

# Wire length, coil geometry, and velocity factor

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- *To:* [tesla-at-pupman-dot-com](mailto:tesla-at-pupman-dot-com)
- *Subject:* Wire length, coil geometry, and velocity factor
- *From:* "Tesla list" <[tesla-at-pupman-dot-com](mailto:tesla-at-pupman-dot-com)>
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Original poster: "Paul Nicholson" <[paul-at-abelian.demon.co.uk](mailto:paul-at-abelian.demon.co.uk)>

Recently Ed Phillips noticed that wire length divided by the free space wavelength of the quarter wave resonance of an unloaded coil was a smooth function of the h/d ratio and largely independent of the turn count.

It is commonly understood (we hope) by coilers that the resonant frequency of a coil is not given by the quarter wave resonance of the straight wire from which the coil was wound. The actual frequency usually exceeds the straight wire prediction by 50% or more in typical TC secondaries.

This implies that signals (EM waves) at TC frequencies traverse the coil faster than they would do if strictly confined to the helical path of the winding. This is not unreasonable because each turn of the coil has some degree of interaction with all the other turns via their E and H fields (inductive and capacitive mutual coupling).

We can indicate numerically the extent to which signals 'leapfrog' the turns of the winding by expressing the apparent or effective speed of propagation along the wire as a ratio with the speed of light. This is the velocity factor of the wire when wound [+].

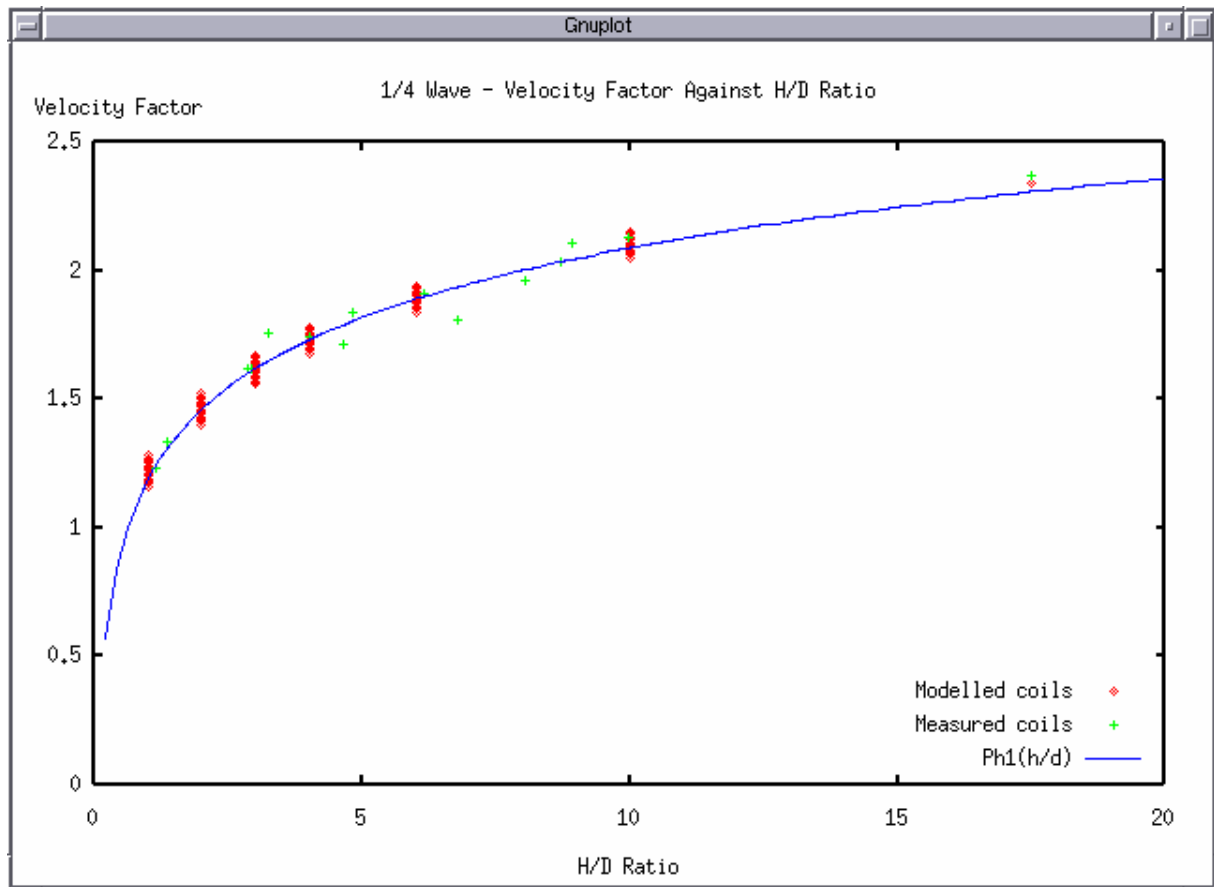
For example, if a particular coil has a velocity factor of 1.72, this in effect means that:

