre of the universe.



If the universe were uniform, and stationary and subject to Newtonian gravitation, the resultant force per unit mass towards the centre would be exactly that given by equation (10.5). More specifically, this force per unit mass would be the resultant of the attractions of all the mass within the sphere on which the point lies, and the resultant of the attractions of all the mass outside this sphere would be zero. Since the model of the universe given by the similarity solution is neither uniform nor stationary it is necessary to determine what the corresponding results would be in the similarity solution if all observers, in their own unique rest frames, experienced Newtonian gravitation.

In this case the resultant force per unit mass towards the centre of the universe, from the attractions of all the mass *within* the sphere on which an observer at $\underline{r}_1 = sin\ \alpha_1\ cosh^2\xi$ lies will be

$$I(\alpha_1) = \int_{\theta=o}^{\pi} \int_{\underline{r}=o}^{sin\,\alpha_1 cosh^2\xi} 2\pi \underline{r}^2\ sin\theta\, d\underline{r}\, d\theta\, \underline{\rho}\, \underline{G}(\alpha_1) cos\phi / \underline{d}^2 \qquad (10.10)$$

where $\underline{d} = cosh^2\xi(sin^2\alpha_1 + sin^2\alpha - 2sin\alpha_1\ sin\alpha\ cos\theta)^{1/2}$ and $cos\ \phi = (\underline{r}_1 - \underline{r}cos\theta)/\underline{d}$

The double integral (10.10) reduces to

$$I(\alpha_1) = \frac{8\pi \underline{G}(\alpha_1)}{3\cosh^4 \xi}\int_{\alpha=0}^{\alpha_1}\left[\int_{\theta=0}^{\pi}\frac{(\sin\alpha_1 - \sin\alpha\cos\theta)\sin\theta d\theta}{(\sin^2\alpha_1 + \sin^2\alpha - 2\sin\alpha_1\sin\alpha\cos\theta)^{3/2}}\right]\sin^2\alpha\cos^7\alpha d\alpha$$

The inner integral is found to be independent of α and equal to $2/sin^2\alpha_1$, and then

$$I(\alpha_1) = \underline{G}(\alpha_1)\ \underline{m}(\alpha_1)/\underline{r}_1^2 \qquad (10.11)$$

in exact agreement with the result (10.5).

It follows however from equation (10.11) that the resultant force per unit mass towards the centre of the universe from the attractions of all the mass *outside* the sphere on which lies the observer at $\underline{r}_1 = sin\alpha_1 cos^2\xi$ is not zero but is equal to

$$I(\pi/2) - I(\alpha_1) = \underline{G}(\alpha_1)[\underline{m}(\pi/2) - \underline{m}(\alpha_1)]/\underline{r}_1^2 \qquad (10.12)$$

and the total resultant force towards the centre of the universe from the attractions of <u>all</u> the mass in the universe is

$$I(\pi/2) = \underline{G}(\alpha_1)\ \underline{m}(\pi/2)/\underline{r}_1^2 \qquad (10.13)$$

The result (10.13) does not agree with the value given by the similarity solution [cf. equation (10.5)] and thus implies that Newtonian gravitation is not universally valid. For an observer near the outer boundary of the universe, however, there is very little difference between $\underline{m}(\alpha)$ and $\underline{m}(\pi/2)$ as shown below.

$\eta = sin\alpha$	0.85	0.875	0.9	0.925	0.95	1.0
$\underline{m}(\alpha)$	.8401	.8454	.8487	.8502	.8508	.8510
$\underline{m}(\alpha)/\underline{m}(\pi/2)$	.9872	.9934	.9973	.9991	.9998	1.0

Newton's 'law' of gravitation is, therefore, an exceedingly good approximation for observers near the edge of the universe, and this, in turn, provides further evidence that we, who observe Newton's 'law' to be valid, are near the outer boundary.

There is also another difference between Newton's 'law' and the concept of gravitation given by the similarity solution. In Newtonian gravitation it is assumed that the 'constant' of gravitation is the same everywhere and that, therefore, the attraction felt by one body for another is mutual. In the similarity solution, however $\underline{G}(\alpha)$ varies with α and, even in the outer reaches of the universe, this variation is significant, since $\underline{G}(\alpha) = 6sin^3\alpha/\underline{m}(\alpha)$ [cf. equation (10.7)]. Locally, however, or between observers with the same value of α, Newtonian gravitation with a constant G is, again, a good approximation to the physical theory of gravitation given by the similarity solution.

Since observers in the universe do not move precisely with the ethereal flow of the similarity solution but are subject to very small perturbations in velocity relative to the mainstream, it follows that the value of $\underline{G}(\alpha)$ for any observer should also exhibit very small variations. This accords with the fact that the 'constant' of gravitation G is the most difficult of all the universal 'constants' to measure accurately; it is, in fact, only known to about one part in two thousand (Kaye and Laby 1973). This gives rise to an interesting question: can G be measured over a prolonged period of time to a sufficient degree of accuracy to detect a monotonic variation due to the Sun's motion around the centre of the galaxy, or even an annual variation due to the Earth's orbit around the Sun?

11. The scale of the universe

In order to determine the scale of the universe, our position in it and the present epoch, by means of the similarity solution, it is necessary to determine five quantities, namely p_1, c_1, L, $cosh\xi_a$ and $sin\alpha_a$, where now the suffix a refers to a terrestrial observer at the present time.

In attempting to do this, it will be assumed here that ξ_a is sufficiently large, say $\xi_a>3$, so that $sinh^2\xi_a$, $cosh^2\xi_a$, $e^{2\xi a}/4$ are practically indistinguishable and $tanh\xi_a$ is practically unity. (When $\xi = 3$, $sinh^2\xi = 100.4$, $cosh^2\xi = 101.4$, $e^{2\xi}/4 = 100.9$, $tanh\xi = 0.995$). Estimates of the Hubble time may then be used as values of t_a. The justification for this is observational, namely that we can 'see' out to red-shifts of order 4 or 5 with axially symmetric instruments, implying that the two principal radii of curvature of waves, arriving even from such large distances are practically indistinguishable; and, so far, no directional differences have been detected.

Our present observed value of the gravitational 'constant' G_a gives, when equation (10.5) is written in full,

$$(L^2p_1/c_1^4)(1 - 9sin^2\alpha_a/5 + 9\, sin^4\alpha_a/7 - sin^6\alpha_a/3) = 27/8\pi G_a \qquad (11.1)$$

The present value of the speed of light gives

$$c_1 \cos\alpha_a/\cosh\xi_a = c_a \quad (11.2)$$

The estimated value of the Hubble time t_H gives

$$L \cosh^2\xi_a/c_1 = 2\sqrt{3}t_a \quad (11.3)$$

The thermodynamics of the ether (Thornhill 1985a) yields a further relation. Thus the homentropy of the similarity solution gives $c_1{}^8/p_1 = c_a{}^8/p_a$, whilst $E = 3p\mathrm{v}$ and $E/\mathrm{v} = A_oT^4$ where the radiation density constant, $A_o = 8\pi^5k^4/15c^3h^3$, is a function of entropy only so that, in a homentropic cosmology A_o and ch are universal constants. (E is specific energy per unit mass; h is Planck's 'constant'; k is Boltzmann's universal constant). Then

$$p_a = E_a/3\mathrm{v}_a = A_oT_a{}^4/3 = 8\pi^5k^4T_a{}^4/45c_a{}^3h_a{}^3$$

so that finally

$$c_1{}^8/p_1 = 45c_a{}^{11}h_a{}^3/8\pi^5k^4T_a{}^4 \quad (11.4)$$

The four relations (11.1-4) are not sufficient to determine the five unknowns p_1, c_1, L, $\sin\alpha_a$ and $\cosh\xi_a$, but they enable the system to be determined completely for any assumed value of our position in the universe as given by $\eta_a = \sin\alpha_a$. Thus, for a given value of $\sin\alpha_a$, the relations (11.2) and (11.3) give

$$Lc_1 = 2\sqrt{3}\, t_ac_a{}^2/\cos^2\alpha_a \quad (11.5)$$

whilst the relations (11.1) and (11.4) give

$$L^2c_1^4 = \frac{1215h_a^3c_a^{11}}{64\pi^6k^4T_a^4G_a(1-9\sin^2\alpha_a/5+9\sin^4\alpha_a/7-\sin^6\alpha_a/3)} \quad (11.6)$$

Finally, the relations (11.5) and (11.6) combine to give

$$c_1^2 = \frac{405h_a^3c_a^7\cos^4\alpha_a}{256\pi^6k^4T_a^4t_a^2G_a(1-9\sin^2\alpha_a/5+9\sin^4\alpha_a/7-\sin^6\alpha_a/3)} \quad (11.7)$$

and then the relation (11.5) gives L, (11.4) gives p_1 and (11.3) gives $\cos\xi_a$. The calculations have been carried out for three values of the Hubble time t_H. The results are given in Tables 1,2 and 3 for $t_H = 10 \times 10^9$ *years* *[H = 97.8 km/(Mpc).s]*, $t_H = 15 \times 10^9$ *years [H = 65.2 km/(Mpc).s]* and $t_H = 20 \times 10^9$ *years [H = 48.9km/(Mpc).s]*.

The physical quantities used in the calculations are as follows.