- \*) The 1/4 wave frequency will be 72% higher than that predicted for its straight wire.
- \*) A signal entering the coil will appear at the other end as if it had travelled the wire at 1.72 times the speed of light.
- \*) If we imagine the EM signal to be spiralling along the coil, the pitch of this spiral would be 72% greater than that of the winding.

Ed's observation recognises that the velocity factor for a coil is a function largely of the overall geometry of the coil and does not depend very much on how many turns are put in. This means that it is worth while defining a geometry factor with which to relate the velocity factor directly to the h/d ratio of the coil.

The graph

<http://www.abelian.demon.co.uk/tmp/ph1.gif>



shows (green crosses) measured velocity factors for a bunch of coils of varying sizes and turns. The blue line is a logarithmic function chosen for a reasonable fit. The function used is

$$\text{Ph1}(h/d) = \ln(h/d) * 0.39 + 1.19$$

where  $\ln()$  is the natural logarithm. The trial function has been tested against a larger set of modelled coils, and the results are plotted in the red dots.

The red dots actually represent 2732 assorted coils ranging from as few as 50 turns up to 3000 turns. Resonant frequencies of some of these coils go up into the Mhz range. We still see a reasonably good fit to  $\text{Ph1}(h/d)$  even when faced with such a wide range of hypothetical coils.

The residual (the difference between  $\text{Ph1}(h/d)$  and the measured or modelled velocity factor) is about +/-5%. Closer inspection of the comparison database reveals that about half of this residual is due to variation of coil base height above the modelled ground plane.

The function  $\text{Ph1}()$  can be used to directly estimate the quarter wave  $F_{res}$  from the  $h/d$  ratio and the wire length:=

$$F_{res} = (0.39 * \ln(h/d) + 1.19) * 75e6/\text{wire\_length} \quad (\text{Hertz})$$

where wire\_length is the total length of wire in metres. It seems we should expect this prediction to be within about +/-5% of the actual frequency for most coils. This is obviously a very much more direct approach to Fres than the conventional route via calculations of inductance and capacitance.

All of the above refers to the fundamental (1/4 wave) free resonance of the unloaded coil. Velocity factors for other frequencies will be different.

The next two overtones, 3/4 wave and 5/4 wave also follow a compact smooth curve when their velocity factor is plotted against h/d ratio. The independence from turns continues. However, these overtones don't seem to follow quite the same sort of logarithmic function of h/d. (A database of modelled coils is available in csv format if anyone wants to try for a fit).

It is clear that the function Ph1 (and the coils) tend towards some velocity factor greater than 2 for large h/d. No measurements are currently available for h/d > 10 so we can only speculate.

One common factor with all the coils modelled and measured in this comparison is that they are all fairly close wound. The smallest modelled spacing ratio is 0.55, and our models continue to refer to close wound behaviour as we let h/d tend to infinity. For this reason we should not be surprised that the velocity tends to some number rather larger than, say, the 0.95 we would expect from a straight wire.

To reach this straight wire velocity factor, our models and calculations would have to allow the pitch to increase to infinity as well as the h/d ratio. This they do not do.

We might anticipate that the compact distribution of velocity factors would be lost if we allowed the pitch to increase substantially. Unfortunately our software models are not qualified for large pitch coils since they rely on some approximations which are only accurate when each turn is almost a circle. Helical antenna modelling software might be tried instead, but those packages tend to bog down when dealing with large numbers of turns. For that reason it might have to be a task for the experimenters to tell us what happens at pitch angles larger than say 5 or 10 degrees.

[+]

The velocity factor of the wire when straight would be about 0.95, depending on thickness. The velocity factor of the winding is given by

$$4 * \text{wire\_length} * \text{Fres} / c$$

where c is 300e6 metres/sec, wire\_length in metres, Fres in Hz.