$c_a = 2.998 \times 10^9 m/s.$ $k = 13.805 \times 10^{-24} (kg)m^2/s^2C^o$
$G_a = 66.70 \times 10^{-12}\ m^3/(kg)s^2$ $h_a = 662.56 \times 10^{-36}(kg)m^2/s$
$T_a = 2.7^oK.$

In the tables, $u_{\alpha a}/c_a$ is the Mach number of our radial flow outwards from the centre of the universe at the present time, which, for the values of $sin\alpha_a$ shown, is therefore both hypersonic and hyperluminal. V_a is the present volume of the universe and M the total mass. $M/\overline{m}$ is the number of ether particles in the universe, the mean mass of ether particles in the ethereal mixture being $\overline{m} = 4kT/3c^2$, (Thornhill 1985a). In Table 1, the quantities in the last four columns are independent of the value chosen for t_H and so apply to all three tables.

$sin\alpha_a$	c_1 m/s	L m	p_1 (kg)/ms^2	$cosh^2\xi_a$	T_1 °K	ρ_1 (kg)/m^3	h_1 (kg)m^2/s	$u_{\alpha a}/c_a$	R_a m	V_a m^3	M (kg)	$M/\overline{m}$
0.95	$10^9 \times$ 20.19	$10^{24} \times$ 48.25	7.449	473.9	$10^3 \times$ 13.12	$10^{-20} \times$ 2.271	$10^{-36} \times$ 9.504	229.4	$10^{27} \times$ 22.86	$10^{84} \times$ 50.06	$10^{57} \times$ 1.628	$10^{96} \times$ 2.949
0.925	29.77	22.89	125.4	1,421	26.57	18.87	6.678	317.9	32.54	144.3	1.446	2.619
0.90	37.63	13.76	818.6	2,990	42.47	77.07	5.282	391.1	41.14	291.7	1.282	2.322
0.875	44.58	9.42	3177	5,176	59.61	213.1	4.458	450.5	48.74	485.0	1.135	2.057
0.85	50.70	6.99	8885	7,926	77.09	460.9	3.921	497.6	55.43	713.2	1.006	1.823
0.825	56.06	5.50	19,844	11,151	94.24	842.0	3.546	534.0	61.28	964.0	0.892	1.616

Table 1: $(t_H)_a = 10 \times 10^9$ years (3.156×10^{17}s)
H = 97.8 km/Mpc.s

$sin\alpha_a$	c_1 m/s	L m	p_1 (kg)/ms^2	$cosh^2\xi_a$	T_1 °K	ρ_1 (kg)/m^3	h_1 (kg)m^2/s	$u_{\alpha a}/c_a$
0.95	$10^9 \times$ 13.94	$10^{24} \times$ 108.6	0.2907	210.6	$10^3 \times$ 5.83	$10^{-20} \times$ 0.1994	$10^{-36} \times$ 14.25	152.64
0.925	19.84	51.51	4.894	631.7	11.81	1.657	10.02	212.0
0.90	25.09	30.96	31.94	1,329	18.88	6.766	7.923	260.7
0.875	29.72	21.19	124.0	2,301	26.50	18.71	6.687	300.3
0.85	33.80	15.73	346.7	3,523	34.26	40.46	5.881	331.8
0.825	37.37	12.37	774.2	4,956	41.88	73.92	5.319	356.0

Table 2: $(t_H)_a = 15 \times 10^9$ years (4.734×10^{17}s)
H = 65.2 km/Mpc.s

$sin\alpha_a$	c_1 m/s	L m	p_1 (kg)/ms^2	$cosh^2\xi_a$	T_1 oK	ρ_1 (kg)/m^3	h_1 (kg)m^2/s	u_{aa}/c_a
0.95	10^9 x 10.46	10^{24} x 193.0	0.2910	118.5	10^3 x 3.279	10^{-20} x 0.3548	10^{-36} x 19.01	114.7
0.925	14.88	91.57	0.4899	355.3	6.643	.2949	13.36	159.0
0.90	18.82	55.04	3.197	747.4	10.62	1.204	10.56	195.5
0.875	22.29	37.66	12.41	1,294	14.90	3.330	8.917	225.2
0.85	25.35	27.97	34.71	1,981	19.27	7.201	7.841	248.8
0.825	28.03	21.98	77.50	2,783	23.56	13.16	7.092	267.0

Table 3: $(t_H)_a$ = 20 x 10^9 years (6.312 x 10^{17}s)
H = 48.9 km/Mpc.s

12. The past, the present and the future

Although sufficient time may now have elapsed for the universe to conform to the complete similarity solution, this cannot lead to any precise information about the very early stages or the initial conditions in the universe. Nevertheless the complete similarity solution does constitute the only available estimate of the comparatively quiescent initial state of the universe. From tables 1, 2 and 3 the initial conditions determined for the similarity solution are: speed of light c_1 of order 3 x 10^{10} m/s; ethereal temperature T_1 of order 50,000°K; Planck's 'constant' h, of order 7 x 10^{-36} (kg) m^2/s.

Observation does, however, allow us to 'see' something of the past state of the universe. Figure 2 shows clearly how an observer P_{aa}, at the present time t_a and travelling in the position $sin\alpha_a$, 'sees' an object travelling in the position $sin\alpha_e$ which was at P_{ee}, at time t_e and is now at P_{ea} at time t_a. To the observer at P_{aa} this object appears to be at A_a, the centre of curvature of nearly spherical waves arriving at P_{aa}, i.e. its apparent position is very close to *where it is now* at P_{ea}. But he 'sees' it *as it was* at P_{ee} at time t_e, when the observer was at P_{ae} and the light set out towards him travelling along a curvilinear ray $P_{ee}P_{aa}$.

When *both* $\underline{t}_a$ and $\underline{t}_e$ are sufficiently large, equation (8.2) gives, for the frequency red-shift $z = t_a/t_e - 1$, and so an observer will 'see' objects as they were at the earlier time $t_e = t_a/(z + 1)$.

Although it is possible at present to identify objects at red-shifts of order 4, it is not yet possible to 'see' them, as they were, in any detail, or to estimate their distances. Indeed, the largest red-shift for which distance can be estimated is about z = 0.015 which corresponds to t_e = 0.985 t_a; even at such distances objects cannot be 'seen' in much detail.

So far as the present is concerned, it is important to realise how small is the fraction of the universe which we can observe in any significant detail at the present time. When $\underline{t}_a$ and $\underline{t}_e$ are sufficiently large, equation (7.6) gives

$$\underline{d}_a \simeq \underline{c}_a(\underline{t}_a - \underline{t}_e)[1 + 3(\underline{t}_a - \underline{t}_e)/4\underline{t}_a + ...]$$

or, with $z = (t_a/t_e - 1)$

$$\underline{d}_a \simeq \underline{c}_a\, \underline{t}_a\, z(1 - z/4 + ...) \qquad (12.1)$$

The ratio of the volume bounded by red-shift z to the volume of the universe is then

$$\underline{d}_a^3/\underline{R}^3 = \underline{c}_a^3 z^3(1 - 3z/4 + ...)/24\sqrt{3} \qquad (12.2)$$

The volume bounded by red-shift z, however, is typical of the spherical annulus contained within spheres of radii (sin $\alpha_a\, cosh^2\xi_a \pm \underline{d}_a$), and the ratio of the volume of this spherical annulus to the volume of the universe is

$$6sin^2\alpha_a\, \underline{d}_a/\underline{R} + 2\underline{d}_a^3/\underline{R}^3$$

$$= \sqrt{3}sin^2\alpha_a \underline{c}_a z(1 - z/4 + ...) + (\underline{c}_a^3 z^3/12\sqrt{3})(1 - 3z/4 + ...) \qquad (12.3)$$

Taking $\underline{c}_a$ about 10^{-2} (see Tables 1, 2, 3) and $sin\ \alpha_a = 0.9$, for example, the ratios (11.2), (11.3) are of the order respectively:

for z = 0.015; 8×10^{-19}, 2×10^{-4}
for z = 0.1; 2×10^{-11}, 1.5×10^{-3}
for z = 1; 10^{-8}, 10^{-2}

Thus if the first order approximations (11.2), (11.3) are reliable up to $z = 1$, the part of the universe we observe out to red-shift z = 1 is still only typical of about 1% of the whole universe.

The future can be predicted more reliably since, if the universe already conforms to the similarity solution, it will continue to do so for all time.

Once the later stages of the universe have been fitted to a similarity solution, c_1, the initial wave-speed at the centre of the similarity solution is fixed. Then at time t_a and position $sin\alpha_a$, $c_a = c_1 cos\alpha_a/cosh\xi_a$, and, in the same position $sin\alpha_a$ at a later time $t_b > t_a$, $c_b = c_1\ cos\alpha_a/cosh\xi_b$. It follows that

$$c_b/c_a = \cosh \xi_a/\cosh \xi_b = (t_a/t_b)^{½} = h_a/h_b \qquad (12.4)$$

since ch is a universal constant.

Further, from the relations (2.2)

$$p_b/p_a = (t_a/t_b)^4;\ \rho_e/\rho_a = (t_a/t_b)^3;\ T_b/T_a = t_a/t_b \qquad (12.5)$$

Thus, for example, when the universe is twice as old as it is at present, conditions in our locality will be (cf. Thornhill 1985a) $c = 2.12 \times 10^8$ *m/s*; $T = 1.35\ ^oK$; $p = 0.209 \times 10^{-15}$ *(kg)/ms*^{2}; $\rho = 0.0248 \times 10^{-30}$ (kg)/m^3; $h = 937 \times 10^{-36}$ *(kg)m*^{2}*/s*.

The ethereal cosmology constructed here effectively presumes that the universe will go on expanding forever, or until it encounters another universe. The consequence, however, of adopting the similarity solution as the asymptotic form of the universal ethereal flow is the impossibility of determining, by means of local observations in its later stages, any knowledge of how the present expansion phase began.