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Paul Nicholson

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# Re: Wire length, coil geometry, and velocity factor

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  - *Subject:* Re: Wire length, coil geometry, and velocity factor
  - *From:* "Tesla list" <[tesla-at-pupman-dot-com](mailto:tesla-at-pupman-dot-com)>
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Original poster: "Paul Nicholson" <[paul-at-abelian.demon.co.uk](mailto:paul-at-abelian.demon.co.uk)>

First, some news of more progress...

We know from measurements that signals traverse a solenoid winding rather faster than they would do if the wire were straight. Ed Phillips has shown that the apparent speed-up factor relates to the h/d ratio in the manner plotted in

<http://www.abelian.demon.co.uk/tmp/ph1.gif>

Malcolm Watts has supplied measurements for a coil h/d=17.5, which is by far the largest h/d coil we've ever looked at. This accounts for the lonely green cross way over on the right. The red dot just beneath it is from the tssp model of Malcolm's coil. Clearly, both model and measurements continue to follow the  $\log(h/d)*0.39 + 1.19$  function. Malcolm measured 1.0697MHz, the formula predicts 1039.6Mhz\*, so only -2.8% error from the formula, even at this extreme.

Now a little more expository waffle. Some of this repeats things already said, but maybe in a different way...

In a straight wire, the H field lines are circular around the wire, and the E field lines are radial. The resulting Poynting vector (which points out the direction in which the conducted energy is actually flowing) is parallel to the wire at every point in the space around the conductor. This means that the signal energy is guided by the conductor and is confined to move in exactly the same direction as the associated current flow.[+]

After the coil is wound, a piece of the wire experiences the E and B fields from some of the rest of the coil in addition to its own field. As a result, the Poynting vectors need no longer be parallel to the wire - which means that signals will propagate along the coil at a speed likely to be different to that of the straight wire.

We usually express this field coupling between remote parts of the coil in terms of mutual inductance and mutual capacitance so that we can apply circuit theory calculations to the coil instead of having to deal directly with the fields using Maxwell's equations. This works well and allows us to confirm by calculation Ed's relationship for the the change in velocity as the coil geometry varies.

Without any prior knowledge, we might have guessed that the greatest

influence on signal velocity might have occurred at small h/d, in which mutual coupling of both kinds is at its strongest. But, interestingly, we see from the graph above that the overall trend is that the greatest speed-up occurs with larger h/d.

For small h/d, it is apparent that the combined effects of mutual reactance, despite being stronger, are cancelling each other out to a large extent, resulting in less deviation of the signal flow from the path of the wire.

At somewhere around h/d = 0.6 the effects seem to balance to give unity velocity factor. This value is not too far from the h/d = 0.45 known to maximise the inductance of a given length of wire.

The velocity 'along the wire' is given by  $1/\sqrt{L \cdot C}$  where L and C are the reactances per unit length of wire. When the wire is wound, the magnetic field around any piece of it is increased by the field from nearby turns carrying current in the same direction. Therefore L is increased quite a lot as a result of winding (no surprise there) and the velocity is reduced accordingly.

Working against this is the electric field effect from the nearby turns - their mutual capacitance. In the straight wire, the E field lines exit the wire radially and head off to infinity, so determining the wire's self capacitance. But in the winding, the proximity of other conductors at nearly the same voltage produces a shielding effect - hindering the outflow of E-field lines. The net result is less charge is needed for a piece of the wire to reach a given voltage, ie there is less total capacitance on each bit of the wire and the velocity thus increases.

Ed's tabulated function shows how these two effects compete with each other as h/d varies. At large h/d, increase of L wins and the velocity is high. At low h/d, the reduction of C seems to overcome the increase in L and the velocity is low.

For the 3/4 wave and 5/4 wave overtones, the graphs are

<http://www.abelian.demon.co.uk/tmp/ph3.gif>  
<http://www.abelian.demon.co.uk/tmp/ph5.gif>

in which the functions roughly fitted to the modelled data are

$$\text{Ph3}(x) = \ln(x + 2.7) * 0.6 + 0.21$$
$$\text{Ph5}(x) = \ln(x + 5) * 0.65 - 0.28$$

[+] In a real wire, there is usually some resistance and so there is a longitudinal voltage gradient, ie an E-field component parallel to the wire. The effect of this is to make the Poynting vectors bend inwards slightly towards the wire. This inflowing energy component represents the power entering the wire itself to be dissipated in the  $I^2R$  loss.

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Paul Nicholson  
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>> \* I am sure Paul meant 1.0396Mhz - Terry <<