It remains, then, an open question as to whether it all began as the re-expansion of a converging mass or as the sudden release of a confined mass at rest.

It remains, too, an open question as to whether matter, as well as ether, has always been present, or whether the universe consisted initially of ether only, out of which matter has been formed during the expansion.

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Appendix

The Universality of the Complete Similarity Solution

The equations of unsteady homentropic flow of an ideal polytropic gas in one space-dimension may be written in the form (cf. Mirels and Mullen 1962, Hubbard 1966),

$$\partial \underline{u} / \partial \underline{t} + \underline{u} \partial \underline{u} / \partial \underline{r} + (1 / \underline{\rho}) \partial \underline{p} / \partial \underline{r} = 0 \tag{A.1}$$

and

$$\partial \underline{\rho} / \partial \underline{t} + \partial (\underline{\rho} \underline{u} / \partial \underline{r} + \sigma \underline{\rho} \underline{u} / \underline{r} = 0 \tag{A.2}$$

where ρ denotes density, u velocity, r distance from the centre of symmetry and $\sigma = 0,1,2$ for plane, cylindrical and spherical symmetry respectively. All quantities with a subscript bar have been non-dimensionalised in terms of a fundamental length, pressure and wave-speed. For homentropy $\underline{p}/\underline{\rho}^{\gamma} = \underline{K}$ so that equation (A.1) may be written as

$$\partial \underline{u} / \partial \underline{t} + \underline{u} \partial \underline{u} / \partial \underline{r} + \underline{K} \gamma \underline{\rho}^{\gamma - 2} \partial \underline{\rho} / \partial \underline{r} = 0 \tag{A.3}$$

For expansion into a vacuum the term in $\partial \underline{\rho} / \partial \underline{r}$ in equation (A.3) becomes negligible as $\underline{t}$ increases so that, for large $\underline{t}$, $\partial \underline{u}/\partial \underline{t} + \underline{u}\, \partial \underline{u}/\partial \underline{r} \rightarrow 0$ asymptotically, and this has the solution $\underline{u} \rightarrow \underline{r}/\underline{t}$. Substituting $\underline{u} \rightarrow \underline{r}/\underline{t}$ in equation (A.2) leads to the solution $\underline{\rho}\underline{t}^{\sigma+1} = f(\underline{r}/\underline{t})$ where f is an arbitrary function. Thus the asymptotic flow in all cases, whatever the initial conditions, is always a similarity solution. The question at issue is whether all solutions, whatever the initial conditions, lead ultimately to the *same* solution asymptotically and that this solution is, therefore, the solution given by the asymptotic form of the complete similarity solution used in the main part of the paper.

First, just as the detonation in air of a piece of high explosive of any shape leads rapidly to a spherically symmetric flow bounded by a spherical Rankine-Hugoniot shock-wave, so the expansion into a vacuum of a mass of gas of any shape and with any distribution of pressure etc. may be expected to lead ultimately to an expanding spherically symmetric flow. Greenspan and Butler (1962) demonstrate the early stages of this process for non-spherical initial conditions. Comparison of the asymptotic solutions for different initial conditions has been attempted in the literature between a sphere of gas at rest, whose initial conditions satisfy the complete similarity solution and an initially uniform sphere of gas at rest. The crucial difference between these two cases is that the flows have different outer boundaries when $\sigma = 1$ or 2.

For the initially uniform sphere (suffix 2) at time $t = 0$, the uniform pressure, density and wave-speed are respectively p_2, ρ_2 and a_2 and the radius is L_2. The asymptotic solution is then

$$u = r/t;\ (\gamma\rho/\rho_2)\ (ta_2/L_2)^{\sigma+1} = f(r/a_2t);\ R_2 = 2a_2t/(\gamma-1) \tag{A.4}$$

So $\quad [\rho(\eta_2,t)/\rho_2]\ (ta_2/L_2)^{\sigma+1} = g(\eta_2,\gamma) \quad$ say

where $\quad \eta_2 = r/R_2 = (\gamma - 1)\ (r/a_2t)/2 \quad$ (A.5)

and

$$[\rho(o,t)/\rho_2]\ (ta_2/L_2)^{\sigma+1} = g(o,\gamma)$$

Then finally

$$\rho(\eta_2,t)/\rho(o,t) = F_2(\eta_2,\gamma) \tag{A.6}$$

is indicative of the 'shape' of the asymptotic solution, and

$$[\rho(o,t)/\rho_2]\ (ta_2/L_2)^{\sigma+1} = D_2(\gamma) \tag{A.7}$$

is indicative of the time-development of the asymptotic solution.

The constant total specific energy is given by

$$E_2 = \int_0^{L_2} \left(\sigma^2 - \sigma + 2\right)\pi^{\sigma(3-\sigma)/2} r^\sigma p_2 dr / \left(\gamma - 1\right)$$

whence $\quad E_2/p_2L_2^{\sigma+1} = (\sigma_2 - \sigma + 2)\ \pi^{\sigma(3-\sigma)/2}/[(\gamma - 1)\ (\sigma + 1)] \quad$ (A.8)

and the constant total specific mass is given by

$$M_2 = \int_0^{L_2} \left(\sigma^2 - \sigma + 2\right)\pi^{\sigma(3-\sigma)/2} r^\sigma \rho_2 dr$$

whence $\quad M_2a_2^2/p_2\, L_2^{\sigma+1} = \gamma(\sigma^2 - \sigma + 2)\ \pi^{\sigma(3-\sigma)/2}/(\sigma + 1) \quad$ (A.9) so that $E_2/M_2a_2^2 = 1/\gamma(\gamma - 1)$ (A.10)

For the complete similarity solution (suffix 1) at time $t = o$, the *central* pressure, density and wave-speed are respectively p_1, ρ_1 and a_1 and the radius is L_1; at the boundary of the flow pressure, density and wave-speed are always zero. The asymptotic solution is then

$$u = r/t;\ R_1 = 2a_1t/[(\gamma - 1)(\sigma + 1)^{½}];\ \eta_1 = r/R_1 = (\gamma - 1)(\sigma + 1)^{½}(r/a_1t) \qquad \text{(A.11)}$$

and

$$\rho(\eta_1, t)/\rho_1 = (1 - \eta_1^2)^{1/(\gamma-1)}[L_1(\gamma - 1)(\sigma + 1)^{½}/2a_1t]^{\sigma+1}$$

So $[\rho(\eta_1, t)/\rho_1](ta_1/L_1)^{\sigma+1} = [\gamma - 1)(\sigma + 1)^{½}/2]^{\sigma+1}(1 - \eta_1^2)^{1/(\gamma-1)}$
and

$$[\rho(o,t)/\rho_1](ta_1/L_1)^{\sigma+1} = [(\gamma - 1)(\sigma + 1)^{½}/2]^{\sigma+1}$$

Then, finally

$$\rho(\eta_1,t)/\rho_1(o,t) = (1 - \eta_1^2)^{1/\gamma-1} = F_1(\eta_1,\gamma) \qquad \text{(A.12)}$$

is indicative of the 'shape' of the solution and

$$[\rho(o,t)/\rho_1][ta_1/L_1]^{\sigma+1} = [(\gamma - 1)(\sigma + 1)^{½}/2]^{\sigma+1} \qquad \text{(A.13)}$$

is indicative of the time development of the asymptotic solution. The constant total specific energy is given by (cf. Thornhill 1958),

$$E_1 = \int_0^{L_1}\left(\sigma^2 - \sigma + 2\right)\pi^{\sigma(3-\sigma)/2} r^{\sigma} p_1\left(1 - r^2/L_1^2\right)^{\gamma/(\gamma-1)} dr/(\gamma - 1)$$

whence $E_1/p_1L_1^{\sigma+1} = (\sigma^2 - \sigma + 2)\,\pi^{\sigma(3-\sigma)/2}\gamma B[(\sigma + 3)/2, \gamma/(\gamma - 1]/[\sigma+1)(\gamma - 1)^2$ (A.14)

where $B(p,q)$ is the Eulerian integral of the first kind and $B(p,q) = \Gamma(p).\Gamma(q)/\Gamma(p + q)$. The constant total specific mass is given by

$$M_1 = \int_0^{L_1}\left(\sigma^2 - \sigma + 2\right)\pi^{\sigma(3-\sigma)/2} r^{\sigma} \rho_1\left(1 - r^2/L_1^2\right)^{1/(\gamma-1)} dr$$

whence $Ma_1^2/p_1L_1^{\sigma+1} = (\sigma^2 - \sigma + 2)\,\pi^{\sigma(3-\sigma)/2}(\gamma/2)\,B[(\sigma + 1)/2, \gamma/(\gamma - 1)]$ (A.15)

so that $E_1/M_1a_1^2 = [2/(\gamma - 1)]/[(\sigma + 3)\gamma - (\sigma + 1)]$ (A.16)

In order to compare the asymptotic solutions for these two expansions into a vacuum, with the same value of γ but with different initial conditions, it is necessary to ensure that $E_1 = E_2, M_1 = M_2$ and $p_1/\rho_1^{\gamma} = p_2/\rho^{\gamma}_2$. Using equations (A.8-10) and (A.14-16) this leads to

$$a_1^2/a_2^2 = (p_1/p_2)^{(\gamma-1)/\gamma} = (\rho_1/\rho_2)^{\gamma-1} = [(\sigma + 3)\ \gamma - (\sigma + 1)]/2\gamma \qquad \text{(A.17)}$$

and

$$\left(\frac{L_2}{L_1}\right)^{\sigma+1} = \frac{\gamma}{(\gamma-1)} B\left(\frac{\sigma+3}{2}, \frac{\gamma}{\gamma-1}\right)\left[\frac{(\sigma+3)\gamma-(\sigma+1)}{2\gamma}\right]^{\gamma/(\gamma-1)} \qquad \text{(A.18)}$$

The time-development of the two asymptotic solutions will be the same if $[\rho(o,t)/\rho_2](a_2t/L_2)^{\sigma+1}$ is the same for both. For the initially uniform sphere

$$[\rho(o,t)/\rho_2](a_2t/L_2)^{\sigma+1} = D_2(\gamma) \qquad \text{(A.7)}$$

For the complete similarity solution

$$[\rho(o,t)/\rho_2](a_2t/L_2)^{\sigma+1} = [\rho(o,t)/\rho_1](a_1t/L_1)^{\sigma+1}\ (\rho_1/\rho_2)(a_2/a_1)^{\sigma+1}\ (L_1/L_2)^{\sigma+1}$$

$$= \left[\frac{(\gamma-1)(\sigma+1)^{½}}{2}\right]^{\sigma+1}\left[\frac{2\gamma}{(\sigma+3)\gamma-(\sigma+1)}\right]^{(\sigma+3)/2}\frac{(\gamma-1)}{\gamma B\left[(\sigma+3)/2, \gamma/(\gamma-1)\right]} \qquad \text{(A.19)}$$

$$= D_1(\gamma), \text{ say}$$

using equation (A.13) and the relations (A.17) and (A.18).

Greifinger and Cole (1966) have evaluated $D_2(\gamma)$, for $\sigma = o$ from the limiting form of an exact analytical solution, and by numerical integration for $\sigma = 1$ and 2. Their values of $D_2(\gamma)$ are compared with the calculated values of $D_1(\gamma)$ in Figure 4. For $\sigma = 0$ the agreement is remarkably good and is satisfactory for small values of $(\gamma - 1)$ when $\sigma = 1, 2$. But the agreement clearly deteriorates as γ increases when $\sigma = 1, 2$ and the flow boundaries are different.

The 'shape' of the two asymptotic solutions will be the same if $F_1(\eta_1, \gamma) = F_2(\eta_2, \gamma)$. Again, Greifinger and Cole have calculated $F_2(\eta_2, \gamma)$, [equation (A.6)] and their results for three values of γ when $\sigma = 1$, are given by Hubbard (1966). These values for $F_2(\eta_2, \gamma)$ are compared with the calculated values of $F_1(\eta_1, \gamma)$ in Figure 5. The relation between η_1 and η_2 is

$$\eta_1/\eta_2 = \{2\gamma(\sigma + 1)/[(\sigma + 3)\gamma - (\sigma + 1)]\}^{½} \qquad (A.20)$$

The agreement is good for $\gamma = 5/4$ but again deteriorates as γ increases.

These comparisons, however, only suggest that the convergence to the asymptotic form of the complete similarity solution takes longer as σ and γ increase. They provide no evidence that the comparisons could not be improved by taking the computations to longer times as σ and γ increase.

Alternatively, since the two sets of initial conditions in these comparisons are so fundamentally different and lead to quite different flow boundaries when $\sigma = 1, 2$, it seems reasonable to assume that the convergence to the complete similarity solution would be quicker and more complete if the initial conditions were not so different; and, in particular, if pressure, density and wave-speed were initially zero at the outer boundary as in the complete similarity solution. The fact that present observations conform so well to the complete similarity solution suggests that this may well have been so and this, in turn, suggests that the present expansion phase began not with a confined quiescent mass of ether but with an unconfined instantaneous stationary minimum consequent upon an earlier convergence. In Newtonian mechanics times does not begin at a mathematical singularity associated with the start of an expansion phase of a particular universe.

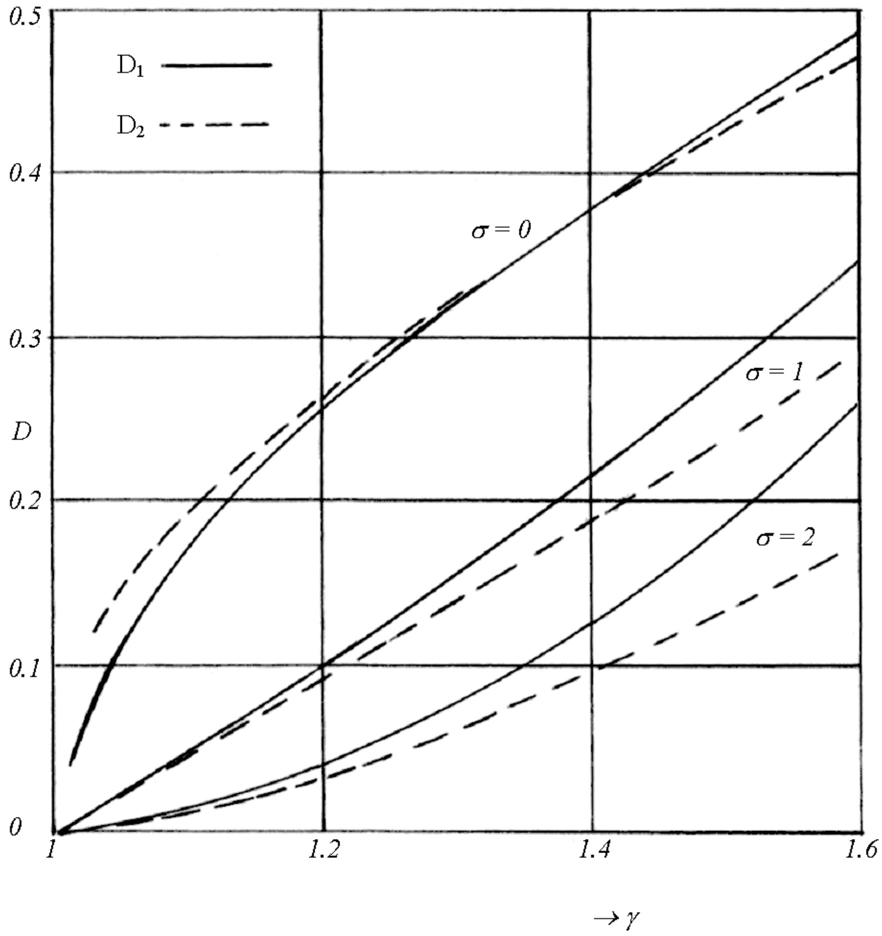

Figure 4

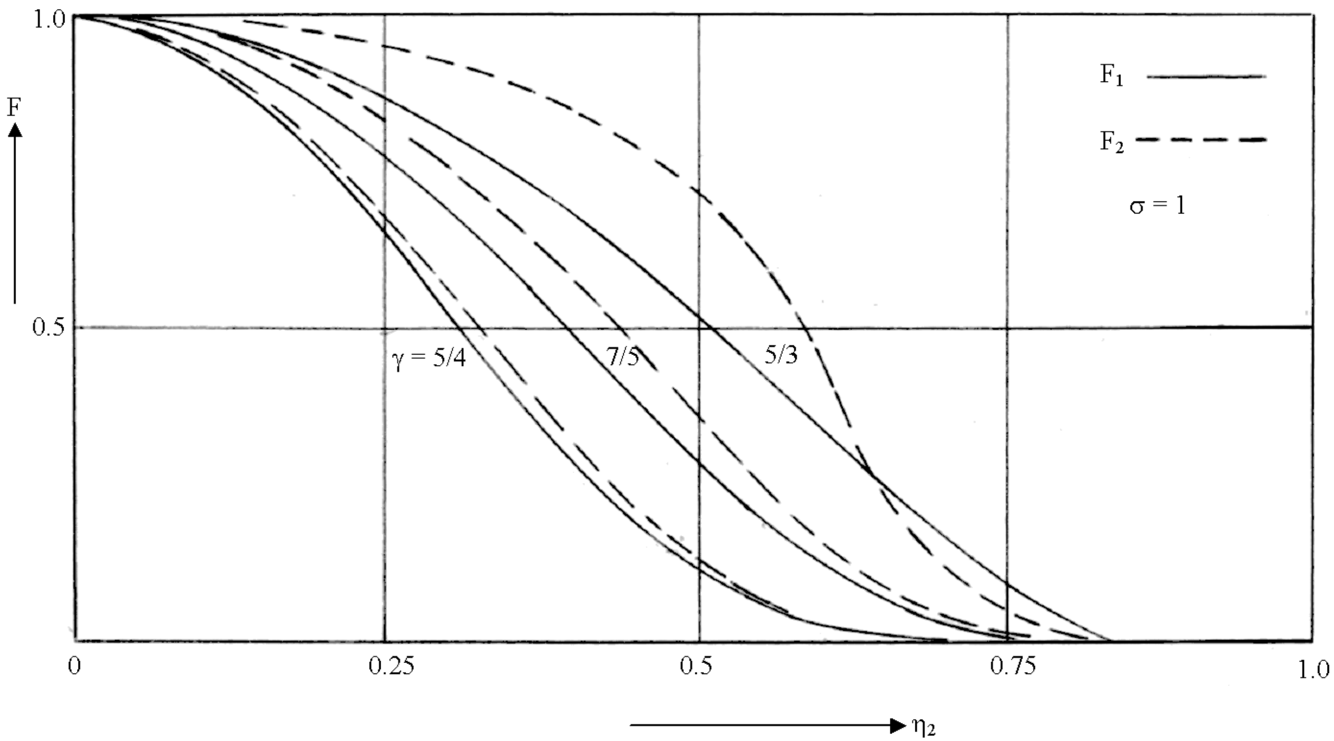

Figure 5